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ROBOTS AND HUMANS: THE ROLE OF FISCAL AND MONETARY POLICIES IN AN ENDOGENOUS GROWTH MODEL

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Robots and humans: the role of fiscal and monetary policies in an endogenous growth model

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In this paper we develop a dynamic general equilibrium growth model in which robots can replace unskilled labor and: *i)* the government uses tax revenues to invest in social capital and compensate those who do not work; *ii)* there is monetary policy with cash-in-advance restrictions that impact, for example, wages; *iii)* social capital increases skilled-labor productivity and facilitates the technological-knowledge progress. Our results confirm that by reducing the unskilled-to-skilled-labor ratio, the robotization process increases the skill premium (and thus wage inequality between skilled and unskilled workers), stimulates economic growth and improves welfare. We also show that fiscal and monetary policies can have important roles in amplifying or mitigating these effects of the robotization process and that implementing specific policies can generate an important efficiency-equity trade-off. Despite the existence of this trade-off, the long-run economic growth is higher with than without the fiscal and monetary policies, which underlines their crucial role in attenuating the negative aspects of Industry 4.0.

Keywords: Robots; Social Capital; Fiscal Policy; Monetary Policy; Growth.

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1 Introduction

A new industrial revolution that emerged in the 21th century, referred to as Industry 4.0, has been generally defined as the digital transformation of traditional manufacturing and industrial production practices through the increasing use of automatation processes and smart tecnologies. As the name Industry 4.0 suggests, the new industrial revolution is the fourth in the series of industrial revolutions since the first one in the 18th century, when the use of water and steam allowed the generation of energy and triggered the technological progress in production. Subsequently, advances in technological knowledge led to the so-called second and third industrial revolutions: with the use of electricity in production in the 19th century and the implementation of mass production systems in the 20th century, the latter being referred to as the "digital revolution" that reformulated production processes. The ongoing revolution – Industry 4.0 – corresponds to the emergence of high-level automation technologies, including cyber-physical systems, internet of things, internet of services and intelligent factory (Hofmann and Rusch 2017) with potential implications so wide that they already bring about major changes in socio-economic structures worldwide. Despite the economic, social, and political concerns arrising in the implementation of Industry 4.0, it is expected to improve the quality of life and enhance economic growth, and change the use of the labor factor. A major challenge in this process relates to the expectation that the extensive and increasing use of robots will destroy the employment of unskilled labor, raising the level of structural unemployment. Another concern in the ongoing production revolution is that it should not – as it might – dig a huge gap in people's access to wealth (e.g., Krueger 1993, Bresnahan and Trajtenberg 1995, Bresnahan et al. 1999, Caselli 1999 and Antonelli 2003).

According to the International Federation of Robotics (IFR),[1] the worldwide use of robots has more than tripled since 2007, mostly in the Asian region and in the automotive and electric / electronic sectors. The IFR also reveals that, according to the latest data (2020 IFR report), Singapore, South Korea, Japan, Germany, and Sweden have the highest robots density in manufacturing, and that it has been increasing in these countries and spreading to others. Given this trend and the challenges associated to the Industry 4.0, it becomes essential to analyze the effects of the robotic revolution on the use of the labor factor, prices of productive inputs, economic growth, and social welfare. Indeed, taking into account the socio-economic consequences of Industry 4.0, in particular the replacement of unskilled labor by robots, it is essential to identify and design institutional mechanisms that allow for achieving good growth rates and the sharing of gains among all interested parties so as to achieve net welfare gains for everyone. Efforts to mitigate the negative effects associated with the structural changes imposed by this creative process are therefore necessary, and this requires the attention of policymakers. The main purpose of this paper is exactly that – to contribute to a better understanding of this issue by investigating the role of fiscal and monetary policies within a general equilibrium structure in which unskilled labor and robots are subject to a specific degree of substitution. While there is certainly also some degree of substitution of skilled labor

[1] https : //ifr.org/downloads/press2018/ExecutiveSummaryWR2018IndustrialRobots.pdf

resulting from the development of artificial intelligence, the evidence shows that robots replace relatively more unskilled labor (Acemoglu and Restrepo 2018, Abbott and Bogenschneider 2018, Salvatore 2019). Thus, as the impact of robots on skilled labor is much less, we can omit this point from our paper.

The impact of replacing human labor by robots has been the subject of a growing number of recent studies (e.g., De Canio 2016, Decker et al. 2017, Acemoglu and Restrepo 2018, Abbott and Bogenschneider 2018, Salvatore 2019). For example, De Canio (2016) finds that for a variety of estimates of the underlying distribution of labor productivity, an elasticity of substitution between robots and human work of around 2.0 leads to wage reductions with the increased introduction of robots.[2]. This finding suggests that in order to mitigate the negative effects of substituting (unskilled) labor, the population should share the returns from robotic work. Another study, by Decker et al. (2017), mentions that although automation initially focused on saving labor costs in standardized manufacturing tasks, it has extended to other less standardized production processes. In addition to the substitutability of manual labor by robots, the same study also addressed the importance of complementarity between robots and human labor. Acemoglu and Restrepo (2018) showed that automation reduced employment and wages in the US between 1990 and 2007; on the other hand, they suggest that the creation of new jobs by automation can also increase wages. Using a dynamic general equilibrium model, Berg et al. (2018) add robots as a substitute for work and a complement to physical capital in production, and show that investment in robots increases long-term growth, but also inequality. Salvatore (2019) noted that industry jobs in advanced economies were lost and that real wages fell short of productivity gains.

The potential gains of Industry 4.0 on the one hand and the increase in the unemployment rate on the other hand, emphasize the role of policies in managing the socioeconomic consequences of the creative destruction processes.With robots substituting human labor we can expect an increase in both wealth and income inequality. In view of these issues, the modeling framework that we propose in this paper emphasizes the role of fiscal policy in the human capital accumulation and in the taxation of the use of robots in production,[3] and the role of monetary policy in paying for human labor.

We develop a dynamic general equilibrium model of a closed economy in which the production function involves, in addition to physical capital, two types of labor: unskilled, which can be replaced by robots, and skilled. Technological knowledge is considered to evolve with both social capital and skilled labor, while social capital evolves dynamically through the government's contribution. Incorporating social capital in the production function has become fundamental for dealing with the potential negative implications of Industry 4.0, such as a further division between the skilled and unskilled groups, and between the employed and unemployed groups. Moreover, departing from the existing literature that investigates the

[2]De Canio (2016) employs 2013 US data; other studies using similar methods but longer time periods report both lower and higher values for the elasticity of substitution between humans and robots (of 1.4, 1.6 and 2.9), as in *e.g.*, Acemoglu and Autor (2012) and Katz and Murphy (1992).

[3] Regarding the introduction of a tax on robots see, for example, Gasteiger and Prettner, 2017; and Abbott and Bogenschneider, 2018.

long-run effects of automation, the proposed model considers that tax revenues include a robot tax on producers, in addition to the usual taxes on labor and capital income, and are then used for accumulating social capital and providing funds, such as universal basic income to those who are either unemployed or remain outside the labor force.

Furthermore, as we intend to focus the analysis on production and its interaction with the monetary sector, the choice of modeling raises the question of whether liquidity is important for production in the real economy and, in fact, the literature provides ample evidence that production requires much of money (Bates et al. 2009, McLean 2011). The literature that introduced demand for money into endogenous growth models essentially considers that money affects the economy because producers need money to make payments, including cash-in-advance (CIA) restrictions – Wang and Yip (1992), Chu and Cozzi (2014), Falato and Sim (2014), Brown and Petersen (2015), Chu et al. (2017). Thus, we analyze the the role of the monetary policy by considering the nominal interest rate as the main policy instrument, consequently affecting the relative magnitude of firms' indebtedness for paying skilled versus unskilled workers and thus affecting the key variables in the model.[4]

We solve the model to achieve a balanced-growth path (BGP) and analyze the effects of robotization on the unskilled-to-skilled-labor ratio, the prices of productive inputs, economic growth, and welfare. We then analyze the effects of fiscal and monetary policies on these key variables and discuss how policies can mitigate the negative implications of Industry 4.0. We confirm that by reducing the unskilled-to-skilled-labor ratio, the robotization process increases the skill premium (and thus wage inequality between skilled and unskilled workers), stimulates economic growth, and improves welfare. Moreover, we show that fiscal and monetary policies can have important roles in amplifying or mitigating these effects of the robotization process, Namely, the rapid increase in wage inequality caused by robotization process can be smoother by a more progressive taxation (achieved by increasing the tax rate on the wages of skilled workers or by decreasing the tax rate on the wages of unskilled workers), or an increase (decrease) in the interest rate when firms are more (less) indebted to pay the wages of skilled workers than the wages of unskilled workers. However, these policies will also imply a negative effect on economic growth and welfare, thereby introducing an important efficiency-equity trade-off in the political decision-making (unlike Berg et al.,

[4] It is known that the framework for the definition and implementation of macroeconomic policies has changed especially since the 1980s. Money supply became the responsibility of independent central banks, and policies were restricted by binding rules: avoid excessive public deficits in the case of fiscal policy and avoid excessive money supply in the case of monetary policy. For example, the effects of excessive public deficits would include an increase in interest rates, leading to possible pressure on central banks to implement a more expansionary monetary policy and consequently to more inflation (De Grauwe 2007), which would, in turn, penalize economic growth (Benhabib and Spiegel 2009). For controlling money supply, two main instruments are available to central banks: the first option is to issue money, however with little effect in the long-run and higher inflation; the second option is to reduce the interest rate, stimulating credit and, therefore, consumption and investment. In most cases, monetary policy materializes in the orientation of the nominal interest rate and, thus, affects the economy.

2018).

We also find that despite the existence of this trade-off, the long-run economic growth is higher with the presence of fiscal and monetary policies than without any policies. In particular, fiscal policy allows for compensating unskilled workers substituted by robots, thus eliminating the negative income effects of unemployment, and for increasing investment in social capital, thus increasing the productivity of skilled workers and stimulating the production of knowledge. In turn, monetary policy contributes to achieving more positive economic results, depending on the firms' relative indebtedness to pay wages of unskilled versus skilled workers.

We structure the remainder of this paper as follows. We present the model in Section 2 and derive the BGP in Section 3. The main results are presented and discussed in Section 4 and conclusions are summarized in Section 5.

