Negotiating Compensation

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Abstract

Workers can often negotiate their compensation. We develop a model showing that optimal negotiations and compensation design depend on whether capital and labor are substitutes or complements. This distinction — closely related to whether labor is low- or high-skilled — further affects whether workers can extract higher compensation by negotiating with fewer firms or by searching for additional job offers. It also affects whether cash constraints inflate or depress compensation when firms compete for workers. By solving for the optimal mechanism for workers to sell their labor, we further show how workers can compare different types of offers and negotiate with firms with different bargaining power.

Keywords: Competition for workers, negotiations, financing wages, compensation structure of non-executive employees, high-skilled employees.

JEL Classification: G32, M52, J54, J33

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"*With workers in high demand, the most costly mistake they can make is leaving the bargaining table without asking for more.*" New York Times, April 15, 2022

1 Introduction

In April 2022, the New York Times ran an article on negotiating pay, recounting the job search experience of Sabrina Hill — a single parent with a degree in social sciences. Within a month, she had conducted multiple interviews and received an offer paying $20.000 more in base salary than her old job as data analyst. At that point "she asked for perks she had never considered in previous negotiations, like restricted stock units" that she also received.

Sabrina Hill's experience showcases a broader phenomenon. Attracting talented workers can mean the difference between firm success and failure but is hard in tight labor markets. In the last decades, this problem has plagued not only the search for top executives (Edmans et al., 2021) but also for rank-and-file employees (BLS, 2021). As a result, the vast majority of hiring managers expect job candidates to negotiate their compensation, with 30%-55% actually doing so (Hall and Krueger, 2012; Brenzel et al., 2014; RobertHalf, 2019). Reflecting this phenomenon, practical advice on how workers can negotiate higher compensation and handle multiple job offers has proliferated (Knight, 2017; DePaul, 2020). Yet academic work has remained largely silent on the topic. Existing work primarily approaches compensation design from the perspective of what is best for firms, focusing on problems such as finding the best workers and incentivizing effort at the lowest cost (Grossman and Hart, 1983). It remains an open question how compensation needs to adapt to be most attractive to *workers*.

In this paper, we study optimal compensation and negotiation design from the perspective of workers trying to maximize their compensation. We show that workers seek different types of compensation and pursue different negotiation tactics depending on whether capital and labor are substitutes or complements — a distinction closely related to whether labor is low- or high-skilled (Krusell et al., 2000; Fonseca and van Doornik, 2022). Three positive predictions stand out. First, while high-skilled workers, such as engineers or scientists, can extract higher compensation by negotiating for higher variable pay (e.g., performance bonuses or stock-based pay), negotiating for higher fixed pay (e.g., base salary or sign-on bonuses) is preferable for lower-skilled workers. Second, while both higher competition and negotiating compensation make workers better off, the benefit from trying to generate additional job offers as opposed to negotiating with the firms that have already shown interest differs. While an engineer is better off negotiating, for a warehouse manager, generating additional competition is a better way to drive up compensation. Third, the interaction between negotiations and compensation design matters from a corporate finance perspective not only because it affects the firms' investor base (when workers are paid in equity) but also because it affects how much external financing firms need. Furthermore, we show that cash-constrained firms compete for high-skilled labor more aggressively than unconstrained firms. By contrast, cash constraints depress wages in firms reliant on less-skilled labor.

Our paper develops a model in which several firms compete to hire a worker whose skills are in short supply. The firms have limited cash but have access to external financing. Competition and negotiations between workers and firms are plagued by information frictions about the potential quality of the match. These information frictions make workers uncertain about how much firms will be willing to pay to hire them. Specifically, while it is commonly known that hiring increases firm value, workers are not quite certain how much value their labor adds.[1] In this environment, a worker needs to decide how best to maximize her compensation. The first alternative is negotiating with the firms that have already shown interest in hiring her. In practice, this requires from applicants to explore what is usual in the industry and try to understand the firms' growth prospects and constraints. Based on this, the workers can make the necessary calculations to demand, for example, a fixed wage of at least $80,000; or pose demands for stock options pay. The alternative to negotiations that we consider is generating an additional job offer. The worker then lets the firms improve sequentially on each other's offers until only one firm remains.

Our paper's starting point is to show that if workers decide to negotiate, they optimally choose different types of compensation depending on whether capital and labor are complements or substitutes. What is specific about the case of complements is that labor creates more value at more productive firms; while in the case of substitutes, the benefit of hiring is lower for firms with more productive capital investments.

Consider negotiations in the case of complements (typical for high-skilled labor). In the simplest type of negotiation, involving one firm only, the best that a worker can do is make a take-it-or-leave-it demand for highly convex variable compensation, such as in call options. Specifically, the worker's problem is that she is uncertain about the highest compensation that she can ask for. Thus, she faces the risk that the firm will reject demands for very high compensation, as it may not be able to afford it. Variable compensation is optimal because it mitigates this problem. Specifically, variable compensation is of little value if the firm's prospects are not very good. Thus, even firms with poorer prospects would be able to meet aggressive demands for such compensation. Simultaneously, variable compensation, and in particular call options, allow the worker to extract more of the upside from firms with better

[1] Before joining a firm, workers typically have less information about its growth prospects, its internal organization, and the quality of internal collaboration, which could give rise to such information asymmetry. We show that the qualitative insights are robust to endowing workers with private information.

prospects that benefit more from hiring.

By contrast, if capital and labor are substitutes (typical for low-skilled labor), workers optimally negotiate for fixed wages. The problem faced by workers is that their labor generates less value at more-productive firms, making such firms less willing to offer higher compensation. Fixed wages are optimal because they achieve two objectives. They reduce the likelihood that more-productive firms reject aggressive wage demands, as fixed wages leave most of the upside of higher productivity to the firm. Simultaneously, fixed wages maximize workers' compensation in low cash flow state. Such protection is important given that it is the less-productive firms that are more interested in hiring. Notably, we show that these results are unaffected by whether firms or workers have private information.

The alternative to negotiations for workers is to search for an additional job offer. Competition, which we model as alternating offers by firms, drives workers' pay up. However, the trade-off is that compensation design is decided by firms. In this case, firms prefer to offer fixed wages, regardless of whether capital and labor are complements or substitutes. Unlike variable pay, the value of which depends on the firm's private information about its productivity, fixed wages are minimally affected by information asymmetries. Thus, by offering fixed pay, firms can minimize the cost that workers undervalue their compensation offers.

Both negotiating compensation and searching for additional job offers can lead to higher expected compensation. However, while negotiating can extract higher pay if capital and labor are complements, the same does not hold in case of substitutes. The explanation for this result is that when capital and labor are complements, variable compensation allows a worker to extract a larger portion of the value generated by her labor than would be possible with the fixed compensation optimally offered by firms. As a result, negotiating for variable pay can dominate attracting additional job offers in fixed pay. Negotiations are particularly beneficial when the firm's willingness to pay for labor is less sensitive to its productivity, as then the risk that negotiations will discourage less productive firms from hiring is lower.

By contrast, attracting additional competition is better than negotiations if capital and labor are substitutes. In this case, the worker can never come close enough to extracting a sufficiently large portion of the value she generates at the firm. The best she can do is negotiate for fixed wages, which is also the type of compensation that firms prefer to offer. But when the choice is between negotiating for a higher fixed payment or searching for an additional firm offering fixed payments, the latter option always dominates (Bulow and Klemperer, 1996). Relating the degree of complementarity to workers' skills (Krusell, et al., 2000), we can summarize our main positive predictions as follows: Low-skilled workers extract higher pay (in particular, base pay and sign-on bonuses) by choosing to generate additional job offers. Negotiations dominate attracting additional competition only for high-skilled workers. In that case, workers negotiate for higher variable compensation, such as higher performance bonuses or equity-based pay.

On the normative side, a key gap in the literature addressed by our paper is to derive the optimal mechanism for workers to "sell" their labor when negotiations involve multiple firms. To deal with the problem that the standard conditions considered by the optimal mechanism design literature are not satisfied, we impose several realistic restrictions. Specifically, we require that only the firm with the highest willingness to pay for labor (if any) hires and pays the worker and, at the time of hiring, does not regret the compensation it offers. Given these restrictions, we show that the best mechanism is for workers to let firms sequentially compete with alternating offers on compensation levels until only one firm remains. The worker negotiates for her preferred compensation structure only with the last-remaining firm. As discussed, the type of compensation demanded by workers will depend then on whether capital and labor are complements or substitutes. This sequential mechanism leads to a substantial improvement to auction-like mechanisms in which the worker can restrict what type of compensation firms should offer.

The main challenge for workers with the optimal mechanism we describe is finding a way to compare compensation levels if firms offer different types of compensation (e.g., fixed wages or equity). We show that compensation contracts can be ranked then efficiently based on the answer to the following question: "*What would be the compensation contracts' expected value if the firm would be indifferent between hiring and not hiring with that contract?*" Such ranking guarantees that when competition forces a firm to raise its offer to the point at which it is indifferent between hiring and not hiring, the worker ranks that final compensation offer fairly. In particular, a firm does not drop out from competing to hire the worker before all firms with a lower willingness to pay for labor have dropped out.

Though workers are hired by the firm willing to pay the most for labor, cash constraints distort the efficient match between workers and firms. We show that cash-constrained firms compete more aggressively to hire a worker if capital and labor are complements but less aggressively if they are substitutes. In the case of complements, the higher aggressiveness arises because financiers overestimate a firm's prospects when it offers (close to) the highest wage that it can afford, as they form their expectations over all types that can afford that wage. Hence, when a firm has raised its wage offer to the point at which it is just indifferent between hiring and not hiring (i.e., it is the lowest type that can afford this wage), it is cross-subsidized by higher types. The resulting cheap financing distorts the highest wage that the firm is willing to offer upward. In the case of substitutes, the effects reverse, as lower types are willing to pay more for labor and, thus, financiers underestimate the firm's type when it makes the highest offer it can afford.

Related Literature. The empirical consensus over the last decades is that labor markets are not perfectly competitive (see Card (2022) for an overview). Yet, to the best of our knowledge, our paper is among the first to investigate the differential impact of competition and negotiations on optimal compensation design and study how that impact depends on whether capital and labor are complements or substitutes. Though we follow the standard approach in labor economics to model competition among firms as a sequential auction (Postel-Vinay and Robin, 2002; Cahuc et al., 2006; Bagger et al., 2014), our key departure is to investigate the impact of asymmetric information between workers and firms. Considering asymmetric information has several main implications.

Fist, compensation structure becomes a central determinant for how surplus in negotiations is shared. In the only other paper that we are aware of that studies the impact of bargaining power on workers' compensation structure, Bova and Yang (2017) argue that the combination of strong product market competition and strong employee bargaining power makes fixed compensation optimal. By making asymmetric information the main friction in our model, we show that worker bargaining power has the opposite effect if capital and labor are complements. Our predictions are qualitatively similar to Bova and Yang (2017) only for the case in which capital and labor are substitutes.

Second, asymmetric information leads to a trade-off between devoting time to negotiations or searching for new job offers, which is not considered in symmetric information models in which workers improve their compensation by searching for new offers (e.g., Postel-Vinay and Robin, 2002). While this trade-off has been studied in the auction literature (Bulow and Klemperer, 1996), the answer in that literature is clear-cut: generating additional competition is always better when bidders are symmetric and payments are in cash. By contrast, we show that when compensation also contains a variable component, negotiations dominate if capital and labor are complements but not when they are substitutes. This insight has broader implications beyond job negotiations, such as in the context of M&A where state-contingent payments and negotiations are also common (Eckbo et al., 2020).

Another contribution of our paper is that it derives the optimal selling mechanism when compensation can be in state-contingent claims. Deriving such a mechanism is challenging since the standard regularity conditions considered by the optimal mechanism design literature are not satisfied. Thus, prior work has focused on finding corresponding conditions for linear instruments, such as equity, which can achieve full rent extraction in special cases (Bernhardt and Liu, 2019; Liu and Bernhardt, 2021). Instead, our approach is to impose restrictions that are typically satisfied in our setting: only the firm willing to pay for labor the most (if any) hires and pays the worker, and it should not regret the contract it has offered at the time of hiring. The appeal of the mechanism we derive — let firms compete on compensation levels and negotiate compensation structure only with the last remaining firm — is that it is simple and familiar. It is a straightforward modification of the optimal mechanism when payments are in cash only (Lopomo, 2000). Furthermore, it is almost "details-free," as it requires workers to learn only about the final firm they negotiate with.

A gap in the literature addressed by this paper is how workers can compare compensation offers when firms choose from different types of compensation and when workers have different bargaining power vis-à-vis different firms. In existing work analyzing auctions with contingent claims, the focus is, instead, on the case in which bidders have the same bargaining power and, in equilibrium, make the same types of offers (Hansen, 1985; DeMarzo et al., 2005). Interestingly, this literature applies the so-called Linkage Principle to show that equity-like instruments stimulate more competition among bidders than cash auctions.[2] By contrast, in our paper, equity-based compensation is optimal (if capital and labor are complements), because it allows workers to extract higher rent, while minimizing the risk that the worker's offers are rejected. Notably, competition and the Linkage Principle are irrelevant for this explanation — indeed, in our setting, variable pay is strictly optimal only once one firm is left; Compensation design is irrelevant while firms are still competing.[3]

2 Model

We study a parsimonious model investigating the key determinants of workers' compensation structure in cases in which the key friction is asymmetric information about firm type and labor is in short supply. The model features $i \in \{1, ..., n\}$ deep-pocketed firms that compete to hire a single worker (she). There are two stages: A hiring stage, $t = 0$, in which compensation offers are extended and negotiated, and the worker decides which firm (if any) to join, and a subsequent production stage, $t = 1$, in which firms undertake their projects, cash flows are realized, and the worker is compensated. All parties are risk-neutral, and there is no discounting.

Projects and firm types. Each firm i seeks to hire the worker to run a project that

[2]The reason is that any given bid translates into a higher payment for higher types — e.g., if a 10% equity offer has an expected value of \$100 to a low type, then matching that offer will extract more than \$100 from a higher type. This effect is the essence of Milgrom and Weber's (1982) Linkage Principle.

[3]More broadly, our paper contributes to the discussion of why firms offer equity-based compensation to workers below the executive level, given that such compensation is unlikely to have any incentive effects (Holmström, 1982). Our perspective on the role of bargaining power and whether capital and labor are complements or substitutes adds to prior work that has argued that equity-based compensation helps: avoid wage renegotiations when the firm's equity value is correlated with the worker' outside options (Oyer, 2004); align the incentives of managers with the interests of investors (Lazear, 2004); exploit the overoptimism of boundedly rational the worker (Bergman and Jenter, 2007); provide a hedge against not being promoted (Chen, 2020); or hedge Knightian uncertainty (Fulghieri and Dicks, 2019).

can either fail, in which case cash flow is low and equal to $x > 0$, or succeed, in which case cash flow is high, $x + \Delta x > x$.[4] The respective probability of success depends on two key firm- or project-level inputs. First, it depends on the quality of the firm's (existing) capital investments – its expected capital productivity – which we denote by θ_i and sometimes refer to as the firm's "type." Capital productivity is each firm's private information, and outsiders only know that it is drawn, independently for each i, from distribution F_i with support normalized to $[0, 1]$. Second, firm i's success probability depends on its human capital, in particular on the outcome h_i of its hiring efforts, where $h_i = H$ denotes that firm i was successful in hiring the worker and $h_i = N$ indicates that firm i could not hire.

