# Lying to Speak the Truth: Selective Manipulation and Improved Information Transmission*

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**Abstract**

We analyze a principal-agent model in which an effort-averse agent can manipulate a publicly observable performance report. The principal cannot observe the agent's cost of effort, her effort choice, and whether she manipulated the report. An optimal contract links compensation to both the eventually realized output and the (possibly manipulated) report, since both are informative about effort provision. We show that the optimal contract may incentivize selective manipulation of an unfavorable report by an agent who exerted a high level of effort. Doing so can convert a "falsely" negative report into a positive one, thereby making the report more informative about the agent's effort choice.

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# 1 Introduction

Financial reporting allows investors to monitor the performance of firms in which they invest. However, financial reporting is noisy, which adds frictions to the design of incentive compensation and may cause suboptimal decisions. Some have argued that investors may benefit from allowing executives some discretion in "managing" financial reports if this reduces the noise in their reporting (e.g., Subramanyam 1996). But such discretion can be abused by managers if their compensation depends on the perceived performance of their firms, and a large literature focusing on agency problems (discussed below) views the manipulation of financial reports as undesirable.[1]

We show that these two views are not necessarily in conflict. Analyzing an optimal contracting model in which performance reports are noisy and managers have the ability to manipulate them, we find that, even if manipulation can easily be prevented, shareholders may not find it optimal to do so. Instead, shareholders may benefit from incentivizing managers who expect their firms to perform well in the long run to manipulate an unfavorable short-term report. By effectively correcting "false negatives,"such selective manipulation can make the report more informative about the manager's performance, thereby lowering the incentive compensation required to motivate the manager to exert costly effort. However, this improved informativeness of the report comes at a cost: Manipulation causes a disutility to the manager, and this makes it more costly to induce effort. Shareholders need to trade off the informational benefits of selective manipulation against this increased cost of incentivizing effort. We show that the benefits outweigh the costs when managerial effort is only moderately productive. In contrast, when effort is highly productive, the optimal contract prevents all manipulation.

The survey results in Graham, Harvey, and Rajgopal (2005) and De Jong et al. (2014) show that it is common for CFOs to manipulate financial reports.[2] Importantly, it seems that CFOs regard such manipulation as being in their firms' best interest. The view that manipulation is benign is also evident from an episode described in Jack Welch's memoir

[1]Different terms are used in the literature to describe various forms of manipulation, such as fraud, irregularities, misconduct, misreporting, or misrepresentation; see Amiram et al. (2018) for an overview.

[2]See Hobson and Stirnkorb (2020) for experimental evidence.

(Welch and Byrne 2003), in which he complains about the managers of one division of GE, who were unwilling to "pitch in" to make up for an unexpected earnings shortfall.[3] It is also consistent with the increasing use of non-GAAP measures in financial reporting, which firms adopt to make their performance look more appealing to investors (e.g., Doyle, Jennings, and Soliman 2013; Curtis, Mcvay, and Whipple 2014; Laurion 2020). The *writing* of financial reports also seems to have predictive power regarding manipulation (e.g., Hoberg and Lewis 2017; Brown, Crowley, and Elliott 2020), and the misclassification of 8-K "current reports" may be another effective manipulation technique (Bird, Karolyi, and Ma 2018).

We demonstrate the optimality of selective manipulation in a simple principal-agent model, in which a manager (the agent) exerts costly effort and shareholders (the principal) can neither observe the manager's effort cost nor her effort choice. Inducing effort is beneficial for shareholders (at least when the manager's effort cost is low) since it increases the firm's chances of earning a high terminal cash flow. Effort also improves the chances that a favorable performance report will be realised at an intermediate date. Both the report and the eventual cash flow are verifiable and can be used to incentivize effort. However, the manager can manipulate the report before it is released: At a personal utility cost, she can convert an unfavorable report into a favorable one. The optimal incentive scheme determines whether the manager exerts effort (depending on her effort cost) and whether or not she manipulates an unfavorable report. Asymmetric information about the cost of effort causes an adverse selection problem that enables the manager to earn an information rent. Allowing for manipulation of the report complicates the firm's optimization problem.

A critical assumption of our model is that shareholders can influence the manager's cost of manipulating the financial report through their choice of corporate governance structures. For example, the board of directors may decide to implement a more elaborate internal control system or to appoint more financial experts to the audit committee, thus making it more

[3] After a negative earnings surprise of $350m was discovered, Welch was pleased by the GE division managers' offers to "pitch in": *"The response of our business leaders to the crisis was typical of the GE culture. [...] many immediately offered to pitch in [...]. Some said they could find an extra $10 million, $20 million, and even $30 million from their businesses to offset the surprise. [...] their willingness to help was a dramatic contrast to the excuses I had been hearing from the Kidder people."* (Welch and Byrne 2003, ch. 15).

difficult for the manager to manipulate the firm's financial reports. To stack the deck in favor of a strong governance system that prevents manipulation by the manager, we assume that the board's choice of governance arrangements—and hence of the manager's manipulation cost—have no impact on the firm's cash flow. In other words, we assume that it is equally costly to the firm to have relaxed financial reporting standards as it is to have strict standards that prevent manipulation. Arguably, stricter standards are likely to be more costly to implement, which could make manipulation optimal at the margin. However, this would be an uninteresting, mechanical explanation for the potential optimality of contracts that induce manipulation.

Our analysis shows that shareholders may benefit from incentivizing the manager to engage in manipulation even if manipulation could be prevented *at no cost* to the firm. To understand this result, which may seem counterintuitive, it is important to realize that the optimal contract incentivizes manipulation only when the manager exerted high effort, but never when she exerted low effort. This is achieved by making the manager's compensation increase in both the reported performance and the realized cash flow in such a way that the manager's marginal benefit from manipulating an unfavorable report increases in the effort level she chose. Shareholders can therefore set the manipulation cost so that the manager's expected benefit from manipulation outweighs her manipulation cost only when she exerted high effort. The resulting *selective manipulation strategy* makes the firm's report more informative about the manager's effort choice. Performance manipulation may therefore not only be unavoidable, as the literature has argued, but it can actually be desirable: Allowing the manager to overstate firm performance enables the principal to design a more efficient compensation scheme.

The improved informativeness of the report comes at a cost though. The expected manipulation cost that the manager incurs under a selective manipulation strategy effectively increases her disutility from exerting high effort, which makes inducing managerial effort more costly to shareholders.

We show that the benefits of a more informative report due to selective manipulation outweigh the agency costs associated with manipulation when managerial effort is only moderately productive. In this case, the optimal contract incentivizes the manager to exert high effort only when her effort cost is low, which means that the manager is unlikely to incur the manipulation cost. In contrast, when managerial effort is very productive (and hence likely to be incentivized in equilibrium), the optimal contract prevents all manipulation because the manager's expected manipulation costs under a selective manipulation strategy would be too large. Thus, managers who argue that their firms benefit from earnings management inadvertently reveal that their ability to add value is not very high.

Our results imply that misreporting can be used to smooth over problems that are temporary and not indicative of fundamental problems. Practitioners often argue that quarterly reporting requirements cause CEOs to become short-termist, suggesting that less disclosure may be favorable. A less radical approach would be to allow some selective manipulation. By allowing CEOs to inflate short-term reports that do not reflect the firm's underlying condition, they can more effectively focus on long-term value creation. For example, in diversified conglomerates, the skill of a CEO may be to gather and manage a portfolio of unrelated operations, focusing on the efficiency of each of the operations. Allowing such a CEO to hide a limited set of bad news may be beneficial, because it allows her to focus on making the best use of her skills.

A variety of explanations for the presence of manipulation have been offered in the literature. First, numerous authors have argued that manipulation is an unavoidable feature of large, widely held firms: It is too costly to completely prevent it, and therefore manipulation can only be managed, not avoided (e.g., Stein 1989; Demski, Frimor, and Sappington 2004; Goldman and Slezak 2006; Crocker and Slemrod 2007; Beyer, Guttman, and Marinovic 2014; Marinovic and Povel 2017; Bertomeu, Darrough, and Xue 2017). This is in stark contrast to our analysis, which shows that some manipulation by the manager may be beneficial to shareholders, even when it could easily be prevented at no cost to the firm. Our model also differs from this literature in that manipulation is related to effort provision: Only managers who exerted high effort are incentivized to engage in manipulation.

Second, if there are limits to communication, contractibility, or commitment, then it may be optimal to let an agent manipulate information (Dye 1988; Arya, Glover, and Sunder 1998; Demski 1998). Our results do not rely on such constraints, as the results are driven by asymmetric information.

Third, current shareholders in a firm may benefit from manipulation if it allows the firm to raise funds from third parties at favorable rates (e.g., Bar-Gill and Bebchuk 2003; Povel, Singh, and Winton 2007; Strobl 2013). This is different from our model, since there is no second period in which funds need to be raised.

Fourth, firms may rely on information generated by investors (and revealed through market prices) when making decisions, and it can then be optimal to allow for some manipulation if it strengthens the incentive to generate such information (e.g., Gao and Liang 2013). There is no such effect in our model.

Our model and results apply to a variety of settings. Venture capital (VC) financing is increasingly "staged", with the next round of financing becoming unavailable if negative news are revealed (Ewens, Nanda, and Rhodes-Kropf 2018). Cornelli and Yosha (2003) argue that entrepreneurs may manipulate interim performance signals, allowing their firms to access the next round of funding, and they show that this manipulation can be prevented by using convertible securities. Our model suggests that, in some cases, the VC may want to tolerate (or, in fact, incentivize) selective manipulation. This intuition is related to Banerjee and Szydlowski (2021), who analyze a cheap-talk model and show that VCs may want to be lax in their monitoring and tolerate some misrepresentation of investment opportunities to solve a risk-choice problem. Similarly, asymmetric information plays a role in M&A negotiations (Higgins and Rodriguez 2006), and it may be optimal to allow selective manipulation to prevent negotiations from being terminated, even if such manipulation does not affect the price that is ultimately paid.[4]

Furthermore, we want to emphasize that the report in our model does not have to be a financial report. It can be interpreted more generally as any observable variable whose

[4] Ahern and Sosyura (2014) show that acquirers manipulate their media coverage during such negotiations.

realization depends on the manager's effort choice, if the manager can, at a cost, improve the chances of a favorable realization. For example, attracting media attention and giving interviews in prestigious television news programs or business reviews may be linked to a CEO's effort. If getting such attention improves a CEO's expected compensation, then she may try to manipulate her chances of being featured in this way. Note that such media appearances do not need to affect the firm's value: It is sufficient if they suggest that a CEO exerted high effort, allowing for a more efficient incentive compensation scheme. For example, industry-level awards that are not observed by the firm's customers and suppliers may affect the CEO's expected compensation and may be manipulated by her, even if they have no effect on the firm's revenue.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 solves for the equilibrium contract. The empirical predictions of the model are derived and discussed in Section 4. Section 5 investigates the robustness of the model's results in various extensions and alternative setups. Finally, Section 6 summarizes our contribution and concludes. All proofs are contained in the Appendix.

## 2 The Model

We study an agency model with two risk-neutral parties, a board of directors and a manager, that takes place over times 0, 1, 2, and 3. At time 0, the board (the principal) chooses the firm's governance system (explained below) and hires a manager (the agent) to run the firm. The board represents the interests of shareholders and offers the manager a contract that maximizes the value of the firm, net of the cost of managerial compensation.[5] At time 1, the manager exerts an unobservable effort to enhance the value of the firm. At time 2, the firm's accounting system produces a public report concerning the manager's performance. A key feature of our model is that this report can be manipulated by the manager. At time 3, the firm's terminal cash flow $v$ is realized and paid out to shareholders.

The firm's cash flow is either high ($v = v_h$) or low ($v = v_\ell < v_h$). The distribution of $v$

[5] We therefore use the terms "shareholders" and "board of directors" synonymously in our analysis.

depends on the manager's effort choice $e \in \{0,1\}$. If the manager exerts high effort ($e = 1$), $v$ is equal to $v_h$ with probability one; if she exerts low effort ($e = 0$), $v$ is equal to $v_h$ with probability $\lambda < 1$ and equal to $v_\ell$ with probability $1 - \lambda$. The manager's private utility cost of exerting high effort, denoted by $c$, is drawn from a uniform distribution over the interval $[0, \bar{c}]$; the cost of low effort is normalized to zero. The manager's effort choice $e$ and effort cost $c$ are her private information and hence cannot be used for contracting purposes. To make the problem interesting, we assume that $\bar{c} > (1 - \lambda)(v_h - v_\ell)$, which ensures that inducing high effort is suboptimal when a high cost of effort $c$ is realized.

Prior to the realization of the cash flow $v$, the firm's accounting system generates a report $r$, providing noisy information to the market about the manager's effort choice (and thus the value of the firm). This report can take on one of two values, $r_h$ or $r_\ell$. Absent any managerial intervention, the report is correlated with the manager's effort choice as follows:

$$\text{prob}\,[r = r_h \mid e = 1] = \text{prob}\,[r = r_\ell \mid e = 0] = \delta, \tag{1}$$

where $\delta \in \left(\frac{1}{2}, 1\right)$. For simplicity, we assume that the report $r$ is independent of the firm's cash flow $v$, conditional on the manager's effort choice $e$. Note that $r$ is nevertheless an informative signal about $v$: A favorable report $r_h$ increases the likelihood of a high effort choice and hence of a high cash flow, whereas an unfavorable report $r_\ell$ decreases it. The parameter $\delta$ captures the quality of the firm's accounting system. It represents various accounting standards and conventions in the economy as well as firm- and auditor-specific factors such as the transparency of the firm's operations and the auditor's experience in the industry.

Although the report is produced by the firm's accounting system, the manager can influence its outcome—for example, by exploiting any leeway in accounting rules or by hiding information from the auditor. Specifically, we assume that, by incurring a utility cost $g$, the manager can turn an unfavorable report $r_\ell$ into a favorable report $r_h$ with probability $\phi$. We allow for the possibility of mixed-strategy equilibria and denote by $m \in [0,1]$ the probability with which the manager takes such an action.

The ability to manipulate information (at a cost) is found in many "costly state falsification" models (e.g., Dye 1988; Stein 1989; Lacker and Weinberg 1989; Maggi and Rodríguez-Clare 1995; Fischer and Verrecchia 2000; Guttman, Kadan, and Kandel 2006; Crocker and Slemrod 2007; Kartik 2009; Kartik, Ottaviani, and Squintani 2007; Beyer and Guttman 2012; Dutta and Fan 2014; Marinovic and Povel 2017). The manipulation cost $g$ may reflect the time spent coming up with creative ways to manage the firm's earnings or the effort involved in convincing an auditor to sign off on a biased report. This cost is influenced by the legal system in which the firm operates, but firm-specific factors are also relevant, such as the rigor of the firm's accounting system and internal controls, the skills and independence of the firm's accounting and internal audit teams, the independence and experience of the board's audit committee, the choice of external auditors, etc. The firm commits to its broader governance system before the manager signs the contract, and the cost $g$ captures the ease or difficulty of manipulating the report $r$. Note that the cost $g$ accrues to the manager, not the firm. However, the firm bears an indirect cost of manipulation: When the equilibrium contract induces selective manipulation, the manager anticipates that she may incur the disutility $g$ after exerting high effort, which makes it more costly to incentivize effort.[6]

To stack the deck against finding equilibria with weak governance, we assume that the board of directors can improve the firm's governance—and hence increase the manager's manipulation cost—at no cost to the firm. That is, at time 0 the board can choose any $g \geq 0$, without having to spend any resources.[7]

The board chooses the firm's governance system and the manager's contract to maximize the value of the firm, net of the cost of managerial compensation. A contract specifies the manager's compensation as a function of the report $r$ and the terminal cash flow $v$. The manager is risk neutral, has no wealth, and is protected by limited liability so that all payments must be nonnegative. Her reservation level of utility is normalized to zero.

The contractual frictions in our model are created by asymmetric information: The board

[6]Allowing for a second cost of manipulation that accrues directly to the firm complicates the analysis, but does not generate any new insights. We discuss this possibility in more detail in Section 5.

[7]For empirical evidence that boards can affect the likelihood of manipulation by changing the firm's governance, see, for example, Beasley (1996), Dechow, Sloan, and Sweeney (1996), Fich and Shivdasani (2007), and Zhao and Chen (2008).

faces an adverse selection problem (the manager's cost of effort is unobservable) and two moral hazard problems (the manager's effort choice and manipulation decision are unobservable). There is no signaling in our model, since the manager's actions are all unobservable. The firm could achieve the first-best outcome if the cost of effort, $c$, and the chosen effort level, $e$, were verifiable: The report $r$ would then have no information value, and the manager would have no incentive to manipulate it. The board would find it optimal to elicit high effort if and only if $v_h - c \geq \lambda v_h + (1-\lambda)v_\ell$, and so the first-best effort level is given by

$$e_{FB} = \begin{cases} 1 & \text{if } c \leq (1-\lambda)(v_h - v_\ell), \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

## 3 Equilibrium Analysis

In this section, we solve for the manager's optimal compensation contract. Our specification of the set of available contracts is without loss of generality in the sense that it is fully consistent with the revelation principle. Thus, we can restrict attention to truthful direct revelation mechanisms. It is important to note that this does not imply that the board will induce the manager to abstain from manipulating an unfavorable report: The manager's decision to manipulate the report $r$ is an action and not a message. Instead, it implies that any allocation that can be achieved through a contract that is contingent on the report and the firm's cash flow can also be achieved through a truthful direct mechanism.