2 Theoretical Setup

In this section we present a dynamic general equilibrium model in an economy where robots can be substituted for unskilled labor in the production process. We first describe the productive sector and define the accumulation rules for robots, technology, physical capital, and social capital. We proceed by presenting the theoretical setup of fiscal and monetary policies. We then describe the households' behavior, by presenting and solving their maximization problem.

2.1 Technology and prices

The inputs of the production process consist of two types of capital: physical and social; and two types of labor: skilled and unskilled. It is assumed that the elasticity of substitution between unskilled (L) and robots (R) is constant. Besides the (L, R) component, skilled labor (H) is entered in the the production function as complementary to the physical capital (K).[5] Hence, we consider the following Cobb-Douglass type production function with a CES component and skilled labor-augmenting technological progress:

$$Y(t) = \left[\alpha \cdot R(t)^{\theta} + (1-\alpha) \cdot L(t)^{\theta}\right]^{\frac{\psi_1}{\theta}} \left[\mathcal{H}(t) \cdot H_Y(t) \cdot A(t)\right]^{\psi_2} K^{1-\psi_1-\psi_2} \tag{2.1}$$

where: $\alpha \in [0,1]$ is the CES distribution parameter between L and R; $\psi_1 \in [0,1]$ stands for the income share of the (R, L) component; $\psi_2 \in [0,1]$ is the income share of the ($\mathcal{H}$, H_Y, A) component; $\theta \in [0,1]$ is the substitution parameter, such that $\frac{1}{1-\theta}$ is the elasticity of substitution between L and R; $\mathcal{H}$ is the social capital; A is the technological knowledge. Hence, skilled labor used to produce Y, $H_Y \leq H$, enters the production function in interaction with, or augmented by, both technology, A, and $\mathcal{H}$. The income

[5] See Griliches (1969), which has since been considered standard to the literature.

share of physical capital is then $1-\psi_1-\psi_2$.[6] The inclusion of robots along with social capital, rather than human capital into (2.1) is one of the novelties of this paper. Different from, and beyond, the human capital concept, $\mathcal{H}$ here stands for *inclusive informal institutions that facilitate coordination and support among people* (e.g., Neyapti 2017). As such, $\mathcal{H}$ stands for not only the level of education, but also cultural norms and societal characteristics with regard to networking or collaborative ability. We will further elaborate on the distinction between $\mathcal{H}$ and A below and the reason to distinguish them from human capital in this model.

The maximization problem of Y, $\max \Pi_Y(t)$, requires at each time t:

$$\max_{R,\, L,\, H,\, K} p_Y \cdot Y - w_L \cdot (1+\Upsilon_L \cdot \iota) \cdot L - w_H \cdot (1+\Upsilon_H \cdot \iota) \cdot H_Y - r_R \cdot (1+\Upsilon_R \cdot \iota) \cdot [1-\tau_R] \cdot R - r_K \cdot (1+\Upsilon_K \cdot \iota) \cdot K, \quad (2.2)$$

where: p_Y is the price of the composite good, Y, and for convenience, in the remainder of the paper we will express all prices, including wages and costs, in real terms denominated in units of the composite good; w_L represents the real wage of unskilled workers; w_H is the real wage of skilled workers, which is an efficiency wage since it is the return on the composite term $\mathcal{H}{\cdot}H$, i.e., the wage paid on H with a premium on social capital $\mathcal{H}$; r_R and r_K are the returns on the two types of capital used in the production, R and K; and τ_R is a tax on the use of R. Moreover, following the literature (e.g., Gil and Iglésias 2019), it is considered that wages and the returns on R and K have an operational and a financial component; that is, $w_L \cdot (1+\Upsilon_L \cdot \iota)$,[7] $w_H \cdot (1+\Upsilon_H \cdot \iota)$, $r_R \cdot (1+\Upsilon_R \cdot \iota)$, and $r_K \cdot (1+\Upsilon_K \cdot \iota)$, where: $\Upsilon_L, \Upsilon_H, \Upsilon_R, \Upsilon_K \in [0,1]$ are the shares of, respectively, the wage of unskilled workers, the wage of skilled workers, the return on robots, and the return on physical capital that requires the borrowing of money from households; and ι is the nominal interest rate. Since the final-goods market is competitive, taking partial derivatives, we obtain the first order conditions, giving us the inverse demand functions for unskilled labor, for skilled labor, for robots, and for physical capital:

$$w_L(t) \cdot (1+\Upsilon_L \cdot \iota) = \psi_1 \cdot (1-\alpha) \cdot \frac{Y(t) \cdot L(t)^{\theta-1}}{\alpha \cdot R(t)^\theta + (1-\alpha) \cdot L(t)^\theta}, \quad (2.3)$$

$$w_H(t) \cdot (1+\Upsilon_H \cdot \iota) = \psi_2 \cdot \frac{Y(t)}{H_Y(t)}, \quad (2.4)$$

$$r_R(t) \cdot (1+\Upsilon_R \cdot \iota) = \frac{\psi_1 \cdot \alpha}{1+\tau_R} \cdot \frac{Y(t) \cdot R(t)^{\theta-1}}{\alpha \cdot R(t)^\theta + (1-\alpha) \cdot L(t)^\theta}, \quad (2.5)$$

$$r_K(t) \cdot (1+\Upsilon_K \cdot \iota) = (1-\psi_1-\psi_2) \cdot \frac{Y(t)}{K(t)}. \quad (2.6)$$

Hence, from the labor-demand perspective, bearing in mind (2.3) and (2.4), the ratio of skilled to unskilled worker wages is:

[6] Augmenting unskilled labor with social capital as well does not change the nature of the results reported below.

[7] For example, final-good firms pay w_L for the unskilled labor, L. However, a cash-in-advance (CIA) constraint is introduced in the production by assuming that firms use money, borrowed from households, subject to the nominal interest rate ι, to pay for a fraction $\Upsilon_L \in [0,1]$ of the input. Since firms cannot repay this amount to households until they earn revenue from production, households are effectively providing credit to these firms (e.g., Feenstra 1986). Hence, w_L has the following operational and a financial component $w_L \cdot (1-\Upsilon_L) + w_L \cdot \Upsilon_L \cdot (1+\iota)$ and, thus, the cost function is $w_L \cdot (1+\Upsilon_L \cdot \iota)$.

$$\frac{w_H(t)}{w_L(t)} = \left(\frac{1+\Upsilon_L \cdot \iota}{1+\Upsilon_H \cdot \iota}\right)\left(\frac{\psi_2}{\psi_1\,(1-\alpha)}\right)\left(\frac{L(t)}{H_Y(t)}\right)\left(\frac{\alpha \cdot R(t)^{\theta} + (1-\alpha) \cdot L(t)^{\theta}}{L(t)^{\theta}}\right) \tag{2.7}$$

and, from (2.5) and (2.6), the rental rate of physical capital to robot labor is:

$$\frac{r_K(t)}{r_R(t)} = \left(\frac{1+\Upsilon_R \cdot \iota}{1+\Upsilon_K \cdot \iota}\right)\left(\frac{(1-\psi_1-\psi_2)\,(1+\tau_R)}{\psi_1 \cdot \alpha}\right)\left(\frac{R(t)}{K(t)}\right)\left(\frac{\alpha \cdot R(t)^{\theta} + (1-\alpha) \cdot L(t)^{\theta}}{R(t)^{\theta}}\right). \tag{2.8}$$

2.2 Accumulation rules

The model's dynamics are based on four accumulation rules, pertaining to R, K, $\mathcal{H}$, and A. The dynamics of robot accumulation and physical capital accumulation are denoted by $\dot{R}(t)$ and $\dot{K}(t)$, respectively:

$$\dot{R}(t) \quad = \quad \beta \cdot I(t) - \kappa \cdot R(t), \tag{2.9}$$

$$\dot{K}(t) \quad = \quad (1-\beta) \cdot I(t) - \kappa \cdot K(t), \tag{2.10}$$

where: β and $(1-\beta)$ are the shares of household savings invested in robots and physical capital, respectively; $\kappa \in (0,1)$ is the depreciation rate, which we assume to be constant and the same for K and R; and $I(t) = Y(t) - T(t) - C(t)$ is total investment in robots and physical capital.

Social capital, $\mathcal{H}$, evolves according to the following process:

$$\dot{\mathcal{H}}(t) \quad = \quad \eta \cdot G(t) - \mu \cdot \mathcal{H}(t), \tag{2.11}$$

which indicates that in the absence of public investment, $\mathcal{H}$ deteriorates by the rate of μ. The underlying issue that led us to define $\mathcal{H}$ as (bridging) social capital rather than human capital is that decreasing trust and increasing social unrest, and even criminality, arising from poverty and/or widening welfare gaps between the working and the non-working population reduces H's productivity.[8] Therefore, the

[8] Among the studies that show generosity and behavioral trust reducing effects of unemployment, see, for example, Helliwell (2006), Winkelmann (2009) and O'Higgins and Stimolo (2015).

government invests in $\mathcal{H}$ by the amount of $\eta \cdot G(t)$ to overcome the social costs of the creative destruction process, which in this paper emanates specifically from the robotic revolution.

Finally, the technological innovation process, denoted by $\dot{A(t)}$, follows:

$$\dot{A}(t) = \delta \cdot \mathcal{H}(t) \cdot H_A(t) \cdot A(t)^{\phi} \tag{2.12}$$

where δ is a positive constant and $\phi > 0$ is similar to the productivity effect of the stock of ideas explained by Jones (1995). Equation (2.12) indicates that technological-knowledge level at time t, $A(t)$, "potentially" spills over to the current period by the factor ϕ. How much innovation takes place in each period, however, depends on the interaction between $A(t)^{\phi}$ and the social capital-augmented skilled labor in this sector, $\mathcal{H}(t) \cdot H_A(t)$. In other words, employed skilled labor H_A is augmented by both $\mathcal{H}$ and A^{ϕ} to generate technological-knowledge progress, where A^{ϕ} is the usual technological-knowledge diffusion term or the scientific knowledge stock carried over to the next period, independent of the institutional framework. While $\mathcal{H}$ evolves according to (2.11), it increases H's productivity due to its bridging nature, that declines with prevailing unemployment. In that regard, A and $\mathcal{H}$ are like the potential and kinetic forms of the "innovation energy". To clarify, consider that $\mathcal{H}$ can be quantified as an index that ranges within unit interval, where larger numbers indicate greater social capital. Hence, $\mathcal{H}$ being less than 1 indicates lack of effective networking and innovation-friendly institutions, which limits the potential of skilled labor to innovate by the extent that is implied by knowledge spillovers A^{ϕ}. Innovation by the skilled labor that faces a given knowledge stock is at its potential when $\mathcal{H} = 1$.