We denote the probability of firm i realizing high cash flow by $p_{h_i}(\theta_i)$, which is strictly increasing in both arguments, i.e., in capital productivity, $\frac{\partial}{\partial \theta} p_h(\theta) > 0$ for all h, and following a successful hire, $p_H(\theta_i) > p_N(\theta_i)$ for all θ_i. Importantly, we allow for both the case in which capital (productivity) and labor are complements, $\frac{\partial}{\partial \theta_i} \left[p_H(\theta_i) - p_N(\theta_i) \right] \geq 0$, as is documented for high-skilled labor, as well as the case in which they are substitutes, $\frac{\partial}{\partial \theta_i} \left[p_H(\theta_i) - p_N(\theta_i) \right] < 0$, which analogously would capture low-skilled labor (Krusell et al., 2000).[5] Thus, in the former (complementarity) case, high productivity firms benefit more from hiring, while in the latter (substitutability) case, the benefit from hiring is higher for low productivity firms. For illustrative purposes, we will repeatedly rely on a simple functional form for the probability of success,

$$p_h(\theta_i) = p_h + \Delta_h \theta_i, \text{ for } h \in \{H, N\}, \ \theta_i \in [0, 1], \tag{1}$$

where $p_h, \Delta_h > 0$, $p_h + \Delta_h \leq 1$, and $p_H + \Delta_H \theta_i > p_N + \Delta_N \theta_i$ for all θ_i. Given the specification in (1), capital and labour are complements if $\Delta_H / \Delta_N \geq 1$ and substitutes if $\Delta_H / \Delta_N < 1$. It is important to note, however, that our qualitative results do not depend on the concrete functional form in (1).[6]

[4] The binary cash flow assumption is taken for illustrative purposes only. All results extend to the case of continuous cash flows, given standard assumptions on the production technology similar to Nachman and Noe (1994).

[5] One could interpret $x + p_N(\theta) \Delta x$ as the expected payoff from the firm's capital investments or, more generally, as the payoff from the firm's "existing business," which consists of all its tangible and intangible investments and current workforce. Hence, we could alternatively interpret θ_i as the size of the overall physical and intangible capital deployed to the project. For completeness, note that complementarity and substitutability are typically defined by the cross partial of the production function with respect to capital and labor, which has the same sign as the cross partial with respect to labor and the productivity of capital.

[6] Our insights extend to a more general formulation of the expected cash flows from production nesting both binary as well as continuous cash flows subject to suitable normalization: $\Pi(K, L, \theta) := \Pi \left(\left(v (\theta K)^\sigma + (1 - v) L^\sigma \right)^{\frac{1}{\sigma}} \right)$, where K and L stand for the level of capital and labor investments, and θ captures the firm's capital productivity. However, since the added generality is not needed for our results, we stick to the simple specification used in the main text.

Compensation Contracts. Upon successful hiring by firm i, its owners split cash flow with the worker according to a compensation contract $\{w_i, \Delta w_i\}$ that stipulates a payment to the worker of w_i in the low cash flow state and of $w_i + \Delta w_i$ in the high cash flow state. We refer to w as fixed pay and to Δw as variable pay, where we drop the subscript i where it does not lead to confusion. The worker has an outside option of $\underline{w} \geq 0$, which can be thought of as expected compensation in her current employment or in unemployment. We stipulate that all n firms are "serious employers," in the sense that their willingness to pay for the worker exceeds the worker's outside option, i.e., $p_H - p_N + (\Delta_H - \Delta_N)\,\theta \geq \underline{w}$ for all θ. Further, we assume that the worker is protected by limited liability and that contracts are monotone, $w, \Delta w \geq 0$. Intuitively, the latter assumption means that the worker should have no incentives to sabotage the project in the high cash flow state (Innes, 1990). Contract offers are determined in a competitive negotiation process which we describe next.

Negotiations and Competition. We differentiate between competition, as captured by the number of firms attempting to hire the worker, and negotiations, which is captured by whether the worker can choose the optimal way to sell her labor, which in turn is linked to the worker's bargaining power. Following Bulow and Klemperer (1996), we consider the following trade-off. Learning to negotiate requires the worker to "devote resources to expanding the market than to collecting the information and making the calculations required to figure out the best mechanism (p. 108)." In practice, the latter involves spending time on learning more about the firms, their growth prospects, and constraints. The alternative for the worker is to spend the same time to conduct additional interviews and attract one more firm willing to hire her. While (similar to Bulow and Klemperer, 1996) we do not explicitly model the process of learning how to negotiate or searching for new offers, we adopt this language in reference to the intuition behind the trade-off between competition and negotiations.

In our baseline model, the worker faces one firm that is interested in hiring her. Learning how to negotiate means that the worker can make a take-it-or-leave-it offer to the firm of the form $\{w, \Delta w\}$ including menus of such contracts. To highlight that negotiations affect not only compensation levels but also compensation structure, we benchmark the results against the case in which the firm can make a take-it-or-leave-it offer to the worker.[7] If the worker chooses not to negotiate but attract competition from one more firm, the worker makes no offers. In this case, the firms compete with alternating offers until only one firm remains. The worker is hired at the compensation offered by the last remaining firm. As we will show, this way of modeling competition among firms will be without loss of generality.

Once we have derived the main results and trade-offs for the case with $n \leq 2$ firms, we

[7] Analyzing intermediate distributions of bargaining power is challenging, as there is no universally accepted solution concept, such as Nash bargaining, when information is asymmetric.

extend this baseline model to the case in which there are multiple firms $n > 2$ trying to hire the worker. We relegate the description of this extension to Section 4, where we solve for the optimal mechanism for workers to sell their human capital. Furthermore, in Section 5 we extend the model to the case in which (some) firms have to raise external financing in order to fund the worker's compensation.

3 Competition, Negotiations, and Compensation Structure

In what follows we derive and compare the worker's expected compensation from negotiations (Section 3.1) and competition (Section 3.2), respectively.

3.1 Compensation Structure if the Worker Negotiates

Suppose that the worker devotes her limited resources to learning how to negotiate, and as a result can make a take-it-or-leave-it offer ("demand") to the firm about the minimum compensation she requires to join. Under symmetric information, the worker would ask for compensation $\{w_i, \Delta w_i\}$ that extracts all the value that she creates at the firm and makes the firm indifferent between hiring and not hiring

$$x - w + p_H(\theta)(\Delta x - \Delta w) = x + p_N(\theta)\Delta x. \tag{2}$$

The problem under asymmetric information is that the worker does not know how much value she creates. This asymmetry would not be a problem if the worker could demand a compensation contract for which expression (2) is satisfied for all θ, as then the worker would extract all value generated by her labor regardless of the firm's productivity type. Such a "first-best" contract $\{w^{fb}, \Delta w^{fb}\}$ would stipulate that

$$\Delta w^{fb} = \left(1 - \frac{\Delta_N}{\Delta_H}\right)\Delta x \tag{3}$$

$$w^{fb} = p_H\left(\frac{\Delta_N}{\Delta_H} - \frac{p_N}{p_H}\right)\Delta x. \tag{4}$$

This first-best contract is feasible if $w^{fb}, \Delta w^{fb} \geq 0$, which requires that $1 \leq \frac{\Delta_H}{\Delta_N} \leq \frac{p_H}{p_N}$. If feasible, this contract is always optimal.

If it is not feasible, let W be a (possibly degenerate) menu of contracts and let $\Omega_W \subseteq [0,1]$ be the set of types accepting a contract from this menu. Denote the contract accepted by type θ with $\{w_\theta, \Delta w_\theta\}$. The worker's problem is to choose W to maximize her expected payoff

$$\int_{\Omega_W} \left(w_\theta + p_H(\theta)\,\Delta w_\theta\right) dF(\theta) + \int_{[0,1]\setminus\Omega_W} \underline{w} dF(\theta) \tag{5}$$

subject to the firm's individual rationality and incentive compatibility constraints and the feasibility restrictions $w_\theta, \Delta w_\theta \geq 0$.

3.1.1 Complementarity Between Capital and Labor

Suppose, first, that labor and capital are complements, i.e. $\frac{\Delta_H}{\Delta_N} \geq 1$. If it also holds that $\frac{\Delta_H}{\Delta_N} > \frac{p_H}{p_N}$, the first-best contract is not feasible, as it would require paying the worker a negative fixed wage, $w^{fb} < 0$ (see (4)). Intuitively, to extract the value generated by her labor, the worker demands compensation with a large "upside" Δw. The rationale is that she creates more value at more productive firms, which are more likely to generate such an upside. However, granting the worker too much of the upside makes hiring unprofitable for the firm (unless the worker's fixed pay is negative). Thus, the worker faces a trade-off between rent extraction and efficiency: more aggressive demands for compensation can extract more of the value she generates for the firm, but such demands are are more like to be rejected. In what follows, we formalize this trade-off and argue that it is mitigated by paying the worker only in variable pay.

Let $\{w, \Delta w\} \in W$ and define $\widetilde{\theta}$ as the type indifferent between hiring and not hiring for this contract. From expression (2), it holds

$$\widetilde{\theta}(w, \Delta w) \equiv \frac{w + p_N \Delta x - p_H(\Delta x - \Delta w)}{\Delta_H(\Delta x - \Delta w) - \Delta_N \Delta x}. \tag{6}$$

Since the firm's expected payoff from hiring compared to non-hiring increases in its type, all types $\theta \geq \widetilde{\theta}$ prefer hiring with $\{w, \Delta w\}$ to not hiring.[8] Hence, all such types will choose a contract from W (as they can always choose $\{w, \Delta w\}$). Taking $\widetilde{\theta}$ to be the lowest type that accepts a contract from W, it will hold that $\Omega_W = [\widetilde{\theta}, 1]$. The rent extraction – efficiency trade-off is now readily apparent: Demanding a higher compensation (higher w or Δw) will raise $\widetilde{\theta}$ and, thus, reduce set of types that accept that offer.

Demanding a contract that pays the worker only in the high cash flow state mitigates this rent-extraction efficiency trade-off, because it allows the worker to extract more of the value she generates at the firm from all types $\theta \geq \widetilde{\theta}$ that accept the worker's offer. To see this, consider the firm's information rent from hiring a worker with a compensation contract

[8] To see this, suppose to a contradiction that $\frac{\partial}{\partial\theta} p_H(\theta)(\Delta x - \Delta w) < \frac{\partial}{\partial\theta} p_N(\theta)\Delta x$ at $\widetilde{\theta}$ and so $\Delta w \geq \left(1 - \frac{\Delta_N}{\Delta_H}\right)\Delta x$. Plugging into (**??**), we obtain that $w \leq p_H\left(\frac{\Delta_N}{\Delta_H} - \frac{p_N}{p_H}\right)\Delta x < 0$, giving a contradiction.

$\{w, \Delta w\}$

$$\begin{aligned} & x - w + p_H(\theta)(\Delta x - \Delta w) - x - p_N(\theta)\Delta x \\ = \; & (\theta - \widetilde{\theta})(\Delta_H(\Delta x - \Delta w) - \Delta_N \Delta x). \end{aligned} \tag{7}$$

This rent is strictly positive for all $\theta > \widetilde{\theta}$, when $\frac{\Delta_H}{\Delta_N} > \frac{p_H}{p_N}$. As expression (7) illustrates, extracting more of the firm's rent, while keeping the set of types $[\widetilde{\theta}, 1]$ that accepts the worker's offer unchanged, requires shifting the worker's compensation from the low to the high cash flow state as much as possible (i.e., minimizing $\Delta x - \Delta w$: see Panel A in Figure 1). A simple contract compensating the worker only in the high cash flow state achieves precisely this goal. No menu of contracts can improve on this offer, as any non-degenerate menu would have to include a compensation contract with $w > 0$ and would leave more of the surplus to the firm. One implementation of such a contract is through call options.

With the rent-extraction efficiency trade-off mitigated, the worker finds it optimal to demand a compensation contract implementing a lower cutoff $\widetilde{\theta}$. Intuitively, variable compensation (and, in particular, call options) is not valuable to firms in which the worker's labor does not create too much value. This makes it easier to design a contract that is acceptable for less productive firms, while at the same time still allowing the worker to extract a large portion of the value from hiring from more productive firms.

3.1.2 Substitutability Between Capital and Labor

If capital and labor are substitutes, i.e., $\frac{\Delta_H}{\Delta_N} < 1$, the worker faces the opposite problem: the surplus from hiring decreases in the firm's productivity. Hence, extracting all surplus from both low- and high-productivity firms requires that the worker's expected compensation also decreases in the firm's productivity. This would require that the worker is paid less in the high than the low cash flow state, $\Delta w^{fb} < 0$, which is infeasible (see (3)).

More precisely, since higher types are willing to pay *less* for labor when capital and labor are substitutes, it holds that $\Omega_W = [0, \widetilde{\theta}]$, where $\widetilde{\theta}$ is the highest type that accepts an offer from W. Rewriting the firm's information rent (7) as

$$(\widetilde{\theta} - \theta)(\Delta_N \Delta x - \Delta_H(\Delta x - \Delta w)). \tag{8}$$

we see immediately that this rent is strictly positive for all $\theta < \widetilde{\theta}$, as $\Delta_H(\Delta x - \Delta w) < \Delta_N \Delta x$ when capital and labor are substitutes.

A compensation contract makes a worker better off if the worker extracts more of the surplus she creates for the firm, reducing the firm's rent. As expression (8) illustrates, for any given set of types $[0, \widetilde{\theta}]$ that accepts the worker's offer, minimizing the firm's rent, requires minimizing the worker's claim on the upside Δw, i.e., shifting compensation from the high to the low cash flow state (Panel B in Figure 1). Hence, a simple fixed wage contract (for which $\Delta w = 0$) is optimal. Once again, no menu of contracts can improve on this offer, as any non-degenerate menu would have to include a compensation contract with $\Delta w > 0$ and would leave more of the surplus to the firm.

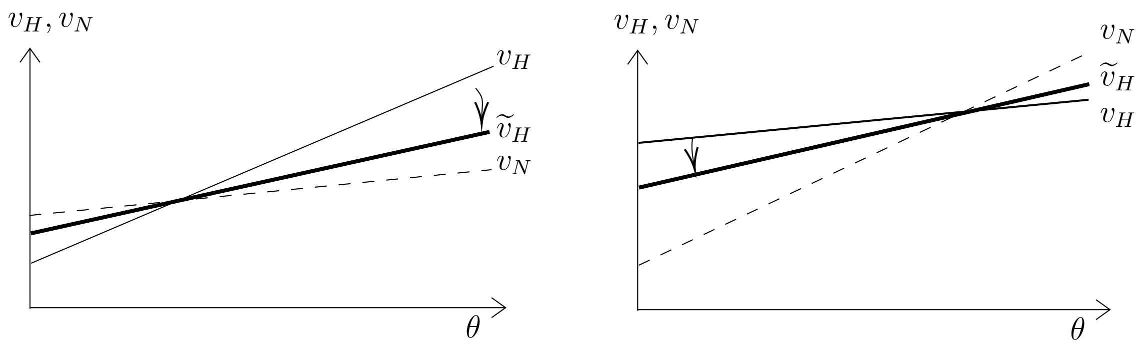
Intuitively, demanding a fixed wage is optimal for two main reasons. First, a fixed wage maximizes the worker's compensation in the low cash flow state. Such protection is needed, as low-productivity firms are those that stand to gain more from hiring the worker when capital and labor are substitutes. Second, high-productivity firms are more likely to reject aggressive compensation demands by the worker, as they benefit less from hiring. Hence, to extract more surplus from such types, while avoiding the risk that they reject her offer, the worker needs to leave them with a larger claim on the upside. Fixed wages achieve precisely this goal, as they minimize the worker's claim on the upside.

Proposition 1 *Suppose that a worker can make the firm a take-it-or-leave-it offer. If the first-best contract* $\left\{w^{fb}, \Delta w^{fb}\right\}$ *is feasible, it is always optimal. If it is not feasible, then:*
(i) If capital and labor are complements, the worker demands only variable and no fixed pay.
(ii) If capital and labor are substitutes, the worker demands only a fixed wage.
No menu can improve on these contracts.