In the ensuing analysis, let $w(r, v|c)$ denote the compensation scheme under the direct mechanism. The fact that the manager has no wealth means that all compensation payments must be nonnegative. This implies that the manager's participation constraint is trivially satisfied: By choosing to exert zero effort and to not manipulate the report, the manager can always achieve a nonnegative payoff.

### 3.1 Preliminary Results

We first show that, under the optimal contract, the manager's effort choice is characterized by a cost threshold $\hat{c}$ such that the manager exerts high effort if and only if $c < \hat{c}$. This follows immediately from incentive-compatibility considerations. Suppose a manager with a cost of effort $c$ finds it optimal to choose the high effort level. A manager with a strictly smaller cost $c' < c$ faces exactly the same feasible actions and continuation payoffs as the manager with a cost $c$: If she also chooses the high effort level, then the continuation payoffs for each feasible action are identical for $c$ and $c'$, but the payoff of the manager with the lower cost $c'$ is larger because her cost of effort is smaller. The continuation payoffs after choosing the low effort level are identical for the two managers, because the cost of exerting low effort is zero. It must therefore be optimal for a manager with a cost $c' < c$ to also choose the high effort level. Conversely, if a manager with a cost of effort $c$ finds it optimal to choose the low effort level, then a manager with a strictly higher cost $c'' > c$ must also find it optimal to choose the low effort level.

**Lemma 1.** *There exists a threshold $\hat{c} \in [0, \bar{c}]$ such that the optimal contract induces high managerial effort (i.e., $e = 1$) for all $c < \hat{c}$ and low managerial effort (i.e., $e = 0$) for all $c > \hat{c}$.*

Note that the manager is never indifferent between the high and the low effort level, except when her cost of effort is exactly at the threshold, $c = \hat{c}$. In equilibrium, under an optimal contract shareholders are also indifferent between inducing high and low managerial effort when $c = \hat{c}$, but not for any other realizations of $c$.[8] Since both shareholders and the manager are indifferent if and only if the zero-probability event $c = \hat{c}$ occurs, we can ignore mixed strategies concerning the manager's effort choice $e$.

Our next result concerns the manipulation decision that the optimal contract induces the manager to take. We demonstrate that this decision depends on the manager's cost of effort only through its effect on the manager's effort choice $e$. This is not surprising, because the cost $c$ has no *direct* effect (besides its effect on effort choice) on the manipulation decision

[8] We analyze the optimal choice of the cost threshold $\hat{c}$ in Proposition 5 below.

that the firm wants to induce: For a given effort choice $e$, the cost $c$ does not affect the firm's cash flow $v$ or the report $r$ and, hence, has no impact on the shareholders' expected payoff.

**Lemma 2.** *For any two effort costs $c$ and $c'$, for which the optimal contract induces the same effort choice $e$, the optimal contract also induces the same manipulation decision $m$.*

Lemma 1 shows that, under the optimal contract, the manager's effort choice is identical for all realizations of the cost parameter $c$ below the threshold $\hat{c}$ and for all realizations above the threshold $\hat{c}$. Together with the result in Lemma 2, this implies that any allocation resulting from an optimal direct mechanism can be implemented through a menu of contracts that pools all managers of type $c < \hat{c}$ and of type $c > \hat{c}$.

**Lemma 3.** *The optimal mechanism can be implemented by offering the manager a menu of contracts that pools all types $c \in [0, \hat{c})$ and all types $c \in (\hat{c}, \bar{c}]$.*

Without loss of generality, we can thus set $w(r, v|c) = w_1(r, v)$ for all $c \in [0, \hat{c})$ and $w(r, v|c) = w_0(r, v)$ for all $c \in (\hat{c}, \bar{c}]$, where the subscript 1 (respectively, 0) indicates the region of parameter values $c$ for which the optimal contract induces high (respectively, low) managerial effort. The optimal compensation scheme can hence be characterized by the menu $\mathcal{W} = \{\boldsymbol{w}_0, \boldsymbol{w}_1\}$, where $\boldsymbol{w}_e = (w_e(r_h, v_h), w_e(r_\ell, v_h), w_e(r_h, v_\ell), w_e(r_\ell, v_\ell))$. For notational convenience, we also define the *manipulation schedule* $\mathcal{M} = (m_0, m_1) \in [0, 1]^2$ as the manipulation choices that the board wants to induce, where $m_e$ is the desired manipulation choice for a given effort choice $e$.

### 3.2 The Principal's Problem

The optimal contract that the board offers the manager maximizes the shareholders' expected payoff, that is, the firm's expected cash flow net of the manager's expected compensation. We solve for the optimal contract in three steps. First, for a given cost threshold $\hat{c}$ and manipulation schedule $\mathcal{M}$, we characterize the compensation scheme $\mathcal{W}$ and manipulation cost $g$ that induce the manager to exert high effort if and only if $c \leq \hat{c}$ and to make the desired manipulation decisions at minimum cost to the firm. Second, for a given cost threshold $\hat{c}$, we compare the firm's profit across different manipulation schedules $\mathcal{M}$. We show that it is never optimal to incentivize the manager to manipulate a low report when she exerted low effort, which allows us to restrict our attention to contracts that may or may not induce manipulation when the manager exerted high effort. Third, for each potentially optimal manipulation schedule, we solve for the cost threshold $\hat{c}$ that maximizes the firm's expected profit. We then compare the expected profits generated by these contracts and determine which contract is optimal for a given set of parameter values.

To simplify the notation, let $\pi_{e,m_e}(r,v)$ denote the probability that a report $r \in \{r_h, r_\ell\}$ and a cash flow $v \in \{v_h, v_\ell\}$ is produced when the manager chooses effort level $e \in \{0,1\}$ and makes the manipulation decision $m_e \in [0,1]$. For example, if the manager exerts high effort ($e=1$), the firm generates a high cash flow with certainty; it generates a high report with probability $\delta$ in case the manager chooses not to manipulate ($m_1 = 0$) and with probability $\delta + (1-\delta)\,\phi$ in case the manager chooses to manipulate ($m_1 = 1$). Thus, we have $\pi_{1,m_1}(r_h, v_h) = \delta + (1-\delta)\,\phi\, m_1$. The probabilities of the other possible outcomes are defined analogously (see the proof of Proposition 1). Based on the results stated in Lemmas 2 to 3, we can then express the manager's expected compensation as

$$\left(\frac{\hat{c}}{\bar{c}}\right) \sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) + \left(1 - \frac{\hat{c}}{\bar{c}}\right) \sum_{r,v} \pi_{0,m_0}(r,v)\, w_0(r,v). \tag{3}$$

For a given cost threshold $\hat{c}$ and manipulation schedule $\mathcal{M}$, the optimal contract $\mathcal{C} = (\mathcal{W}, g)$ minimizes the expected payment to the manager subject to the nonnegativity constraints

$$g \geq 0, \quad w_0(r,v) \geq 0, \quad w_1(r,v) \geq 0, \qquad \forall r \in \{r_h, r_\ell\}, v \in \{v_h, v_\ell\}, \tag{4}$$

and the following incentive compatibility (IC) constraints that ensure that the manager takes the desired actions: First, to induce the manager to follow the manipulation schedule $\mathcal{M}$, the compensation scheme has to satisfy the constraints

$$\phi\left[w_1(r_h, v_h) - w_1(r_\ell, v_h)\right] - g \begin{cases} \leq 0 & \text{if } m_1 = 0, \\ = 0 & \text{if } m_1 \in (0,1), \\ \geq 0 & \text{if } m_1 = 1, \end{cases} \tag{5}$$

$$\phi\left[\lambda\big(w_0(r_h, v_h) - w_0(r_\ell, v_h)\big) + (1-\lambda)\big(w_0(r_h, v_\ell) - w_0(r_\ell, v_\ell)\big)\right] - g \begin{cases} \leq 0 & \text{if } m_0 = 0, \\ = 0 & \text{if } m_0 \in (0,1), \\ \geq 0 & \text{if } m_0 = 1. \end{cases} \tag{6}$$

These constraints ensure that the manager's expected benefit from manipulation, which turns an unfavorable report $r_\ell$ into a favorable report $r_h$ with probability $\phi$, outweighs (respectively, does not outweigh) her manipulation cost $g$ when $m_e = 1$ (respectively, when $m_e = 0$).

Second, for the manager to exert high effort when $c < \hat{c}$ and to exert low effort when $c > \hat{c}$, we must have

$$\sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) - \hat{c} - (1-\delta)gm_1 \geq \max_{m \in [0,1]} \sum_{r,v} \pi_{0,m}(r,v)\, w_1(r,v) - \delta gm, \tag{7}$$

$$\sum_{r,v} \pi_{0,m_0}(r,v)\, w_0(r,v) - \delta gm_0 \geq \max_{m \in [0,1]} \sum_{r,v} \pi_{1,m}(r,v)\, w_0(r,v) - \hat{c} - (1-\delta)gm. \tag{8}$$

The above IC constraints take into account the fact that the manager's effort choice affects the distribution of the firm's report $r$ and hence the likelihood that the manager will incur the manipulation cost $g$. The probability of an unmanipulated low report is $1-\delta$ when the manager exerts high effort and $\delta$ when the manager exerts low effort.

Finally, to ensure that the manager truthfully reports her effort cost $c$, it must be that

$$\sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) - c - (1-\delta)gm_1 \geq$$
$$\max_{e \in \{0,1\}, m \in [0,1]} \sum_{r,v} \pi_{e,m}(r,v)\, w_0(r,v) - e\big(c + (1-\delta)gm\big) - (1-e)\delta gm, \quad \forall c \in [0, \hat{c}), \tag{9}$$

$$\sum_{r,v} \pi_{0,m_0}(r,v)\, w_0(r,v) - \delta g m_0 \geq$$

$$\max_{e\in\{0,1\}, m\in[0,1]} \sum_{r,v} \pi_{e,m}(r,v)\, w_1(r,v) - e\big(c + (1-\delta)gm\big) - (1-e)\delta g m, \quad \forall c \in (\hat{c}, \bar{c}]. \quad (10)$$

For a given cost threshold $\hat{c}$ and manipulation schedule $\mathcal{M}$, the principal's optimization problem is thus to minimize the manager's expected compensation in (3), subject to the constraints in (4)–(10).

### 3.3 No Manipulation vs. Selective Manipulation

In this section, we derive the optimal contract for various manipulation schedules $\mathcal{M} \in [0,1]^2$, taking the cost threshold $\hat{c}$ as given. We demonstrate that it is never optimal to incentivize the manager to manipulate a low report when she exerted low effort.

We begin our analysis by characterizing the optimal no-manipulation contract, that is, the optimal contract that induces the manager to never manipulate the report, irrespective of her chosen effort level. The following proposition shows that the optimal no-manipulation contract rewards the manager only when both the firm's cash flow and its report are high (i.e., when $v = v_h$ and $r = r_h$), which allows for the strongest inference that the manager exerted high effort. The cost of manipulation, $g^n$, is set such that manipulation is never optimal for the manager.

**Proposition 1.** *For any cost threshold $\hat{c} \in [0, \bar{c}]$, the optimal no-manipulation contract $\mathcal{C}^n$ consists of a compensation scheme*

$$w_0^n(r,v) = w_1^n(r,v) = \begin{cases} \frac{\hat{c}}{\delta - \lambda(1-\delta)} & \text{if } r = r_h \text{ and } v = v_h, \\ 0 & \text{otherwise,} \end{cases} \quad (11)$$

*and a manipulation cost*

$$g^n \geq \frac{\phi\, \hat{c}}{\delta - \lambda(1-\delta)}. \quad (12)$$

*This contract induces the manager to exert high effort if $c \leq \hat{c}$ and low effort if $c > \hat{c}$, and to* *follow the manipulation schedule $m_0 = m_1 = 0$, at minimum cost.*

Despite the fact that the manager has private information about her cost of effort $c$, shareholders cannot benefit from offering the manager a menu of type-specific contracts with different compensation schemes depending on the (truthfully reported) cost of effort. The reason is that both the principal and the agent are risk neutral in our setting: Both parties care only about the expected value of payments, contingent on the manager's actions $e$ and $m$. Thus, any compensation scheme that leads to the same expected contingent payments as the one in (11) is optimal, as long as it satisfies the incentive compatibility constraints. For example, the compensation scheme could include lotteries after $r$ and $v$ have been realized or it could offer a fixed payment if the manager announces a cost $c > \hat{c}$ (instead of a payment that is contingent on $r$ and $v$). It also means that setting the compensation scheme $\boldsymbol{w}_0$ equal to $\boldsymbol{w}_1$ is optimal: This choice of $\boldsymbol{w}_0$ (i) incentivizes a manager with a cost $c > \hat{c}$ to exert low effort, and (ii) ensures that the expected compensation of a low-effort manager is equal to the minimum amount required by the truth-telling constraint in (10). Intuitively, there are no real effects if a manager with a cost $c > \hat{c}$ falsely reports a cost below $\hat{c}$, as long as she thereafter chooses the desired effort level $e = 0$ and does not manipulate.

We next turn to the optimal contract that prompts the manager to implement the manipulation schedule $m_0 = 0$ and $m_1 = 1$, that is, that induces the manager to manipulate a low report if she exerted high effort, but not if she exerted low effort. We refer to such a contract as a selective-manipulation contract.

**Proposition 2.** *For any cost threshold $\hat{c} \in [0, \bar{c}]$, the optimal selective-manipulation contract $\mathcal{C}^s$ consists of a compensation scheme*

$$w_0^s(r, v) = w_1^s(r, v) = \begin{cases} \frac{\hat{c}}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} & \text{if } r = r_h \text{ and } v = v_h, \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

*and a manipulation cost*

$$g^s = \frac{\lambda\phi\,\hat{c}}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}. \tag{14}$$

*This contract induces the manager to exert high effort if $c \leq \hat{c}$ and low effort if $c > \hat{c}$, and to follow the manipulation schedule $m_0 = 0$ and $m_1 = 1$, at minimum cost.*

As in the no-manipulation case, the manager only receives a compensation when the outcome $v = v_h$ and $r = r_h$ is observed, which allows for the strongest inference that the manager exerted high effort. The cost of manipulation, $g^s$, is chosen such that only a manager who exerted high effort has an incentive to manipulate a low report $r_\ell$. It should not be chosen larger than necessary, because the manager anticipates that she may have to incur the cost $g^s$ if she exerts high effort, and an increase in $g^s$ therefore requires an increase in the promised compensation. Thus, the principal optimally sets $g^s$ equal to $\lambda \phi w^s(r_h, v_h)$, the minimum amount required to prevent the manager from manipulating a low report when she exerted low effort.

Our next result shows that it is never optimal for shareholders to incentivize the manager to manipulate a low report when she exerted low effort or to play a mixed manipulation strategy when she exerted high effort.

**Proposition 3.** *An optimal contract (i) does not induce manipulation after low effort and (ii) does not induce a randomized manipulation decision after high effort.*

A contract that incentivizes a manager who exerted high effort to use mixed strategies when making her manipulation decision cannot be optimal for two reasons. First, compared to the selective-manipulation contract $\mathcal{C}^s$, which always induces manipulation of a low report $r_\ell$ after high effort, inducing such behavior with a probability of less than one makes the public report $r$ less informative about the manager's effort choice. Second, inducing $m_1 \in (0, 1)$ requires a higher manipulation cost $g > g^s$ because the cost must make a manager who chose the high effort level (and hence correctly anticipates a high cash flow $v_h$) indifferent between manipulating and not manipulating. In contrast, under the selective-manipulation contract $\mathcal{C}^s$ a manager who exerted low effort is kept indifferent, whereas a manager who exerted high effort strictly prefers manipulation. Inducing mixed strategies over the choice of $m_1$ thus causes two inefficiencies for shareholders: The link between effort and compensation is weakened, and the required increase in the manipulation cost $g$ makes it more costly for shareholders to incentivize high managerial effort. Similarly, incentivizing manipulation by a manager who exerted low effort has a negative effect: It reduces the informativeness of the report about the manager's effort choice and hence increases the compensation payment required to induce high managerial effort.

Proposition 3 implies that, for any desired cost threshold $\hat{c}$, the optimal contract is either the no-manipulation contract $\mathcal{C}^n$ defined in Proposition 1 (which prevents manipulation entirely) or the selective-manipulation contract $\mathcal{C}^s$ defined in Proposition 2 (which permits manipulation only after high effort). The following proposition compares the manager's expected compensation under these two contracts (taking the threshold $\hat{c}$ as given).