Hence, the term $\delta \cdot \mathcal{H}(t) \cdot H_A(t) \cdot A(t)^{\phi}$ stands for productivity increase due to the combination of knowledge spillover and employed social capital-augmented skilled labor. Given this structure, skilled labor is thus augmented by social capital and technological-knowledge in final goods production (2.1), while technological knowledge, in turn, evolves with both skilled labor and social capital employed in the R&D sector. Thus, the aggregate amount of skilled labor devoted to R&D is $H_A(t) = \dot{A}(t) \cdot \frac{1}{\delta \cdot \mathcal{H}(t)} \cdot A(t)^{-\phi} \leq H = H_Y(t) + H_A(t)$. We consider that $H_Y = \vartheta \cdot H$ and $H_A = (1 - \vartheta) \cdot H$, where $\vartheta \in (0, 1)$.

The existing stock of designs spills over (designs are non-rival goods) and, as the existing stock of designs becomes larger, a lower level of skilled labor is required for the invention ("shoulders of giants" effect). Since R&D activities require $\frac{1}{\delta \cdot \mathcal{H}(t)} \cdot A(t)^{-\phi}$ units of labor to invent a new design, and since in equilibrium the wage of skilled labor engaged in R&D has to be equal to the wage of skilled labor engaged in the final-goods sector, the invention cost is $\frac{1}{\delta \cdot \mathcal{H}(t)} \cdot A(t)^{-\phi} \cdot w_H(t)$.

2.3 Fiscal and monetary policies

Fiscal policy

In the framework of the model, the government collects taxes from all the factor returns, such that its tax revenue is

$$T(t) = \tau_L \cdot w_L(t) \cdot L(t) + \tau_H \cdot w_H(t) \cdot H(t) + \tau_R \cdot r_R(t) \cdot R(t) + \tau_K \cdot r_K(t) \cdot K(t), \tag{2.13}$$

where the superscripts on the tax rates, τ, indicate that tax rates are potentially income specific. Tax revenue, T, is used for two purposes: to augment $\mathcal{H}$ and to provide income for those who are either not in the work force or unemployed. In addition to the investment in human capital via education spending, investment in $\mathcal{H}$ also takes the form of legislating and enforcing laws and regulations, such as patent rights that would facilitate value generation, scientific collaboration and technological innovation. Moreover, we consider that the provision of public goods, such as arts health, education, and sports platforms and other recreational activities, contributes to the stock of $\mathcal{H}$, in the form of bridging social capital (e.g., Putnam 2000). The remaining part of T is spent merely as income transfer to those who do not work.[9] Hence, the balanced budget identity of the government, $G(t) = T(t)$, is given by:

$$T(t) = \eta \cdot G(t) - (1 - \eta) \cdot G(t), \tag{2.14}$$

where G is the government spending and η and $(1 - \eta)$ represent the shares of G spent on $\mathcal{H}$ and on income compensation, respectively $(0 \leq \eta \leq 1)$. Denoting N as the population size and u $(0 \leq u < 1)$ as the share of the population consisting of either the unemployed or people outside the labor force, a number uN of individuals receives an amount of fiscal transfers equal to $(1 - \eta) \cdot G$.

Monetary policy

We consider that the monetary authority adopts an inflation-targeting framework and its monetary policy instrument is the nominal interest rate, $\iota(t) = \iota$ (e.g., Bernanke and Mishkin 1997). This allows the monetary authority to affect the macroeconomic variables by operating through the cash-in-advance (CIA) constraints (e.g., Chu and Cozzi 2014; Gil and Iglésias 2019). That is, with a change in the nominal interest rate, the CIA constraints generate forces that transmit different inflation costs to the economy, which distort the incentives and the use of economic resources in the different sectors, as we will show below.

In this framework we follow the literature and assume that the nominal interest rate is exogenously chosen by the monetary authority.[10] Thus, the inflation rate, $\pi(t)$, which corresponds to the growth rate of the nominal price of the composite good, $\pi(t) \equiv \frac{\dot{p_Y}(t)}{p_Y(t)}$, is itself endogenously determined, according to the Fisher equation. In fact, in line with the empirical observations that indicate that the inflation rate is only indirectly controlled through the monetary policy instrument, i.e., the nominal interest rate

[9] Compensation of the unemployed and non-working population may be in the form of universal basic income, the idea for which goes back to Thomas More's Utopia and has been increasingly receiving attention for the general purpose of poverty alleviation following the Great Recession. Recent experimental implementations have been carried out in several countries, such as Canada, Finland, India, and Kenya (e.g., Standing 2008, Marinescu 2018, McGaughey 2018).

[10] In addition to Chu and Cozzi (2014) and Gil and Iglésias (2019), recent papers along these lines are Chu and Ji (2016), Chu et al. (2017), and Chu et al. (2019), among others.

(Bernanke and Mishkin 1997), the Fisher equation reveals that the inflation rate is endogenous in the sense that it depends on the real macroeconomic conditions reflected in the endogenous real interest rate (besides being regulated by the exogenous choice of the nominal interest rate by the monetary authority).

Denoting the nominal money supply by $M(t)$, the real money supply/balances is $m(t) = \frac{M(t)}{p_Y(t)}$ and, in terms of growth rates, it results, respectively, that $\frac{\dot{m}(t)}{m(t)} = \frac{\dot{M}(t)}{M(t)} = -\pi(t)$. Hence, knowing the value of $\pi(t)$ from the Fisher equation, the growth rate of the nominal money supply will be endogenously determined: $\frac{\dot{M}(t)}{M(t)} = \frac{\dot{m}(t)}{m(t)} + \pi(t) = \frac{\dot{m}(t)}{m(t)} + \iota - r(t)$. As usual in the literature, we consider that to balance its budget, the monetary authority returns the seigniorage revenues to households as a lump-sum transfer, i.e., $\tau(t) = \frac{\dot{M}(t)}{p_Y(t)} = \frac{(m(t) \cdot \dot{p_Y(t)})}{p_Y(t)} = \frac{\dot{m}(t) \cdot p_Y(t) + \dot{p_Y}(t) \cdot m(t)}{p_Y(t)} = \dot{m}(t) + \pi(t) \cdot m(t)$.

As we will show later below, the steady-state equilibrium relationships entail a positive correlation between the inflation rate, π, and the nominal interest rate, ι, meaning that we can extend all the comparative-statics results pertaining to shifts in ι also to shifts in the steady-state inflation rate. We could, as an alternative, consider the growth rate of money supply or even the inflation rate as the policy variable directly controlled by the monetary authority. The consideration of the nominal interest rate as the policy instrument, however, simplifies the analytical derivation of the steady-state equilibrium without changing the comparative-statics results.

2.4 Households' problem

The economy is populated by a number of infinitely-lived households. Following Bertinelli et al. (2013) and Neto et al. (2019), there is a representative infinitely-lived household, which, at time $t = 0$, maximizes the discounted intertemporal lifetime utility, which subject to the flow budget constraint, has perfect foresight concerning the macroeconomic variables overtime. The utility of a household depends positively on its consumption and negatively on the labor level supplied, such that:

$$U = \int_0^\infty \left(\frac{C(t)^{1-\xi_1} - 1}{1 - \xi_1} + \frac{[-L(t)]^{1-\xi_2}}{1 - \xi_2} + \frac{[-H(t)]^{1-\xi_2}}{1 - \xi_2} \right) e^{-\rho t} dt, \tag{2.15}$$

where: $\rho > 0$ is the subjective discount rate (whereby U is bounded away from infinity if the consumption of the composite good, $[C(t)]_{t \geq 0}$, is stable over time); $\xi_1 > 0$ is the inverse of the inter-temporal elasticity of substitution; $\xi_2 > 0$ is the inverse of the Frisch elasticity[11]; terms $\frac{[-L(t)]^{\xi_2}}{1-\xi_2}$ and $\frac{[-H(t)]^{1-\xi_2}}{1-\xi_2}$ introduce a disutility from working.

Hence, the representative household solves the following dynamic optimization problem

[11]The Frisch elasticity of labor supply is a useful indicator that gives the percentage increase in labor supply resulting from a 1% increase in the wage rate, while maintaining the marginal utility of wealth constant.

$$\max_{L(t),H(t),R(t),K(t),C(t)} U$$

$$s.t\ \dot{R}(t) = \beta \cdot I(t) - \kappa \cdot R(t); \quad \dot{K}(t) = (1-\beta) \cdot I(t) - \kappa \cdot K(t); \quad b(t) \le m(t) = \Upsilon_L \cdot w_L \cdot L + \Upsilon_H \cdot w_H \cdot H_Y + \Upsilon_R \cdot r_R \cdot R + \Upsilon_K \cdot r_K \cdot K \tag{2.16}$$

given the initial values $K(0)$ and $R(0)$. Hence, the utility maximization problem of the household is subject to the consumer's problem (2.16) such that households consume and collect income from (i) labor supply (wages), (ii) the government, as income compensation, and (iii) investments. Investments spending is allocated between R and K with the respective shares of β and $(1-\beta)$ from where money balances operate.