A key implication of the preceding analysis is that firms in which labor complements capital and, thus, hiring can lead to large leaps in productivity will offer compensation in call options. Instead, firms where hiring workers will even out the differences between high- and low-productivity firms, as capital and labor are substitutes, will offer fixed compensation. These predictions imply a positive association between the use of call options and firm productivity that is unrelated to incentive effects.

Corollary 1 *The use of call options when capital and labor are complements and fixed wages when capital and labor are substitutes will lead to a positive association between the use of call options and firm productivity that is unrelated to effort incentives.*

As noted in the Introduction, it is worth noting that the use of call options and fixed pay in Proposition 1 is *unrelated* to stimulating competition among firms (through the so-called Linkage Principle), which is the explanation for the use of call options in auctions (DeMarzo et al., 2005). Instead, the rationale comes from the objective of extracting a higher compensation, while minimizing the probability that the last remaining firm will reject the worker' demands.



Panel A: Complements. **Panel B**: Substitutes.

Figure 1: **Rent extraction through compensation design.** The dashed line represents the firm's expected payoff if it does not hire. The thick solid line shows how the worker can extract more of the value generated by her labor if she asks for call options (in the case of complements) or a fixed wage (in the case of substitutes), respectively.

3.1.3 Robustness: Compensation Structure Does Not Depend on Who Has Private Information

The qualitative result from Proposition 1 does *not* depend on whether the worker or the firm is better informed. In Proposition B.1 in Appendix B.1, we solve for a version of the model in which the worker has private information about the firm's true productivity type — possibly because she is an expert in the field, and the firm lacks such experts.[9] We then have a game of signaling in which the worker's choice of compensation can signal information about the quality of the worker-firm match. We show that in the equilibrium of this game, the worker chooses compensation in call options if capital and labor are complements and fixed wages if capital and labor are substitutes. We relegate a detailed discussion of the intuition behind these results as well as all formal derivations to Appendix B.1.

3.2 Competition and Compensation

To see that negotiations affect not only compensation levels but also compensation structure, we benchmark the results from Proposition 1 against the case in which the firm makes a take-it-or-leave-it-offer to the worker. This offer needs to compensate the worker for forgoing her outside option $\underline{w}$

$$w + \int_0^1 p_H(\theta)\, d\widetilde{F}(\theta)\, \Delta w - \underline{w} \geq 0. \tag{9}$$

[9]Alternatively, we could assume that the worker's information, θ, is about her ability. The analysis will then correspond to the case in which capital and labor are complements, as it is unlikely that the worker's type affects the firm's outside option of not hiring.

where $\widetilde{F}$ is the worker's posterior distributions over θ after receiving an offer $\{w, \Delta w\}$. Since we have a game of incomplete information, our equilibrium concept is perfect Bayesian equilibrium. That is, on the equilibrium path, $\widetilde{F}$ is formed using Bayes rule. To deal with potential multiplicity of equilibria, out-of-equilibrium beliefs are refined using Cho and Kreps' (1987) Intuitive Criterion.

The firm's residual payoff, net of its outside option, is

$$x - w + p_H(\theta)(\Delta x - \Delta w) - x - p_N(\theta)\Delta x, \tag{10}$$

and the firm's problem is to design the offer $\{w, \Delta w\}$ in a way that maximizes this payoff, subject to (9) and $w, \Delta w \geq 0$.

The main insight from this benchmark is that it is optimal for the firm to offer a fixed wage that the firm guarantees paying in all cash flow states. When a firm chooses the worker's compensation contracts, the firm's choices are interpreted as signals about the quality θ of its projects. In the only perfect Bayesian equilibrium that arises, the firm offers fixed wages guaranteed by external financing because such compensation avoids misvaluation by the worker. Clearly, the firm will offer the lowest fixed wage satisfying the worker's participation constraint, i.e., $w = \underline{w}$ ($\Delta w = 0$).

Lemma 1 *If there is only one firm that makes the worker a take-it-or-leave-it offer, the firm hires the worker by promising her a fixed wage of* $\underline{w}$.

Competition. Suppose, now, that the worker searches for an additional offer instead of negotiating. Recall that when firms compete, they compete with alternating offers. That is, they sequentially improve on each other's offers until only one firm remains. When a firm makes an offer, it commits to dropping out if the worker rejects its offer. The worker can reject the offer or accept conditional on not finding a better offer. In the latter case, the firm has the option to make a new offer after the other firms have made their offers. If there are no such offers, the worker takes the last-standing offer.

Crucially, when firms compete, they do so on fixed wages. To see this, observe that the only change to Lemma 1 is that the worker's outside option is endogenous, and $\underline{w}$ needs to be replaced by the minimum that the worker would accept above the last-standing offer. With such competition, each firm will have a weakly dominant strategy to keep increasing its compensation offer until its final offer extracts all surplus from hiring. Offering more is clearly suboptimal. Dropping out at an earlier point is also suboptimal, as the firm could have hired the worker and generated positive surplus. With such competition, the worker will be hired with a fixed wage by the firm willing to pay for labor the most, and the worker's compensation will correspond to the valuation of the firm with the second-highest valuation of its labor. Comparing these insights to Proposition 1, we obtain:

Proposition 2 *(i) When firms compete, they do so on fixed wages. (ii) Attracting additional competition affects the level of fixed compensation, while negotiating also affects the structure of compensation.*

3.3 Choosing Between Negotiations and Competition

Our main result in this section is that the choice between competition and negotiations depends on whether capital and labor are complements or substitutes. In particular, if capital and labor are complements, the workers can extract higher expected compensation by negotiating for variable pay than from attracting competition from one more firm that competes in fixed wages. By contrast, competition always dominates negotiations when capital and labor are substitutes and the worker negotiates for fixed compensation.

Consider the case in which capital and labor are substitutes. In this case, workers choose to negotiate for fixed wages, which is the same type of compensation that also firms offer when competing to hire the worker (Proposition 1-2). With compensation structure being the same in both cases, attracting one more firm is always better for the workers than negotiation. The result goes back to Bulow and Klemperer (1996): In expectation, the value increase that comes from adding one more firm offering fixed compensation is higher than what can be extracted from optimal negotiations on fixed compensation.

The result no longer holds, however, if capital and labor are complements. In this case, the ability to ask for a different compensation structure magnifies the worker's ability to extract value from the firm. This benefit dominates adding one more firm offering fixed compensation, as variable compensation allows the worker to extract a large portion of the value generated by her labor. In particular, if the first-best contract is feasible, $\Delta_H/\Delta_N \in [1, \frac{p_H}{p_L}]$, negotiations always leads to a higher expected compensation. In general, the reason negotiations dominate for lower degrees of complementarity is that then the value-added from hiring for the firm is less sensitive to the firm's true productivity. As a result, aggressive demands for call options will be accepted even by lower types, while still allowing the worker to extract large surplus from more productive firms.

Proposition 3 *(i) If capital and labor are complements and $\Delta_H/\Delta_N \in [1, T)$, where $T \in (p_H/p_N, \infty)$, negotiating leads to higher expected compensation than adding competition from one more firm. (ii) If capital and labor are substitutes, $\Delta_H/\Delta_N \in [0, 1)$, generating additional competition always leads to higher expected compensation than negotiations.*

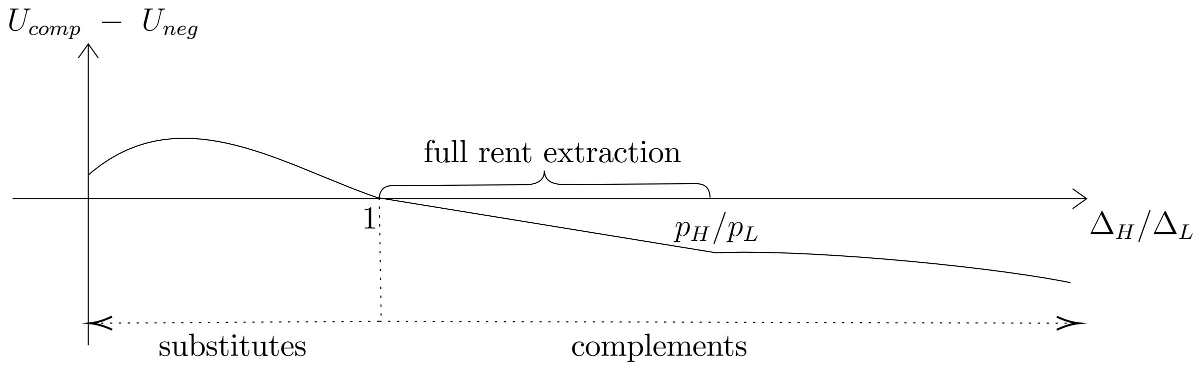


Figure 2: **Competition vs. negotiations.** The figure plots the difference in the worker's expected compensation between competition and negotiations.

Example 1. To see that negotiations can lead to a much higher expected compensation, consider the example in which θ is uniformly distributed. If the worker negotiates and $\frac{\Delta_H}{\Delta_N} \in \left[1, \frac{p_H}{p_N}\right]$, her expected compensation is

$$U_{neg} = \left(p_H - p_N + \frac{1}{2}(\Delta_H - \Delta_N)\right)\Delta x.$$

If, instead, the worker attracts competition from a second firm, the worker's compensation corresponds to the valuation of the firm with the second-higher willingness to pay. Thus, her expected compensation is

$$U_{comp} = (p_H - p_N + \mathrm{E}[\min(\theta_1, \theta_2)](\Delta_H - \Delta_N))\,\Delta x = \left(p_H - p_N + \frac{1}{3}(\Delta_H - \Delta_N)\right)\Delta x.$$

Hence, the expected compensation from negotiating is higher than from competition. If $p_H \approx p_N$ negotiations lead to 50% higher expected compensation than competition for $\frac{\Delta_H}{\Delta_N} \in \left[1, \frac{p_H}{p_N}\right]$. Figure 2 illustrates Proposition 3 using this example by comparing the expected compensation following competition and optimal negotiations. ▲

4 Optimal Negotiations With Multiple Firms ($n > 2$)

In the preceding analysis, we modeled competition as an alternating offers game in which the firm willing to pay for labor the most hires the worker. This approach is without loss of generality for any number of firms. It is a standard result that all mechanisms in which the firm with the highest valuation outcompetes all others and pays in cash lead to the same expected payoff for the firm and the workers if the firms' valuations are private and independently drawn from the same distribution (Myerson, 1981).[10] It is also worth noting that a sequential competition can finish after only two offers. In particular, another payoff-equivalent equilibrium of the alternating offer game we studied in the previous section is that the first firm offers the same compensation it would offer as in a first-price auction. Since such an offer will reveal the first firm's valuation of labor, the second firm will either counter with an offer equal to that valuation if its own valuation is higher or it will drop out.

The open question to which we turn next is what is the best way for workers to sell their human capital when they can negotiate with multiple firms. In what follows, we describe the mechanism, show with an example that it improves on mechanisms discussed by prior work, and then formally derive its optimality.

4.1 The Optimal Negotiation Mechanism

The best known implementation of Myerson's (1981) optimal mechanism when offers are in cash only is an English auction followed by a take-it-or-leave-it offer to the last remaining firm (Lopomo, 2000). The main modification to this mechanism we propose involves the offer made in the final negotiation stage.

Specifically the mechanism is as follows: The worker asks firm $i = 1$ to make an offer higher than her outside option $\underline{w}$. If the firm makes such an offer, the worker can come back to it again if she does not find a better offer; if the firm does not make such an offer, it drops out. The worker then approaches firm $i = 2$ and asks her to improve on the last-standing offer (i.e., the worker's new outside option) or drop out. For now, we assume that all firms offer fixed compensation — crucially, we relax this assumption in the next section and show that it is without loss of generality. Once all remaining firms have either made an offer that is higher than the last-standing offer or have dropped out, the game proceeds to the next round. In that round, all remaining firms need to decide again sequentially on whether to improve on the last-standing offer or drop out. The game continues in this fashion until only one firm remains. We denote the last remaining firm's offer (that no other firm is willing to match) with $\underline{w}^*$. The worker then negotiates with the last remaining firms as dictated by Proposition 1, where the only modification we need to apply to this proposition is that the worker updates her beliefs about the type of the last remaining firm. In particular, the worker's posterior beliefs are formed using Bayes rule over $\left[\frac{\underline{w}^* - (p_H - p_N)\Delta x}{(\Delta_H - \Delta_N)\Delta x}, 1\right]$, since the last remaining firm's type must be at least equal to that of the last firm that has dropped out.

Note that, just as in the case of competition, each firm will have a weakly dominant strategy to keep increasing its compensation offer until its final offer extracts all surplus

[10] Offering fixed compensation will also be optimal in mechanisms in which firms make a single offer, such as in a first-rice auction, and are free to choose the structure of pay (DeMarzo et al., 2005).

	Expected compensation	
	Fixed wages	Call Options
Sequential competition + final TIOLI offer	$0.25\Delta x$	$0.28\Delta x$
Two sequential TIOLI offers	$0.24\Delta x$	$0.24\Delta x$
Each firm makes a single offer; highest offer wins	$0.20\Delta x$	$0.24\Delta x$
Sequential competition w/o final TIOLI offer	$0.20\Delta x$	$0.24\Delta x$

Table 1: **Expected Compensation for Different Mechanisms.**

from hiring. That is, $\underline{w}^*$ corresponds to the value-added from hiring of the firm with the second-highest willingness to pay for the worker. As the following example shows, the simple sequential mechanism we outline presents a substantial improvement on mechanisms in which the worker tries to maximize her compensation by restricting the types of offers that firms can make.

Example 2. Consider the following example in which capital and labor are complements. Let $p_H = p_N = \frac{c}{c+1}$, $\Delta_H = \frac{1}{c+1}$, and $\Delta_N = 0$. Hence, it holds that $p_H(\theta) = \frac{c+\theta}{c+1}$, $p_N = \frac{c}{c+1}$, and hiring creates positive surplus for all $\theta \in [0,1]$. For this example, assume that there are two firms trying to hire the worker, θ is uniformly distributed on $[0,1]$, and $c = 0.7$. We compare the expected compensation from the sequential mechanism we have proposed with two mechanisms where the worker restricts firms to offer certain types of contracts — we consider the examples of fixed wages or call options. In the first of these mechanisms, the firms compete with alternating offers using the type of contracts that are chosen by the worker until only one firm remains (effectively an English auction); in the second mechanism, the worker asks each firm to make a single offer from a certain type, with the worker choosing the higher higher offer (i.e., the worker effectively runs a first-price auction). These mechanisms are also compared to a simple approach of sequentially making a TIOLI offer to the first firm and then to the second firm. If the second firm accepts, the worker joins that firm. Otherwise, the worker joins the first firm, as long as it has accepted the initial offer. We derive the optimal strategies for each of these mechanisms in Appendix B.3. ▲

The key insight from Example 2 is that the choice of the selling mechanism is at least as important as that of compensation structure. The sequential mechanism we have described always leads to the highest expected compensation. In particular, it dominates the first-price auction in call options, which prior work has shown to lead to the highest expected compensation among all standard mechanisms, such as the English, second, and first-price auctions (DeMarzo et al., 2005).[11] In fact, in this example, the proposed sequential mechanism dom-

[11] As it is standard, the English and the second-price auctions are payoff equivalent. While the first-price

inates the first price auction in call options *regardless* of whether the worker negotiates for fixed compensation or call options with the last remaining firm.

Conditions for Optimality. Deriving the optimal mechanism when workers can choose their compensation structure is challenging, as some of the main assumptions made in the mechanism design literature are not satisfied.[12] However, we can show that the mechanism we have described is optimal conditional on several realistic restrictions on the set of admissible mechanism: (i) the firm willing to pay the most for the worker's labor (if any firm) is the one hiring the worker and paying her compensation; (ii) that firm does not regret hiring — i.e., at the time of hiring (as opposed to before any offers are made), the offered compensation is incentive compatible and individually rational for the firm. Notably, we do not require that the mechanism be efficient, as it can be that the worker is not hired by any of the firms.