**Proposition 4.** *Let* $\kappa = \frac{(1-\delta)(1-\lambda)}{\delta-\lambda(1-\delta)} \in (0,1)$. *Then,*

*(i) for any cost threshold $\hat{c} \in (0, \kappa\bar{c})$, the expected compensation under the no-manipulation contract $\mathcal{C}^n$ defined in Proposition 1 is strictly higher than the expected compensation under the selective-manipulation contract $\mathcal{C}^s$ defined in Proposition 2;*

*(ii) for any cost threshold $\hat{c} \in (\kappa\bar{c}, \bar{c}]$, the expected compensation under the no-manipulation contract $\mathcal{C}^n$ defined in Proposition 1 is strictly lower than the expected compensation under the selective-manipulation contract $\mathcal{C}^s$ defined in Proposition 2.*

Proposition 4 shows that the board offers a selective-manipulation contract if it wants to implement a low cost threshold $\hat{c}$, and a no-manipulation contract if it prefers to induce a high cost threshold $\hat{c}$. To understand this result, we need to analyze the expected payments to the manager under the two contracts. Selective manipulation has two effects. First, it makes the report $r$ more informative about the manager's effort choice: It increases the likelihood that a high-effort manager generates a favorable report $r_h$ from $\delta$ to $\delta + (1-\delta)\,\phi$, while leaving the likelihood that a low-effort manager produces such an outcome unchanged. This improved informativeness allows for a more efficient compensation contract: It reduces the payment required to induce a high level of effort. Second, the selective-manipulation contract must prevent a manager who exerted low effort from manipulating. This is achieved by setting a sufficiently high cost of manipulation $g^s$. However, this cost must be borne by a manager who exerted high effort and, due to bad luck, generated an unfavorable report $r_\ell$. Anticipating this possibility, the manager hence becomes more hesitant to exert high effort in the first place: Manipulating a low report selectively when $e = 1$ effectively increases the manager's cost of exerting high effort by the amount of her expected manipulation cost, $(1-\delta)g^s$. This makes it more costly for shareholders to incentivize effort provision. The increase in the payment $w^s(r_h, v_h)$ necessary to induce high effort partly undoes the reduction in $w^s(r_h, v_h)$ made possible by the improved informativeness of the report $r$.

For a manager with a cost $c > \hat{c}$, the net effect of switching to a selective-manipulation contract is easy to determine. The probability of receiving a compensation payment is the same under both contracts, $\lambda(1-\delta)$, but the payment is lower under the selective-manipulation contract. Thus, a low-effort manager earns a lower expected compensation under the selective-manipulation contract. For a manager with a cost $c < \hat{c}$, in contrast, the expected compensation is increased. The promised payment is lower, but selective manipulation increases the probability of receiving a payment. The increased probability more than offsets the reduction in the payment and, as a result, a high-effort manager's expected compensation under the selective-manipulation contract exceeds that under the no-manipulation contract:[9]

$$(\delta + (1-\delta)\phi)\left(\frac{\hat{c}}{\delta - \lambda(1-\delta) + (1-\delta)(1-\lambda)\phi}\right) > \delta\left(\frac{\hat{c}}{\delta - \lambda(1-\delta)}\right). \tag{15}$$

This does not mean, however, that a high-effort manager receives a higher expected utility under the selective-manipulation contract. On the contrary, the increase in the manager's expected compensation is more than offset by the cost of manipulation that she expects to incur. This is intuitive. The effort IC constraint in (7) is binding under both contracts, so if switching from a no-manipulation contract to a selective-manipulation contract reduces the low-effort manager's expected payoff (it does, because of the improved informativeness of $r$), it must also reduce the high-effort manager's expected payoff (net of the expected cost of manipulation).

[9]The expressions on the left- and right-hand side of inequality (15) are identical for $\phi = 0$, and the expression on the left-hand side is increasing in $\phi$.

When considering whether to offer a selective-manipulation contract, the firm trades off the reduction in expected compensation due to the improved informativeness of the report against the increased cost of inducing effort caused by the expected cost of manipulation. Which of these two effects dominates depends on how likely the firm is to face either a low-cost or a high-cost manager, which in turn depends on the choice of the cost threshold $\hat{c}$. For low values of $\hat{c}$, the manager is unlikely to exert high effort and hence to manipulate the report. In this case, shareholders prefer the selective-manipulation contract, because the deadweight loss due to the manager's manipulation cost is small compared to the reduction in the expected compensation due to improved information transmission. For high values of $\hat{c}$, the opposite is the case. The manager is likely to exert high effort and hence to incur the manipulation cost. Shareholders thus prefer the no-manipulation contract, because the deadweight loss due to the manager's manipulation cost is large compared to the reduction in the expected compensation due to improved information transmission.

An inspection of $\kappa\bar{c}$, the maximum value of the cost threshold $\hat{c}$ for which shareholders prefer the selective-manipulation contract to the no-manipulation contract, shows that it decreases in both $\delta$ and $\lambda$. This is consistent with the intuition described above. A more informative unmanipulated report (higher $\delta$) reduces the benefit of selective manipulation, thereby making the selective-manipulation contract relatively less attractive. A higher $\lambda$ improves the chances of a low-effort manager generating a high cash flow $v_h$, which makes it more beneficial for her to manipulate a low report $r_\ell$. Since manipulation by a low-effort manager is never optimal, this means that the manipulation cost $g^s$ must be increased, which in turn requires an increase in the compensation payment $w^s(r_h, v_h)$ (to incentivize a low-cost manager to exert high effort). Thus, an increase in $\lambda$ makes selective manipulation less attractive for the firm.

### 3.4 Optimal Contract

Our analysis in Section 3.3 shows that the optimal contract to implement a given cost threshold $\hat{c}$ is either a no-manipulation contract or a selective-manipulation contract. We now endogenize the board's choice of the threshold $\hat{c}$ and analyze which of these two contracts is optimal in different situations. We show that the board's decision depends on the size of the ratio $\frac{v_h - v_\ell}{\bar{c}}$. The numerator, $v_h - v_\ell$, is the increase in cash flow that effort can generate; it is divided by $\bar{c}$, which captures the average cost of effort (since $c$ is uniformly distributed over the interval $[0, \bar{c}]$). We interpret this ratio as a measure of the productivity of effort. This is intuitive if the ratio is multiplied by $(1-\lambda)$, since $(1-\lambda)(v_h - v_\ell)$ is the expected value of the incremental cash flow when high effort is exerted instead of low effort. We show that when effort is only moderately productive, the board chooses a low threshold $\hat{c}$ and implements it using a selective-manipulation contract. In contrast, when effort is highly productive, the board implements a higher threshold $\hat{c}$ using a no-manipulation contract. As a first step, we derive the optimal value of the threshold $\hat{c}$ for each type of contract.

**Proposition 5.** *Under the no-manipulation contract $\mathcal{C}^n$ defined in Proposition 1, firm value is maximized at a cost threshold of*

$$\hat{c}_n = \max\left\{\frac{1}{2}\left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta - \lambda(1-\delta)}\right), 0\right\}. \tag{16}$$

*In contrast, under the selective-manipulation contract $\mathcal{C}^s$ defined in Proposition 2, firm value is maximized at a cost threshold of*

$$\hat{c}_s = \max\left\{\frac{1}{2}\left(\frac{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}{\delta-(1-\delta)(\lambda-\phi)}\right)\left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}\right), 0\right\}. \tag{17}$$

Under both types of contract, the board may optimally choose not to incentivize effort provision: If the expected value-added of high effort, $(1-\lambda)(v_h - v_\ell)$, is small, the optimal cost threshold $\hat{c}$ is equal to zero (which is implemented by setting all compensation payments equal to zero). The board implements a positive cost threshold $\hat{c} > 0$ (and hence induces high effort provision by a manager with a cost $c < \hat{c}$) only if effort is sufficiently productive. An inspection of (16) and (17) reveals that $\hat{c}_n = 0$ if $\hat{c}_s = 0$, but not vice versa. This means that, for some parameter values, the board incentivizes effort provision by the manager only under the selective-manipulation contract. The expression for $\hat{c}_s$ in (17) immediately implies the following result.

**Corollary 1.** *The optimal contract implements a cost threshold $\hat{c} > 0$ (i.e., incentivizes the manager to exert high effort with a strictly positive probability) if and only if*

$$\frac{v_h - v_\ell}{\bar{c}} > \frac{\lambda(1-\delta)}{(1-\lambda)\left[\delta - (1-\delta)(\lambda - \phi + \lambda\phi)\right]}. \tag{18}$$

Having determined the optimal cost threshold $\hat{c}$ under the no-manipulation and the selective-manipulation contract, we can now solve for the optimal contract by analyzing which of these two contracts generates a higher firm value when the cost threshold is chosen optimally (i.e., when $\hat{c}$ is set to $\hat{c}_n$ under the no-manipulation contract and to $\hat{c}_s$ under the selective-manipulation contract).

**Proposition 6.** *If the condition*

$$\frac{v_h - v_\ell}{\bar{c}} \leq \frac{1-\delta}{\delta - \lambda(1-\delta)}\left(\frac{1}{1-\lambda} + \sqrt{\frac{\delta - (1-\delta)(\lambda - \phi)}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}}\right) \tag{19}$$

*is satisfied, then the optimal contract is the selective-manipulation contract $\mathcal{C}^s$ defined in Proposition 2. If the above condition is not satisfied, then the optimal contract is the no-manipulation contract $\mathcal{C}^n$ defined in Proposition 1.*

As discussed above, the ratio $\frac{v_h - v_\ell}{\bar{c}}$ determines which contract is optimal. There are three distinct regions. If $\frac{v_h - v_\ell}{\bar{c}}$ is so small that condition (18) is violated, the board optimally offers a contract that induces low effort for any cost of effort $c > 0$ (by setting $\hat{c} = 0$). In this case, both the selective-manipulation contract (with $w^s(r_h, v_h) = 0$ and $g^s = 0$) and the no-manipulation contract (with $w^n(r_h, v_h) = 0$ and $g^n = 0$) generate the same firm value. For intermediate values of $\frac{v_h - v_\ell}{\bar{c}}$ such that both conditions (18) and (19) are satisfied, the board optimally offers a selective-manipulation contract that induces high effort if $c < \hat{c}_s$ and incentivizes manipulation of a report $r_\ell$ if the manager exerted high effort. For high values of $\frac{v_h - v_\ell}{\bar{c}}$ such that condition (18) is satisfied but condition (19) is violated, the board optimally offers a no-manipulation contract that induces high effort if $c < \hat{c}_n$ and prevents all manipulation.

These results about the optimal contract are consistent with the intuition we provided in Section 3.3, comparing the costs of implementing a *given* threshold $\hat{c}$ using either a no-manipulation contract or a selective-manipulation contract. As discussed above, a selective-manipulation contract is preferred by the board when $\hat{c}$ is low, since the improved informativeness under such a contract decreases the expected compensation of a low-effort manager (with $c > \hat{c}$), whom the board is more likely to face when $\hat{c}$ is low. In contrast, a no-manipulation contract is preferred by the board when $\hat{c}$ is high, since selective manipulation makes inducing high effort more costly and, as a result, increases the expected compensation of a high-effort manager (with $c < \hat{c}$), whom the board is more likely to face when $\hat{c}$ is high. An inspection of (16) and (17) shows that both $\hat{c}_n$ and $\hat{c}_s$ are increasing in $\frac{v_h - v_\ell}{\bar{c}}$. Furthermore, it can be shown that $\hat{c}_n > \hat{c}_s$ when the condition in (19) is violated.[10] We thus obtain the intuitive result that a no-manipulation contract is used when it is optimal to implement a high threshold $\hat{c}_n$, which is the case when managerial effort is more productive (i.e., when $\frac{v_h - v_\ell}{\bar{c}}$ is high). In contrast, a selective-manipulation contract is used when it is optimal to implement a low threshold $\hat{c}_s$, which is the case when managerial effort is less productive (i.e., when $\frac{v_h - v_\ell}{\bar{c}}$ is low).

Having identified conditions for the optimality of the selective-manipulation contract and the no-manipulation contract, we conclude this section by analyzing the compensation payments $w^s(r_h, v_h)$ and $w^n(r_h, v_h)$ that the manager hopes to earn, depending on which contract type is optimal.

**Proposition 7.** *The compensation $w(r_h, v_h)$ that the manager earns in case of a successful outcome is increasing in the payoff difference $v_h - v_\ell$. This result continues to hold when the increase in $v_h - v_\ell$ causes the board to switch from a selective-manipulation contract to a no-manipulation contract: $w^s(r_h, v_h) < w^n(r_h, v_h)$ when the condition in (19) is satisfied with equality.*

[10] The proof of Proposition 7 shows that $\hat{c}_n > \hat{c}_s$ when the inequality in (19) in binding. Since $\hat{c}_n$ increases in $\frac{v_h - v_\ell}{\bar{c}}$ at a faster rate than $\hat{c}_s$ (because $\frac{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}{\delta - (1-\delta)(\lambda - \phi)} < 1$), this implies that $\hat{c}_n > \hat{c}_s$ also when $\frac{v_h - v_\ell}{\bar{c}}$ exceeds the right-hand side of (19) or, in other words, when the condition in (19) is violated.

The result that the compensation $w(r_h, v_h)$ increases in the payoff difference $v_h - v_\ell$ is intuitive: An increase in $v_h - v_\ell$ makes effort more desirable, and an increase in $w(r_h, v_h)$ induces more effort. It is straightforward to show that the result in Proposition 7 holds when an increase in $v_h - v_\ell$ does not change the type of contract that the board chooses to offer: Both $w^s(r_h, v_h)$ and $w^n(r_h, v_h)$ are increasing in $\hat{c}$ (Propositions 1 and 2), and both $\hat{c}_n$ and $\hat{c}_s$ are increasing in $v_h - v_\ell$ (Proposition 5).

The result in Proposition 7 also holds when an increase in $v_h - v_\ell$ causes the board to switch from a selective-manipulation contract to a no-manipulation contract, that is, when the condition in (19) is no longer satisfied. This means that the payment $w^n(r_h, v_h)$ that the manager earns in case of a successful outcome when the board optimally offers a no-manipulation contract (because the payoff difference $v_h - v_\ell$ is high) exceeds the payment that she earns when the board optimally offers a selective-manipulation contract (because $v_h - v_\ell$ is low). The reason is twofold. First, the improved informativeness of the report about the manager's effort choice due to selective manipulation requires a smaller payment to incentivize high managerial effort: As shown in Section 3.3, $w^s(r_h, v_h) < w^n(r_h, v_h)$ for any desired cost threshold $\hat{c}$. Second, the board optimally offers a selective-manipulation contract only when it wants to implement a low cost threshold $\hat{c}$ (Proposition 4), which requires relatively weaker incentives.

## 4 Empirical Predictions

We now turn to the empirical predictions of our model and discuss a few existing studies that document findings consistent with these predictions. Much of the reporting done by companies is financial reporting. It is therefore not surprising that the empirical work that relates to our predictions is mostly based on financial reporting.

A key result of our analysis is that firms may find it optimal to incentivize manipulation when the manager exerted high effort, but not when she exerted low effort. Our model therefore predicts that manipulation is positively related to effort provision. Furthermore, since managers are induced to exert high effort only if the cost $c$ is sufficiently low, manipulation is negatively related to the cost of effort.

**Prediction 1.** *Earnings manipulation is expected to occur more frequently when managers exerted high effort and when managers have low costs of effort.*

Models of CEO "effort" are not necessarily to be taken literally: Most CEOs are extremely dedicated to their jobs and spend every hour they can working. Models with unobservable effort are often meant to capture a CEO's willingness to make difficult decisions or perform tedious tasks that add value, instead of pursuing more pleasant or exciting but less value-adding tasks. Our model thus predicts that managers who are more focused on adding long-term value are more likely to manipulate financial reports than managers who spend time socializing with celebrities or with non-core entrepreneurial activities.

A low cost of effort may be captured by a CEO's "ability", which may be unobservable at the time a CEO is appointed but may become observable during the CEO's tenure. Baik, Choi, and Farber (2020) use a measure of ability (from Demerjian, Lev, and McVay 2012) and find that higher-ability managers smooth earnings more and have more informative earnings and stock prices. This seems to be consistent with Prediction 1.

Effort and manipulation are positively correlated in our model under certain conditions (i.e., when the conditions in (18) and (19) are satisfied). However, this does not imply that more high-powered incentive contracts go along with more manipulation. The incentive to manipulate also depends on the cost of manipulation $g$, which is set higher when the condition in (19) is violated (to prevent all manipulation) than when it is satisfied (to induce a high-effort manager to manipulate a low report). The power of incentive contracts depends on the payments that a manager can potentially realize. The payments $w^n(r_h, v_h)$ and $w^s(r_h, v_h)$ represent the *maximum* compensation that the manager can earn. Because the *minimum* compensation is zero, these payments also reflect the difference between the highest and lowest possible compensation levels and hence can be used as measures of how high-powered the incentive compensation is under the two contracts.

Proposition 7 shows that $w^s(r_h, v_h) < w^n(r_h, v_h)$ when the condition in (19) is satisfied with equality, that is, when the board is indifferent between offering a selective-manipulation contract and a no-manipulation contract. By continuity, this result extends to parameter values in the vicinity of the boundary defined by (19): If the distance between two sets of parameter values is not too large, the payment $w^n(r_h, v_h)$ offered for parameter values violating the condition in (19) (and hence making the no-manipulation contract optimal) exceeds the payment $w^s(r_h, v_h)$ offered for parameter values satisfying the condition in (19) (and hence making the selective-manipulation contract optimal).[11] Since manipulation is incentivized only under a selective-manipulation contract, our model therefore predicts that manipulation is negatively correlated with the incentive power of compensation contracts across similar firms.

**Prediction 2.** *Earnings manipulation is expected to occur more frequently if incentive compensation is low-powered.*

The empirical evidence about the relation between manipulation and incentive compensation is mixed, with some authors finding a positive relation for some (but not all) components of incentive compensation (Bergstresser and Philippon 2006; Burns and Kedia 2006), and others finding a negative relation (Armstrong, Jagolinzer, and Larcker 2010) or no relation (Erickson, Hanlon, and Maydew 2006). Our paper contributes to this literature by offering a simple explanation for why manipulation may correlate *negatively* with the power of incentive compensation.