In order to specify the Hamiltonian, we start by looking at the aggregate consistency condition, which is given by $Y(t) = C(t) + I(t) + G(t)$, where: Y is the aggregate output; C is the aggregate consumption; $I(t) = Y(t) - T(t) - C(t)$ is the aggregate investment that is equal to the aggregate savings, $I = S$; and aggregate public expenditures G are equal to aggregate taxes T – see (2.14). Thus, at each time t, the aggregate investment / savings are given by $S(t) = Y(t) - T(t) - C(t)$ and the household budget constraint is $C(t) = Y(t) - T(t) - S(t)$, with $Y(t) - T(t)$ representing disposable income:

$$Y - T = (1-\tau_L) \cdot w_L \cdot L + (1-\tau_H) \cdot w_H \cdot H + r_R \cdot R + (1-\tau_K) \cdot r_K \cdot K + (1-\eta) \cdot G + \tau - \pi \cdot m - \dot{m} + \iota \cdot b. \tag{2.17}$$

Equation (2.17) shows that the households: (i) pay taxes on all the factor returns except for the return on robots, since, as suggested by the literature, the robot tax is paid by firms;[12] (ii) receive $(1-\eta)$ portion of $G(t)$ as compensation, which can be either in the form of universal basic income to be shared among uN individuals or in the form of improved public services or social platforms; (iii) receive seigniorage revenues as a lump-sum transfer, $\tau = \dot{m} + \pi \cdot m$, from the monetary authority to balance their budget; (iv) lend the amount of money b to finance production in exchange for the return ι; thus, the CIA constraints imply that $b \le m$.[13]

Hence, bearing in mind the equations of accumulation for R and K in (2.9) and (2.10), respectively,

[12] Abott and Bogenschneider (2018), for example, argue that the automation tax on firms can be considered as a way of internalizing the social costs of automation, and to contribute to the unemployment compensation mechanism. Similarly, Obertson (2017) suggests that as firms substitute labor with robots, they are the agents that can be taxed on the use of robots.

[13] In the case of an additional CIA constraint on consumption, the CIA constraint would become $\sigma C + b \le m$, where σ denotes the strength of the CIA on consumption (e.g., Chu and Cozzi, 2014). We abstract from the CIA constraint on consumption or even from a money-in-utility or a liquidity/pecuniary-transaction-costs specification in the household's optimization problem (e.g., Freenstra 1986) since we wish to focus on the worldwide production and technology sectors of the model and their interaction with the monetary sector. See Gil and Iglésias (2019) for a detailed analysis of a model with CIA constraints on R&D and manufacturing combined with either a money-in-utility or a liquidity/pecuniary-transaction-costs specification.

we specify the Hamiltonian and find the solution for the household's problem in Appendix A, summarized by the no-arbitrage condition between real money balances and real financial assets (this amounts to the well-known Fisher equation),

$$\iota = (1 - \tau_K)\, r_K(t) - \kappa + \beta \cdot \pi(t), \tag{2.18}$$

the optimal path of consumption (the household's Euler equation),

$$\frac{\dot{C}(t)}{C(t)}, = \frac{(1 - \tau_K)\, r_K(t) - \rho}{\theta_1}, \tag{2.19}$$

the optimal relative skilled-human-labor supply,

$$\frac{w_H}{w_L} = \left(\frac{L}{H}\right)^{-\xi_2} \left(\frac{1 - \tau_L}{1 - \tau_H}\right), \tag{2.20}$$

and the optimal relative return on R, which must be equal to the after-tax return on K:

$$\frac{r_R}{r_K} = (1 - \tau_K)\,, \tag{2.21}$$

considering in the analysis above that $\beta = 0.5$.

3 Balanced growth path

In this section we examine the dynamic general equilibrium that results from optimal decentralized behavior. We first show that the Balanced Growth Path (BGP) exists and is unique, and proceed by computing and analyzing the steady-state economic growth rate. We then calculate the equilibrium in the factor market, as well as the BGP welfare level.

3.1 Existence and uniqueness

According to Uzawa (1961) and Grossman et.al. (2017), the balanced growth path (BGP) exists if, in equilibrium, each factor income share is uniquely determined and strictly positive. From (2.3), (2.4), (2.5), and (2.6), we find that the income shares of each factor are:

$$\frac{w_L(t) \cdot L(t)}{Y(t)} = \left(\frac{1}{1+\Upsilon_L \cdot \iota}\right) \frac{\psi_1 (1-\alpha)}{\alpha \cdot \left[\frac{R(t)}{L(t)}\right]^{\theta} + (1-\alpha)} \tag{3.1}$$

$$\frac{w_H(t) \cdot H(t)}{Y(t)} = \left(\frac{1}{1+\Upsilon_H \cdot \iota}\right) \frac{\psi_2}{\vartheta} \tag{3.2}$$

$$\frac{(1+\tau_R) \cdot r_R(t) \cdot R(t)}{Y(t)} = \left(\frac{1}{1+\Upsilon_R \cdot \iota}\right) \frac{\psi_1 \cdot \alpha}{\alpha + (1-\alpha) \cdot \left[\frac{L(t)}{R(t)}\right]^{\theta}} \tag{3.3}$$

$$\frac{r_K(t) \cdot K(t)}{Y(t)} = \left(\frac{1}{1+\Upsilon_K \cdot \iota}\right) (1-\psi_1-\psi_2) \tag{3.4}$$

Hence, for the existence and uniqueness of the BGP of the economy, Proposition 1 must hold:

Proposition 1. *The BGP exists and is unique iff (i) the ratio of unskilled labor over robot labor,* $\frac{L}{R}$*, is constant and positive, and (ii)* $\alpha, \psi_1, \psi_2, \vartheta \in (0,1)$ *and* $\psi_1 + \psi_2 \in (0,1)$*.*

Proof. It follows directly from equations (3.1), (3.2), (3.3), and (3.4).

□

Moreover, the unique BGP is defined as stated in Proposition 2:

Proposition 2. *The BGP is defined as:* $g^* = g_Y = g_R = g_L = g_{\mathcal{H}} = g_K = g_C = g_G$*.*

Proof. Since from Proposition 1 the ratio $\frac{L}{R}$ must be constant for the existence of BGP, we have $g_L = g_R$. Moreover, given that along the BGP the growth rates of physical capital, robot labor, and social capital are constant, and the fraction of household savings invested in robots and physical capital β, is also constant, from (2.9), (2.10), and (2.11), the set of equations:

$$\frac{\dot{R}(t)}{R(t)} = \beta \cdot \left[\frac{Y(t)}{R(t)} - (1-\eta)\frac{G(t)}{R(t)} - \frac{C(t)}{R(t)}\right] - \kappa \tag{3.5}$$

$$\frac{\dot{K}(t)}{K(t)} = (1-\beta) \cdot \left[\frac{Y(t)}{K(t)} - (1-\eta)\frac{G(t)}{K(t)} - \frac{C(t)}{K(t)}\right] - \kappa \tag{3.6}$$

$$\frac{\dot{\mathcal{H}}(t)}{\mathcal{H}(t)} = \eta \cdot \frac{G(t)}{\mathcal{H}(t)} - \mu \tag{3.7}$$

leads to $g^* = g_Y = g_R = g_L = g_{\mathcal{H}} = g_K = g_C = g_G$, considering that κ and μ are constant.

□

3.2 Economic growth rate

We next obtain the explicit solution for g^* in per capita terms. Noting that $L + H + uN = N$, and denoting $\frac{L}{N}$ by l, and $\frac{H}{N}$ by h, we can write $l + h = 1 - u$. In addition, from Proposition 2 we know that $g_R = g_L$ must hold for the existence of the BGP. Using the above relationships, we then solve for the per capita BGP growth rate $g_y = g_Y - n$, where $y = \frac{Y}{N}$.

Proposition 3. *The per capita BGP growth rate of the economy is given by*

$$g_y = \frac{n\,(3-\phi)}{\frac{l}{h}\,(2-\phi) - 1} = \frac{n\,(3-\phi)}{\frac{L}{H}\,(2-\phi) - 1}, \tag{3.8}$$

where the constant population growth rate, n, is positive.

Proof. Since the population growth rate is n ($g_N = n$), the BGP in *per capita* terms is defined as: $g_y = g_Y - n = g_r = g_R - n = g_l = g_L - n = g_{\mathcal{H},pc} = g_{\mathcal{H}} - n = g_k = g_K - n = g_c = g_C - n = g_{g,pc} = g_G - n$, where: $y = \frac{Y}{N}$, $r = \frac{R}{N}$, $l = \frac{L}{N}$, $\mathcal{H}_{pc} = \frac{\mathcal{H}}{N}$, $k = \frac{K}{N}$, $c = \frac{C}{N}$, and $G_{pc} = \frac{G}{N}$.

From (2.1), the natural logarithm form of production function is:

$$\ln Y(t) = \frac{\psi_1}{\theta} \ln\left[\alpha \cdot R(t)^\theta + (1-\alpha) \cdot L(t)^\theta\right] + \psi_2 \ln \mathcal{H}(t) + \psi_2 \ln H_Y(t) + \psi_2 \ln A(t) + (1 - \psi_1 - \psi_2) \ln K(t). \tag{3.9}$$

By expressing the growth rate of $x = Y, R, L, \mathcal{H}, H, A, K$ as the time derivative of the natural logarithm, $g_x = \frac{d}{dt}(\ln x(t)) = \frac{\dot{x}(t)}{x(t)}$, from (3.9) we obtain:

$$g_Y = \psi_1 \frac{\alpha \cdot g_R \cdot R(t)^\theta + (1-\alpha) \cdot g_L \cdot L(t)^\theta}{\alpha \cdot R(t)^\theta + (1-\alpha) \cdot L(t)^\theta} + \psi_2\,(g_{\mathcal{H}} + g_{H_Y} + g_A) + (1 - \psi_1 - \psi_2)\,g_K, \tag{3.10}$$

which, given that $g_L = g_R = g_Y$ and $g_{H_Y} = g_H$, leads to:

$$g_Y = \psi_1 g_Y + \psi_2 \left(g_{\mathcal{H}} + g_H + g_A\right) + \left(1 - \psi_1 - \psi_2\right) g_Y. \tag{3.11}$$

Moreover, from (2.12) technological-knowledge progress grows at the rate $g_A = \frac{\dot{A}}{A} = \delta \cdot \mathcal{H} \cdot H_A \cdot A^{\phi - 1}$, which, since g_A is constant along the BGP and $g_H = g_{H_Y}$, implies that:

$$g_{\mathcal{H}} + g_H = (1 - \phi)\, g_A \Leftrightarrow g_A = \frac{g_{\mathcal{H}} + g_H}{1 - \phi} \tag{3.12}$$

Given that g_A is undefined for $\phi = 1$ and $\phi > 1$ is strongly rejected by evidence (e.g., Neves and Sequeira, 2018), the feasible range for ϕ is $[0, 1)$, which guarantees that technological-knowledge progress grows at a positive rate.