Proposition 4 *Letting firms compete on compensation levels by sequentially improving on each other's offers, followed by a negotiation with the last remaining firm about compensation structure, yields higher expected compensation than any incentive compatible mechanism in which the firm with the highest type (if any) hires and pays the worker and does not regret it. The compensation negotiations with the last remaining firm follow Proposition 1, with the worker's posterior beliefs being that the last-remaining firm's type is a draw from* $\left[\frac{\underline{w}^* - (p_H - p_N)\Delta x}{(\Delta_H - \Delta_N)\Delta x}, 1\right]$.

The conditions imposed by Proposition 4 are the same as those required that an English auction followed by a final take-it-or-leave-it offer implements when offers are in cash only is an optimal selling mechanism (Lopomo, 2000). We show that by modifying only the final step (in which the worker negotiates with the last-remaining firm), we can show optimality among a much broader set of mechanisms that allow for offers in any type of state-contingent claims. We believe that the simplicity and familiarity of our mechanism are key to its appeal. Another advantage is that the mechanism is almost details free, as the worker only needs to learn about the last remaining firm.

Though the modification we propose is simple, its implications go beyond improving on mechanisms discussed in prior work (Example 2). As Proposition 3 shows, which we extend to $n \geq 2$, the change overturns Bulow and Klemperer's (1996) classical result that

auction is slightly better in the case of call options, the difference is inidistinguishable (when rounding at the two decimal level).

[12] Specifically, the bidders' virtual valuation is a complicated object that depends on the contract's structure and is, in general, not monotone in θ. For recent advances, restricting attention to equity auctions, see Sogo et al. (2016) and Liu (2016). Prior approaches, such as DeMarzo et al. (2005), restrict attention to symmetric mechanisms without optimally designed reserve prices. Note that the final take-it-or-leave-it offer in our mechanism can be interpreted as a reserve price, optimally chosen once only one firm remains.

competition is better than negotiations. In what follows, we extend the mechanism to allow firms to make any type of compensation offers while trying to outcompete other firms.

4.2 Identifying the Firm Willing to Pay Most for Labor

The main challenge for workers that remains is finding a way to rank compensation levels when firms compete by making different types of offers, such as when some firms offer fixed wages while others equity-based pay. In this section, we show that there is a simple way for making such comparisons. The analysis starts with the case in which a worker has a strong bargaining position vis-à-vis all firms but allows these firms to offer any type of compensation they wish. Subsequently, we tackle the case in which the worker cannot dictate terms to all firms.

The worker's objective after any given offer is to optimally set the minimum requirements for new offers. For example, following an offer for \$80,000 and options for 0.3% of the firm's equity, the worker may require that new fixed wage offers must be for at least \$100,000, and equity compensation must offer at least 0.1% of the firm's equity. The crucial step lies in setting these minimum requirements. The difficulty is that unlike ranking fixed-wage offers, ranking offers that include, for example, equity or options is far from trivial.

This problem can be solved as follows. Let the last standing compensation offer by "Firm A" be $\{w_a, \Delta w_a\}$. For Firm B to beat Firm A's offer, the worker requires Firm B to choose from the set of all contracts $\{w_b, \Delta w_b\}$ that satisfy

$$w_b + p_H\left(\widetilde{\theta}\left(w_b, \Delta w_b\right)\right)\Delta w_b > w_a + p_H\left(\widetilde{\theta}\left(w_a, \Delta w_a\right)\right)\Delta w_a, \tag{11}$$

where $\widetilde{\theta}\left(w, \Delta w\right)$ is defined in (6). If Firm B rejects the offer, it drops out; if it accepts, the worker may come back to it. The worker then goes to Firm A, and the game proceeds until only one firm remains. Intuitively, condition (11) states that the worker ranks compensation offers based on the answer to the following question:

> *"What would be the compensation contract's expected value if the firm would be indifferent between hiring and not hiring at that compensation?"* (Q)

This ranking effectively undervalues all contracts for which the firm makes a profit from hiring but ranks those for which the firm is indifferent between hiring and not hiring based on their true value. Hence, a firm drops out from competing to hire the worker only after all firms with lower valuations have dropped out. Furthermore, the worker extracts the maximum possible information about the firms' types, as she can perfectly infer the types of all firms that drop out, knowing that the last remaining firm's type is higher. The worker's optimal final offer is, then, given by Proposition 4. Summarizing:

Proposition 5 *The worker can identify the firm willing to pay the most for her labor, while extracting the maximum information about its valuation, by demanding that firms improve on each other's offers as dictated by condition (11).*

A key aspect of Propositions 4 and 5 is that they do not require the worker to have the same bargaining power with all firms. As discussed in Section 3.2, firms with strong bargaining power will compete by offering fixed wages, and competition will proceed in effectively the same way as under the optimal mechanism without the final negotiation stage between the worker and the last-remaining firm. That is, dealing with firms with differing bargaining power is simple: The optimal mechanism for the worker is the same as before, with the only difference that the worker may not be able to negotiate with the last remaining firm if that firm has strong bargaining power in the sense that it can commit not to negotiate (for reasons outside our model).

5 Cash Constraints and Wage Distortions

Suppose that firms have no cash at hand, which forces them to secure external financing if they want to promise a fixed wage $w > x$. If firm i secures external financing to guarantee its ability to pay its wage promises, it does so at competitive terms. Specifically, the firm makes a take-it-or-leave-it offer to financiers, together with the offer it makes to the worker.[13] An external financing contract $\{S_i, \Delta S_i\}$ stipulates that firm i pays the financiers S_i in the low cash flow state and $S_i + \Delta S_i$ in the high cash flow state at $t = 1$. $\{S_i, \Delta S_i\}$ are the promised payments to the financier net of the transfers needed to guarantee the worker' compensation. A negative value for S_i or $S_i + \Delta S_i$ means that there is a transfer from the financiers to the firm. The financing contract is commonly observable. As before, the subscript i, denoting the firm's identity, is dropped whenever doing so does not lead to confusion. As it is standard, we assume that all parties are protected by limited liability and that all contracts are monotone (Nachman and Noe, 1994). Formally, it should hold that $0 \leq w + S \leq x$ and $0 \leq \Delta w + \Delta S \leq \Delta x$. Financiers require to break even when guaranteeing the worker' compensation:

$$S + \int_0^1 p_H(\theta)\, d\widetilde{F}(\theta)\, \Delta S \geq 0, \tag{12}$$

[13] A previous working paper version shows that the results do not qualitatively depend on whether financing is arranged before or after the firm hires the workers.

where $\widetilde{F}$ is the financiers' posterior distributions over θ after receiving an offer $\{S, \Delta S\}$. Note that since financiers and the worker have the same information, they share the same posterior distribution $\widetilde{F}$.

In what follows, we show that cash constraints distort the firms' willingness to pay for labor *upward* if capital and labor are complements but *downward* if they are substitutes. We limit our discussion to the case in which firms compete to hire the workers. In the case in which workers negotiate their compensation, they ask for variable compensation that does not require external financing if capital and labor are complements (Proposition 4). And if capital and labor are substitutes, workers prefer searching for additional offers to negotiations (Proposition 3).

The argument proceeds in two steps. The first is to establish that offering fixed wages secured by external financing is an equilibrium strategy for the firms. Indeed, this is the most beneficial equilibrium for the firm that can be supported. Despite the complication that there are two types of investors (external financiers and the worker who invests by forgoing her outside option), the analysis of this step is largely standard and relegated to the appendix. A sketch of the intuition is as follows. Let $\underline{w}'$ be the minimum that the worker will accept over her outside option. The firm whose turn it is to make an offer offers the worker a fixed wage with $w = \underline{w}'$ (and $\Delta w = 0$), with external financing filling the gap of $\underline{w}' - x$. The reason that such an equilibrium can be supported is that deviations to other types of financing or compensation are more disadvantageous to high types. Thus, when out-of-equilibrium beliefs are refined with the Intuitive Criterion, such deviations make the worker and financiers believe that they are facing a low type firm and are rejected. The main difference from the standard analysis (Nachman and Noe, 1994) is that the firm does not actually need to raise capital at $t = 0$. It can sign an insurance contract allowing it to raise capital at $t = 1$ in the low cash flow state in return for paying a premium in the high cash flow state. One interpretation of this insurance contract is as a credit line.

We should note that there are multiple equilibria of this game, where the worker co-finances her compensation of $\underline{w}'$. Since the worker and financiers share the same information, the financing terms they effectively offer are the same. In practice, this means that the worker may agree to be paid a fixed wage of $w \in [x, \underline{w}']$ in return for a variable component paid in the high cash flow states. All that matters for our equilibrium characterization is the joint claim $R = S + w$ and $\Delta R = \Delta S + \Delta w$ offered to the workers and financiers, where in any equilibrium, it should hold that $R = x$.

The second step is to show the distortive effects of cash constraints. A firm stays in the competition to hire the worker until its final fixed wage offer, $w(\theta)$, exhausts its benefit from hiring the high-skilled worker

$$(p_H(\theta) - p_N(\theta))\,\Delta x - w(\theta) - (S + p_H(\theta)\,\Delta S) = 0. \tag{13}$$

Since in the discussed equilibrium, a firm seeking financing for $w(\theta)$ cannot signal its type through its choice of contracts (since all types offer a fixed wage and seek credit lines), financing will entail cross-subsidization from high to low types.

Consider, now, the case in which capital and labor are complements. When type θ^* makes the highest compensation offer it can afford, i.e., $w(\theta^*)$, the financiers overvalue its type, as they form their expectation over all types $\left[\theta^*, \overline{\theta}\right]$ that make a weakly positive profit from hiring at this wage. In particular, the financier's break-even condition is

$$\int_{\Omega} (S + \theta \Delta S)\, d\widetilde{F}(\theta) \geq 0, \tag{14}$$

where $\Omega = [\theta^*, 1]$. As is standard, it is assumed that in a competitive capital market, condition (14) is satisfied with equality, implying that $S + \theta^* \Delta S < 0$. Because type θ^*'s highest compensation offer, $w(\theta^*)$, is cross-subsidized by higher types, this offer is higher than the expected surplus from hiring. Specifically, from $S + \theta^* \Delta S < 0$ and expression (13), it holds that $w(\theta^*) > (p_H(\theta^*) - p_N(\theta^*))\,\Delta x$.

If capital and labor are substitutes, the argument is reversed. In this case, the set of types that can afford $w(\theta^*)$ is $\Omega = [0, \theta^*]$. Hence, type θ^* is pooled with lower types, implying that $S + \theta^* \Delta S < 0$ and $w(\theta^*) < (p_H(\theta^*) - p_N(\theta^*))\,\Delta x$. These distortions do not depend on whether the workers co-finance their compensation.

Proposition 6 *Cash constraints distort the highest fixed wage that the firm is prepared to offer upward if capital and labor are complements but downward if capital and labor are substitutes.*

6 Empirical Implications

In what follows, we summarize the main empirical predictions of our analysis and relate them to empirical evidence.

Negotiation vs. Competition. The relative importance of competition versus bargaining power for the level of workers' wages is debated in the literature, with some studies finding that competition is more important (Cahuc et al., 2006), while others finding that bargaining power plays a substantial role (Bagger et al., 2014; Di Addario et al., 2021).

Our model sheds light on such differing findings by arguing that the importance of negotiations (and, thus, bargaining power) versus competition will endogenously depend on whether capital and labor are substitutes or complements. Based on the evidence that capital and low-skilled labor are typically substitutes, while capital and high-skilled labor complements (Krusell et al., 2000; Larrain, 2015; Fonseca and van Doornik, 2022), we predict:

Implication 1 *The benefit from searching for one more job offer as opposed to negotiating with the firms that have already shown interest differs depending on whether workers are low- or high-skilled. While low-skilled workers are always better off attracting more competition, for high-skilled workers negotiations can be better.*

The majority of hiring managers expect workers to negotiate their compensation, with the fraction of workers that do so steadily increasing over the last decades and presently being at over 50% (Hall and Krueger, 2012; Brenzel et al., 2014; RobertHalf, 2019). In such cases, workers are typically asked by employers about their salary expectations, and the standard advice is to respond by offering a salary range. Offering such range can be seen as the first offer made by workers to firms. A firm then typically responds with an offer, and workers can then negotiate based on this offer or use the offer to ask for higher compensation at another firm interested in hiring them. Possibly motivated by such anecdotal evidence, the literature often models competition as sequential auctions (Cahuc et al., 2006; Bagger et al., 2014). Our results highlight that such sequential negotiations are not only practical for workers but also maximize their compensation.[14] In particular, a key normative implication of our model is that the following almost details-free mechanism is optimal:

Implication 2 *The optimal way for workers to negotiate with firms is to let firms sequentially improve on each other's compensation levels and then (if capital and labor are complements) negotiate only with the last remaining firm about compensation structure.*

Compensation Structure. Larger firms with rigid salary scales may be limited to offering compensation only within these scales. For this reason, compensation consultants advise negotiating also over types of compensation about which might have more flexibility, such as performance bonuses, equity-based pay, or signing bonuses. Our next implication is that the type of compensation that can maximize workers' compensation depends on whether capital and labor are complements or substitutes (i.e., whether labor is low- or high-skilled).

[14] As discussed, sequential negotiations can finish in only a handful of rounds. Especially when firms have strong bargaining power, jump offers constitute a payoff-equivalent equilibrium.

Implication 3 *(i) High-skilled workers can increase their compensation by negotiating for variable pay, such as equity-based pay or performance bonuses. (ii) By contrast, firms prefer increasing workers' fixed pay, such as base pay and signing bonuses, when competing for workers. For low-skilled workers, this type of compensation is also preferable when negotiating higher pay.*

Implication 3 is consistent with anecdotal evidence that high-skilled workers try to increase their compensation by negotiating for stock options (see, for example the New York Times article cited in the Introduction), while lower-skilled workers, such as warehouse managers, grocery store employes, or truck drivers who have seen pay increases during hot labor markets in the form of higher hourly wages and sign-on bonuses.[15] Empirically isolating the effects of negotiations and competition is challenging. In practice, workers may be more likely to negotiate their pay when the level of competition for workers (n in our model) is high, as then their outside options of restarting the job search ($\underline{w}$ in our model) are better and workers may find it easier to learn how to negotiate when they can observe multiple competing offers.[16] To the extent that a higher level of competition for labor makes negotiations more likely, the evidence is consistent with Implication 3. For example, Kedia and Rajgopal (2005) find that firms are more likely to offer equity-based compensation when they compete against similar nearby firms that also offer equity-based compensation — arguably when worker's bargaining power is stronger. Furthermore, Giannetti and Metzger (2015) find that long-term compensation in banks, which includes stock and stock options, is higher when there is more competition for talent and workers' bargaining power is stronger; and Mehran and Tracy (2001) find that more competition for workers in the 1990s is associated with an increase in stock-based compensation.[17]

Our explanation of equity-based pay as means of *attracting* workers with better negotiating positions complements prior work that has discussed the retention benefits of equity-based pay (Oyer, 2004). To differentiate between the two explanations, one could test whether the evidence we cite above is stronger for firms about whose growth options there is

[15] See "See "Companies you'd never expect are offering signing bonuses to new employees" (CNN, June 7, 2021)," The Guardian, September 16, 2019. Our competition-driven explanation for the use of signing bonuses complements other motivations pointed out in the literature, such as firms signaling their belief in the quality of the match with workers (Van Wesep, 2010) or compensating employees for forgoing pay at their current employers.