We now analyze how some key parameters affect the possible optimality of manipulation by studying how these parameters affect the boundary between the no-manipulation region and the selective-manipulation region (Proposition 6). The key parameters of interest are $\lambda$, $\delta$, and $\phi$. Letting $\Gamma$ denote the term on the right-hand side of the inequality in (19), our focus is on the signs of $\frac{d\Gamma}{d\lambda}$, $\frac{d\Gamma}{d\delta}$, and $\frac{d\Gamma}{d\phi}$.[12]

In our model, managerial effort is more productive when $\lambda$ is small, because the expected increase in the firm's cash flow due to high effort is $(1-\lambda)(v_h - v_\ell)$. One would thus expect that

[11] As shown in Proposition 7, the result also holds for any values of $v_h - v_\ell$ if all other parameters are held constant.

[12] As we discuss below, our results remain unchanged if the boundary in (18) that separates the selective-manipulation region from the region in which no incentive contract is offered is taken into account.

inducing high effort is more beneficial when $\lambda$ is small and, consequently, that manipulation is less likely when $\lambda$ is small (because a no-manipulation contract is optimal when $\hat{c}$ is large). Indeed, (19) immediately implies that $\frac{d\Gamma}{d\lambda} > 0$, which means that an increase in $\lambda$ increases the set of parameter values for which the selective-manipulation contract is optimal. This leads to our next prediction.

**Prediction 3.** *Earnings manipulation is expected to occur more frequently in firms with less productive managerial effort.*

Our model predicts that if top executives are not the key value drivers in a firm or industry, then it is more likely that (selective) manipulation is optimal. In contrast, when CEO talent and focus on creating value are essential to a firm's success, it is more likely that firms make manipulation prohibitively costly. Unfortunately, existing empirical studies provide little guidance on how to proxy for the productivity of managerial effort.

A second parameter of interest is $\delta$, which measures the quality of the firm's accounting system in the absence of manipulation: A higher $\delta$ makes the unmanipulated report more informative. From (19) it follows that $\frac{d\Gamma}{d\delta} < 0$. Thus, a decrease in $\delta$ increases the region of parameter values for which the selective-manipulation contract is optimal, which leads to the following prediction.

**Prediction 4.** *Earnings manipulation is expected to occur more frequently in firms with less informative accounting systems (in the absence of manipulation).*

This prediction, which distinguishes our model from most of the existing literature (e.g., Strobl 2013), is driven by the fact that (selective) manipulation is more valuable when the accounting system is less informative because it can offset the noise inherent in the accounting system. A key difference between our model and other models of manipulation is that in our model only a manager who exerted *high* effort may have an incentive to manipulate financial reports. One should therefore not interpret evidence of manipulation as evidence of poor performance or of severe agency problems that are not being addressed.

Consistent with Prediction 4, the empirical literature reports that misconduct is more frequent when firms are financially less transparent (Gerety and Lehn 1997; Dechow, Ge, et al. 2011), when a board does not have an audit committee (Dechow, Sloan, and Sweeney 1996), when there are fewer audit committee meetings (Farber 2005), when there is no formal internal audit function (Beasley et al. 2000; Coram, Ferguson, and Moroney 2008), or when a firm uses a smaller auditing firm (Lennox and Pittman 2010). There is also a large literature on *earnings quality*, focusing on various measures of the informativeness of financial statements. Consistent with our prediction, Chalmers, Naiker, and Navissi (2012) find that securities fraud class action lawsuits follow periods of low earnings quality.

In our model, manipulation is more effective when $\phi$ is large: A larger $\phi$ means that a manipulation attempt by the manager is more likely to succeed and thus to produce a favorable report. It is therefore not surprising that an increase in $\phi$ increases the set of parameter values for which a selective-manipulation contract is optimal. In fact, from (19) we have $\frac{d\Gamma}{d\phi} > 0$.

**Prediction 5.** *Earnings manipulation is expected to occur more frequently in firms in which it has a stronger effect on reported earnings.*

Our model predicts that firms are more likely to tolerate manipulation when manipulation techniques are more effective. This seems to be consistent with the empirical findings by Carcello and Nagy (2004): Manipulation occurs more frequently during the first three years of an auditor's tenure, when the auditor arguably still lacks relevant firm-specific knowledge and managers can hence manipulate financial reports more effectively. It is important to keep in mind though that only "good" managers who exerted high effort are incentivized to manipulate financial reports in our model. Prediction 5 is, however, also consistent with the traditional view in the literature that manipulation is largely unavoidable and can at best be managed. Hence, if it is more effective, it is more likely to be used.

We conclude this section with a technical point. Predictions 3 –5 are based on how $\Gamma$, the threshold of $\frac{v_h - v_\ell}{\bar{c}}$ that separates the selective-manipulation region from the no-manipulation region (i.e., the right-hand side of the inequality in (19)), responds to changes in one parameter of the model. However, such a parameter change also affects the threshold of $\frac{v_h - v_\ell}{\bar{c}}$ that separates the selective-manipulation region from the region in which no incentive contract is offered (since $w(r, v) = 0$ for all $r$ and $v$), which is the case when the condition in (18) is violated. When determining whether one contract type becomes more prevalent in response to a parameter change, both thresholds should therefore be examined. Our discussion above has focused on the threshold $\Gamma$, because this makes it easier to provide intuitive explanations for the model's predictions. However, our results remain unchanged if we incorporate the second threshold. Letting $\Lambda$ denote the term on the right-hand side of the inequality in (18), it is straightforward to show that $0 < \frac{d\Lambda}{d\lambda} < \frac{d\Gamma}{d\lambda}$, $\frac{d\Gamma}{d\delta} < \frac{d\Lambda}{d\delta} < 0$, and $\frac{d\Lambda}{d\phi} < 0 < \frac{d\Gamma}{d\phi}$. An increase in $\lambda$ causes both thresholds $\Gamma$ and $\Lambda$ to increase, but $\Gamma$ increases by more than $\Lambda$ so that the region of $\frac{v_h - v_\ell}{\bar{c}}$ for which the selective-manipulation contract is optimal becomes larger, consistent with Prediction 3. Similarly, an increase in $\delta$ reduces both thresholds, but the reduction is larger for $\Gamma$ than for $\Lambda$ so that the selective-manipulation region shrinks in size, consistent with Prediction 4. Finally, an increase in $\phi$ moves the two thresholds in opposite directions and pushes them further apart, thereby increasing the size of the selective-manipulation region, consistent with Prediction 5.

# 5 Robustness

In this section, we assess the robustness of our results that serve as the basis for the paper's empirical predictions and discuss some additional implications of our model. Specifically, we examine the implications of introducing a manipulation cost that accrues directly to the firm and of changing the timing of the contracting problem, and argue that the optimality of the selective-manipulation contract is largely unaffected by these changes. We also clarify that our manipulation technology is standard, and that the benefit of selective manipulation is a feature of the optimal contract and not a feature of the manipulation technology. Finally, we discuss how manipulation would affect stock prices if, contrary to our assumption, manipulation were observable, and we propose an extension of our model that, consistent with the empirical evidence, can generate negative "announcement returns" associated with the discovery of manipulation.

## 5.1 Manipulation Cost Incurred by the Firm

Our model assumes that manipulation is costly to the manager, but not to the firm: Manipulating the report imposes a utility cost of $g$ on the manager, but it does not affect the firm's cash flow. Although this is a standard assumption in the literature, it is useful to discuss the implications of a manipulation cost that is directly incurred by the firm. Arguably, the manipulation of financial reports may cause a waste of resources or lead to inefficient decisions based on inaccurate information, thereby reducing the firm's cash flow.

Such a cost would not alter our results qualitatively: It would not affect the manager's expected compensation in (3), the nonnegativity constraints in (4), or the incentive constraints in (5)–(10). Thus, the optimal contracts $\mathcal{C}^n$ and $\mathcal{C}^s$ (specified in Propositions 1 and 2, respectively) would remain unchanged. However, a manipulation cost that accrues directly to the firm would lower the value of the firm under the selective-manipulation contract and hence would reduce the set of parameter values for which the selective-manipulation contract $\mathcal{C}^s$ is optimal.

We also want to emphasize that the firm *indirectly* bears a cost of manipulation in our model. Under a selective-manipulation contract, a manager with a low cost $c < \hat{c}_s$ anticipates that she may incur a disutility $g^s$ if she exerts high effort, so a higher compensation payment is required to restore incentive compatibility. As discussed in Section 3.3, for a given threshold $\hat{c}$ the expected compensation of a high-effort manager is higher under a selective-manipulation contract than under a no-manipulation contract. In other words, part of the manipulation cost that the manager incurs under a selective-manipulation contract is ultimately borne by the firm's shareholders.

## 5.2 Ex-Ante Contracting

The manager knows her effort cost $c$ at the contracting stage in our model. However, our results are robust to the alternative assumption that the contract is signed at the *ex-ante* stage before the manager discovers her cost of effort $c$.

Having the board of directors and the manager contract at the ex-ante stage would not change the incentive constraints, since these constraints are specific to either the cost level $c$ or the effort choice $e$ and, hence, need to be satisfied, unchanged, by any optimal direct mechanism, independent of the timing assumption. Ex-ante contracting would, however, change the manager's participation constraint: Whereas the ex-post participation constraints in our model must be satisfied for any cost level $c$, an ex-ante participation constraint would require that the manager earns her reservation utility level of zero in expectation, given the distribution of possible cost levels. In some contracting models, replacing the ex-post participation constraints with an ex-ante participation constraint affects the principal's optimal trade-off between rent extraction and efficiency. In our setup, however, the participation constraints play no role because they are trivially satisfied: The manager can achieve a nonnegative payoff under any contract by not exerting effort and not manipulating. The limited-liability constraints in (4) are sufficient to satisfy the participation constraints, and the participation constraints are therefore not part of the principal's optimization problem. The same is true in the case of ex-ante contracting. The limited-liability constraints, which have to hold independently of the timing assumption, ensure that the ex-ante participation constraint is satisfied. Thus, our results would be unchanged if the contract were signed at the ex-ante stage.

### 5.3 Manipulation and Information Quality

Our model is a model of manipulation, although in equilibrium manipulation may (for some parameter values) become a tool to improve information transmission: Selective manipulation makes the firm's report more informative about the manager's effort choice. We want to emphasize, however, that improved information transmission is a feature of the equilibrium contract, not a feature of the manipulation technology itself.

Manipulation enables a manager to adjust a performance measure upwards, with the intention of improving her compensation. Intuitively, one might expect manipulation to make the possibly-manipulated performance measure less informative for the principal. This is indeed the case in our model, depending on the manipulation schedule $\mathcal{M} = (m_0, m_1)$ being implemented. Conditional on observing a high report $r_h$, the probability that the manager exerted high effort $e = 1$ is given by

$$\text{prob}\left[e=1 \mid r=r_h, m_0, m_1\right] = \frac{\frac{\hat{c}}{\bar{c}}\left[\delta+(1-\delta)\phi m_1\right]}{\frac{\hat{c}}{\bar{c}}\left[\delta+(1-\delta)\phi m_1\right]+\left(1-\frac{\hat{c}}{\bar{c}}\right)\left[1-\delta+\delta\phi m_0\right]}. \tag{20}$$

It is easily verified that

$$\begin{aligned} &\text{prob}\left[e=1 \mid r=r_h, m_0=1, m_1=0\right] < \text{prob}\left[e=1 \mid r=r_h, m_0=1, m_1=1\right] \\ &\quad < \text{prob}\left[e=1 \mid r=r_h, m_0=0, m_1=0\right] < \text{prob}\left[e=1 \mid r=r_h, m_0=0, m_1=1\right]. \end{aligned} \tag{21}$$

That is, compared with the no-manipulation case, a high report $r = r_h$ becomes a worse predictor of a high effort choice $e = 1$ when the manager always manipulates (i.e., when $m_0 = m_1 = 1$) or when she selectively manipulates after exerting low effort (i.e., when $m_0 = 1$ and $m_1 = 0$), and it becomes a better predictor when the manager selectively manipulates after exerting high effort (i.e., when $m_0 = 0$ and $m_1 = 1$). This demonstrates that the manager can use the manipulation technology to make the report more informative or less informative about her effort choice. In equilibrium, the manager chooses to manipulate only if doing so increases her expected utility, and shareholders incentivize manipulation only if it improves firm value. The result that manipulation improves the information quality of the report is therefore a feature of the equilibrium and reflects the optimality of the manager's compensation contract rather than any limitations of the manipulation technology. Our manipulation technology is standard, but the equilibrium contract makes use of this technology in a novel way.

### 5.4 Manipulation Technology and Announcement Returns

In our model, manipulation only occurs when shareholders choose the selective-manipulation contract, and it is limited to managers who exert high effort. If, contrary to our assumption, manipulation were observable (but not contractible), shareholders would therefore revise upwards their beliefs about the firm's future cash flow upon observing a manipulated report: Manipulation is good news in the eyes of shareholders because it implies that the manager exerted high effort and hence that the firm will generate a high cash flow $v_h$ with certainty. This positive effect conflicts with findings in the empirical literature: Discoveries or allegations of misconduct typically cause stock prices to drop, because investors update their beliefs about the firm's actual, unmanipulated performance, and possibly also because of reputation effects (see, e.g., Karpoff, Lee, and Martin 2008).[13] We can reconcile our results with these empirical findings by relaxing our assumption that the board has full control over the manager's cost of manipulation.

We expand our model by introducing a second manipulation technology. As described in Section 2, the manager can incur a cost $g$ and convert a low report $r_\ell$ into a high report $r_h$ with probability $\phi$. Alternatively, the manager can choose a less costly but also less effective manipulation technology that converts a low report $r_\ell$ into a high report $r_h$ with probability $\psi < \phi$. For simplicity, we assume that the manager incurs no cost when using this alternative manipulation technology. When the manager discovers that, in the absence of manipulation, a report $r = r_\ell$ will be issued but does not find it optimal to incur the manipulation cost $g$ to change the outcome, she will then employ the zero-cost manipulation technology: There is no cost to using it, and it increases the likelihood of receiving the payment $w(r_h, v_h)$. The motivation for this assumption is that although shareholders can, through their choice of governance arrangements, make manipulation costly to the manager, they cannot thwart all manipulation attempts. We denote the manager's decision to manipulate the report (using either the costly or the costless technology) by $\widetilde{m} \in \{0, 1\}$.

Adding this alternative manipulation technology to our model does not change the results derived in Section 3 qualitatively. As long as $\psi < \phi$, a selective-costly-manipulation contract that induces the manager to incur the manipulation cost $g$ only when she exerted high effort (and to choose the costless manipulation technology when she exerted low effort) still makes the report more informative about the manager's effort choice. Thus, shareholders optimally choose such a contract when managerial effort is only moderately productive (i.e., when $v_h - v_\ell$ is not too large). In contrast, when managerial effort is very productive, shareholders find it optimal to offer a no-costly-manipulation contract under which the manager only uses the

[13] For more references, see Section 4.2.1 of Amiram et al. (2018).

costless manipulation technology. This means that, in the expanded model, the manager may manipulate the report irrespective of her cost of effort, $c$, and her effort choice, $e$. It is therefore not obvious whether observing manipulation is good news or bad news for the firm's shareholders.

Suppose that $v_h - v_\ell$ is not too large so that a selective-costly-manipulation contract is optimal. Suppose further that shareholders observe $(r, \widetilde{m}) = (r_h, 1)$. The posterior probability that a high cash flow $v_h$ will be realized is then

$$\text{prob}\left[v = v_h \mid r = r_h, \widetilde{m} = 1\right] = \frac{\frac{\hat{c}}{\bar{c}}\,(1-\delta)\,\phi + \left(1 - \frac{\hat{c}}{\bar{c}}\right)\lambda\,\delta\,\psi}{\frac{\hat{c}}{\bar{c}}\,(1-\delta)\,\phi + \left(1 - \frac{\hat{c}}{\bar{c}}\right)\delta\,\psi}. \tag{22}$$

Alternatively, suppose that shareholders observe $(r, \widetilde{m}) = (r_h, 0)$. In this case, the posterior probability that a high cash flow $v_h$ will be realized is

$$\text{prob}\left[v = v_h \mid r = r_h, \widetilde{m} = 0\right] = \frac{\frac{\hat{c}}{\bar{c}}\,\delta + \left(1 - \frac{\hat{c}}{\bar{c}}\right)\lambda\,(1-\delta)}{\frac{\hat{c}}{\bar{c}}\,\delta + \left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta)}. \tag{23}$$

Given the observed report $r = r_h$ and manipulation $\widetilde{m} \in \{0, 1\}$, the firm's valuation, net of compensation promised to the manager, is

$$\text{prob}\left[v = v_h \mid r = r_h, \widetilde{m}\right]\,\big(v_h - w(r_h, v_h)\big) + \left(1 - \text{prob}\left[v = v_h \mid r = r_h, \widetilde{m}\right]\right) v_\ell.$$

Shareholders thus reduce their valuation of the firm after observing manipulation ($\widetilde{m} = 1$) if $\text{prob}\left[v = v_h \mid r = r_h, \widetilde{m} = 1\right] < \text{prob}\left[v = v_h \mid r = r_h, \widetilde{m} = 0\right]$, which holds if $\psi > \left(\frac{1-\delta}{\delta}\right)^2 \phi$. So if $\left(\frac{1-\delta}{\delta}\right)^2 \phi < \psi < \phi$ and $v_h - v_\ell$ is not too large, then observing manipulation of a good report is bad news for shareholders, consistent with the empirical evidence.