Taking now the time derivatives of the natural logarithm of the expression $N = L + H + uN$ yields:

$$g_H = \frac{n\,(1-u)}{h} - g_L \frac{l}{h} \tag{3.13}$$

Plugging (3.12) and (3.13) into (3.11), and replacing g_L and g_H by g_Y, we obtain, after some algebra, $g_Y = \frac{(1-u)(\phi-2)n}{l(\phi-2)+h}$. Dividing the denominator and numerator by h and using the expression $l + h = 1 - u$ yields $g_Y = \frac{\left(1+\frac{l}{h}\right)(\phi-2)n}{\frac{l}{h}(\phi-2)+1} = \frac{\left(1+\frac{l}{h}\right)(2-\phi)n}{\frac{l}{h}(2-\phi)-1}$. Finally, expressing the growth rate into *per capita* terms, $g_y = g_Y - n$, we obtain the BGP expression for the baseline model in *per capita* terms given in (3.8).

From (3.8), the following corollary emerges: □

Corollary 4. *Given $n > 0$ and $0 \leq \phi < 1$, g_y : (i) is positive if $\frac{L}{H} > \frac{1}{2-\phi}$; (ii) decreases in $\frac{L}{H}$; (iii) is positively associated with n and ϕ.*

Proof. Given that $n > 0$ and $0 \leq \phi < 1$:

(i) the numerator of equation (3.8) is positive. Then, a positive g_y requires that the inequality $\frac{L}{H}(2-\phi) - 1 > 0$ holds. Solving the inequality for $\frac{L}{H}$, we obtain $\frac{L}{H} > \frac{1}{2-\phi}$, which implies that the minimum value of $\frac{L}{H}$ approaches to $\frac{1}{2}$ as ϕ approaches to 0, and to 1 and ϕ approaches to 1.

(ii) the derivative $\frac{\partial g_y}{\partial \frac{L}{H}} = -\frac{(2-\phi)(3-\phi)}{\left[\frac{L}{H}(2-\phi)-1\right]^2} < 0$ and this also implies that $\frac{\partial g_y}{\partial L} < 0$ and $\frac{\partial g_y}{\partial H} > 0$.

(iii) the derivative $\frac{\partial g_y}{\partial n} = \frac{(3-\phi)}{\frac{L}{H}(2-\phi)-1} > 0$ when the denominator is positive, which holds for the positive values of g_y, i.e., for $\frac{L}{H} > \frac{1}{2-\phi}$; moreover, the derivative $\frac{\partial g_y}{\partial \phi} = \frac{n\left(1+\frac{L}{H}\right)}{\left[\frac{L}{H}(2-\phi)-1\right]^2} > 0$.

Thus, as in the case of the semi-endogenous growth models (e.g., Jones, 1995), the existence of sustained positive economic growth is only possible if there is population growth. Also in line with Jones (1995), the magnitude of the knowledge spillovers, ϕ, influences positively the steady-state growth rate.

Furthermore, the steady-state growth rate depends crucially on the ratio between unskilled and skilled workers, $\frac{L}{H}$. Provided that $\frac{L}{H} > \frac{1}{2-\phi}$, the lower the relative number of unskilled workers, the higher the growth rate. The reason for this is simple: lower levels of $\frac{L}{H}$ mean that skilled labor is relatively more abundant; and given that only skilled labor is used in the production of knowledge, more abundant skilled labor favors technological progress and, consequently, stimulates economic growth.

□

3.3 Factor market

We can now proceed with the analysis of the factor market in BGP, bearing in mind from Proposition 2 that $g_R = g_L = g_K$. By considering together the demand or firms' side (2.8) and the supply or households' side (2.21), we obtain

$$\frac{K}{R} = \left(\frac{1+\Upsilon_R \cdot \iota}{1+\Upsilon_K \cdot \iota}\right) \frac{(1-\psi_1-\psi_2)(1+\tau_R)(1-\tau_K)}{\psi_1 \cdot \alpha} \left[\alpha + (1-\alpha)\left(\frac{L}{R}\right)^{\theta}\right], \tag{3.14}$$

which solved for L as a function of K and R is:

$$\frac{L}{R} = \left\{\frac{\alpha}{1-\alpha}\left[\left(\frac{1+\Upsilon_K \cdot \iota}{1+\Upsilon_R \cdot \iota}\right) \frac{\psi_1}{(1-\psi_1-\psi_2)(1+\tau_R)(1-\tau_K)} \frac{K}{R} - 1\right]\right\}^{\frac{1}{\theta}}, \tag{3.15}$$

i.e., the ratio $\frac{L}{R}$ is positively related to the ratio $\frac{K}{R}$, indicating that increasing robot use substitutes away both unskilled labor and physical capital.

In turn, by substituting (2.7) from the demand side into (2.20) from the supply side, we have:

$$\frac{L}{H} = \left[\left(\frac{1}{\vartheta}\right)\left(\frac{1-\tau_H}{1-\tau_L}\right)\left(\frac{1+\Upsilon_L \cdot \iota}{1+\Upsilon_H \cdot \iota}\right)\left(\frac{\psi_2}{\psi_1(1-\alpha)}\right)\left(\alpha \cdot \left(\frac{R}{L}\right)^{\theta} + (1-\alpha)\right)\right]^{\frac{1}{-\xi_2-1}}. \tag{3.16}$$

Having determined the ratios in terms of endowments, $\frac{K}{R}$, $\frac{L}{R}$, and $\frac{L}{H}$, and since the relative return of R against K is given by, for example, (2.21), it is now important to determine the skill premium (the ratio between the wage of skilled and unskilled workers). For this purpose, using, for example, equation (2.20) and equation (3.16) we obtain:

$$\frac{w_H}{w_L} = \left[\left(\frac{1}{\vartheta}\right)\left(\frac{1+\Upsilon_L \cdot \iota}{1+\Upsilon_H \cdot \iota}\right)\left(\frac{\psi_2}{\psi_1(1-\alpha)}\right)\left(\alpha \cdot \left(\frac{R(t)}{L(t)}\right)^{\theta} + (1-\alpha)\right)\right]^{\frac{-\xi_2}{-\xi_2-1}} \left(\frac{1-\tau_H}{1-\tau_L}\right)^{\frac{-\xi_2}{-\xi_2-1}}. \tag{3.17}$$

Moreover, since $g_Y = g_C$ from Proposition 2, $g_Y = \frac{(1+\frac{L}{H})(2-\phi)n}{\frac{L}{H}(2-\phi)-1}$ from the proof of Proposition , and $g_C = \frac{(1-\tau_K)r_K(t)-\rho}{\theta_1}$ from the Euler equation (2.19) or $g_C = \frac{r_R(t)-\rho}{\theta_1}$ bearing also in mind (2.21), we find that the returns on physical capital and robots are:

$$r_K = \frac{1}{1-\tau_K}\left\{\theta_1 \frac{\left(1+\frac{L}{H}\right)(2-\phi)\,n}{\frac{L}{H}(2-\phi)-1} + \rho\right\}\ and\ r_R = \theta_1 \frac{\left(1+\frac{L}{H}\right)(2-\phi)\,n}{\frac{L}{H}(2-\phi)-1} + \rho. \tag{3.18}$$

3.4 Welfare analysis

We can also compute the welfare measure, Z, along the BGP from the lifetime utility function – see Appendix B:

$$Z_S = \frac{1}{1-\xi_1}\left\{\frac{C(0)^{1-\xi_1}}{[\rho-(1-\xi_1)\,g_Y]} - \frac{1}{\rho}\right\} - \frac{L(0)^{1-\xi_2}+H(0)^{1-\xi_2}}{(1-\xi_2)\,[\rho-(1-\xi_2)\,n]}. \tag{3.19}$$

3.5 With and without policies

In the previous case we found that a strictly positive BGP is possible under fairly broad conditions, specifically in case of positive population growth rate and when $\frac{L}{H} > \frac{1}{2-\phi}$, which implies that the lower bound for the unskilled to skilled labor ratio has to be $\frac{1}{2}$. We now turn to the solution of BGP for the above model in case fiscal and monetary policies are inactive.

4 Effects of robotization and fiscal and monetary policies

Based on the expressions obtained in Section 3, we now analyze the effects of the robotization process and of fiscal and monetary policies on prices of productive inputs, economic growth, and welfare in steady-state. Given that along the BGP all these variables depend crucially on the unskilled-to-skilled-labor ratio, we first analyze the effects of robotization and policies on $\frac{L}{H}$. The results are summarized in the following propositions.

4.1 Effects of robotization

Proposition 5. *The robotization process affects: (i) negatively the unskilled-to-skilled labor ratio;* $\frac{L}{H}$ *(ii) positively the skill premium,*w_H/w_L*, the returns on capital and robots,* r_K *and* r_R*, the long-run economic growth rate,* g_y*, and welfare,* Z_S*.*

Proof. Taking the partial derivatives of (3.16), (3.17), (3.18), (3.8), and (3.19) with respect to R, and considering that $0 \leq \phi < 1$ and $L/H > \frac{1}{2-\phi}$, we have:

$$\frac{\partial L/H}{\partial R} = \frac{L}{H} \cdot \left(\alpha \left(\frac{R}{L}\right)^{\theta} + (1-\alpha)\right)^{-1} \cdot \frac{\alpha\theta}{-\xi_2 - 1} \cdot \left(\frac{R}{L}\right)^{\theta} \cdot R^{-1} < 0;$$
$$\frac{\partial w_H/w_L}{\partial R} = \frac{w_H}{w_L} \cdot \frac{-\xi_2}{-\xi_2-1} \cdot \left(\alpha \left(\frac{R}{L}\right)^{\theta} + (1-\alpha)\right)^{-1} \cdot \frac{\alpha\theta}{R} \cdot \left(\frac{R}{L}\right)^{\theta} > 0;$$
$$\frac{\partial r_K}{\partial L/H} = \frac{\theta_1}{1-\tau_K} \cdot \frac{n(2-\phi)}{L/H \cdot (2-\phi)-1} \cdot \frac{(\phi-2)}{L/H \cdot (2-\phi)-1} < 0 \text{ and } \frac{\partial L/H}{\partial R} < 0;$$
$$\frac{\partial r_R}{\partial L/H} = \frac{\theta_1 n(2-\phi)}{L/H \cdot (2-\phi)-1} \cdot \frac{(\phi-2)}{L/H \cdot (2-\phi)-1} < 0 \text{ and } \frac{\partial L/H}{\partial R} < 0;$$
$$\frac{\partial g_y}{\partial L/H} = -n \cdot \frac{(3-\phi)\cdot(2-\phi)}{(L/H \cdot (2-\phi)-1)^2} < 0;$$
$$\frac{\partial Z_S}{\partial g_y} = C(0)^{1-\xi_1} \cdot \left(\rho - (1-\xi_1) \cdot (g_y + n)\right)^{-2} > 0 \text{ and } \frac{\partial g_y}{\partial L/H} < 0.$$ □

Given that robots replace unskilled labor, an immediate consequence of the process of robotization in the economies is the decline in the ratio $\frac{L}{H}$. This happens because as more and more robots are being used in the productive process, firms reduce their demand for L, which results in a decline in the number of employed unskilled workers. The contraction in the demand for L also reduces the unskilled wages, leading to an increase in the skill premium. Therefore, another consequence of robots is the increase of wage inequality, namely the widening of the wage gap between skilled and unskilled workers.