[16] Our model has nothing to say about how the level of competition (n in our model) correlates with workers' willingness to negotiate. Instead, Implication 2 studies whether for a *given* level of competition, n, negotiations are better than attracting interest from one more firm.

[17] Interestingly, in a model without information frictions (which is the key friction in our model), Bova and Yang (2017) show that strong worker bargaining power will be associated with less equity-based pay and point to evidence from union negotiations. We obtain a qualitatively similar prediction to Bova and Yang for the case in which capital and labor are substitutes (part (ii) of Implication 3).

more information asymmetry. Younger firms and firms about which there is more dispersion in analysts' forecasts are likely to fit this description. Notably, we show that asymmetric information should have the *opposite* effect when incorporated into models in which firms can dictate terms — then, information asymmetry will lead to less equity-based compensation (Lemma 1). Furthermore, one could test whether firms with decreasing stock prices also offer equity-based compensation to workers with stronger bargaining power. Such evidence would be less-consistent with alternative explanations that equity-based compensation helps retain workers when stock prices increase (Oyer, 2004). Extending Kedia and Rajgopal's (2005) empirical analysis offers support for these predictions.[18]

Another prediction emerging from Implication 3 is that there will be a positive association between offering equity-based pay to workers below the executive level and firm performance that is unrelated to incentive effects. Indeed, the evidence for such an association (Hochberg and Lindsey, 2010) is somewhat surprising from a theory stand point, as equity-based pay is unlikely to have strong incentive effects for rank-and-file employees (Holmström, 1982). In our model, this association arises because firms in which hiring can lead to large productivity increases (because capital and labor are complements) offer equity-based compensation to attract workers, while firms in which hiring evens out difference between more and less productive firms offer fixed pay (Corollary 1).

Distorted Matching Between Workers and Firms. Another insight from the paper is that, when external financing for fixed wages is easily available, it distorts the firms' willingness to pay for labor upward when capital and labor are complements but downward when they are substitutes (Proposition 6). That is, if capital and labor are complements, there is a mitigating force to concerns that compensation may be lower if external financing is hard to come by — as, for example, during a financial crisis. By contrast, the effects are reinforcing each other if capital and labor are substitutes.[19]

Implication 4 *Cash-constrained firms compete more aggressively for high-skilled workers and less aggressively for low-skilled workers than unconstrained firms.*

As a secondary implication, the paper also offers new predictions for how workers' compensation structure will correlate with within-firm wage inequality. Specifically, since equity-

[18] The empirical analysis is contained in the paper's working paper version and is available upon request.

[19] Our results that competition among financially constrained firms can lead to overly aggressive or depressed compensation depending on whether capital and labor are complements or substitutes provides a new angle to the literature investigating the effect of such constraints on employment. Prior work has focused, instead, on three other aspects: that financing constraints may prevent efficient retention (Falato and Liang, 2016; Caggese et al., 2018); that higher leverage may strengthen firms' bargaining power in negotiations with unions (Perotti and Spier, 1993; Matsa, 2010; Chava, Danis, and Hsu, 2020); and that high labor protection may increase operating leverage, thus, crowding out financial leverage (Simintzi, Vig, and Volpin, 2018; Woods, Tan, Faff, 2019).

based compensation allows workers to extract a higher share of the surplus generated by their labor, the compensation difference between skilled workers and management ("the firm" in the paper) will be lower. This perspective differs from that of the prior literature (Terviö 2008; Gabaix and Landier 2008; Edmans et al., 2009). This literature has typically focused on how the manager's compensation affects within-firm wage inequality but not on how that inequality correlates with the worker' compensation structure.[20]

Implication 5 *Within-firm wage inequality between skilled the worker and management will be lower in firms compensating skilled workers with more call options.*

7 Conclusion

Workers are often in a position to compare offers from different potential employers. Yet the literature is silent on how such competition affects financing and compensation structure and how workers can negotiate to extract better compensation. This paper addresses these gaps. From a corporate finance perspective, workers' compensation structure matters because it affects how much external financing firms need, the type of financing they use, and whether workers become part of the firm's investor base.

The main positive predictions in our analysis are that if capital and labor are complements (typical for high-skilled labor), workers obtain higher expected compensation by negotiating for highly-convex variable compensation, such as in call options. However, if capital and labor are substitutes (typical for low-skilled labor), workers' compensation is higher when they negotiate for fixed compensation. The degree of complementarity between capital and labor further matters for whether workers can extract higher compensation by focusing their energy on trying to attract one more job offer or negotiating with firms that have already shown interest. While increasing competition is preferable if capital and labor are substitutes, negotiations can lead to higher pay if capital and labor are complements. Whether capital and labor are complements also matters for the impact of cash constraints on compensation. We show that firms in which capital and labor are complements compete more aggressively because external financing distorts their willingness to pay for workers upward. The prediction is opposite for firms in which capital and labor are substitutes.

A key normative prediction from our paper concerns the best way for workers to negotiate their compensation. We show that the optimal mechanism is simple and almost details free. It involves firms competing on compensation levels by sequentially improving on each

[20] Another strand of the literature explains within-firm wage inequality with the organization of knowledge hierarchies within firms (Garicano and Rossi-Hansberg, 2006). For recent surveys, see Edmans et al. (2017) and Garicano and Rossi-Hansberg (2015).

other's offers, with the workers negotiating about compensation structure only with the last remaining firm. Notably, if firms have different bargaining power and make different types of offers, workers can still efficiently compare offers and successfully identify the firm willing to pay the most for their labor. Overall, the paper highlights the importance of competition and negotiations for corporate financing decisions, the structure of non-executive compensation, and the efficient matching between workers and firms.

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Appendix A Proofs

Proof of Lemma 1. In the only perfect Bayesian equilibrium, the firm offers a fixed wage. To see this, suppose that there is an equilibrium for which the worker's participation constraint is satisfied with equality and in which there is a type θ' that offers a compensation contract with $\Delta w > 0$. If multiple types offer the same contract, take θ' to be the highest type in the pool. By deviating to a fixed wage contract offering the worker $w = \underline{w}$, type θ' benefits from avoiding being pooled with lower types. The worker accepts the deviation, as the deviation contract is a fixed wage (which does not depend on out-of-equilibrium beliefs) that gives her the same as their outside option.

To see that an equilibrium in which all types offer fixed wages and the worker break even can be supported, suppose that the worker observes a deviation to a compensation contract with $\Delta\widetilde{w} > 0$ and $\widetilde{w} \leq \underline{w}$. Since the original fixed wage contract avoids misvaluation of firms, for any deviation that makes the firm better off, the worker must be worse off compared with the original contract. Thus, for any out-of-equilibrium beliefs that put positive probability only on types that can benefit from deviating, the worker rejects the deviation, and the equilibrium can be supported. **Q.E.D.**

Proof of Proposition 1. Suppose initially that a worker offers a single take-it-or-leave-it contract. At the end of the proof, we show that offering a menu will always be dominated. Let

$$\begin{aligned} u\left(\boldsymbol{\omega},\theta\right) &= w + p_H\left(\theta\right)\Delta w \\ v\left(\boldsymbol{\omega},\theta\right) &= x - w + p_H\left(\theta\right)\left(\Delta x - \Delta w\right) \end{aligned}$$

be a worker's and the firm's gross expected payoffs when the firm hires that worker with a compensation contract $\boldsymbol{\omega} = \{w, \Delta w\}$.

Claim 1: *If capital and labor are complements and first-best is not feasible, the worker optimally demands a contract with variable compensation only. That is, it must be that* $w = 0$.

Proof. Suppose to a contradiction that a compensation contract $\boldsymbol{\omega} = \{w, \Delta w\}$ with $w > 0$ were optimal and accepted by types $\theta \geq \widetilde{\theta}$. Construct now an alternative compensation contract $\widetilde{\boldsymbol{\omega}} = \left\{ w - \zeta, \Delta w + \frac{\zeta}{p_H(\widetilde{\theta})} \right\}$. By construction, the set of types that would be better off accepting than rejecting the deviation offer is unchanged. For the deviating worker, it holds $u(\widetilde{\boldsymbol{\omega}}, \theta) - u(\boldsymbol{\omega}, \theta) = -\zeta + p_H\left(\theta\right) \frac{\zeta}{p_H(\widetilde{\theta})}$. This expression is positive if and only if $\theta > \widetilde{\theta}$, which coincides with the set of types that accept the worker's deviation offer. Since this is the same set of types that accepts the original offer, the worker is better off with the proposed deviation. To guarantee that $\widetilde{\boldsymbol{\omega}}$ is feasible, we only need to choose ζ small enough that $\widetilde{w} \geq 0$.

For completeness, observe that by expressing $\Delta w = \frac{\left(p_H\left(\widetilde{\theta}\right) - p_N\left(\widetilde{\theta}\right)\right)\Delta x - w}{p_H\left(\widetilde{\theta}\right)}$ (using expression (6)), the optimal choice of $\widetilde{\theta}$

$$\arg\max_{\widetilde{\theta} \geq y} \int_y^{\widetilde{\theta}} w \frac{dF(\theta)}{1 - F(y)} + \int_{\widetilde{\theta}}^1 \left(w + p_H(\theta) \frac{\left(p_H\left(\widetilde{\theta}\right) - p_N\left(\widetilde{\theta}\right)\right)\Delta x - w}{p_H\left(\widetilde{\theta}\right)} \right) \frac{dF(\theta)}{1 - F(y)}$$

is increasing in w, where y is the lowest feasible type of the firm. In the baseline model, $y = 0$ (in later sections, y is the type of the firm with the second highest willingness to pay for labor). The cross partial of the above expression with respect to w and $\widetilde{\theta}$ is

$$\frac{1}{1 - F(y)} \left(\int_{\widetilde{\theta}}^1 p_H(\theta) \frac{\Delta_H}{\left(p_H(\widetilde{\theta})\right)^2} dF(\theta) + f\left(\widetilde{\theta}\right) \right) > 0.$$

Hence, by monotonic selection arguments, the (interior) optimum $\widetilde{\theta}$ increases in w. Thus, choosing a call options contract (for which $w = 0$) leads to the lowest cutoff $\widetilde{\theta}$ as claimed in the main text. **Q.E.D.**

Claim 2: *If capital and labor are substitutes, the worker optimally demands a fixed wage.*
Proof. Suppose to a contradiction that a compensation contract $\boldsymbol{\omega} = \{w, \Delta w\}$ with $\Delta w > 0$ were optimal. The firm accepts this contract if $\theta \leq \widetilde{\theta}$. Construct now an alternative compensation contract $\widetilde{\boldsymbol{\omega}} = \left\{ w + \zeta, \Delta w - \frac{\zeta}{p_H\left(\widetilde{\theta}\right)} \right\}$. By construction, the set of types that accepts this alternative contract is unchanged. For the worker, it holds $u(\widetilde{\boldsymbol{\omega}}, \theta) - u(\boldsymbol{\omega}, \theta) = \zeta - p_H(\theta) \frac{\zeta}{p_H\left(\widetilde{\theta}\right)}$. This expression is positive if and only if $\theta < \widetilde{\theta}$, which coincides with the types that accept the worker's deviation offer. Since this is the same set of types that accepts the original offer, the worker is better off with the proposed deviation. To guarantee that $\widetilde{\boldsymbol{\omega}}$ is feasible, we only need to choose ζ small enough that $\Delta\widetilde{w} \geq 0$. **Q.E.D.**

Claim 3: *Offering a menu is not optimal.*
Proof. Consider any non-degenerate menu W. Let $\widetilde{\boldsymbol{\omega}} \in W$ be the contract chosen by the (i) lowest type, $\widetilde{\theta}$, that accepts the worker's offer if capital and labor are complements; or (ii) the highest type $\widetilde{\theta}$ that accepts the worker's offer if capital and labor are substitutes.

Consider now a deviation that drops all other contracts except $\widetilde{\boldsymbol{\omega}}$ from the menu. Note that if a type prefers $\widetilde{\omega}$ over to its outside option of not hiring, the same holds for all: (i) higher types if capital and labor are complements; (ii) lower types if capital and labor are substitutes. Thus, the set of types, Ω_W, that accepts the worker's offer remains unchanged. But then, by revealed preference of the firm for contracts other than $\widetilde{\boldsymbol{\omega}}$, the worker must be better off dropping these contracts. In particular, if there existed a contract $\boldsymbol{\omega} \in W$ such that $v(\boldsymbol{\omega}, \theta) > v(\widetilde{\boldsymbol{\omega}}, \theta)$ for one of the types that accepts the worker's offer, this would necessarily imply that $u(\boldsymbol{\omega}, \theta) < u(\widetilde{\boldsymbol{\omega}}, \theta)$. Thus, the worker is better off offering only contract $\widetilde{\omega}$. **Q.E.D.**

Proof of Proposition 3. We show the proof for the general case in which the worker needs to choose between negotiating with n firms or attracting competition from one more firm. The optimal negotiation mechanism is the one from Proposition 4. For the case with $n = 2$, negotiations are as dictated by Proposition 1.

(i) Consider, first, the workers' expected payoff when $n + 1$ firms compete to hire the workers by incrementally increasing their fixed-wage offers until only one firm remains. In such competition, the workers' expected payoff is equal to the expected valuation of the firm with the second-highest productivity:

$$\begin{aligned} U_{comp} &= \int_0^1 \left(p_H\left(\theta\right) - p_N\left(\theta\right)\right) \Delta x dF_2\left(\theta\right) \\ &= \int_0^1 \left(p_H\left(\theta\right) - p_N\left(\theta\right)\right) \Delta x \left(n+1\right) n \left(1 - F\left(\theta\right)\right) F\left(\theta\right)^{n-1} f\left(\theta\right) d\theta, \end{aligned} \tag{A.1}$$

where we use that the distribution of the second-highest-order statistics with $n + 1$ firms is

$$\begin{aligned} F_{2,n+1}\left(y\right) &= F\left(y\right)^{n+1} + \left(n+1\right) F\left(y\right)^{n} \left(1 - F\left(y\right)\right). \\ f_{2,n+1}\left(y\right) &= \left(n+1\right) n \left(1 - F\left(y\right)\right) f\left(y\right) F\left(y\right)^{n-1} \end{aligned}$$

Since values are private and independent, and firms are symmetric, it holds that the worker's expected payoff would be the same in any alternative mechanism in which the highest-productivity firm hires the worker and makes fixed payments (Myerson, 1981).