Now consider a setting in which $v_h - v_\ell$ is sufficiently large so that the no-costly-manipulation contract is optimal. The manager will then choose the costless manipulation technology if $r = r_\ell$, irrespective of her chosen effort level. If shareholders observe $(r, \widetilde{m}) = (r_h, 1)$, then the posterior probability that a high cash flow $v_h$ will be realized is

$$\text{prob}\left[v = v_h \mid r = r_h, \widetilde{m} = 1\right] = \frac{\frac{\hat{c}}{\bar{c}}\,(1-\delta)\,\psi + \left(1 - \frac{\hat{c}}{\bar{c}}\right)\lambda\,\delta\,\psi}{\frac{\hat{c}}{\bar{c}}\,(1-\delta)\,\psi + \left(1 - \frac{\hat{c}}{\bar{c}}\right)\delta\,\psi}. \tag{24}$$

If shareholders observe $(r, \widetilde{m}) = (r_h, 0)$, the posterior probability that a high cash flow $v_h$ will be realized is (as before)

$$\text{prob}\left[v = v_h \mid r = r_h, \widetilde{m} = 0\right] = \frac{\frac{\hat{c}}{\bar{c}}\,\delta + \left(1 - \frac{\hat{c}}{\bar{c}}\right)\lambda\,(1-\delta)}{\frac{\hat{c}}{\bar{c}}\,\delta + \left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta)}. \tag{25}$$

In this case, shareholders always reduce their valuation of the firm after observing manipulation ($\widetilde{m} = 1$): $\text{prob}\left[v = v_h \mid r = r_h, \widetilde{m} = 1\right] < \text{prob}\left[v = v_h \mid r = r_h, \widetilde{m} = 0\right]$ because $\delta > \frac{1}{2}$. So if $v_h - v_\ell$ is sufficiently large, then observing manipulation of a good report is bad news for shareholders, consistent with the empirical evidence.

Shareholders may also be able to observe whether the manager *unsuccessfully* tried to manipulate a report $r_\ell$ (i.e., a manipulation attempt that resulted in an unchanged report $r_\ell$). However, this possibility would not change the shareholders' beliefs in the extended model. The reason is that when the manager expects a report $r_\ell$, she will manipulate the report with certainty (using either the costly or the costless manipulation technology). Observing a low report $r_\ell$ therefore reveals to shareholders that the manager tried and failed to manipulate the report. Shareholders do not need the manipulation attempt to be observable to reach this conclusion. In other words, the observation that the manager did indeed (unsuccessfully) try to manipulate the report does not add any information.

In sum, our model can easily be extended to account for the negative stock price reaction that empirical research has found following allegations or discoveries of manipulation.

## 6 Conclusion

Practitioners have long argued that manipulation may be helpful in that it can eliminate some of the noise inherent in financial reports, in particular when unfavorable reports shed a wrong (negative) light on a firm's performance. However, an obvious drawback of manipulation is that it may be used opportunistically by managers to increase their own compensation. The literature has suggested several possible explanations for the presence of manipulation (as discussed in Section 1), but has not yet provided an answer to the question whether tolerating some manipulation can be beneficial for firms even when faced with possibly opportunistic behavior by managers, because the informational benefits of manipulation outweigh the costs.

In this paper, we present a simple principal-agent model that incorporates both of these features: Manipulation can improve the information content of a noisy performance measure, but managers can use manipulation to improve their expected compensation. We show that an optimally designed compensation contract may incentivize manipulation of unfavorable reports by managers who exerted a high level of effort (and hence expect their firms to perform well), but never by managers who exerted a low level of effort. This type of selective manipulation makes the report more informative about the manager's effort choice, thereby strengthening the link between effort choice and compensation. However, incentivizing selective manipulation is not always optimal: Since manipulation is costly to the manager, selectively manipulating a report after high effort effectively makes exerting high effort more costly to the manager. We find that the increased cost of incentivizing high effort outweighs the benefit of a more informative performance signal when managerial effort is highly productive.

# Appendix

**Proof of Lemma 1.** We prove this result by contradiction. Suppose the result does not hold. Then, there must exist a cost $c_0 > 0$ that induces effort choice $e = 0$ and a cost $c_1 > c_0$ that induces effort choice $e = 1$. Thus, letting $U(e, m, c)$ denote the manager's expected utility if she chooses effort $e$ and manipulation strategy $m$ when facing a cost of effort $c$ (that she reports truthfully), we must have

$$U(0, m_0, c_0) \geq U(1, m_1, c_0), \tag{A1}$$

$$U(1, m_1, c_1) \geq U(0, m_0, c_1), \tag{A2}$$

where $m_e$ denotes the manager's optimal manipulation choice for a given effort choice $e$. Furthermore, let $\hat{U}(e, m, c, c')$ denote a type-$c$ manager's expected utility from choosing $e$ and $m$ when she mimics the behavior of a type-$c'$ manager (i.e., claims to be of type $c'$ and chooses $e$ and $m$ accordingly). Since a type-$c_0$ manager prefers not to mimic the behavior of a type-$c_1$ manager, we have

$$U(0, m_0, c_0) \geq \hat{U}(1, m_1, c_0, c_1) > U(1, m_1, c_1), \tag{A3}$$

where the last inequality follows from the fact that $c_1 > c_0$. Similarly, since a type-$c_1$ manager prefers not to mimic the behavior of a type-$c_0$ manager, we have

$$U(1, m_1, c_1) \geq \hat{U}(0, m_0, c_1, c_0) = U(0, m_0, c_0), \tag{A4}$$

where the equality follows from the fact that the effort cost does not directly affect the manager's expected utility if she chooses low effort $e = 0$. Clearly, the two inequalities in (A3) and (A4) are inconsistent with each other, proving that such a case cannot exist. The result must therefore be true. ■

**Proof of Lemma 2.** For a given effort choice $e$, the manager's cost of effort does not affect the distribution of the firm's cash flow $v$ or the report $r$. Thus, if the manager chooses the same effort level $e$ when her effort cost is either $c$ or $c'$, her continuation payoffs and hence her incentives to engage in manipulation are the same in both cases. Furthermore, since the firm's cash flow $v$ depends on the manger's effort cost only through its effect on the manager's effort choice $e$, if shareholders find it optimal to induce the manager to manipulate an unfavorable report $r_\ell$ with probability $m$ when her effort cost is $c$, doing so must also be optimal when the manager's effort cost is $c'$, as long as the manager's optimal effort choice is the same for $c$ and $c'$. ■

**Proof of Lemma 3.** From Lemma 1, it follows that all manager types $c \in [0, \hat{c})$ choose the same effort $e = 1$ and hence make the same manipulation decision $m_1$ (Lemma 2). Thus, these types face the same probability of generating outcome $(r, v)$, for all $r \in \{r_h, r_\ell\}$ and $v \in \{v_h, v_\ell\}$. This means that, under an incentive-compatible mechanism, these types must all receive the same expected compensation. Otherwise, they would all report to be of the type that generates the highest expected compensation. Without loss of generality, we can therefore set $w(r, v|c) = w_1(r, v)$, for all $c \in [0, \hat{c})$. An analogous argument holds for all manager types $c \in (\hat{c}, \bar{c}]$, so that, without loss of generality, we can set $w(r, v|c) = w_0(r, v)$, for all $c \in (\hat{c}, \bar{c}]$. ■

**Proof of Proposition 1.** We derive the optimal no-manipulation contract by first considering a simplified optimization problem and then showing that the solution to this simplified problem is also a solution to the full optimization problem in (3)–(10).

To simplify the notation, let $\pi_{e,m_e}(r, v)$ denote the probability that a report $r \in \{r_h, r_\ell\}$ and a cash flow $v \in \{v_h, v_\ell\}$ is produced when the manager chooses effort level $e \in \{0, 1\}$ and follows the manipulation schedule $m_e \in [0, 1]$. That is,

$$\pi_{1,m_1}(r_h, v_h) = \delta + (1 - \delta)\,\phi\, m_1, \tag{A5}$$
$$\pi_{0,m_0}(r_h, v_h) = \lambda(1 - \delta + \delta\,\phi\, m_0), \tag{A6}$$
$$\pi_{1,m_1}(r_\ell, v_h) = (1 - \delta)(1 - \phi\, m_1), \tag{A7}$$
$$\pi_{0,m_0}(r_\ell, v_h) = \lambda\,\delta(1 - \phi\, m_0), \tag{A8}$$
$$\pi_{1,m_1}(r_h, v_\ell) = 0, \tag{A9}$$
$$\pi_{0,m_0}(r_h, v_\ell) = (1 - \lambda)(1 - \delta + \delta\,\phi\, m_0), \tag{A10}$$
$$\pi_{1,m_1}(r_\ell, v_\ell) = 0, \tag{A11}$$
$$\pi_{0,m_0}(r_\ell, v_\ell) = (1 - \lambda)\delta(1 - \phi\, m_0). \tag{A12}$$

Also, define $\Delta\pi_{m_0,m_1}(r, v) = \pi_{1,m_1}(r, v) - \pi_{0,m_0}(r, v)$.

We begin by rewriting the principal's objective function in (3). Setting $e = 0$ and $m = m_0$ on the right-hand side of (9) yields

$$\sum_{r,v} \pi_{1,m_1}(r, v)\, w_1(r, v) \geq \sum_{r,v} \pi_{0,m_0}(r, v)\, w_0(r, v) + \hat{c} + G(m_0, m_1), \tag{A13}$$

where $G(m_0, m_1)$ denotes the difference in the manager's expected manipulation cost when she exerts high rather than low effort, that is, $G(m_0, m_1) = [(1 - \delta)m_1 - \delta m_0]\, g$. Similarly, setting $e = 1$ and $m = m_1$ on the right-hand side of (10), we have

$$\sum_{r,v} \pi_{0,m_0}(r,v)\, w_0(r,v) \geq \sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) - \hat{c} - G(m_0, m_1). \tag{A14}$$

An inspection of (A13) and (A14) shows that both constraints must be binding, and the principal's objective function can therefore be written as

$$\min_{\boldsymbol{w}_0, \boldsymbol{w}_1, g} \sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \left(\hat{c} + G(m_0, m_1)\right). \tag{A15}$$

We next consider a simplified optimization problem. In particular, we solve for the optimal compensation scheme $\boldsymbol{w}_1$ that implements an effort choice characterized by the threshold $\hat{c} \in (0, \bar{c}]$ for a given manipulation schedule $m_0 = m_1 = 0$ and (temporarily) ignore the contracting variables $\boldsymbol{w}_0$ and $g$, the effort-choice constraint in (8) (for the case when $c > \hat{c}$), and the truth-telling constraints in (9) and (10). Since $G(m_0, m_1) = 0$ when $m_0 = m_1 = 0$, the simplified problem is thus given by

$$\min_{\boldsymbol{w}_1} \sum_{r,v} \pi_{1,0}(r,v)\, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \hat{c} \tag{A16}$$

$$\text{s.t.} \sum_{r,v} \Delta\pi_{0,0}(r,v)\, w_1(r,v) \geq \hat{c} \tag{A17}$$

$$w_1(r,v) \geq 0, \quad \forall r \in \{r_h, r_\ell\}, v \in \{v_h, v_\ell\} \tag{A18}$$

Denoting the Lagrangian multiplier of the constraint in (A17) by $\nu$ and the respective multipliers of the limited liability constraints in (A18) by $\xi_{r,v}$, we derive the first order condition of the above optimization problem with respect to $w_1(r,v)$ as

$$\pi_{1,0}(r,v) - \nu\, \Delta\pi_{0,0}(r,v) - \xi_{r,v} = 0, \tag{A19}$$

with the complementary slackness condition $\xi_{r,v} w_1(r,v) = 0$. We first show that the IC constraint in (A17) must be binding. For the constraint to be satisfied for any $\hat{c} > 0$, the payment $w_1(r_h, v_h)$ or $w_1(r_\ell, v_h)$ must be strictly positive because $\Delta\pi_{0,0}(r_h, v_\ell) < 0$ and $\Delta\pi_{0,0}(r_\ell, v_\ell) < 0$. (Note that $\Delta\pi_{0,0}(r_\ell, v_h)$ may be positive or negative, whereas $\Delta\pi_{0,0}(r_h, v_h)$ is always positive.) If the constraint in (A17) were not binding for any $\hat{c} > 0$, the expected compensation in (A16) could therefore be reduced by lowering one of these positive payments without violating any constraints. Optimality thus requires that the IC constraint in (A17) be binding and that $\nu > 0$. Since $\pi_{1,0}(r,v) = 0$ and $\Delta\pi_{0,0}(r,v) < 0$ for the two outcomes $(r_h, v_\ell)$ and $(r_\ell, v_\ell)$ and since $\nu > 0$, the first order condition in (A19) implies that $\xi_{r_h,v_\ell} > 0$ and $\xi_{r_\ell,v_\ell} > 0$. Thus, complementary slackness requires that $w_1(r_h, v_\ell) = w_1(r_\ell, v_\ell) = 0$.

Furthermore, for the IC constraint in (A17) to hold for $\hat{c} > 0$, at least one of the two remaining payments, $w_1(r_h, v_h)$ and $w_1(r_\ell, v_h)$, must be positive. However, they cannot both be positive: If $\xi_{r_h,v_h} = \xi_{r_\ell,v_h} = 0$, the first order condition in (A19) would require that

$$\frac{\delta}{\delta - \lambda(1-\delta)} = \frac{\pi_{1,0}(r_h, v_h)}{\Delta\pi_{0,0}(r_h, v_h)} = \nu = \frac{\pi_{1,0}(r_\ell, v_h)}{\Delta\pi_{0,0}(r_\ell, v_h)} = \frac{1-\delta}{1-\delta-\lambda\delta}, \tag{A20}$$

which cannot hold since $\delta > \frac{1}{2}$ and $\lambda > 0$. Consequently, the IC constraint in (A17) implies that either

$$w_1(r_h, v_h) = \frac{\hat{c}}{\Delta\pi_{0,0}(r_h, v_h)} = \frac{\hat{c}}{\delta - \lambda(1-\delta)} \quad \text{and} \quad w_1(r_\ell, v_h) = 0 \tag{A21}$$

or

$$w_1(r_h, v_h) = 0 \quad \text{and} \quad w_1(r_\ell, v_h) = \frac{\hat{c}}{\Delta\pi_{0,0}(r_\ell, v_h)} = \frac{\hat{c}}{1-\delta-\lambda\delta}. \tag{A22}$$

The latter case is only feasible if $1-\delta-\lambda\delta > 0$, since the payment $w_1(r_\ell, v_h)$ would otherwise be negative and hence violate the limited liability constraint in (A18). However, even if the payment scheme $w_1(r_h, v_h) = 0$ and $w_1(r_\ell, v_h) > 0$ is feasible, it is never optimal. To see this, consider an increase in $w_1(r_h, v_h)$ to $\varepsilon_1 > 0$ and a decrease in $w_1(r_\ell, v_h)$ by $\varepsilon_2 > 0$ such that the IC constraint in (A17) remains binding, that is,

$$\varepsilon_2 = \frac{\Delta\pi_{0,0}(r_h, v_h)}{\Delta\pi_{0,0}(r_\ell, v_h)}\,\varepsilon_1 = \frac{\delta - \lambda(1-\delta)}{1-\delta-\lambda\delta}\,\varepsilon_1. \tag{A23}$$

Such a change in payments would change the manager's expected compensation by

$$\pi_{1,0}(r_h, v_h)\,\varepsilon_1 - \pi_{1,0}(r_\ell, v_h)\,\varepsilon_2 = \delta\,\varepsilon_1 - (1-\delta)\,\frac{\delta-\lambda(1-\delta)}{1-\delta-\lambda\delta}\,\varepsilon_1 = -\frac{\lambda(2\delta-1)}{1-\delta-\lambda\delta}\,\varepsilon_1, \tag{A24}$$

which is negative since $\delta > \frac{1}{2}$ and $1-\delta-\lambda\delta > 0$. A positive payment $w_1(r_\ell, v_h)$ can therefore not be optimal. The optimal compensation scheme is hence given by $w_1(r_h, v_h) = \frac{\hat{c}}{\delta-\lambda(1-\delta)}$ and $w_1(r_\ell, v_h) = w_1(r_h, v_\ell) = w_1(r_\ell, v_\ell) = 0$. This is intuitive: The expected compensation in (A16) is minimized if the manager receives a positive payment only in the state of nature with the highest likelihood ratio $\frac{\pi_{1,0}(r,v)}{\pi_{0,0}(r,v)}$, which is state $(r_h, v_h)$ in which both the report and the terminal cash flow signal high managerial effort.

Now consider the "no-manipulation" contract $\mathcal{C}^n = (\boldsymbol{w}_0^n, \boldsymbol{w}_1^n, g^n)$ with $w_1^n(r_h, v_h) = \frac{\hat{c}}{\delta-\lambda(1-\delta)}$ and $w_1^n(r_\ell, v_h) = w_1^n(r_h, v_\ell) = w_1^n(r_\ell, v_\ell) = 0$ as above, $w_0^n(r, v) = w_1^n(r, v)$ for all $r \in \{r_h, r_\ell\}$ and $v \in \{v_h, v_\ell\}$, and $g^n \geq \phi\, w_1^n(r_h, v_h)$. Since $\boldsymbol{w}_0$ and $g$ are not part of the simplified problem, this contract is clearly a solution to the simplified problem in (A16)–(A18). Furthermore, since the objective functions in (A15) and (A16) are identical when $m_0 = m_1 = 0$ and since the constraints in (A17) and (A18) are implied by the constraints in (7) and (4), respectively, the contract $\mathcal{C}^n$ is also a solution to the full optimization problem characterized in Section 3.2 if it satisfies the additional constraints in (4)–(10).