The robotization process also has important growth effects. The decline in the $\frac{L}{H}$ makes skilled labor (relatively) more abundant, which favors innovation and enhances economic growth, g_y. Moreover, given that r_K, r_R, and Z_S are directly determined by g_y, robotization also contributes to increasing the returns on capital and robots, as well as total welfare in steady-state.

4.2 Fiscal and monetary policy effects

Proposition 6. *The BGP unskilled-to-skilled-labor ratio, L/H, depends:* i) *positively (negatively) on fiscal policy targeting skilled (unskilled) labour, τ_H (τ_L); ii) positively (negatively) on monetary policy, ι, if paying the wage of unskilled workers requires firms borrowing less (more) from households than paying the wages of skilled workers,* i.e., *$\gamma_L < \gamma_H$ ($\gamma_L > \gamma_H$).*

Proof. $\frac{\partial L/H}{\partial \tau_H} = -\frac{L}{H} \cdot \frac{1}{-\xi_2-1} \cdot (1-\tau_H)^{-1} > 0$; $\frac{\partial L/H}{\partial \tau_L} = \frac{L}{H} \cdot \frac{1}{-\xi_2-1} \cdot (1-\tau_L)^{-1} < 0$; and $\frac{\partial L/H}{\partial i} = \frac{L}{H} \cdot \frac{1}{-\xi_2-1} \cdot \left(\frac{\gamma_L}{1+\gamma_L \cdot i} - \frac{\gamma_H}{1+\gamma_H \cdot i}\right)^{-1} < 0$ if $\gamma_L > \gamma_H$ and > 0 if $\gamma_L < \gamma_H$.

□

The ratio $\frac{L}{H}$ in steady-state is crucially determined by the design of fiscal policy. In particular, an increase in the tax rate on the wages of unskilled workers, τ_L, lowers $\frac{L}{H}$, since higher taxes lower the net income of unskilled workers, which reduces unskilled labor supply. Conversely, an increase in the tax rate on the wages of skilled workers, τ_H, raises the $\frac{L}{H}$ ratio.

As for the monetary policy, ι, changes in interest rates can have different effects on $\frac{L}{H}$ depending on the shares of skilled and unskilled wages that require borrowing from households. More specifically, if the monetary authority raises the interest rate in order to lower inflation, this will increase the ratio $\frac{L}{H}$, if $\gamma_H > \gamma_L$, *i.e.*, if paying the wages of skilled workers requires borrowing more than paying the wages of unskilled workers. In this case, higher interest rates will lead to an increase in the financial costs associated with the remuneration of skilled labor, this increase being more pronounced than the increase in the financial costs associated with the remuneration of unskilled labor. Put differently, the costs of employing skilled labor will increase more than the costs of employing unskilled labor, which will lead to a reduction in the relative demand for skilled labor and thereby to an increase in the ratio $\frac{L}{H}$. The opposite occurs if $\gamma_L > \gamma_H$: given that paying unskilled labor requires borrowing more than paying skilled labor, firms will be less willing to employ unskilled workers in the face of an increase in the interest rates, which will lead to a reduction in the ratio $\frac{L}{H}$.

Thus, fiscal and monetary policies can either attenuate or amplify the effects of the robotization process on $\frac{L}{H}$. For example, the decline in $\frac{L}{H}$ brought up by robotization can be attenuated by a more progressive taxation (reflected in an increase in τ_H) or by a restrictive monetary policy when paying the wages of skilled workers requires borrowing more than paying those of unskilled workers. This can be of particular importance when there is high probability of rapidly increasing unemployment of unskilled workers due to the robotization process. Conversely, the decline in $\frac{L}{H}$ can be amplified if fiscal policy increases τ_L or if the monetary authority raises the interest rate when paying the wages of unskilled workers requires borrowing more than paying those of skilled workers. Policy intervention in this direction can be useful, for example, when the productive structure of the economy is compatible with a more intensive use of skilled labor.

Proposition 7. *The BGP skill-premium, $\frac{w_H}{w_L}$, depends:* i) *positively (negatively) on fiscal policy targeting unskilled (skilled) labor, τ_L (τ_H); ii) positively (negatively) on monetary policy if paying the wage of unskilled workers requires firms borrowing more (less) from households than paying the wages of skilled workers,* i.e., *$\gamma_L > \gamma_H$ ($\gamma_L < \gamma_H$).*

Proof. $\frac{\partial w_H/w_L}{\partial \tau_H} = -\frac{w_H}{w_L} \cdot \frac{-\xi_2}{-\xi_2-1} \cdot (1-\tau_H)^{-1} < 0$;

$\frac{\partial w_H/w_L}{\partial \tau_L} = -\frac{w_H}{w_L} \cdot \left(-\frac{-\xi_2}{-\xi_2-1}\right) \cdot (1-\tau_H)^{-1} > 0$;

$\frac{\partial w_H/w_L}{\partial i} = \frac{w_H}{w_L} \cdot \frac{-\xi_2}{-\xi_2-1} \cdot \left(\frac{\gamma_L}{1+\gamma_L \cdot i} - \frac{\gamma_H}{1+\gamma_H \cdot i}\right) > 0$ if $\gamma_L > \gamma_H$ and < 0 if $\gamma_L < \gamma_H$.

□

The skill premium, w_H/w_L, depends crucially on the direction of fiscal policy. An increase in the tax rate on the wage of skilled workers, τ_H, reduces their net income, thereby reducing the skill premium.

The opposite effect occurs when the fiscal authority raises the tax rate on the wage of unskilled workers, τ_L.

The skill premium is also influenced by monetary policy through the relative demand of unskilled labor. As mentioned in Proposition 6, an increase in the interest rate, ι, raises the ratio $\frac{L}{H}$ when $\gamma_H > \gamma_L$ and reduces it when $\gamma_L > \gamma_H$. In the first case, the skill premium will fall, while in the second case it will rise.

Thus, policy-makers can use fiscal and monetary policies to mitigate the increase in wage inequality caused by robotization processes. They can, for example, adopt a more progressive taxation (by raising τ_H or reducing τ_L). They can also raise interest rates when the payment of wages of skilled workers requires borrowing more than the payment of wages of unskilled workers, or reduce the interest rates in the contrary case.

Proposition 8. *The BGP returns on capital and robots, r_K* and *r_R, depend:* i) *positively (negatively) on fiscal policy targeting unskilled (skilled) labor, τ_L (τ_H); ii) positively (negatively) on monetary policy if paying the wage of unskilled workers requires firms borrowing more (less) from households than paying the wages of skilled workers,* i.e.*, $\gamma_L > \gamma_H$ ($\gamma_L < \gamma_H$). In addition, the BGP return on capital, r_K, depends positively on fiscal policy targeting the use of capital, τ_K.*

Proof. From Proposition 5 $\frac{\partial r_K}{\partial L/H} < 0$, $\frac{\partial r_R}{\partial L/H} < 0$, and from Proposition 6 $\frac{\partial L/H}{\partial \tau_H} > 0$, $\frac{\partial L/H}{\partial \tau_L} < 0$, $\frac{\partial L/H}{\partial i} < 0$ if $\gamma_L > \gamma_H$ and > 0 if $\gamma_L < \gamma_H$.

$\frac{\partial r_K}{\partial \tau_K} = r_K \cdot (1 - \tau_K)^{-1} > 0$.

□

Fiscal and monetary policies have both a direct and an indirect effect on the returns on capital and robots, r_K and r_R. The direct effect corresponds to the impact of τ_K on r_K. Namely, keeping in mind that in equilibrium the net return on robots, r_R, must equal the after-tax return on capital, $(1 - \tau_K) \cdot r_K$, then in order to keep this equality a higher tax on τ_K must lead to an increase in r_K. The indirect effects correspond to the impact that τ_L, τ_H, and ι have on r_K and r_R, via $\frac{L}{H}$. Given that for the empirically relevant values of ϕ, both factor returns depend negatively on the ratio $\frac{L}{H}$, policies that increase $\frac{L}{H}$ tend to reduce r_K and r_R.

Proposition 9. *The BGP per capita long-run growth rate, g_y, and the welfare measure, Z_S, depend:* i) *positively (negatively) on fiscal policy targeting unskilled (skilled) labor, τ_L (τ_H); ii) positively (negatively) on monetary policy if paying the wage of unskilled workers requires firms borrowing more (less) from households than paying the wages of skilled workers,* i.e.*, $\gamma_L > \gamma_H$ ($\gamma_L < \gamma_H$).*

Proof. From Proposition 5 $\frac{g_y}{\partial L/H} < 0$, $\frac{\partial Z_S}{\partial L/H} < 0$, and from Proposition 6 $\frac{\partial L/H}{\partial \tau_H} > 0$, $\frac{\partial L/H}{\partial \tau_L} < 0$, $\frac{\partial L/H}{\partial i} < 0$ if $\gamma_L > \gamma_H$ and > 0 if $\gamma_L < \gamma_H$.