Consider, next, the worker's expected payoff when she negotiates with n firms. If $\frac{\Delta_H}{\Delta_N} \in \left[1, \frac{p_H}{p_N}\right]$, the worker can extract all surplus created by her labor. The distribution of the highest order statistics with n firms is $F_{1,n}\left(y\right) = F\left(y\right)^n$. Hence, when n firms compete to hire the worker, the worker' expected compensation when that compensation extracts the full surplus from the firm willing to pay for labor the most is

$$U_{neg} = \int_0^1 (p_H(\theta) - p_N(\theta))\,\Delta x n F^{n-1}(\theta) f(\theta)\,d\theta \tag{A.2}$$

Let $\widehat{\theta}$ be defined by $F(\widehat{\theta}) = \frac{n}{n+1}$. The difference $U_{neg} - U_{comp}$ is

$$\begin{aligned}
D_1 &\equiv \int_0^1 (p_H(\theta) - p_N(\theta))\,\Delta x F^{n-1}(\theta)\,(1 - (n+1)(1 - F(\theta)))\,n f(\theta)\,d\theta \\
&= n(n+1)\Bigg(\int_0^{\widehat{\theta}} (p_H(\theta) - p_N(\theta))\,\Delta x F^{n-1}(\theta) \underbrace{\left(F(\theta) - \frac{n}{n+1}\right)}_{-} f(\theta)\,d\theta \\
&\quad + \int_{\widehat{\theta}}^1 (p_H(\theta) - p_N(\theta))\,\Delta x F^{n-1}(\theta) \underbrace{\left(F(\theta) - \frac{n}{n+1}\right)}_{+} f(\theta)\,d\theta\Bigg) \\
&> n(n+1)\left(p_H\left(\widehat{\theta}\right) - p_N\left(\widehat{\theta}\right)\right)\Delta x \int_0^1 F^{n-1}(\theta)\left(F(\theta) - \frac{n}{n+1}\right) f(\theta)\,d\theta \qquad \text{(A.3)} \\
&= n(n+1)\left(p_H\left(\widehat{\theta}\right) - p_N\left(\widehat{\theta}\right)\right)\frac{1}{n}\left(\frac{1}{n+1} - \int_0^1 F^n(\theta) f(\theta)\,d\theta\right) = 0
\end{aligned}$$

where we the inequality follows from the fact that capital and labor are complements, i.e., $(p_H(\theta) - p_N(\theta))\,\Delta x$ increases in θ. The equalities in the last line follow from integration by parts, as

$$\begin{aligned}
&\int_0^1 F^{n-1}(\theta)\left(F(\theta) - \frac{n}{n+1}\right) f(\theta)\,d\theta \\
&= \frac{1}{n+1} - \int_0^1 \left(nF^n(\theta) - \frac{n}{n+1}(n-1)\,F^{n-1}(\theta)\right) f(\theta)\,d\theta
\end{aligned}$$

and

$$\int_0^1 F^n(\theta) f(\theta)\,d\theta = 1 - n\int_0^1 F^n(\theta) f(\theta)\,d\theta.$$

Observe, now, that for $\frac{\Delta_H}{\Delta_N} = \frac{p_H}{p_N}$, the first best contract is $\left\{w^{fb}, \Delta w^{fb}\right\} = \left\{0, \left(1 - \frac{\Delta_N}{\Delta_H}\right)\Delta x\right\}$, implying that the transitioning to a contract that pays only in the high cash flow state (as dictated by Proposition 1) is smooth. In particular, for $\frac{\Delta_H}{\Delta_N} \geq \frac{p_H}{p_N}$, we obtain that the difference in the worker's expected payoff between negotiations with n firms and competition among $n+1$ firms is

$$D_2 \equiv \int_0^1 \left(\left(\int_y^{\widetilde{\theta}} \underline{w} dF(\theta|\theta \geq y) + \int_{\widetilde{\theta}}^1 p_H(\theta) \Delta w \Delta x dF(\theta|\theta \geq y) \right) dF_{2,n}(\theta) - \int_0^1 (p_H(y) - p_N(y)) \Delta x dF_{2,n+1}(y) \right) \quad \text{(A.4)}$$

where Δw and $\widetilde{\theta}$ are defined as in Proposition 1 and it holds that $D_2 = D_1 > 0$ for $\frac{\Delta_H}{\Delta_N} = \frac{p_H}{p_N}$. But then by continuity of D_2 in Δ_H and Δ_N, it must be that if $D_2 \leq 0$ then this must be above some threshold T for $\frac{\Delta_H}{\Delta_N}$, for which it holds that $T > \frac{p_H}{p_N}$.

(ii) If capital and labor are substitutes, the worker optimally demands fixed compensation. Since the firm's payments are then in cash both when the firm negotiates and when it searches for one more offer, the analysis is the same as in Bulow and Klemperer (1996). **Q.E.D.**

Proof of Proposition 4. In Proposition 1, we have shown that the mechanism is optimal for $n = 1$. Consider the case with $n > 1$ and let $q(\theta_i, \boldsymbol{\theta}_{-i})$ denote the probability that type θ_i hires the workers when the other $n-1$ firms' type realizations are $\boldsymbol{\theta}_{-i}$. Since there are n firms with independent types, we can state the worker's problem as finding the optimal menu $W = \{q(\theta_i, \boldsymbol{\theta}_{-i}), w(\theta_i, \boldsymbol{\theta}_{-i}), \Delta w(\theta_i, \boldsymbol{\theta}_{-i})\}$ that maximizes the expected payment by each firm

$$\max_W \sum_{i=1}^n \int_{[0,1]^n} q(\theta_i, \boldsymbol{\theta}_{-i}) (w(\theta_i, \boldsymbol{\theta}_{-i}) + p_H(\theta_i) \Delta w(\theta_i, \boldsymbol{\theta}_{-i})) dF(\theta_i, \boldsymbol{\theta}_{-i}), \quad \text{(A.5)}$$

subject to the ex ante incentive constraints

$$\begin{aligned} &\int_{[0,1]^{n-1}} \Big(q(\theta_i, \boldsymbol{\theta}_{-i}) (x - w(\theta_i, \boldsymbol{\theta}_{-i}) + p_H(\theta_i)(\Delta x - \Delta w(\theta_i, \boldsymbol{\theta}_{-i}))) \quad \text{(A.6)} \\ &\qquad + (1 - q(\theta_i, \boldsymbol{\theta}_{-i})) p_N(\theta_i) \Delta x \Big) dF(\boldsymbol{\theta}_{-i}) \\ \geq &\int_{[0,1]^{n-1}} \Big(q(z_i, \boldsymbol{\theta}_{-i}) (x - w(z_i, \boldsymbol{\theta}_{-i}) + p_H(\theta_i)(\Delta x - \Delta w(z_i, \boldsymbol{\theta}_{-i}))) \\ &\qquad + (1 - q(z_i, \boldsymbol{\theta}_{-i}))(x + p_N(\theta_i) \Delta x) \Big) dF(\boldsymbol{\theta}_{-i}) \end{aligned}$$

for each firm and $w, \Delta w \geq 0$. Next to these conditions, we further impose the no-regret participation and incentive constraints that for a *given* realization of $\boldsymbol{\theta}_{-i}$, for type θ_i it holds

$$w(\theta_i, \boldsymbol{\theta}_{-i}) + p_H(\theta_i)\Delta w(\theta_i, \boldsymbol{\theta}_{-i}) \leq (p_H(\theta_i) - p_N(\theta_i))\Delta x \tag{A.7}$$

and

$$\begin{aligned} &\int \Big(q(\theta_i, \boldsymbol{\theta}_{-i})(x - w(\theta_i, \boldsymbol{\theta}_{-i}) + p_H(\theta_i)(\Delta x - \Delta w(\theta_i, \boldsymbol{\theta}_{-i}))) \\ &\qquad + (1 - q(\theta_i, \boldsymbol{\theta}_{-i}))\, p_N(\theta_i)\Delta x \Big)\, dF(\boldsymbol{\theta}_{-i}) \\ \geq &\int \Big(q(z_i, \boldsymbol{\theta}_{-i})(x - w(z_i, \boldsymbol{\theta}_{-i}) + p_H(\theta_i)(\Delta x - \Delta w(z_i, \boldsymbol{\theta}_{-i}))) \\ &\qquad + (1 - q(z_i, \boldsymbol{\theta}_{-i}))\Big(x + p_N(\theta_i)\Delta x\Big)\Big)\, dF(\boldsymbol{\theta}_{-i}). \end{aligned} \tag{A.8}$$

Clearly, the ex post incentive constraints (A.8) imply the ex ante incentive constraints (A.6). Furthermore, we require that only the firm with the highest valuation (if any) hires and pays the worker

$$\begin{aligned} &q(\theta_i, \boldsymbol{\theta}_{-i}) = 0 \text{ and } \{w(\theta_i, \boldsymbol{\theta}_{-i}), \Delta w(\theta_i, \boldsymbol{\theta}_{-i})\} = \{0, 0\} \\ &\text{if } (p_H(\theta_i) - p_N(\theta_i))\Delta x < \max_{\theta_j \in \boldsymbol{\theta}_{-i}} (p_H(\theta_j) - p_N(\theta_j))\Delta x. \end{aligned} \tag{A.9}$$

Without loss of generality, we assume that ties are resolved with a coin toss.

Observe, now, that conditions (A.7), (A.8), and (A.9) imply that the worker's program when capital and labor are complements can be restated as

$$\int_{[0,1]^{n-1}} \max_W \left(\int_{[0,1]} (w(\theta_i, \boldsymbol{\theta}_{-i}) + p_H(\theta_i)\Delta w(\theta_i, \boldsymbol{\theta}_{-i}))\, dF(\theta_i | \theta_i \geq \max \boldsymbol{\theta}_{-i}) \right) dF(\boldsymbol{\theta}_{-i}), \tag{A.10}$$

subject to (A.7), (A.8) and $w, \Delta w \geq 0$. If capital and labor are substitutes, we need to replace the conditional distribution with $F(\theta_i | \theta_i \leq \min \boldsymbol{\theta}_{-i})$. Conditional on knowing $\boldsymbol{\theta}_{-i}$, solving for the optimal offer follows the same steps as Proposition 1, where we only need to replace the worker's prior in that proposition with the conditional distribution from program (A.10).

Finally, we argue that the mechanism stated in the proposition satisfies the incentive constraints (A.8). In particular, we argue that the proposed mechanism can be represented as a message game in which (i) each firm reports its type, θ_i; (ii) the worker selects the player that reports the type with the highest willingness to pay: $(p_H(\theta_i) - p_N(\theta_i))\Delta x >$

$\max_{\theta_j \in \boldsymbol{\theta}_{-i}} (p_H(\theta_j) - p_N(\theta_j))\,\Delta x$; (iii) the worker commits to demand compensation $\{w(y), \Delta w(y)\}$ from that firm as prescribed by Proposition 1.

To see that the proposed mechanism is incentive compatible, observe that, conditional on hiring the worker, no firm has a strict incentive to misreport her type to affect the compensation she pays to the worker, since that compensation does not depend on her report. Furthermore, no firm has an incentive to misreport her type to affect her probability of hiring the worker. Consider the case in which capital and labor are complements. Reporting $z_i < \theta_i$ is suboptimal, as it reduces the firm's probability of hiring without affecting its compensation it will have to pay the worker if it hires her. Reporting $z_i > \theta_i$ is also suboptimal. Such a report matters only if it increases the firm's probability of hiring. This occurs if there is a firm with type $\theta_j > \theta_i$ that would have otherwise hired the worker. But then, for θ_i to outcompete $\widetilde{\theta}_j$ it must offer compensation for which it holds that $\widetilde{\theta}_i \geq \theta_j$ (where $\widetilde{\theta}_i$ is defined by (6)), violating the firm's participation constraint (A.7). The argument for the case in which capital and labor are substitutes is analogous. **Q.E.D.**

Proof of Proposition 5. We show the proof for the case in which capital and labor are complements. The proof for the case of substitutes follows similar steps.

To see that the ranking proposed in the proposition identifies the firm willing to pay most for labor, while extracting the maximum information about its valuation, start by considering the firms' perspective. A firm is willing to stay in the race to hire the worker as long as the compensation it needs to offer does not exhaust all surplus from hiring. It is suboptimal for the firm to refuse to choose a contract from W_b if there is a contract in this menu for which it holds

$$x - w_b + p_H(\theta)(\Delta x - \Delta w_b) \geq x + p_N(\theta)\,\Delta x, \tag{A.11}$$

as the firm would lose the possibility of hiring at a compensation for which it is still better off compared to not hiring. Conversely, choosing a contract from W_b even though there is no contract in this set for which condition (A.11) is satisfied is also suboptimal, as the firm only retains the option to hire the worker at a compensation for which hiring leads to a negative net present value.

Observe that, for any menu of contracts W_b, the firm (Firm B) prefers to choose only among the contracts for which $\widetilde{\theta}(w_b, \Delta w_b)$ is lowest. This is because if the other firm (Firm A) drops out on the next move, Firm B would otherwise unnecessarily give away that its type is higher than $\widetilde{\theta}(w_b, \Delta w_b)$. This would allow the worker to extract more surplus with her final take-it-or-leave-it offer. Firm B is indifferent among all offers that have the same cutoff $\widetilde{\theta}(w_b, \Delta w_b)$, since all these offers communicate the same information to the worker (i.e., that $\theta \geq \widetilde{\theta}(w_b, \Delta w_b)$).

Consider now the worker' perspective. It is shown in what follows that there is no other way of ranking offers (compared to that proposed in the Proposition) that allows the worker to extract higher compensation with her final take-it-or-leave-it offer. Without loss of generality, restrict attention to two firms — Firm A whose type is θ_A and Firm B whose type is θ_B, where $\theta_B > \theta_A$. With the proposed strategy, the worker can infer Firm A's type θ_A and makes Firm B a final take-it-or-leave-it offer $\{0, \Delta w\}$ that maximizes

$$\begin{aligned} &\max_{\Delta w} \frac{1}{1-F(\theta_A)} \left(\int_{\theta_A}^{\theta^*} \underline{w} dF(\theta) + \int_{\theta^*}^{1} p_H(\theta) \Delta w dF(\theta) \right) \qquad \text{(A.12)} \\ s.t. \qquad & x + p_H(\theta^*)(\Delta x - \Delta w) = x + p_N(\theta^*) \Delta x \text{ and } \theta^* \geq \theta_A \end{aligned}$$

where the worker's posterior beliefs are formed using Bayes rule over $[\theta_A, 1]$.

Consider now the possibility that a worker pursues alternative ways to rank the firms' offers. Clearly, if for such alternatives, Firm B drops out before Firm A, the worker is worse off, as $\theta_B > \theta_A$. The worker is also worse off if Firm A drops out before Firm B, but the worker is only able to learn that Firm A's type is at least θ' where $\theta' < \theta_A$ (instead of at least θ_A), as then the worker's final offer is based on less information. Thus, it only remains to consider the case in which there is a strategy that allows the worker to extract from Firm B that its type is at least $\theta' > \theta_A$ (instead of at least θ_A). Extracting this information from Firm B requires that the worker ranks one of Firm A's offers relative to the new offer(s) picked by Firm B in a way that firm B stays only if its type is above θ'. To see that this strategy cannot make a worker better off, consider an idealized scenario for the worker in which she knows that Firm A's type is θ_A and only has to deal with the problem that she does not know Firm B's type. In this idealized scenario, let $\{w_a, \Delta w_a\}$ be the highest offer that Firm A would choose before dropping out. By ranking $\{w_b, \Delta w_b\}$ the same as $\{w_a, \Delta w_a\}$ even though

$$w_b + p_H\left(\widetilde{\theta}(w_b, \Delta w_b)\right) \Delta w_b > w_a + p_H\left(\widetilde{\theta}(w_a, \Delta w_a)\right) \Delta w_a,$$

the worker can "test" whether Firm B's type is at least $\theta' = \widetilde{\theta}(w_b, \Delta w_b)$. The worker optimally chooses $\{w_b, \Delta w_b\}$, trading off the risk that Firm B drops out if its type is below θ' against the benefit that the worker can subsequently make a more-informed take-it-or-leave-it final offer $\{0, \Delta w\}$ to Firm B if it does not drop out

$$\max_{w_b,\Delta w_b,\Delta w} \frac{1}{1-F(\theta_A)} \left(\int_{\theta_A}^{\theta'} \underline{w} dF(\theta) + \int_{\theta'}^{1} \left(\int_{\theta'}^{\theta^*} \underline{w} \frac{f(\theta)}{1-F(\theta')} d\theta + \int_{\theta^*}^{1} p_H(\theta) \Delta w \frac{f(\theta)}{1-F(\theta')} d\theta \right) dF(\theta) \right.$$

$$\begin{aligned} s.t. \quad & x - w_b + p_H(\theta')(\Delta x - \Delta w_b) = x + p_N(\theta') \Delta x \text{ and } \theta' \geq \theta_A \\ & x + p_H(\theta^*)(\Delta x - \Delta w) = x + p_N(\theta^*) \Delta x \text{ and } \theta^* \geq \theta'. \end{aligned}$$

This problem reduces to program (A.12), proving that the worker cannot do better than with the proposed strategy. **Q.E.D.**

Proof of Proposition 6. The proof proceeds in two steps. Step 1 shows that the proposed equilibrium can be supported. Step 2 shows that cash constraints distort the highest fixed wage the firm is prepared to offer. The proof that no other equilibrium survives is relegated to Appendix B.