The contract $\mathcal{C}^n$ clearly satisfies the nonnegativity constraints in (4). Furthermore, any $g^n \geq \phi\, w_1^n(r_h, v_h)$ satisfies the manipulation incentive constraints in (5) and (6) when $m_0 = m_1 = 0$.

Since $g^n \geq \phi\, w_1^n(r_h, v_h)$, the right-hand side of (7) is maximized by setting $m = 0$: The expected gain from manipulating, $\lambda\delta\phi\, w_1^n(r_h, v_h)$, is lower than the expected cost, $\delta g^n$. The constraint in (7) then becomes identical to the constraint in (A17) and is binding. The right-hand side of (8) is also maximized by setting $m = 0$: the expected gain from manipulating, $(1-\delta)\phi\, w_0^n(r_h, v_h)$, cannot exceed the expected cost, $(1-\delta)g^n$, when $g^n \geq \phi\, w_1^n(r_h, v_h)$. Since $\boldsymbol{w}_0^n = \boldsymbol{w}_1^n$, this means that the expression on the right-hand side of (8) is identical to the expression on the left-hand side of (7) when $m_1 = 0$. Furthermore, the expression on the left-hand side of (8) is identical to the expression on the right-hand side of (7) when $m_0 = 0$ because the right-hand side of (7) is maximized by setting $m = 0$, as demonstrated above. Thus, the result that (7) is binding implies that (8) is also binding.

The truth-telling constraint in (9) is implied by the constraint in (7) when $e = 0$ on the right-hand side of (9). To see this, note that, for $c = \hat{c}$, (7) is identical to (9) when $e = 0$ because $\boldsymbol{w}_0^n = \boldsymbol{w}_1^n$. Thus, (9) must be satisfied for all $c \leq \hat{c}$ when $e = 0$. When $e = 1$, the constraint in (9) is (weakly) more restrictive when $m = 0$ on the right-hand side: the expected gain from manipulating is $(1-\delta)\phi\, w_0^n(r_h, v_h)$ and hence cannot exceed the expected cost of $(1-\delta)g^n$ since $g^n \geq \phi\, w_1^n(r_h, v_h)$. This means that the constraint is trivially satisfied when $e = 1$ because, for $m = 0$ (and $m_1 = 0$), the expression on the left-hand side equals the expression on the right-hand side. Similarly, the truth-telling constraint in (10) is implied by the constraint in (8) when $e = 1$ on the right-hand side of (10). To see this, note that, for $c = \hat{c}$, (8) is identical to (10) when $e = 1$ because $\boldsymbol{w}_0^n = \boldsymbol{w}_1^n$. Thus, (10) must be satisfied for all $c \geq \hat{c}$ when $e = 1$. When $e = 0$, the constraint in (10) is (weakly) more restrictive when $m = 0$ on the right-hand side: the expected gain from manipulating is $\lambda\delta\phi\, w_1^n(r_h, v_h)$ and hence is lower than the expected cost of $\delta g^n$ since $g^n \geq \phi\, w_1^n(r_h, v_h)$. This means that the constraint is trivially satisfied when $e = 0$ because, for $m = 0$ (and $m_0 = 0$), the expression on the left-hand side equals the expression on the right-hand side. ∎

**Proof of Proposition 2.** The derivation of the optimal contract that induces manipulation by the manager when she exerted high effort but not when she exerted low effort (i.e., when $c < \hat{c}$) is similar to that of the optimal no-manipulation contract. We again first consider a simplified optimization problem that minimizes the cost of implementing an effort choice characterized by the threshold $\hat{c}$ for a given manipulation schedule $m_0 = 0$ and $m_1 = 1$ and then show that its solution is also a solution to the full optimization problem in (3)–(10). The simplified problem consists of the objective function in (A15) (ignoring the contracting variable $\boldsymbol{w}_0$), which is equivalent to the objective function in (3) as demonstrated in the proof of Proposition 1, the effort-choice constraint in (7) for the case when $c < \hat{c}$ (both for $m = 0$ and $m = 1$ on the right-hand side), and the nonnegativity constraint for $\boldsymbol{w}_1$ in (4). Since $G(m_0, m_1) = (1-\delta)g \geq 0$ when $m_0 = 0$ and $m_1 = 1$, the simplified problem is thus given by

$$\min_{\boldsymbol{w_1},g} \sum_{r,v} \pi_{1,1}(r,v)\, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right)(\hat{c} + (1-\delta)g) \tag{A25}$$

$$\text{s.t.} \sum_{r,v} \Delta\pi_{0,1}(r,v)\, w_1(r,v) \geq \hat{c} + (1-\delta)g \tag{A26}$$

$$\sum_{r,v} \Delta\pi_{1,1}(r,v)\, w_1(r,v) \geq \hat{c} + (1-2\delta)g \tag{A27}$$

$$w_1(r,v) \geq 0, \quad \forall r \in \{r_h, r_\ell\}, v \in \{v_h, v_\ell\} \tag{A28}$$

Denoting the Lagrangian multiplier of the constraint in (A26) by $\nu$, the multiplier of the constraint in (A27) by $\mu$, and the respective multipliers of the limited liability constraints in (A28) by $\xi_{r,v}$, we derive the first order condition of the above optimization problem with respect to $w_1(r,v)$ as

$$\pi_{1,1}(r,v) - \nu\, \Delta\pi_{0,1}(r,v) - \mu\, \Delta\pi_{1,1}(r,v) - \xi_{r,v} = 0, \tag{A29}$$

with the complementary slackness condition $\xi_{r,v} w_1(r,v) = 0$, and the first order condition with respect to $g$ as

$$-\left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta) + \nu(1-\delta) + \mu(1-2\delta) = 0. \tag{A30}$$

We first show that it is optimal to set $w_1(r_h, v_\ell) = w_1(r_\ell, v_\ell) = 0$. Suppose this is not the case (i.e., $w_1(r, v_\ell) > 0$ for $r = r_h$ or $r = r_\ell$). If $w_1(r, v_\ell) > 0$, complementary slackness requires that $\xi_{r,v_\ell} = 0$. But since $\pi_{1,1}(r,v) = 0$, $\Delta\pi_{0,1}(r,v) < 0$, and $\Delta\pi_{1,1}(r,v) < 0$ for the two outcomes $(r_h, v_\ell)$ and $(r_\ell, v_\ell)$, this implies that the first order condition in (A29) can only be satisfied if $\nu = \mu = 0$ (the multipliers have to be nonnegative), which means that the IC constraints in (A26) and (A27) are not binding. This, in turn, implies that it is uniquely optimal to set $w_1(r_h, v_h) = w_1(r_\ell, v_h) = 0$ because $\pi_{1,1}(r_h, v_h) > 0$ and $\pi_{1,1}(r_\ell, v_h) > 0$. But this makes it impossible to elicit high effort for any nonzero $\hat{c}$: since $\Delta\pi_{0,1}(r_h, v_\ell) < 0$ and $\Delta\pi_{0,1}(r_\ell, v_\ell) < 0$, (A26) is violated if $w_1(r_h, v_h) = w_1(r_\ell, v_h) = 0$. Thus, we must have that $w_1(r_h, v_\ell) = w_1(r_\ell, v_\ell) = 0$.

We next argue that the IC constraints in (A26) and (A27) must both be binding. Suppose this is not the case. If the constraint in (A26) is slack, we must have $\nu = 0$. The first order condition in (A30) then implies that $\mu < 0$ (since $\delta > \frac{1}{2}$). But this violates the condition that the multiplier $\mu$ has to be nonnegative at the optimum. Thus, the constraint in (A26) must be binding. Similarly, if the constraint in (A27) is slack, we must have $\mu = 0$. Since a payment $w_1(r,v)$ can only be strictly positive if $\xi_{r,v} = 0$, the first order condition in (A29) then implies that $\nu = \frac{\pi_{1,1}(r,v)}{\Delta\pi_{0,1}(r,v)} = \frac{\pi_{1,1}(r,v)}{\pi_{1,1}(r,v)-\pi_{0,0}(r,v)}$. However, this expression either exceeds one (if $\pi_{1,1}(r,v) > \pi_{0,0}(r,v) > 0$) or it is nonpositive (if $\pi_{1,1}(r,v) < \pi_{0,0}(r,v)$). In both cases, it violates the first order condition in (A30) when $\mu = 0$, which requires that $\nu = 1 - \frac{\hat{c}}{\bar{c}} \in (0,1]$ for any nonzero $\hat{c}$. Thus, the constraint in (A27) must be binding.

Since both IC constraints in (A26) and (A27) must be binding at the optimum, we obtain the following expression for $g$ by subtracting (A27) from (A26) (and using the fact that $w_1(r_h, v_\ell) = w_1(r_\ell, v_\ell) = 0$):

$$g = \frac{1}{\delta} \sum_{r,v} \left( \Delta\pi_{0,1}(r,v) - \Delta\pi_{1,1}(r,v) \right) \, w_1(r,v) \tag{A31}$$

$$= \frac{1}{\delta} \sum_{r,v} \left( \pi_{0,1}(r,v) - \pi_{0,0}(r,v) \right) \, w_1(r,v) \tag{A32}$$

$$= \lambda\phi \left( w_1(r_h, v_h) - w_1(r_\ell, v_h) \right). \tag{A33}$$

Note that, with this choice of $g$, the two IC constraints in (A26) and (A27) become identical. We can therefore drop one of the constraints. Substituting $g$ into the objective function in (A25) and the constraint in (A26), we can rewrite the optimization problem as

$$\min_{\boldsymbol{w}_1} \sum_{r,v} \pi_{1,1}(r,v) \, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \left[\hat{c} + (1-\delta)\lambda\phi \left( w_1(r_h, v_h) - w_1(r_\ell, v_h) \right)\right] \tag{A34}$$

$$\text{s.t.} \sum_{r,v} \Delta\pi_{0,1}(r,v) \, w_1(r,v) = \hat{c} + (1-\delta)\lambda\phi \left( w_1(r_h, v_h) - w_1(r_\ell, v_h) \right) \tag{A35}$$

$$w_1(r_h, v_h) \geq 0, w_1(r_\ell, v_h) \geq 0, w_1(r_h, v_\ell) = 0, w_1(r_\ell, v_\ell) = 0 \tag{A36}$$

As before, denote the Lagrangian multiplier of the constraint in (A35) by $\nu$ and the multipliers of the limited liability constraints by $\xi_{r_h,v_h}$ and $\xi_{r_\ell,v_h}$. The first order conditions with respect to $w_1(r_h, v_h)$ and $w_1(r_\ell, v_h)$ are then

$$\delta + (1-\delta)\phi - \left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta)\lambda\phi - \nu \left[\delta - (1-\delta)(\lambda - \phi + \lambda\phi)\right] - \xi_{r_h,v_h} = 0, \tag{A37}$$

$$(1-\delta)(1-\phi) + \left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta)\lambda\phi - \nu \left[(1-\delta)(1 - \phi + \lambda\phi) - \lambda\delta\right] - \xi_{r_\ell,v_h} = 0, \tag{A38}$$

where we have substituted in the expressions for $\pi_{0,0}(r,v)$ and $\pi_{1,1}(r,v)$ from (A5)–(A8). For the IC constraint in (A35) to hold for $\hat{c} > 0$, at least one of the payments $w_1(r_h, v_h)$ and

$w_1(r_\ell, v_h)$ must be positive. However, they cannot both be positive. If they were, complementary slackness would require that $\xi_{r_h,v_h} = \xi_{r_\ell,v_h} = 0$. But then the first order conditions in (A37) and (A38) would imply that

$$\frac{\delta + (1-\delta)\phi - \left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta)\lambda\phi}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} = \frac{(1-\delta)(1-\phi) + \left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta)\lambda\phi}{(1-\delta)(1-\phi+\lambda\phi) - \lambda\delta}, \tag{A39}$$

or, equivalently, that

$$\frac{\hat{c}}{\bar{c}} = 1 + \frac{2\delta - 1}{(1-\delta)(1-\lambda)\phi}, \tag{A40}$$

which cannot be the case because $\delta > \frac{1}{2}$ and $\hat{c} \leq \bar{c}$. Consequently, the IC constraint in (A35) implies that either

$$w_1(r_h, v_h) = \frac{\hat{c}}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} \quad \text{and} \quad w_1(r_\ell, v_h) = 0 \tag{A41}$$

or

$$w_1(r_h, v_h) = 0 \quad \text{and} \quad w_1(r_\ell, v_h) = \frac{\hat{c}}{(1-\delta)(1-\phi+\lambda\phi) - \lambda\delta}. \tag{A42}$$

In the former case, the payment $w_1(r_h, v_h)$ is positive because $\delta > \frac{1}{2}$. In the latter case, the payment $w_1(r_\ell, v_h)$ is positive only if $(1-\delta)(1-\phi+\lambda\phi) - \lambda\delta > 0$. Thus, the latter payment scheme may not be feasible because it may violate the limited liability constraint in (A36). However, even if it is feasible, this payment scheme is never optimal. To see this, consider an increase in $w_1(r_h, v_h)$ to $\varepsilon_1 > 0$ and a decrease in $w_1(r_\ell, v_h)$ by $\varepsilon_2 > 0$ such that the IC constraint in (A35) remains binding, that is,

$$\varepsilon_2 = \frac{\Delta\pi_{0,1}(r_h, v_h) - (1-\delta)\lambda\phi}{\Delta\pi_{0,1}(r_\ell, v_h) + (1-\delta)\lambda\phi}\,\varepsilon_1 = \frac{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}{(1-\delta)(1-\phi+\lambda\phi) - \lambda\delta}\,\varepsilon_1. \tag{A43}$$

Such a change in payments would change the manager's expected compensation by

$$\begin{aligned} &\left[\pi_{1,1}(r_h, v_h) - \left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta)\lambda\phi\right]\varepsilon_1 - \left[\pi_{1,1}(r_\ell, v_h) + \left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta)\lambda\phi\right]\varepsilon_2 \\ &\quad = \left[\delta + (1-\delta)\left(\phi - \left(1 - \frac{\hat{c}}{\bar{c}}\right)\lambda\phi\right)\right]\varepsilon_1 - \left[(1-\delta)\left(1 - \phi + \left(1 - \frac{\hat{c}}{\bar{c}}\right)\lambda\phi\right)\right]\varepsilon_2 \end{aligned} \tag{A44}$$

$$= -\frac{\lambda\left[2\delta - 1 + \left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta)(1-\lambda)\phi\right]}{(1-\delta)(1-\phi+\lambda\phi) - \lambda\delta}\,\varepsilon_1, \tag{A45}$$

which is negative since $\delta > \frac{1}{2}$ and $(1-\delta)(1-\phi+\lambda\phi) - \lambda\delta > 0$. A positive payment $w_1(r_\ell, v_h)$ can therefore not be optimal. The optimal solution to the problem in (A25)–(A28) is thus given by the compensation scheme $w_1(r_h, v_h) = \frac{\hat{c}}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}$, $w_1(r_\ell, v_h) = w_1(r_h, v_\ell) = w_1(r_\ell, v_\ell) = 0$, and the manipulation cost $g = \lambda\phi\, w_1(r_h, v_h)$.

Now consider the selective-manipulation contract $\mathcal{C}^s = (\boldsymbol{w}_0^s, \boldsymbol{w}_1^s, g^s)$ with $w_1^s(r_h, v_h) = \frac{\hat{c}}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}$, $w_1^s(r_\ell, v_h) = w_1^s(r_h, v_\ell) = w_1^s(r_\ell, v_\ell) = 0$, $g^s = \lambda\phi\, w_1^s(r_h, v_h)$ as above, and $w_0^s(r, v) = w_1^s(r, v)$ for all $r \in \{r_h, r_\ell\}$ and $v \in \{v_h, v_\ell\}$. Since $\boldsymbol{w}_0$ is not part of the simplified problem, this contract is clearly a solution to the simplified problem in (A25)–(A28). Furthermore, since the objective functions in (A15) and (A25) are identical when $m_0 = 0$ and $m_1 = 1$ and since the constraints in (A26), (A27), and (A28) are implied by the constraints in (7) and (4), the contract $\mathcal{C}^s$ is also a solution to the full optimization problem characterized in Section 3.2 if it satisfies the additional constraints in (4)–(10).

The contract $\mathcal{C}^s$ clearly satisfies the nonnegativity constraints in (4). Furthermore, $g^s = \lambda\phi\, w_1^s(r_h, v_h)$ satisfies the manipulation incentive constraints in (5) and (6) when $m_0 = 0$ and $m_1 = 1$ ((5) is slack and (6) is binding).

Since $g^s = \lambda\phi\, w_1^s(r_h, v_h)$, the right-hand side of (7) is the same for $m = 0$ and $m = 1$: the expected gain from manipulating, $\lambda\delta\phi\, w_1^s(r_h, v_h)$, is equal to the expected cost, $\delta g^s$. The constraint in (7) then becomes identical to the constraint in (A26) and is binding. The right-hand side of (8) is maximized by setting $m = 1$: the expected gain from manipulating, $(1-\delta)\phi\, w_0^s(r_h, v_h)$, exceeds the expected cost, $(1-\delta)g^s = (1-\delta)\lambda\phi\, w_1^s(r_h, v_h)$. Since $\boldsymbol{w}_0^s = \boldsymbol{w}_1^s$, this means that the expression on the right-hand side of (8) is identical to the expression on the left-hand side of (7) when $m_1 = 1$. Furthermore, the expression on the left-hand side of (8) is identical to the expression on the right-hand side of (7) when $m_0 = 0$ because the right-hand side of (7) is maximized by setting $m = 0$, as demonstrated above. Thus, the result that (7) is binding implies that (8) is also binding.