□

Both fiscal and monetary policies affect the steady-state growth rate and welfare,[14] through the influence they exert on $\frac{L}{H}$. Since the growth rate is inversely related to $\frac{L}{H}$, policies that reduce $\frac{L}{H}$ will have positive effects on economic growth. From the previous analysis, these policies include a less progressive taxation (by raising τ_L or reducing τ_H), a more restrictive monetary policy when the payment of wages of unskilled workers requires borrowing more than the payment of wages of skilled workers, or an expansionary monetary policy in the contrary case. However, all these policies are opposite to the policies that contribute to reducing wage inequality, *i.e.*, the skill premium. For policy-makers there is thus a clear trade-off between efficiency and equity: if their main target is to mitigate the increase in wage inequality brought up by the robotization process, the adopted policies will have a negative effect on growth; on the contrary, if their main target is to stimulate growth and increase welfare, the adopted policies will aggravate the existing wage inequality.

The effects of policies on long-run economic growth and welfare can also be analyzed by comparing the steady-state growth rate presented in (3.8) with the growth rate that would be obtained if there were no policies.

Proposition 10. *In the absence of fiscal or monetary policies, i.e. when* $G = T = \Upsilon_L = \Upsilon_H = \Upsilon_R = \Upsilon_K = 0$*, the per capita BGP growth rate of the economy is given by:*

$$g_y^{new} = \frac{n - u(2-\phi)}{1 - \phi + \frac{L}{H}(2-\phi)}. \tag{4.1}$$

Proof. Consider the growth accounting formula in (3.10), where we now have $g_{\mathcal{H}} = -u$ due to (2.11), under the assumption that $G = T = 0$. Using $g_H + g_{\mathcal{H}} = g_H - u$ from the proof of Proposition 3, we write $g_A = \frac{g_H - u}{1-\phi}$. Combining this expression with the growth accounting equation ((3.11)), we have $g_Y = \frac{2-\phi}{1-\phi}(g_H - u)$. Rewriting $g_H = \frac{n(1-u)}{h} - g_L \frac{l}{h}$ from the proof of Proposition 3 we obtain, using $H + L + u = N$, in *per capita* terms $g_h = \frac{n(1-u)}{h} - (g_L - n)\frac{l}{h} - n$. Substituting this expression in the formula for $g_Y - n = g_y$, we get (4.1).

□

[14] Welfare depends directly on the economy's growth rate, so the analysis of the policies' effects on growth also applies to welfare.

From Proposition 10, two Corollaries emerge.

Corollary 11. *The new BGP of the economy, given in (4.1), implies that now long-run growth: (i) is positive if* $u < \frac{n}{2-\phi}$*; (ii) decreases with the unemployment rate, unlike the benchmark case; (iii) is also positively related to* n *and* ϕ*, and negatively related to* $\frac{L}{H}$*.*

Proof. Given that $n > 0$ and $0 \leq \phi < 1$:

(i) the denominator of equation (4.1) is positive. Then, a positive g_y requires that the inequality $n - u(2-\phi) > 0 \Longleftrightarrow u < \frac{n}{2-\phi}$ holds.

(ii) $\frac{\partial g_y^{new}}{\partial u} = -\frac{(2-\phi)}{1-\phi+\frac{L}{H}(2-\phi)} < 0$.

(iii) $\frac{\partial g_y^{new}}{\partial n} = \frac{1}{1-\phi+\frac{L}{H}(2-\phi)} > 0$, $\frac{\partial g_y^{new}}{\partial \phi} = \frac{u(1-\phi+\frac{L}{H}(2-\phi))+(1+\frac{L}{H})(n-u(2-\phi))}{(1-\phi+\frac{L}{H}(2-\phi))^2} > 0$, $\frac{\partial g_y^{new}}{\partial \frac{L}{H}} = -\frac{(2-\phi)(n-u(2-\phi))}{(1-\phi+\frac{L}{H}(2-\phi))^2} > 0$, for $u < \frac{n}{2-\phi}$.

□

In the absence of fiscal or monetary policy, the long-run growth would decline in the unemployment rate. Therefore, the expected increase in unemployment generated by the replacement of unskilled labor by robots would harm long-term growth. The reason for this is that without fiscal policy, the government would not provide any compensation to the unemployed. As a result, the purchasing power of the unemployed labor force would be significantly reduced, which would harm aggregate consumption, production, and growth. This result contrasts with the benchmark case, in which fiscal policy is actively used to redistribute tax revenues to compensate the unemployed. In this situation, the government can absorb the effects of the replacement of unskilled labor by robots, and therefore the steady-state growth does not depend on the unemployment rate.

In addition, given that the usual rate of n is smaller than 2% in developed economies, in the absence of policies u would have to be very small for a positive g_y^{new}. In particular, the unemployment rate would have to be at most half of the population growth rate, *i.e.*, $u < 1\%$. In view of the global increase in unemployment rates due to structural reasons, as well as due to the effects of the COVID19 pandemic, which will probably persist for an unpredictable period of time, the absence of policies would result in negative steady-state growth rates. The results obtained are thus consistent with the emphasis on the role of fiscal and monetary policy to achieve positive growth.

Comparing both BGPs, with and without policies, we can state:

Corollary 12. $g_y > g_y^{new}$ *holds for all the parameter values for which positive growth is obtained in both scenarios.*

Proof. Given that $0 \leq \phi < 1$, we have that $2 - \phi > 0$ and $3 - \phi > 1$; as a result, the numerator of (4.1) is lower than the numerator of (3.8). Moreover, with $1 - \phi > 0$ and $L/H > \frac{1}{2-\phi}$, the denominator of (4.1) is greater than than the denominator of (3.8). Therefore, $g_y > g_y^{new}$.

□

Generally, in the context of increasing robotization, the long-run economic growth is higher with than without the presence of fiscal and monetary polices. In particular, monetary policy can contribute positively to economic growth, as changes in the nominal interest rates can provide better incentives for firms' borrowing to pay wages for a specific type of labor. As for fiscal policy, it is growth-enhancing for two reasons. First, as mentioned above, it prevents the purchasing power of the unskilled workers substituted by robots from falling abruptly, thereby eliminating the negative income effects of unemployment. Second, it contributes to increase investments in social capital, which is an important determinant of skilled-labor productivity and technological innovation, which, in turn, enhance economic growth.

5 Concluding remarks

In this paper we have developed a dynamic general equilibrium model in which robots can replace unskilled labor. We then investigate the implications of the redistributive role of fiscal policy and the action of monetary policy. The government collects taxes on all production inputs, including the tax on the use of robots, and uses these revenues to redistribute to the non-working population and to invest in social capital. By incorporating social capital into the model as a factor that increases the productivity of skilled labor and contributes to technological-knowledge progress, social capital can also be considered to be a type of essential public good.

Solving the model for the balanced-growth path we show that the long-run growth rate increases with the population growth rate and with the magnitude of spillovers in the knowledge production function, as in the semi-endogenous growth models. Our results also indicate that the long-run growth rate decreases with the ratio between the unskilled and skilled workers. Given that this ratio is negatively affected by the increasing use of robots, robotization processes increase the long-run economic growth rate. We also find that robotization increases the wage inequality between skilled and unskilled workers (the skill-premium) and the returns on capital and robots.

Our results also indicate that fiscal and monetary policies can have important roles in amplifying or mitigating the effects of the robotization. In particular, increasing the tax rate on the wages of skilled workers or decreasing the tax rate on the wages of unskilled workers can smooth the rapid increase in wage inequality caused by robotization processes. A similar effect can be obtained by implementing specific monetary policy: when firms are more indebted to pay the wages of skilled workers than the wages of unskilled workers, increasing the interest rate can contain the increase in the skill-premium; decreasing the interest rate can yield the same effect in the opposite situation. However, the implementation of policies in this direction may hamper economic growth and reduce welfare, which represents an important efficiency-equity trade-off in the political decision-making.

Finally, based on the model results we conclude that despite the existence of this trade-off, the presence of fiscal and monetary policy induces a higher long-run economic growth rate than the growth rate that would be obtained without any policies. This happens firstly because when unskilled workers are substituted by robots, redistribution through fiscal policy allows eliminating the negative income effects of unemployment. Secondly, taxes allow the government to increase investment in social capital, which increases the skilled workers' productivity and stimulates the production of knowledge. Thirdly, depending on the relative firms' indebtedness to pay wages of unskilled versus skilled workers, changes in the nominal interest rates can also contribute to achieving more positive economic outcomes. These results are particularly important as they emphasize the crucial role that policies may have in attenuating the negative impacts and strengthening the positive features of Industry 4.0.

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Appendix A

From the standard Optimal Control Theory, we consider the auxiliary Hamiltonian function, assuming the (static) CIA constraint is binding as in the baseline model; i.e., $b(t) = m_I(t)$,

$$
\begin{gathered}
H = \left[\frac{C^{1-\xi_1}-1}{1-\xi_1} + \frac{[-L]^{1-\xi_2}}{1-\xi_2} + \frac{[-H]^{1-\xi_3}}{1-\xi_3}\right] e^{-\rho t} + \lambda_1 \left[b - m\right] + \\
+\lambda_2 \left\{ \beta \underbrace{\left[(1-\tau_L)\, w_L L + (1-\tau_H)\, w_H H + r_R R + (1-\tau_K)\, r_K K + (1-\eta)\, G + \tau - \pi \cdot m + \iota \cdot b - C\right]}_{=\dot{R}+\dot{m}} - \kappa R \right\} + \\
+\lambda_3 \left\{ (1-\beta) \underbrace{\left[(1-\tau_L)\, w_L L + (1-\tau_H)\, w_H H + r_R R + (1-\tau_K)\, r_K K + (1-\eta)\, G + \tau - \pi \cdot m + \iota \cdot b - C\right]}_{=\dot{K}+\dot{m}} - \kappa K \right\}
\end{gathered}
$$

where: R, K, and m are the state variables; λ_1, λ_2,and λ_3 are the costate variables; C, L, H, and b are the control variables. Then, the necessary conditions under the Maximum Principle are:

$$\frac{\partial Ham}{\partial C(t)} = 0 \Leftrightarrow C(t)^{-\xi_1} e^{-\rho t} - \lambda_2(t) \cdot \beta - \lambda_3(t) \cdot (1-\beta) = 0 \tag{5.1}$$

$$\frac{\partial Ham}{\partial b(t)} = 0 \Leftrightarrow \lambda_1(t) + \lambda_2(t) \cdot \beta \cdot \iota + \lambda_3(t) \cdot (1-\beta) \cdot \iota = 0 \tag{5.2}$$