Claim 1: *There is a perfect Bayesian equilibrium with a compensation contract $\{w, \Delta w\}$ and financing contract $\{S, \Delta S\}$ for which it holds that $S + w = x$. This equilibrium survives refining out-of-equilibrium beliefs with the Intuitive Criterion.*

Proof. Suppose that $\{w, \Delta w\}$ and $\{S, \Delta S\}$ are such that the worker's and financiers' participation constraints are satisfied. Define $R = w + S$ and $\Delta R = \Delta w + \Delta S$ as the sum of the claims offered to the worker and the financiers. Note that the Intuitive Criterion has no bite for deviations that benefit all types. In such cases, we can assume that the worker's and financiers' out-of-equilibrium beliefs place probability one on the lowest type, prompting the worker and financiers to reject the deviation. It remains to consider deviations that benefit only some types. Since $R = x$ (and feasibility dictates that $R \leq x$), consider a deviation to $\widetilde{R} = R - \zeta$ and $\Delta\widetilde{R} = \Delta R + \delta$ ($\zeta, \delta > 0$). Define the threshold type $\widehat{\theta}$ as

$$x - \widetilde{R} + p_H(\widehat{\theta})(\Delta x - \Delta\widetilde{R}) - x + R - p_H(\widehat{\theta})(\Delta x - \Delta R) = 0.$$

If the firm deviates to $\{\widetilde{R}, \Delta\widetilde{R}\}$, the difference in expected payoffs between the equilibrium and deviation contract for the firm is

$$\begin{aligned} & \zeta + p_H(\theta)(\Delta x - \Delta\widetilde{R}) - p_H(\theta)(\Delta x - \Delta R) \\ = \; & -\left(p_H(\theta) - p_H(\widehat{\theta})\right)\delta. \end{aligned} \tag{A.13}$$

Expression (A.13) is positive for any $\theta < \widehat{\theta}$ and negative otherwise. Suppose that at least some types benefit from this deviation (otherwise, it can be discarded). Expression (A.13) is then positive for the lowest type for which hiring with the original contract is better than not hiring. Hence, it is consistent with the Intuitive Criterion to assume that outsiders place probability one on the deviation coming from that type. Given that the worker and financiers just break even with the original contracts (which pool that type with higher types), they are strictly better off rejecting the deviation for such out-of-equilibrium beliefs. Hence, there is no profitable deviation to $\widetilde{R} < x$, and the proposed equilibrium candidate with $R = x$ can be supported. Furthermore, it survives refining out-of-equilibrium beliefs with the Intuitive Criterion. **Q.E.D.**

Claim 2. *The external financing contract from Step 1 distorts the highest fixed wage that the firm is prepared to offer upward if capital and labor are complements; the distortion is downward if capital and labor are substitutes.*

Proof. Consider the case in which capital and labor are complements. Let $w(\theta)$ be the fixed wage at which type θ is indifferent between hiring and not hiring, and let $v(\theta^*) \equiv x + p_N(\theta^*)\Delta x$. Using that $S = x - w$ and $\Delta S = \frac{w-x}{\int_{\theta^*}^1 p_H(\theta)\frac{f(\theta)}{1-F(\theta^*)}d\theta}$ to plug into (13) and rearranging terms, it holds

$$
\begin{aligned}
w(\theta^*) &= x + \int_{\theta^*}^1 \frac{p_H(\theta)}{p_H(\theta^*)}\frac{f(\theta)}{1-F(\theta^*)}d\theta\,(p_H(\theta^*)\Delta x - v(\theta^*)) \\
&\geq x + p_H(\theta^*)\Delta x - v(\theta^*).
\end{aligned}
$$

The inequality follows from the fact that $p_H(\theta^*)\Delta x - v(\theta^*)$ must be positive whenever external financing is needed, i.e., $w(\theta^*) > x$. Furthermore, the inequality is strict for any $\theta^* < 1$.

Next, consider the case in which capital and labor are substitutes. Since $\Omega = [0, \theta^*]$, it holds that $\Delta S = \frac{w-x}{\int_0^{\theta^*} p_H(\theta)\frac{f(\theta)}{F(\theta^*)}d\theta}$ and we have

$$
\begin{aligned}
w(\theta^*) &= x + \int_0^{\theta^*} \frac{p_H(\theta)}{p_H(\theta^*)}\frac{f(\theta)}{F(\theta^*)}d\theta\,(p_H(\theta^*)\Delta x - v(\theta^*)) \\
&\leq x + p_H(\theta^*)\Delta x - v(\theta^*).
\end{aligned}
$$

The last inequality is strict for any $\theta^* > 0$. **Q.E.D.**

Appendix B Internet Appendix

B.1 Robustness of Proposition 1: It Does Not Matter Who Has Private Information

If capital and labor are complements, the privately informed worker will offer an information insensitive claim to the uninformed firm and demand the most information sensitive claim (i.e., call options). The intuition is that a worker who knows that the quality of the match between her and the firm is high wants to minimize the cost coming from the firm's uncertainty about that match — this uncertainty causes the firm to treat the match with the worker as average, leading it to reject aggressive compensation demands. The best way for the worker to minimize the "undervaluation" cost arising from this concern is by offering to be paid with variable compensation only, as such compensation exposes the worker most (and the firm least) to the true worker-firm match quality. No other type of compensation (involving fixed and variable pay) can survive in equilibrium, as a worker matching with a more productive firm will deviate by offering to be paid with variable pay only and, thus, convincingly indicate (when refining beliefs using the Intuitive Criterion) the high quality of the match.

The key difference when labor and capital are substitutes is that low-productivity firms are willing to pay more to hire the worker. Thus, the strongest incentive to deviate from equilibria pooling high and low types is now for a worker that has matched with a *low* (rather than high) productivity firm. Such a worker can do so by offering a fixed wage contract that benefits workers facing a low type more those facing higher types. Intuitively, workers that are matched with a high-productivity firm have more to lose from giving up variable pay, Δw. Thus, contracts offering variable pay cannot be sustained in equilibrium, as workers facing lower-productivity firms will deviate by offering fixed wage contracts that are more valuable to the worker when she has matched with a low productivity compared to high productivity firm. Such deviations will be successful, as they convince the firm (using the Intuitive Criterion as refinement) of its low capital productivity and, thus, the high value of hiring.

Proposition B.1 *Suppose that the worker is privately informed about the firm's productivity and can make a take-it-or-leave-it offer to the firm. (i) If capital and labor are complements, the worker still demands compensation in call options; (ii) If capital and labor are substitutes, the worker demands compensation in fixed wages.*

Proof of Proposition B.1. We prove the Proposition in three steps.

Claim 1: *In any candidate equilibrium in which types $\theta', \theta'' \in [0,1]$, where $\theta'' > \theta'$, offer different compensation contracts, it must be that $\Delta w_{\theta''} \geq \Delta w_{\theta'}$.*

Proof. From incentive compatibility, it must be that

$$\begin{aligned} w_{\theta'} + p_H(\theta')\,\Delta w_{\theta'} &\geq w_{\theta''} + p_H(\theta')\,\Delta w_{\theta''} \\ w_{\theta''} + p_H(\theta'')\,\Delta w_{\theta''} &\geq w_{\theta'} + p_H(\theta'')\,\Delta w_{\theta'}, \end{aligned}$$

from which we obtain that

$$(p_H(\theta'') - p_H(\theta'))\,\Delta w_{\theta''} \geq (p_H(\theta'') - p_H(\theta'))\,\Delta w_{\theta'},$$

proving the claim. Note that there can be no separating equilibrium in which two types offer a contract with the same Δw but different w, as such contracts are not incentive compatible. Hence, if there is a type $\underline{\theta}$ that makes the same contract offer as θ'', then all types in $[\underline{\theta}, \theta'']$ make the same offer. **Q.E.D.**

Claim 2: *Let θ' and θ'' be the lowest and highest type that get hired and suppose that these types offer different compensation contracts. If capital and labor are substitutes, there is no equilibrium, satisfying the Intuitive Criterion, in which types $[\theta', \theta'')$ offer to be paid in a contract that is different from a fixed wage.*

Proof. Let $\{w_{\theta'}, \Delta w_{\theta'}\}$ and $\{w_{\theta''}, \Delta w_{\theta''}\}$ be the contracts offered by types θ' and θ'', respectively. First, we show that type θ' must offer a contract with $\Delta w_{\theta'} = 0$. Extending the argument to all types $\theta < \theta''$ is then straightforward.

If there is a type that makes the same offer as θ'', let $\underline{\theta}$ be the lowest among these types. By Claim 1, it holds then that all types in $[\underline{\theta}, \theta'']$ make the same offer ($\underline{\theta} = \theta''$ corresponds to the case in which type θ'' fully separates). Let $\theta^* \in [\underline{\theta}, \theta'']$ be the highest type that offers $\{w_{\theta''}, \Delta w_{\theta''}\}$ for which it holds that

$$x - w_{\theta''} + p_H(\theta^*)(\Delta x - \Delta w_{\theta''}) \geq x + p_N(\theta^*)\,\Delta x.$$

Such a type always exists, as the firm must at least break even in equilibrium.

Suppose to a contradiction that $\Delta w_{\theta'} > 0$, which (by Claim 1) also implies that all contracts offered in equilibrium have a strictly positive variable component. Construct a deviation contract $\{\widetilde{w}, \Delta\widetilde{w}\}$ with $\Delta\widetilde{w} = 0$ such that

$$\widetilde{w} = w_{\theta''} + p_H(\theta^*)\,\Delta w_{\theta''}.$$

By incentive compatibility for type θ^*, it further holds that

$$\widetilde{w} = w_{\theta''} + p_H(\theta^*)\,\Delta w_{\theta''} \geq w_\theta + p_H(\theta^*)\,\Delta w_\theta$$

for any contract $\{w_\theta, \Delta w_\theta\}$ offered by types $\theta \neq \theta^*$ in equilibrium (if all types offer the same contract, the inequality is weak). But since $p_H(\theta)$ is increasing in θ and $\Delta w_\theta > 0$ for all contracts offered in equilibrium, it holds that $\widetilde{w} > w_\theta + p_H(\theta)\,\Delta w_\theta$ for all types $\theta < \theta^*$, with the inequality being reversed for types $\theta > \theta^*$. Hence, any out-of-equilibrium beliefs satisfying the Intuitive Criterion should put a positive mass only on types $\theta \leq \theta^*$.

Consider, now, the firm's expected payoff. By construction, it holds that

$$x - \widetilde{w} + p_H(\theta)\,\Delta x \geq x + p_N(\theta)\,\Delta x \text{ for } \theta = \theta^* \tag{B.1}$$

Since capital and labor are substitutes (i.e., $\frac{\partial}{\partial\theta}(p_H(\theta) - p_N(\theta)) < 0$), the inequality in (B.1) is strict for all types $\theta \leq \theta^*$. Hence, we obtain that for *any* out of equilibrium beliefs satisfying the Intuitive Criterion, the firm is better off hiring with the deviation contract than not hiring and accepts the deviation. Hence, we obtain a contradiction, and it must be that $\Delta w_{\theta'} = 0$.

Suppose, next, that there is some type $\widehat{\theta} > \theta'$ that offers a contract $\{w_{\widehat{\theta}}, \Delta w_{\widehat{\theta}}\}$ with $\Delta w_{\widehat{\theta}} > 0$ and let $\widehat{\theta}$ be the lowest such type. By Claim 1, it must be that all types $[\theta', \widehat{\theta})$ offer the same contract with $\Delta w_\theta = 0$. We can construct now a profitable deviation for type $\widehat{\theta}$ following the same steps as above to show that there is a profitable deviation from $\Delta w_{\widehat{\theta}}$ to a contract with $\Delta\widetilde{w} = 0$ that will be accepted by the firm. We can proceed iteratively to show that all contracts offered in equilibrium by types $\theta < \theta''$ must have $\Delta w = 0$. **Q.E.D.**

Claim 3: *If capital and labor are substitutes, there is an equilibrium in which all worker types offer to be paid a fixed wage offer.*

Proof. A fixed wage offer for which the firm at least breaks from hiring

$$\int_0^1 (x + p_H(\theta)\,\Delta x - (x + p_N(\theta)\,\Delta x))\, dF(\theta) = w \tag{B.2}$$

can be supported as an equilibrium. Clearly, the worker cannot deviate to an alternative fixed wage offer for which she is better off, as then all types will benefit (note that the Intuitive Criterion has then no bite). Thus, for out-of-equilibrium beliefs placing probability one on the highest type, the firm would be making a loss from hiring and will not accept. This follows from the assumption that capital and labor are substitutes, implying from (B.2) that $x + p_H(\theta)\,\Delta x - (x + p_N(\theta)\,\Delta x) < w$ for $\theta = 1$.

Consider a deviation to a contract with $\Delta\widetilde{w} > 0$. If there is at least one type θ^* that weakly benefits from this deviation, then it must be that all $\theta > \theta^*$ benefit as well, implying that the firm is worse off with the deviation contract for all these types. Hence, for out-of-equilibrium beliefs placing probability one on the highest type, $\theta = 1$, the firm will reject the deviation, as it will hold that

$$x + p_H(\theta)\Delta x - (x + p_N(\theta)\Delta x) < w < \widetilde{w} + p_H(\theta)\Delta\widetilde{w} \quad \text{for } \theta = 1.$$

Q.E.D.

We omit the case in which capital and labor are complement and first-best is not attainable, as it follows from a straightforward modification of Claims 2 and 3 above. In Claim 2, this modification requires constructing a deviation to a contract with $w = 0$ for which higher (instead of lower) types benefit from deviating. For a very similar game with continuous cash flows, we refer the reader to Nachman and Noe (1994). **Q.E.D.**

B.2 Equilibria With External Financing

Claim: *In any equilibrium of the hiring and financing game in Proposition 6, it must hold that* $w + S = x$*.*

Proof. We argue to a contradiction. Suppose that there is an equilibrium in which type θ' hires the worker and raises financing with contracts $\{w_{\theta'}, \Delta w_{\theta'}\}$ and $\{S_{\theta'}, \Delta S_{\theta'}\}$ for which it holds that $R_{\theta'} = w_{\theta'} + S_{\theta'} < x$ and $\Delta R_{\theta} = \Delta S_{\theta'} + \Delta w_{\theta'} > 0$. For any two types $\theta'' > \theta'$ offering contracts $\{S_{\theta''}, \Delta S_{\theta''}\}$ and $\{w_{\theta''}, \Delta w_{\theta''}\}$, respectively, incentive compatibility requires that

$$\begin{aligned} x - R_{\theta''} + p_H(\theta'')(\Delta x - \Delta R_{\theta''}) &\geq x - R_{\theta'} + p_H(\theta'')(\Delta x - \Delta R_{\theta'}) \\ x - R_{\theta''} + p_H(\theta')(\Delta x - \Delta R_{\theta''}) &\leq x - R_{\theta'} + p_H(\theta')(\Delta x - \Delta R_{\theta'}), \end{aligned}$$

which implies that $\Delta R_{\theta''} < \Delta R_{\theta'}$ and $R_{\theta''} > R_{\theta'}$ (where the inequalities are weak if the two types offer the same contract).

Note that the only thing that matters for incentive compatibility is the joint claim offered to the worker and financiers. Thus, we consider contracts that have the same R and ΔR as equivalent for the firm. Furthermore, note that there can be no separating equilibrium in which two types offer contracts with the same ΔR but different R, as such contracts are not incentive compatible. Hence, if there is a type $\overline{\theta}$ that makes the same contract offer as θ', then all types in $\left[\theta', \overline{\theta}\right]$ make the same offer.