The truth-telling constraint in (9) is implied by the constraint in (7) when $e = 0$ on the right-hand side of (9). To see this, note that, for $c = \hat{c}$, (7) is identical to (9) when $e = 0$ because $\boldsymbol{w}_0^s = \boldsymbol{w}_1^s$. Thus, (9) must be satisfied for all $c \le \hat{c}$ when $e = 0$. When $e = 1$, the constraint in (9) is more restrictive when $m = 1$ on the right-hand side: the expected gain from manipulating is $(1-\delta)\phi\, w_0^s(r_h, v_h)$ and hence exceeds the expected cost of $(1-\delta)g^s$ since $g^s = \lambda\phi\, w_1^s(r_h, v_h)$. This means that the constraint is trivially satisfied when $e = 1$ because, for $m = 1$ (and $m_1 = 1$), the expression on the left-hand side equals the expression on the right-hand side. Similarly, the truth-telling constraint in (10) is implied by the constraint in (8) when $e = 1$ on the right-hand side of (10). To see this, note that, for $c = \hat{c}$, (8) is identical to (10) when $e = 1$ because $\boldsymbol{w}_0^s = \boldsymbol{w}_1^s$. Thus, (10) must be satisfied for all $c \ge \hat{c}$ when $e = 1$. When $e = 0$, the constraint in (10) is (weakly) more restrictive when $m = 0$ on the right-hand side: the expected gain from manipulating is $\lambda\delta\phi\, w_1^s(r_h, v_h)$ and hence equals the expected cost of $\delta g^s$ since $g^s = \lambda\phi\, w_1^s(r_h, v_h)$. This means that the constraint is trivially satisfied when $e = 0$ because, for $m = 0$ (and $m_0 = 0$), the expression on the left-hand side equals the expression on the right-hand side. ■

**Proof of Proposition 3.** We prove this result by showing (i) that any contract that induces manipulation decisions $m_0 > 0$ and $m_1 = 0$ is dominated by the no-manipulation contract $\mathcal{C}^n$ derived in Proposition 1, (ii) that any contract that induces manipulation decisions $m_0 > 0$ and $m_1 = 1$ is dominated by the selective-manipulation contract $\mathcal{C}^s$ derived in Proposition 2, and (iii) that any contract that induces manipulation decisions $m_0 \geq 0$ and $m_1 \in (0, 1)$ is dominated by the no-manipulation contract $\mathcal{C}^n$ as well.

As shown in the proof of Proposition 1, the manager's expected compensation can be written as

$$\sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \left(\hat{c} + (1-\delta)gm_1 - \delta g m_0\right). \tag{A46}$$

Since $g \geq 0$, the manager's expected compensation if $m_0 > 0$ can therefore not be lower than

$$\sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \left(\hat{c} + (1-\delta)gm_1\right), \tag{A47}$$

the expected compensation if $m_0 = 0$.

First, consider the case where $m_0 > 0$ and $m_1 = 0$. The IC constraint in (7) then requires that

$$\sum_{r,v} \Delta\pi_{0,0}(r,v)\, w_1(r,v) \geq \hat{c}. \tag{A48}$$

This constraint is identical to the IC constraint in (A17) of the simplified problem analyzed in the proof of Proposition 1. Furthermore, the objective function of that problem in (A16) is identical to (A47) if $m_1 = 0$. The optimal no-manipulation contract $\mathcal{C}^n$ thus minimizes (the lower bound of) the manager's expected compensation in (A47) (with $m_1 = 0$) subject to the IC constraint in (A48) and the limited liability constraints $w_1(r,v) \geq 0$. But these constraints also have to be satisfied by any contract that implements the manipulation decisions $m_0 > 0$ and $m_1 = 0$. Furthermore, the additional constraints in (4)–(10) cannot reduce the manager's expected compensation. Hence, any contract that implements the cost threshold $\hat{c}$ and the manipulation decisions $m_0 > 0$ and $m_1 = 0$ is dominated by the no-manipulation contract $\mathcal{C}^n$.

Next, consider the case where $m_0 > 0$ and $m_1 = 1$. The IC constraint in (7) then requires that

$$\sum_{r,v} \Delta\pi_{0,1}(r,v)\, w_1(r,v) \geq \hat{c} + (1-\delta)g, \tag{A49}$$

and that

$$\sum_{r,v} \Delta\pi_{1,1}(r,v)\, w_1(r,v) \geq \hat{c} + (1-2\delta)g. \tag{A50}$$

These constraints are identical to the IC constraints in (A26) and (A27) of the simplified problem analyzed in the proof of Proposition 2. Furthermore, the objective function of that problem in (A25) is identical to (A47) if $m_1 = 1$. The optimal selective-manipulation contract $\mathcal{C}^s$ thus minimizes (the lower bound of) the manager's expected compensation in (A47) (with $m_1 = 1$) subject to the IC constraints in (A49) and (A50) and the limited liability constraints $w_1(r, v) \geq 0$. But these constraints also have to be satisfied by any contract that implements the manipulation decisions $m_0 > 0$ and $m_1 = 1$. Furthermore, the additional constraints in (4)–(10) cannot reduce the manager's expected compensation. Hence, any contract that implements the cost threshold $\hat{c}$ and the manipulation decisions $m_0 > 0$ and $m_1 = 1$ is dominated by the selective-manipulation contract $\mathcal{C}^s$.

Finally, consider the case where $m_0 \geq 0$ and $m_1 \in (0, 1)$. In this case, a manager who chose the high effort level must be indifferent between choosing $m_1 = 0$ and $m_1 = 1$. Thus, the IC constraints in (5) and (7) require that

$$g = \phi \left( w_1(r_h, v_h) - w_1(r_\ell, v_h) \right) \tag{A51}$$

and that

$$\sum_{r,v} \Delta\pi_{0,m_1}(r, v)\, w_1(r, v) \geq \hat{c} + (1 - \delta) g m_1. \tag{A52}$$

Since $\Delta\pi_{0,m_1}(r_h, v_h) = \delta + (1-\delta)\phi\, m_1 - \lambda(1-\delta)$ and $\Delta\pi_{0,m_1}(r_\ell, v_h) = (1-\delta)(1-\phi\, m_1) - \lambda\delta$, substituting (A51) into (A52) yields

$$\sum_{r,v} \Delta\pi_{0,0}(r, v)\, w_1(r, v) \geq \hat{c}, \tag{A53}$$

which is identical to the effort IC constraint in (A17) of the simplified problem considered in the proof of Proposition 1. Furthermore, using (A51) we can write the lower bound of the manager's expected compensation in (A47) as

$$\sum_{r,v} \pi_{1,m_1}(r, v)\, w_1(r, v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \Big[\hat{c} + (1 - \delta)\, \phi \left( w_1(r_h, v_h) - w_1(r_\ell, v_h) \right) m_1 \Big], \tag{A54}$$

which is equivalent to

$$\sum_{r,v} \pi_{1,0}(r, v)\, w_1(r, v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \hat{c} + \frac{\hat{c}}{\bar{c}} (1 - \delta)\, \phi \left( w_1(r_h, v_h) - w_1(r_\ell, v_h) \right) m_1, \tag{A55}$$

because $\pi_{1,m_1}(r_h, v_h) = \delta + (1-\delta)\phi\, m_1$ and $\pi_{1,m_1}(r_\ell, v_h) = (1-\delta)(1-\phi\, m_1)$. Since $g \geq 0$ and hence $w_1(r_h, v_h) \geq w_1(r_\ell, v_h)$, the manager's expected compensation can therefore not be lower than

$$\sum_{r,v} \pi_{1,0}(r, v)\, w_1(r, v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \hat{c}, \tag{A56}$$

the expected compensation in the no-manipulation case given by (A16). The optimal no-manipulation contract $\mathcal{C}^n$ thus minimizes (the lower bound of) the manager's expected compensation subject to the IC constraint in (A53) and the limited liability constraints $w_1(r,v) \geq 0$. But these constraints also have to be satisfied by any contract that implements the manipulation decisions $m_0 \geq 0$ and $m_1 \in (0,1)$. Furthermore, the additional constraints in (4)–(10) cannot reduce the manager's expected compensation. Hence, any contract that implements the cost threshold $\hat{c}$ and the manipulation decisions $m_0 \geq 0$ and $m_1 \in (0,1)$ is dominated by the no-manipulation contract $\mathcal{C}^n$. ■

**Proof of Proposition 4.** From the objective function in (A15) and the compensation scheme in Proposition 1, it follows that, for any cost threshold $\hat{c} \in [0, \bar{c}]$, the expected compensation required to induce the manager to exert high effort if and only if $c \leq \hat{c}$ and to follow the no-manipulation schedule $m_0 = m_1 = 0$ is given by

$$\mathbb{E}w^n(\hat{c}) = \pi_{1,0}(r_h, v_h)\, w_1^n(r_h, v_h) - \left(1 - \frac{\hat{c}}{\bar{c}}\right)\hat{c} = \left(\frac{\lambda(1-\delta)}{\delta - \lambda(1-\delta)} + \frac{\hat{c}}{\bar{c}}\right)\hat{c}. \tag{A57}$$

Similarly, from (A15) and Proposition 2, it follows that the expected compensation necessary to induce the manager to exert high effort if and only if $c \leq \hat{c}$ and to follow the selective-manipulation schedule $m_0 = 0$ and $m_1 = 1$ is given by

$$\mathbb{E}w^s(\hat{c}) = \pi_{1,1}(r_h, v_h)\, w_1^s(r_h, v_h) - \left(1 - \frac{\hat{c}}{\bar{c}}\right)\left(\hat{c} + (1-\delta)g^s\right)$$

$$= \left(\frac{\delta + (1-\delta)\phi - \left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta)\lambda\phi}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} - 1 + \frac{\hat{c}}{\bar{c}}\right)\hat{c} \tag{A58}$$

$$= \left(\frac{\left[1 + \left(\frac{\hat{c}}{\bar{c}}\right)\phi\right](1-\delta)\lambda}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} + \frac{\hat{c}}{\bar{c}}\right)\hat{c}. \tag{A59}$$

For any cost threshold $\hat{c} > 0$, the expressions in (A57) and (A59) imply that $\mathbb{E}w^s(\hat{c}) \underset{(>)}{<} \mathbb{E}w^n(\hat{c})$ if and only if

$$\frac{\left[1 + \left(\frac{\hat{c}}{\bar{c}}\right)\phi\right](1-\delta)\lambda}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} \underset{(>)}{<} \frac{\lambda(1-\delta)}{\delta - \lambda(1-\delta)}, \tag{A60}$$

or, equivalently, if and only if

$$\frac{\hat{c}}{\bar{c}} \underset{(>)}{<} \frac{(1-\delta)(1-\lambda)}{\delta - \lambda(1-\delta)}. \tag{A61}$$

■

**Proof of Proposition 5.** For a given cost threshold $\hat{c} \in [0, \bar{c}]$, the value of the firm (net of the cost of managerial compensation) under the optimal no-manipulation contract $\mathcal{C}^n$ specified in Proposition 1 is given by

$$V_n(\hat{c}) = \left(\frac{\hat{c}}{\bar{c}}\right) v_h + \left(1 - \frac{\hat{c}}{\bar{c}}\right) \big(\lambda\, v_h + (1-\lambda)\, v_\ell\big) - \mathbb{E}w^n(\hat{c}), \tag{A62}$$

where the expected compensation $\mathbb{E}w^n(\hat{c})$ is given by (A57) in the proof of Proposition 4. Substituting the expression in (A57) into the above equation yields

$$V_n(\hat{c}) = V_0 + (1-\lambda)(v_h - v_\ell)\left(\frac{\hat{c}}{\bar{c}}\right) - \left(\frac{\lambda(1-\delta)}{\delta - \lambda(1-\delta)} + \frac{\hat{c}}{\bar{c}}\right)\hat{c}, \tag{A63}$$

where $V_0 = \lambda v_h + (1-\lambda)v_\ell$. Note that $V_n$ is a strictly concave function of $\hat{c}$ with $V_n'(\bar{c}) < (1-\lambda)(v_h - v_\ell)/\bar{c} - 2 < 0$ because, by assumption, $(1-\lambda)(v_h - v_\ell) < \bar{c}$. Thus, if $V_n'(0) \geq 0$, the optimal cost threshold that maximizes $V_n$ is uniquely determined by the first order condition

$$\hat{c}_n = \frac{1}{2}\left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta - \lambda(1-\delta)}\right). \tag{A64}$$

If $V_n'(0) < 0$, the above expression is negative and the optimal cost threshold is zero.

Similarly, the value of the firm under the optimal selective-manipulation contract $\mathcal{C}^s$ specified in Proposition 2 is given by

$$V_s(\hat{c}) = \left(\frac{\hat{c}}{\bar{c}}\right) v_h + \left(1 - \frac{\hat{c}}{\bar{c}}\right) \big(\lambda\, v_h + (1-\lambda)\, v_\ell\big) - \mathbb{E}w^s(\hat{c}), \tag{A65}$$

where the expected compensation $\mathbb{E}w^s(\hat{c})$ is given by (A59). Substituting the expression in (A59) into the above equation yields

$$V_s(\hat{c}) = V_0 + (1-\lambda)(v_h - v_\ell)\left(\frac{\hat{c}}{\bar{c}}\right) - \left(\frac{\left[1 + \left(\frac{\hat{c}}{\bar{c}}\right)\phi\right]\lambda(1-\delta)}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} + \frac{\hat{c}}{\bar{c}}\right)\hat{c}, \tag{A66}$$

where, as before, $V_0 = \lambda v_h + (1-\lambda)v_\ell$. Similarly to $V_n$, $V_s$ is a strictly concave function of $\hat{c}$ with $V_s'(\bar{c}) < (1-\lambda)(v_h - v_\ell)/\bar{c} - 2 < 0$. Thus, if $V_s'(0) \geq 0$, the optimal cost threshold that maximizes $V_s$ is uniquely determined by the first order condition

$$\hat{c}_s = \frac{1}{2}\left(\frac{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}{\delta - (1-\delta)(\lambda - \phi)}\right)\left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}\right). \tag{A67}$$

If $V_s'(0) < 0$, the above expression is negative and the optimal cost threshold is zero. ■

**Proof of Corollary 1.** From Proposition 3, we know that, for any cost threshold $\hat{c}$, the optimal contract is either the no-manipulation contract $\mathcal{C}^n$ defined in Proposition 1 or the selective-manipulation contract $\mathcal{C}^s$ defined in Proposition 2. Proposition 5 shows that the optimal cost threshold under the no-manipulation contract, $\hat{c}_n$, is zero whenever the optimal cost threshold under the selective-manipulation contract, $\hat{c}_s$, is zero. Thus, a necessary and sufficient condition for the optimal contract to induce high effort is that $\hat{c}_s > 0$, which is equivalent to the condition in (18). ■

**Proof of Proposition 6.** From Proposition 3, we know that, for any cost threshold $\hat{c}$, the optimal contract is either the no-manipulation contract $\mathcal{C}^n$ defined in Proposition 1 or the selective-manipulation contract $\mathcal{C}^s$ defined in Proposition 2. Furthermore, Proposition 5 shows that firm value under the no-manipulation contract (respectively, the selective-manipulation contract) is maximized at a cost threshold of $\hat{c}_n$ (respectively, $\hat{c}_s$). Thus, to prove the result it suffices to show that $V_s(\hat{c}_s) \geq V_n(\hat{c}_n)$ if and only if (19) is satisfied, where, as in the proof of Proposition 5, $V_n(\hat{c})$ denotes the firm value under the no-manipulation contract and $V_s(\hat{c})$ the firm value under the selective-manipulation contract.

The result that $V_s(\hat{c}_s) \geq V_n(\hat{c}_n)$ trivially holds if $\hat{c}_n = 0$ because $\max\{V_s(\hat{c}_s), V_s(0)\} \geq V_s(0) = V_n(0)$. Furthermore, since the right-hand side of (19) exceeds the right-hand side of (18), it follows that $\hat{c}_s > 0$ if (19) is not satisfied. But if $\hat{c}_s > 0$, the fact that $V_s(\hat{c}_s) < V_n(\hat{c}_n)$ implies that $\hat{c}_n > 0$ as well. Thus, we are left to show that $V_s(\hat{c}_s) \geq V_n(\hat{c}_n)$ if and only if (19) is satisfied in case $\hat{c}_s > 0$ and $\hat{c}_n > 0$.