$$\frac{\partial Ham}{\partial L(t)} = 0 \Leftrightarrow -\left(-L(t)\right)^{-\xi_2} \cdot e^{-\rho t} + \lambda_2(t) \cdot \beta \cdot (1-\tau_L) \cdot w_L(t) + \lambda_3(t) \cdot (1-\beta) \cdot (1-\tau_L) \cdot w_L(t) = 0 \tag{5.3}$$

$$\frac{\partial Ham}{\partial H(t)} = 0 \Leftrightarrow -\left(-H(t)\right)^{-\xi_2} \cdot e^{-\rho t} + \lambda_2(t) \cdot \beta \cdot (1-\tau_L) \cdot w_L(t) + \lambda_3(t) \cdot (1-\beta) \cdot (1-\tau_L) \cdot w_L(t) = 0 \tag{5.4}$$

$$\frac{\partial Ham}{\partial R(t)} = -\dot{\lambda_2}(t) \Leftrightarrow \lambda_2 \cdot \beta \cdot r_R(t) - \lambda_2 \cdot \kappa + \lambda_3 \cdot (1-\beta) \cdot r_R(t) = \dot{\lambda_2}(t) \tag{5.5}$$

$$\frac{\partial Ham}{\partial K(t)} = -\dot{\lambda_3}(t) \Leftrightarrow \lambda_2 \cdot \beta \cdot (1-\tau_K) \cdot r_K + \lambda_3 \cdot (1-\beta) \cdot (1-\tau_K) \cdot r_K - \lambda_3 \cdot \kappa = \dot{\lambda_3}(t) \tag{5.6}$$

$$\frac{\partial Ham}{\partial m(t)} = -\dot{\lambda_2}(t) \Leftrightarrow -\lambda_1(t) - \lambda_2(t) \cdot \beta \cdot \pi(t) = -\dot{\lambda_2}(t) \tag{5.7}$$

$$\frac{\partial Ham}{\partial m(t)} = -\dot{\lambda_3}(t) \Leftrightarrow -\lambda_1(t) - \lambda_3(t) \cdot (1-\beta) \cdot \pi(t) = -\dot{\lambda_3}(t) \tag{5.8}$$

$$\frac{\partial Ham}{\partial \lambda_1(t)} = 0 \tag{5.9}$$

$$\frac{\partial Ham}{\partial \lambda_2(t)} = \dot{R}(t) + \dot{m}(t) \tag{5.10}$$

$$\frac{\partial Ham}{\partial \lambda_3(t)} = \dot{K}(t) + \dot{m}(t) \tag{5.11}$$

$$\lim_{t\to+\infty} \lambda_2(t) \cdot R(t) = 0; \ \lim_{t\to+\infty} \lambda_2(t) \cdot K(t) = 0; \ \lim_{t\to+\infty} \lambda_2(t) \cdot m(t) = 0; \tag{5.12}$$

$$\lim_{t\to+\infty} \lambda_3(t) \cdot R(t) = 0; \ \lim_{t\to+\infty} \lambda_2(t) \cdot K(t) = 0; \ \lim_{t\to+\infty} \lambda_3(t) \cdot m(t) = 0 \tag{5.13}$$

In solving the problem, we consider that $\lambda_2 = \lambda_3$. First, by considering together (5.2), (5.7), and (5.8), we have that $\frac{\dot{\lambda_2}(t)}{\lambda_2(t)} = \beta \cdot \pi(t) - \iota$ and $\frac{\dot{\lambda_2}(t)}{\lambda_2(t)} = (1-\beta) \cdot \pi(t) - \iota$, which implies that $\beta = 0.5$. In turn, by considering (5.5) and (5.6), we have that $\dot{\lambda_2} = -\lambda_2 \cdot \kappa + \lambda_2 \cdot r_R$ and $\dot{\lambda_2} = -\lambda_2 \cdot \kappa + \lambda_2 \cdot (1-\tau_K) \cdot r_K$, which implies that $r_R = (1-\tau_K) \cdot r_K$. Hence, by considering together $\frac{\dot{\lambda_2}(t)}{\lambda_2(t)} = \beta \cdot \pi(t) - \iota$ or $\frac{\dot{\lambda_2}(t)}{\lambda_2(t)} = (1-\beta) \cdot \pi(t) - \iota$ and $\dot{\lambda_2} = -\lambda_2 \cdot \kappa + \lambda_2 \cdot r_R$ or $\dot{\lambda_2} = -\lambda_2 \cdot \kappa + \lambda_2 \cdot (1-\tau_K) \cdot r_K$, we obtain the Fisher equation:

$$\iota = (1-\tau_K)\, r_K(t) - \kappa + \beta \cdot \pi(t). \tag{5.14}$$

Now, by rearranging (5.1), we have that $C(t)^{-\xi_1} e^{-\rho t} = \lambda_2(t)$, which implies that $\frac{\dot{\lambda_2}(t)}{\lambda_2(t)} = -\xi_1 \frac{\dot{C}(t)}{C(t)} - \rho$. Then, bearing in mind for example (5.6), we find the Euler equation for consumption (2.19):

$$\frac{\dot{C}(t)}{C(t)} = \frac{(1-\tau_K)\, r_K(t) - \rho}{\xi_1}.$$

From (5.1) and (5.3), we obtain $w_L(1-\tau_L) = \frac{C^{\xi_1}}{(-L)^{-\xi_2}}$ and from (5.1) and (5.4) we obtain $w_H(1-\tau_H) = \frac{C^{-\xi}}{(-H)^{-\xi_3}}$. Thus, in the solution of the household's problem, the wage ratio of the skilled versus unskilled workers can be written as:

$$\frac{w_H}{w_L} = \left(\frac{L}{H}\right)^{-\xi_2} \left(\frac{1-\tau_L}{1-\tau_H}\right). \tag{5.15}$$

In turn, by combining (5.5) and (5.6) we reach the following relationship:

$$\frac{r_R}{r_K} = (1-\tau_K). \tag{5.16}$$

Appendix B

Bearing in mind the utility function $U = \int_0^\infty \left(\frac{C(t)^{1-\xi_1}-1}{1-\xi_1} + \frac{[-L(t)]^{1-\xi_2}}{1-\xi_2} + \frac{[-H(t)]^{1-\xi_2}}{1-\xi_2}\right) e^{-\rho t}dt$ and assuming that $C(0)$, $L(0)$, and $H(0)$ are known, we now determine the expression for the welfare along the BGP. The welfare utility is $Z = \frac{1}{1-\xi_1}\int_0^\infty \left\{[C(0)e^{g_Y t}]^{(1-\xi_1)} - 1\right\} e^{-\rho t}dt + \frac{1}{1-\xi_2}\int_0^\infty \left\{[-L(0)e^{g_L t}]^{(1-\xi_2)}\right\} e^{-\rho t}dt + \frac{1}{1-\xi_2}\int_0^\infty \left\{[-H(0)e^{g_H t}]^{(1-\xi_2)}\right\} e^{-\rho t}dt$, since along the BGP $C(t) = C(0)e^{g_Y t}$, $L(t) = L(0)e^{g_L t}$, and $H(t) = H(0)e^{g_H t}$. To put it simply, and considering that $g_L = g_H = n$, it comes: $Z = \frac{C(0)^{1-\xi_1}}{1-\xi_1}\int_0^\infty e^{[(1-\xi_1)g_Y-\rho]t}dt - \frac{1}{1-\xi_1}\int_0^\infty e^{-\rho t}dt - \frac{L(0)^{1-\xi_2}}{1-\xi_2}\int_0^\infty e^{[(1-\xi_2)n-\rho]t}dt - \frac{H(0)^{1-\xi_2}}{1-\xi_2}\int_0^\infty e^{[(1-\xi_2)n-\rho]t}dt$. Thus, we need to calculate:

$$\begin{aligned} Z = &\underbrace{\frac{C(0)^{1-\xi_1}}{(1-\xi_1)[(1-\xi_1)g_Y-\rho]}\left[\lim_{t\to\infty} e^{[(1-\xi_1)g_Y-\rho]t}-1\right]}_{=\mathcal{A}} + \underbrace{\frac{1}{(1-\xi_1)\rho}\left[\lim_{t\to\infty} e^{-\rho t}-1\right]}_{=\mathcal{B}} + \\ &- \underbrace{\frac{L(0)^{1-\xi_2}}{(1-\xi_2)[(1-\xi_2)n-\rho]}\left[\lim_{t\to\infty} e^{[(1-\xi_1)n-\rho]t}-1\right]_0^\infty}_{=\mathcal{C}} - \underbrace{\frac{H(0)^{1-\xi_2}}{(1-\xi_2)[(1-\xi_2)n-\rho]}\left[\lim_{t\to\infty} e^{[(1-\xi_2)n-\rho]t}-1\right]_0^\infty}_{=\mathcal{D}}. \end{aligned} \tag{5.17}$$

In (5.17), $[(1-\xi_1)g_Y-\rho]$, can be less than, equal to, or greater than zero, resulting, respectively, in $\mathcal{A} = \frac{C(0)^{1-\xi_1}}{(1-\xi_1)[\rho-(1-\xi_1)g_Y]}$, $\mathcal{A} = 0$, or $\mathcal{A}$ divergent. In turn, $-\rho$ is less than zero, resulting in $\mathcal{B} = \frac{1}{(\xi_1-1)\rho}$. Moreover, $[(1-\xi_2)n-\rho]$ can be less than, equal to, or greater than zero, resulting, respectively, in: $\mathcal{C} = \frac{L(0)^{1-\xi_2}}{(1-\xi_2)[\rho-(1-\xi_2)n]}$, $\mathcal{C} = 0$, or $\mathcal{C}$ divergent; $\mathcal{D} = \frac{H(0)^{1-\xi_2}}{(1-\xi_2)[\rho-(1-\xi_2)n]}$, $\mathcal{D} = 0$, or $\mathcal{D}$ divergent. Supposing, as expected from the data, that $[(1-\xi_1)g_Y-\rho] < 0$ and $[(1-\xi_2)n-\rho < 0]$, we have the infinite-horizon welfare:

$$Z_S = \frac{1}{1-\xi_1}\left\{\frac{C(0)^{1-\xi_1}}{[\rho-(1-\xi_1)g_Y]} - \frac{1}{\rho}\right\} - \frac{L(0)^{1-\xi_2}+H(0)^{1-\xi_2}}{(1-\xi_2)[\rho-(1-\xi_2)n]}.$$