Consider the case in which there is a type $\overline{\theta} \geq \theta'$ that makes the same offer as θ' and let $\overline{\theta}$ be the highest among these types. From above, it holds then that all types in $\left[\theta', \overline{\theta}\right]$ make the same offer. Let $\theta^* \in \left[\theta', \overline{\theta}\right]$ be the lowest type in this set for which it holds that

$$w_{\theta'} + p_H(\theta)\Delta w_{\theta'} \geq w \quad \text{(B.3)}$$

$$S_{\theta'} + p_H(\theta)\Delta S_{\theta'} \geq 0. \quad \text{(B.4)}$$

Such a type always exists, as the investor and the worker must at least break even in equilibrium. Crucially, note that the inequalities (B.3) and (B.4) hold for all types $\theta > \theta^*$, as $\frac{\partial}{\partial\theta}p_H(\theta) > 0$ and $\Delta w_{\theta'} \geq 0$ and $\Delta S_{\theta'} > 0$.

Construct a deviation contract $\left\{\widetilde{R}, \Delta\widetilde{R}\right\}$ with $\widetilde{R} = x > R_{\theta'}$ and $\Delta\widetilde{R} < \Delta R_{\theta'}$ (where $\Delta\widetilde{w} \leq \Delta w_{\theta'}$ and $\Delta\widetilde{S} < \Delta S_{\theta'}$) such that

$$\begin{aligned} \widetilde{w} + p_H(\theta^*)\Delta\widetilde{w} &= w_{\theta'} + p_H(\theta^*)\Delta w_{\theta'} \\ \widetilde{S} + p_H(\theta^*)\Delta\widetilde{S} &= S_{\theta'} + p_H(\theta^*)\Delta S_{\theta'}. \end{aligned}$$

Note that by continuity of the worker's and investor's payoffs in $w, \Delta w$ and $S, \Delta S$, respectively, the deviation can be constructed such that it continues to hold that

$$\begin{aligned} \widetilde{w} + p_H(\theta)\Delta\widetilde{w} &\geq w \\ \widetilde{S} + p_H(\theta)\Delta\widetilde{S} &\geq 0. \end{aligned}$$

for all types $\theta > \theta^*$. Thus, for the deviation to be successful (i.e., accepted by the worker and the financier), it is sufficient to argue that the worker and investor will place probability zero on the deviation coming from types $\theta < \theta^*$. To see that this is the case, observe that by construction of $\left\{\widetilde{R}, \Delta\widetilde{R}\right\}$ and incentive compatibility, it holds that

$$x - \widetilde{R} + p_H(\theta^*)\left(\Delta x - \Delta\widetilde{R}\right) = x - R_{\theta'} + p_H(\theta^*)(\Delta x - \Delta R_{\theta'}) \geq x - R_\theta + p_H(\theta^*)(\Delta x - \Delta R_\theta)$$

for all contracts $\{R_\theta, \Delta R_\theta\}$ offered in equilibrium by types $\theta \geq \theta^*$ (where the last inequality is weak in case these types offer the same contracts). But then for any such contracts, for which $R_\theta < x$, it holds that $x - \widetilde{R} + p_H(\theta)\left(\Delta x - \Delta\widetilde{R}\right) > x - R_\theta + p_H(\theta)(\Delta x - \Delta R_\theta)$ for $\theta > \theta^*$, as $\Delta\widetilde{R} < \Delta R_{\theta'} \leq \Delta R_\theta$, and $x - \widetilde{R} + p_H(\theta^*)\left(\Delta x - \Delta\widetilde{R}\right) < x - R_{\theta'} + p_H(\theta^*)(\Delta x - \Delta R_{\theta'})$ for $\theta < \theta^*$. Hence, for any beliefs satisfying the Intuitive Criterion, the worker and the financier should place positive probability only on types $\theta > \theta^*$. Hence, the deviation is accepted, leading to the desired contradiction. Since this deviation can be constructed for any equilibrium in which types $\theta < 1$ do not offer $R = x$, all types $\theta < 1$ must offer $R = x$. **Q.E.D.**

B.3 Derivation of Optimal Strategies in Example 2

In this Appendix, we derive the optimal strategies for the worker and the firms for the four different mechanisms discussed in Section 4. In the calculations of all examples, we assume that $c = 0.7$.

B.3.1 Sequential Offers

Fixed wage: If firms sequentially improve on each other's offers until only one firm remains, it is a weakly dominating strategy for each firm to continue increasing its offers until the final compensation it offers extracts all surplus from hiring, $(p_H(\theta) - p_N(\theta))\,\Delta x = \frac{\theta}{c+1}\Delta x$. Hence, the worker's expected compensation will be equal to the expected surplus it creates at the firm with the second-highest willingness to pay for labor. If the firms compete by offering fixed compensation, the worker's expected payoff is

$$\begin{aligned} \mathrm{E}\left[\min\left(\theta_1, \theta_2\right)\right] \frac{\Delta x}{c+1} &= \int_0^1 \left(y \frac{\Delta x}{c+1}\right) \left(2\left(1-y\right)\right) dy \\ &= \frac{\Delta x}{3\left(c+1\right)} = 0.196 \Delta x. \end{aligned}$$

Call options: If the firms compete by offering call options, their maximum offer will be $\Delta w^s = \frac{\theta}{c+\theta}\Delta x$. Hence, the worker's expected compensation

$$\begin{aligned} &\mathrm{E}\left[p_H\left(\theta\right) \min\left(\frac{\theta_1}{c+\theta_1}, \frac{\theta_2}{c+\theta_2}\right) |\theta > \min\left(\theta_1, \theta_2\right)\right] \Delta x \\ = &\int_0^1 \int_y^1 \left(\frac{c+\theta}{c+1}\right) \frac{1}{1-y} d\theta \left(\frac{y}{c+y}\right) \left(2\left(1-y\right)\right) dy \Delta x = 0.242\, \Delta x \end{aligned}$$

B.3.2 Sequential Offers, Followed by TIOLI Offer

We consider now a final take-it-or-leave-it offer made by the worker to the last remaining firm. If the firm rejects, the workers take their initial outside option of $\underline{w}$. We compute the examples with $\underline{w} = 0$.

Fixed wage: Note that if the worker offers a fixed wage contract, which corresponds to type θ's maximum willingness to pay, $\frac{\theta}{c+1}\Delta x$, this offer's probability of acceptance is $\frac{1-\theta}{1-y}$, where y is the type of the firm with the lower willingness to pay. Hence, the worker maximizes

$$\max_{\theta} \frac{\theta}{c+1} \Delta x \min\left(\frac{1-\theta}{1-y}, 1\right) + \theta \underline{w},$$

where the min-operator indicates that the minimum θ that the worker will choose is y, in which case the firm's probability of acceptance is one. Hence, the worker's optimal choice will be $\theta^* = \max\left(y, \frac{1}{2} + \frac{\underline{w}}{2}\frac{(c+1)(1-y)}{\Delta x}\right)$. Let $\widehat{y}$ be the value of y for which the two terms in the max operator are equal. The worker's ex ante expected payoff (before y is revealed) becomes

$$\int_0^{\widehat{y}} \frac{\left(\underline{w}\left(c+1\right)\left(1-y\right)+\Delta x\right)^2}{4\left(c+1\right)\left(1-y\right)\Delta x}\left(2\left(1-y\right)\right)dy + \int_{\widehat{y}}^1 y\left(\frac{\Delta x}{c+1}+\underline{w}\right)\left(2\left(1-y\right)\right)dy$$

and for $\underline{w}=0$, we have

$$\left(\int_0^{0.5} \frac{1}{4\left(c+1\right)\left(1-y\right)}\left(2\left(1-y\right)\right)dy + \int_{0.5}^1 \frac{y}{c+1}\left(2\left(1-y\right)\right)dy\right)\Delta x = \frac{5}{12}\frac{\Delta x}{c+1} = 0.245\Delta x.$$

Call options: Note that if the worker offers a call options contract $\{0, \Delta w^s\} = \left\{0, \frac{\theta}{c+\theta}\Delta x\right\}$, which corresponds to type θ's maximum willingness to pay, this offer has a probability of acceptance $\frac{1-\theta}{1-y}$, where y is the type of the firm with the lower willingness to pay. Hence, the worker maximizes

$$\begin{aligned} & \max_\theta \int_\theta^1 \frac{c+z}{c+1}\left(\frac{\theta}{c+\theta}\Delta x\right)\frac{1}{1-y}dz + \theta\underline{w} \\ = \ & \frac{1}{2}\frac{\left(\theta-\theta^2\right)\left(2c+\theta+1\right)}{\left(c+\theta\right)\left(c+1\right)\left(1-y\right)}\Delta x + \theta\underline{w} \end{aligned}$$

and for the case where $\underline{w}=0$, we have $\theta^* = \max\left(y, \frac{1}{3}\sqrt{4c^2+6c+3} - \frac{2}{3}c\right)$. And the ex ante expected payoff is

$$\begin{aligned} & \int_0^{\widehat{y}} \left(\frac{1}{2}\frac{\left(\theta^*-\theta^2\right)\left(2c+\theta^*+1\right)}{\left(c+\theta^*\right)\left(1-y\right)}\right)\left(2\left(1-y\right)\right)dy\frac{\Delta x}{c+1} \\ + \ & \int_{\widehat{y}}^1 \left(\frac{1}{2}\frac{\left(\widehat{y}-\theta^2\right)\left(2c+\widehat{y}+1\right)}{\left(c+\widehat{y}\right)\left(1-y\right)}\right)\left(2\left(1-y\right)\right)dy\frac{\Delta x}{c+1} = 0.282\Delta x \end{aligned}$$

B.3.3 First Price Auction

Fixed wage: If the firms offer fixed wages, the workers' expected compensation is the same as with sequential offers.

Call options: If the offers call options, let $\beta(\theta)$ denote the equilibrium payment in the high cash flow state offered by type θ. We restrict attention to symmetric equilibria in which the highest type wins. That is, type θ's probability of hiring is θ. Each firm maximizes

$$\max_b \left(\frac{c+\theta}{c+1}\left(\Delta x - b\right) - \frac{c}{c+1}\Delta x\right)\beta^{-1}(b)$$

By optimality, it needs to hold that

$$-p_H(\theta)\,\beta^{-1}(b) + (p_H(\theta)(\Delta x - b) - p_N \Delta x)\frac{1}{\beta'\left(\beta^{-1}(b)\right)} = 0$$

Furthermore, in a symmetric equilibrium, it must hold that $b = \beta(\theta)$. Using that $\frac{\partial}{\partial\theta}(\theta\beta(\theta)) = \beta(\theta) + \theta\beta'(\theta)$, we obtain

$$\frac{\partial}{\partial\theta}(\theta\beta(\theta)) = \frac{p_H(\theta) - p_N}{p_H(\theta)}\Delta x$$

and so

$$\begin{aligned}\beta(\theta) &= \beta(0) + \frac{1}{\theta}\int_0^\theta \left(1 - \frac{p_N}{p_H(y)}\right)\Delta x dy \\ &= \left(1 - \frac{c}{\theta}\ln\left(\frac{\theta + c}{c}\right)\right)\Delta x.\end{aligned}$$

where we use that it is weakly optimal for the lowest type to bid $\beta(0) = \frac{p_H(0) - p_N}{p_H(0)}\Delta x = 0$.[21] Since there are two firms, the worker' expected compensation is twice the ex ante expected payment made by any given firm

$$\begin{aligned}2\int_0^1 p(\theta)\,\beta(\theta)\,\theta d\theta &= 2\int_0^1 \left(\frac{c+\theta}{c+1}\left(1 - \frac{c}{\theta}\ln\left(\frac{\theta+c}{c}\right)\right)\Delta x\theta\right)d\theta \\ &= \left(\frac{2}{3} + \frac{3}{2}c + c^2 - c(c+1)^2\ln\frac{c+1}{c}\right)\frac{\Delta x}{c+1} = 0.242\,\Delta x.\end{aligned}$$

B.3.4 Two Sequential TIOLI Offers

Finally, we consider a mechanism in which the worker makes a take-it-or-leave-it offer to the first firm, followed by a take-it-or-leave-it-offer to the second firm. If the second firm rejects, the worker goes to work for the first firm if that firm had accepted its original offer. Otherwise, the worker takes its initial outside option of $\underline{w} = 0$.

Fixed wage: The worker's TIOLI offer (which is to demand the valuation of some type θ^*, i.e., $x + p(\theta)\Delta x - \underline{v} = \theta^* \frac{\Delta x}{c+1}$) to the second firm, given an outside option of $\underline{w}_1$ is

$$\max \frac{\theta_2^*}{c+1}\Delta x\left(1 - \theta_2^*\right) + \underline{w}_1 \theta_2^*.$$

[21] Note that we present only a heuristic sketch of the argument. For a more-complete derivation of the optimal strategies in first-price auctions, see for example DeMarzo et al. (2005).

Hence, the optimal offer at $t = 2$, depending on whether the first firm rejects the worker's first offer is

$$\theta_2^* = \frac{1 + (c+1)\frac{\underline{w}_1}{\Delta x}}{2} = \begin{cases} \frac{1+\theta_1^*}{2} & \text{if first offer is accepted} \\ \frac{1}{2} & \text{if first offer is rejected} \end{cases},$$

where we use that $\underline{w}_1 = \theta^* \frac{\Delta x}{c+1}$ if the first offer is accepted and $\underline{w}_1$ if it is rejected. The expected payoff to the worker is

$$\left(\frac{1 + (c+1)\frac{\underline{w}_1}{\Delta x}}{2}\right)^2 \frac{\Delta x}{c+1}.$$

The worker's TIOLI offer to the first firm is

$$\max_{\theta_1} \left(\left(\frac{1+\theta_1}{2}\right)^2 (1-\theta_1) + \left(\frac{1}{2}\right)^2 \theta_1\right) \frac{\Delta x}{c+1}$$

Taking the first-order condition and solving for θ_1^*, we obtain

$$\theta_1^* = \frac{1}{3}\left(\sqrt{7} - 1\right)$$

and the worker's expected compensation is

$$\left(\frac{7}{54}\sqrt{7} + \frac{7}{108}\right)\frac{\Delta x}{c+1} = 0.240\Delta x.$$

Call options: Type θ's maximum willingness to pay is $\Delta w_2 = \frac{\theta}{c+\theta}\Delta x$. Hence the worker's expected payoff at $t = 2$ is

$$\int_{\theta_2^*}^1 \frac{c+\theta}{c+1}\frac{\theta_2^*}{c+\theta_2^*}\Delta x d\theta + \theta_2^* \left(\frac{1}{1-\theta_1^*}\int_{\theta_1^*}^1 \frac{c+\theta}{c+1}\frac{\theta_1^*}{c+\theta_1^*}\Delta x d\theta\right),$$

And the problem at $t = 0$ is

$$\max_{\theta_1^*, \theta_2^*(a), \theta_2^*(r)} \left(\left(\int_{\theta_2^*(a)}^1 (c+\theta)\frac{\theta_2^*(a)}{c+\theta_2^*(a)} d\theta + \theta_2^*(a)\left(\frac{1}{1-\theta_1^*}\int_{\theta_1^*}^1 \frac{c+\theta}{c+1}\frac{\theta_1^*}{c+\theta_1^*} d\theta\right)\right)(1-\theta_1^*) \right.$$
$$\left. + \left(\int_{\theta_2^*(r)}^1 (c+\theta)\frac{\theta_2^*(r)}{c+\theta_2^*(r)} d\theta\right)\theta_1^*\right)\frac{\Delta x}{1+c}.$$

Solving this problem numerically, we obtain that the worker's expected compensation is $0.240\Delta x$.