If $\hat{c}_n > 0$, it follows from (16) and (A63) that

$$V_n(\hat{c}_n) = V_0 + \frac{1}{\bar{c}}\left[\left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta - \lambda(1-\delta)}\right)\hat{c}_n - \hat{c}_n^2\right] = V_0 + \frac{\hat{c}_n^2}{\bar{c}}. \tag{A68}$$

Similarly, if $\hat{c}_s > 0$, from (17) and (A66) we have

$$V_s(\hat{c}_s) = V_0 + \frac{1}{\bar{c}}\left[\left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}\right)\hat{c}_s - \left(\frac{\delta - (1-\delta)(\lambda - \phi)}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}\right)\hat{c}_s^2\right] \tag{A69}$$

$$= V_0 + \left(\frac{\delta - (1-\delta)(\lambda - \phi)}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}\right)\frac{\hat{c}_s^2}{\bar{c}}. \tag{A70}$$

Thus, $V_s(\hat{c}_s) \geq V_n(\hat{c}_n)$ if and only if

$$\frac{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}{\delta - (1-\delta)(\lambda - \phi)}\left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}\right)^2 \geq$$

$$\left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta - \lambda(1-\delta)}\right)^2. \quad \text{(A71)}$$

Since $\hat{c}_n$ and $\hat{c}_s$ are positive, we can rewrite this condition as

$$\left(1 - \sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}\right)\frac{(1-\lambda)(v_h-v_\ell)}{\bar{c}} \geq$$

$$\frac{\lambda(1-\delta)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} - \frac{\lambda(1-\delta)}{\delta-\lambda(1-\delta)}\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}, \quad \text{(A72)}$$

or, since the term under the square root sign is greater than one, as

$$\frac{(1-\lambda)(v_h-v_\ell)}{\bar{c}} \leq \frac{\frac{\lambda(1-\delta)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} - \frac{\lambda(1-\delta)}{\delta-\lambda(1-\delta)}\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}{1-\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}. \quad \text{(A73)}$$

The term on the right-hand side of (A73) can be rearranged as follows:

$$\frac{\frac{\lambda(1-\delta)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} - \frac{\lambda(1-\delta)}{\delta-\lambda(1-\delta)}\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}{1-\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}$$

$$= \frac{1-\delta}{\delta-\lambda(1-\delta)} + \frac{\frac{\lambda(1-\delta)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} - \frac{1-\delta}{\delta-\lambda(1-\delta)} + \left(\frac{1-\delta}{\delta-\lambda(1-\delta)} - \frac{\lambda(1-\delta)}{\delta-\lambda(1-\delta)}\right)\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}{1-\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}} \quad \text{(A74)}$$

$$= \frac{1-\delta}{\delta-\lambda(1-\delta)} + \frac{\frac{\lambda(1-\delta)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} - \frac{1-\delta}{\delta-\lambda(1-\delta)} + \frac{(1-\delta)(1-\lambda)}{\delta-\lambda(1-\delta)}\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}{1-\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}} \quad \text{(A75)}$$

$$= \frac{1-\delta}{\delta-\lambda(1-\delta)} + \frac{(1-\delta)(1-\lambda)}{\delta-\lambda(1-\delta)}\left(\frac{\frac{\frac{\lambda(1-\delta)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} - \frac{1-\delta}{\delta-\lambda(1-\delta)}}{\frac{(1-\delta)(1-\lambda)}{\delta-\lambda(1-\delta)}} + \sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}{1-\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}\right) \quad \text{(A76)}$$

$$= \frac{1-\delta}{\delta-\lambda(1-\delta)} + \frac{(1-\delta)(1-\lambda)}{\delta-\lambda(1-\delta)}\left(\frac{\frac{\lambda}{1-\lambda}\left(\frac{\delta-\lambda(1-\delta)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} - \frac{1}{\lambda}\right) + \sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}{1-\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}\right) \quad \text{(A77)}$$

$$= \frac{1-\delta}{\delta - \lambda(1-\delta)} + \frac{(1-\delta)(1-\lambda)}{\delta - \lambda(1-\delta)} \left( \frac{-\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} + \sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}{1-\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}} \right) \tag{A78}$$

$$= \frac{1-\delta}{\delta - \lambda(1-\delta)} + \frac{(1-\delta)(1-\lambda)}{\delta - \lambda(1-\delta)} \sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}. \tag{A79}$$

Thus, $V_s(\hat{c}_s) \geq V_n(\hat{c}_n)$ if and only if

$$\frac{v_h - v_\ell}{\bar{c}} \leq \frac{1-\delta}{\delta - \lambda(1-\delta)} \left( \frac{1}{1-\lambda} + \sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}} \right). \tag{A80}$$

■

**Proof of Proposition 7.** The result that the payments $w^n(r_h, v_h)$ and $w^s(r_h, v_h)$ increase in the payoff difference $v_h - v_\ell$ follows immediately from Propositions 1, 2, and 5: Propositions 1 and 2 show that both $w^n(r_h, v_h)$ and $w^s(r_h, v_h)$ increase in the cost threshold $\hat{c}$, and Proposition 5 shows that the optimal cost threshold under both the no-manipulation contract and the selective-manipulation contract, $\hat{c}_n$ and $\hat{c}_s$, increase in $v_h - v_\ell$. Thus, the compensation that the manager earns in case of a successful outcome $r = r_h$ and $v = v_h$ increases in $v_h - v_\ell$ for any $v_h - v_\ell$ that renders a no-manipulation contract optimal (i.e., that violates the condition in (19)) and for any $v_h - v_\ell$ that renders a selective-manipulation contract optimal (i.e., that satisfies the condition in (19)). We are therefore left to show that $w^s(r_h, v_h) < w^n(r_h, v_h)$ when the condition in (19) is satisfied with equality and the board is indifferent between offering a no-manipulation contract and a selective-manipulation contract.

As in the proof of Proposition 5, let $V_n(\hat{c})$ denote the firm value under the no-manipulation contract (given by (A63)) and $V_s(\hat{c})$ the firm value under the selective-manipulation contract (given by (A66)). The functions $V_s(\hat{c})$ and $V_n(\hat{c})$ intersect in $\hat{c} = 0$ (both contracts then implement neither effort nor manipulation, so they generate the same outcome). Both $V_s(\hat{c})$ and $V_n(\hat{c})$ are quadratic, concave functions of the cost threshold $\hat{c}$ (see the proof of Proposition 5), so they intersect at some $\hat{c} \neq 0$. When the condition in (19) holds with equality and hence $V_s(\hat{c}_s) = V_n(\hat{c}_n)$, this second intersection must be at some $\hat{c}$ such that $\min\{\hat{c}_s, \hat{c}_n\} \leq \hat{c} \leq \max\{\hat{c}_s, \hat{c}_n\}$. From Proposition 4, we know that this intersection is at $\hat{c} = \kappa\bar{c}$, with $\kappa \in (0, 1)$. Furthermore, we have $V_s(\hat{c}) > V_n(\hat{c})$ for all $\hat{c} \in (0, \kappa\bar{c})$, and $V_s(\hat{c}) < V_n(\hat{c})$ for all $\hat{c} \in (\kappa\bar{c}, \bar{c}]$. The optimality of $\hat{c}_n$ and $\hat{c}_s$ therefore implies that $\hat{c}_n > \kappa\bar{c}$ and $\hat{c}_s < \kappa\bar{c}$ (because $V_s(\hat{c}_s) > V_n(\hat{c}_s)$ and $V_s(\hat{c}_n) < V_n(\hat{c}_n)$). Hence, $\hat{c}_n > \hat{c}_s$ when the condition in (19) holds with equality.

From Propositions 1 and 2, we have that, for a given threshold $\hat{c}$,

$$w^n(r_h, v_h) = \frac{\hat{c}}{\delta - \lambda(1-\delta)} > \frac{\hat{c}}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} = w^s(r_h, v_h). \tag{A81}$$

Since the payments $w^n(r_h, v_h)$ and $w^s(r_h, v_h)$ are increasing in $\hat{c}$, this implies that $w^n(r_h, v_h) > w^s(r_h, v_h)$ if under the optimal contract $\hat{c}_n > \hat{c}_s$, which is the case when the condition in (19) holds with equality. ■

## References


Ahern, K. R. and D. Sosyura, 2014, "Who Writes the News? Corporate Press Releases during Merger Negotiations," *Journal of Finance* 69, pp. 241–291.

Amiram, D., Z. Bozanic, J. D. Cox, Q. Dupont, J. M. Karpoff, and R. Sloan, 2018, "Financial Reporting Fraud and Other Forms of Misconduct: A Multidisciplinary Review of the Literature," *Review of Accounting Studies* 23, pp. 732–783.

Armstrong, C. S., A. D. Jagolinzer, and D. F. Larcker, 2010, "Chief Executive Officer Equity Incentives and Accounting Irregularities," *Journal of Accounting Research* 48, pp. 225–271.

Arya, A., J. Glover, and S. Sunder, 1998, "Earnings Management and the Revelation Principle," *Review of Accounting Studies* 3, pp. 7–34.

Baik, B., S. Choi, and D. B. Farber, 2020, "Managerial Ability and Income Smoothing," *Accounting Review* 95, pp. 1–22.

Banerjee, S. and M. Szydlowski, 2021, "Friends Don't Lie: Monitoring and Communication with Risky Investments," Working Paper, University of California, San Diego.

Bar-Gill, O. and L. A. Bebchuk, 2003, "Misreporting Corporate Performance," Working Paper, Harvard Law School.

Beasley, M. S., 1996, "An Empirical Analysis of the Relation Between the Board of Director Composition and Financial Statement Fraud," *Accounting Review* 71, pp. 443–465.

Beasley, M. S., J. V. Carcello, D. R. Hermanson, and P. D. Lapides, 2000, "Fraudulent Financial Reporting: Consideration of Industry Traits and Corporate Governance Mechanisms," *Accounting Horizons* 14, pp. 441–454.

Bergstresser, D. and T. Philippon, 2006, "CEO Incentives and Earnings Management," *Journal of Financial Economics* 80, pp. 511–529.

Bertomeu, J., M. N. Darrough, and W. Xue, 2017, "Optimal Conservatism with Earnings Manipulation," *Contemporary Accounting Research* 34, pp. 252–284.

Beyer, A. and I. Guttman, 2012, "Voluntary Disclosure, Manipulation, and Real Effects," *Journal of Accounting Research* 50, pp. 1141–1177.

Beyer, A., I. Guttman, and I. Marinovic, 2014, "Optimal Contracts with Performance Manipulation," *Journal of Accounting Research* 52, pp. 817–847.

Bird, A., S. A. Karolyi, and P. Ma, 2018, "Strategic Disclosure Misclassification," Working Paper, Carnegie Mellon University.

Brown, N. C., R. M. Crowley, and W. B. Elliott, 2020, "What Are You Saying? Using *Topic* to Detect Financial Misreporting," *Journal of Accounting Research* 58, pp. 237–291.

Burns, N. and S. Kedia, 2006, "The Impact of Performance-Based Compensation on Misreporting," *Journal of Financial Economics* 79, pp. 35–67.

Carcello, J. V. and A. L. Nagy, 2004, "Audit Firm Tenure and Fraudulent Financial Reporting," *Auditing* 23, pp. 55–69.

Chalmers, K., V. Naiker, and F. Navissi, 2012, "Earnings Quality and Rule 10b-5 Securities Class Action Lawsuits," *Journal of Accounting and Public Policy* 31, pp. 22–43.

Coram, P., C. Ferguson, and R. Moroney, 2008, "Internal Audit, Alternative Internal Audit Structures and the Level of Misappropriation of Assets Fraud," *Accounting & Finance* 48, pp. 543–559.

Cornelli, F. and O. Yosha, 2003, "Stage Financing and the Role of Convertible Securities," *Review of Economic Studies* 70, pp. 1–32.

Crocker, K. J. and J. Slemrod, 2007, "The Economics of Earnings Manipulation and Managerial Compensation," *RAND Journal of Economics* 38, pp. 698–713.

Curtis, A. B., S. E. Mcvay, and B. C. Whipple, 2014, "The Disclosure of Non-GAAP Earnings Information in the Presence of Transitory Gains," *Accounting Review* 89, pp. 933–958.

De Jong, A., G. Mertens, M. van der Poel, and R. van Dijk, 2014, "How Does Earnings Management Influence Investor's Perceptions of Firm Value? Survey Evidence from Financial Analysts," *Review of Accounting Studies* 19, pp. 606–627.

Dechow, P. M., W. Ge, C. R. Larson, and R. G. Sloan, 2011, "Predicting Material Accounting Misstatements," *Contemporary Accounting Research* 28, pp. 17–82.

Dechow, P. M., R. G. Sloan, and A. P. Sweeney, 1996, "Causes and Consequences of Earnings Manipulation: An Analysis of Firms Subject to Enforcement Actions by the SEC," *Contemporary Accounting Research* 13, pp. 1–36.

Demerjian, P., B. Lev, and S. McVay, 2012, "Quantifying Managerial Ability: A New Measure and Validity Tests," *Management Science* 58, pp. 1229–1248.

Demski, J. S., 1998, "Performance Measure Manipulation," *Contemporary Accounting Research* 15, pp. 261–285.

Demski, J. S., H. Frimor, and D. E. M. Sappington, 2004, "Efficient Manipulation in a Repeated Setting," *Journal of Accounting Research* 42, pp. 31–49.

Doyle, J. T., J. N. Jennings, and M. T. Soliman, 2013, "Do Managers Define Non-GAAP Earnings to Meet or Beat Analyst Forecasts?" *Journal of Accounting and Economics* 56, pp. 40–56.

Dutta, S. and Q. Fan, 2014, "Equilibrium Earnings Management and Managerial Compensation in a Multiperiod Agency Setting," *Review of Accounting Studies* 19, pp. 1047–1077.

Dye, R. A., 1988, "Earnings Management in an Overlapping Generations Model," *Journal of Accounting Research* 26, pp. 195–235.

Erickson, M., M. Hanlon, and E. L. Maydew, 2006, "Is There a Link Between Executive Equity Incentives and Accounting Fraud?" *Journal of Accounting Research* 44, pp. 113–143.

Ewens, M., R. Nanda, and M. Rhodes-Kropf, 2018, "Cost of Experimentation and the Evolution of Venture Capital," *Journal of Financial Economics* 128, pp. 422–442.

Farber, D. B., 2005, "Restoring Trust After Fraud: Does Corporate Governance Matter?" *Accounting Review* 80, pp. 539–561.

Fich, E. M. and A. Shivdasani, 2007, "Financial Fraud, Director Reputation, and Shareholder Wealth," *Journal of Financial Economics* 86, pp. 306–336.

Fischer, P. E. and R. E. Verrecchia, 2000, "Reporting Bias," *Accounting Review* 75, pp. 229–245.

Gao, P. and P. J. Liang, 2013, "Informational Feedback, Adverse Selection, and Optimal Disclosure Policy," *Journal of Accounting Research* 51, pp. 1133–1158.

Gerety, M. and K. Lehn, 1997, "The Causes and Consequences of Accounting Fraud," *Managerial and Decision Economics* 18, pp. 587–599.

Goldman, E. and S. L. Slezak, 2006, "An Equilibrium Model of Incentive Contracts in the Presence of Information Manipulation," *Journal of Financial Economics* 80, pp. 603–626.

Graham, J. R., C. R. Harvey, and S. Rajgopal, 2005, "The Economic Implications of Corporate Financial Reporting," *Journal of Accounting and Economics* 40, pp. 3–73.

Guttman, I., O. Kadan, and E. Kandel, 2006, "A Rational Expectations Theory of Kinks in Financial Reporting," *Accounting Review* 81, pp. 811–848.

Higgins, M. J. and D. Rodriguez, 2006, "The Outsourcing of R&D Through Acquisitions in the Pharmaceutical Industry," *Journal of Financial Economics* 80, pp. 351–383.

Hoberg, G. and C. Lewis, 2017, "Do Fraudulent Firms Produce Abnormal Disclosure?" *Journal of Corporate Finance* 43, pp. 58–85.

Hobson, J. L. and S. Stirnkorb, 2020, "Managing Earnings to Appear Truthful: The Effect of Public Scrutiny on Exactly Meeting a Threshold," Working Paper, University of Illinois at Urbana-Champaign.

Karpoff, J. M., D. S. Lee, and G. S. Martin, 2008, "The Cost to Firms of Cooking the Books," *Journal of Financial and Quantitative Analysis* 43, pp. 581–611.

Kartik, N., 2009, "Strategic Communication with Lying Costs," *Review of Economic Studies* 76, pp. 1359–1395.

Kartik, N., M. Ottaviani, and F. Squintani, 2007, "Credulity, Lies, and Costly Talk," *Journal of Economic Theory* 134, pp. 93–116.

Lacker, J. M. and J. A. Weinberg, 1989, "Optimal Contracts Under Costly State Falsification," *Journal of Political Economy* 97, pp. 1345–1363.

Laurion, H., 2020, "Implications of Non-GAAP Earnings for Real Activities and Accounting Choices," *Journal of Accounting and Economics* 70, pp. 1–22.

Lennox, C. and J. A. Pittman, 2010, "Big Five Audits and Accounting Fraud," *Contemporary Accounting Research* 27, pp. 209–247.

Maggi, G. and A. Rodríguez-Clare, 1995, "Costly Distortion of Information in Agency Problems," *RAND Journal of Economics* 26, pp. 675–689.

Marinovic, I. and P. Povel, 2017, "Competition for Talent Under Performance Manipulation," *Journal of Accounting and Economics* 64, pp. 1–14.

Povel, P., R. Singh, and A. Winton, 2007, "Booms, Busts, and Fraud," *Review of Financial Studies* 20, pp. 1219–1254.

Stein, J. C., 1989, "Efficient Capital Markets, Inefficient Firms: A Model of Myopic Corporate Behavior," *Quarterly Journal of Economics* 104, pp. 655–669.

Strobl, G., 2013, "Earnings Manipulation and the Cost of Capital," *Journal of Accounting Research* 51, pp. 449–473.

Subramanyam, K. R., 1996, "The Pricing of Discretionary Accruals," *Journal of Accounting and Economics* 22, pp. 249–281.

Welch, J. and J. A. Byrne, 2003, *Jack: Straight from the Gut*, New York: Warner Books.

Zhao, Y. and K. H. Chen, 2008, "Staggered Boards and Earnings Management," *Accounting Review* 83, pp. 1347–1381.