cef.up
Centro de Economia e Finanças da UPorto

cef.up working paper 2021-02

AUTOMATION, EDUCATION, AND POPULATION: DYNAMIC EFFECTS IN AN OLG GROWTH AND FERTILITY MODEL

Catarina Peralta
Pedro Mazeda Gil

Centro de Economia e Finanças da UP
www.fep.up.pt

Automation, Education, and Population: Dynamic Effects in an OLG Growth and Fertility Model*

Catarina Peralta[†]& Pedro Mazeda Gil[‡]

July 2021

Abstract

We address two main structural changes occurring in developed countries: the rise of automation and population ageing. We use an R&D-based growth model in an OLG framework with endogenous education and fertility, and automation in the production process. Our model is able to combine the growth of real wages over time and either a fall or an increase in birth rates, consistent with recent data regarding the birth rate by skill group. Moreover, our model allows for the study of the interplay between the effects of population ageing and those of automation. The results show a dynamics consistent with the US trends for the period covering 1970 to 2019.

Keywords: Ageing, Automation, Economic growth, Endogenous fertility

JEL Codes: J11, J23, J24, O3, O4

*This research has been financed by Portuguese public funds through FCT - Fundação para a Ciência e a Tecnologia, I.P., in the framework of the project UIDB/04105/2020 (CEF.UP - Center for Economics and Finance at University of Porto). We also acknowledge the financial support of FCT and ESF - European Social Fund, throught individual Ph.D. scholarship SFRH/BD/144630/2019.

†Faculdade de Economia, Universidade do Porto. Corresponding author: up201600179@edu.fep.up.pt

‡Faculdade de Economia, Universidade do Porto, and CEF.UP.

1 Introduction

Developed countries have been facing two main structural changes: the rise of automation and the ongoing process of population ageing. Specifically, in the United States, the stock of operation robots used in industries has increased 425% from 1993 to 2019 (International Federation of Robotics, 2020). At the same time, the old-age dependency ratio has increased 34% during the same period (World Bank, 2020a). Moreover, according to Nerlich & Schroth (2018), population ageing is expected to intensify drastically over the following years. Simultaneously, global automation adoption shows no signs of slowing down, with recent events, such as the current pandemic, supporting the incentives for modernization and digitalization of production on the way to recovery. It is widely known that population ageing induces many challenges at the macroeconomic level, through changes in, namely, the savings rate (e.g. Hansen, 1939; Gehringer & Prettner, 2019), human capital accumulation (e.g. Cervellati & Sunde, 2005), labor supply (e.g. Maestas et al., 2016) and innovation processes (e.g. B. F. Jones, 2010). Additionally, automation has been receiving much atention, particulary, regarding its impact on wage inequality and whether this new wave of technology will make labor redudant (e.g. Brynjolfsson & McAfee, 2014; David, 2015; Acemoglu & Restrepo, 2018a,b,c; Prettner, 2019; Prettner & Strulik, 2020).

To address the issues brought by automation and population ageing, we follow an OLG framework with endogenous education and fertility decisions and with automation that allows one to study the interplay between the effects of population ageing and those of automation. In particular, we build an R&D-based model extended to include automation in the production function, where robots are substitutes to low-skilled workers but complementary to high-skilled workers, as in Prettner & Strulik (2020), and a demographic structure, by introducing fertility choice (Baldanzi, Bucci, & Prettner, 2019) and a survival probability from young to old age (Baldanzi, Prettner, & Tscheuschner, 2019) in the household sector of the economy.

Only a few studies have analyzed the implications of automation and demographic changes combined. In an attempt to explain the positive correlation between GDP growth and population ageing, Acemoglu & Restrepo (2017, 2021) show that countries experiencing a significant aged population tend to adopt more automation in their production process, resulting in a productivity boost which counteracts the effect of an aged labor force. Following their contribution, a new wave of research has arisen in the literature regarding this issue (Abeliansky & Prettner, 2017; Abeliansky et al., 2020; Acemoglu & Restrepo, 2017, 2021; Irmen, 2020; Leitner & Stehrer, 2019; Basso & Jimeno, 2021; Zhang et al., 2021; Stähler, 2021). Some of these studies show that demographic change can affect robot adoption since the incentives to become more technological dependent increase. For instance, as population growth decreases, the incentives to automate increase as the labor force relative supply decreases. Additionally, as the population ages, the labor force also becomes older, implying a reduction in productivity due to deskilling effects, thus increasing the incentives to substitute labor for new and more productive inputs. These studies focus on the effects of demography on automation; however, in a recent contribution, Prettner & Bloom (2020) refer a possible reverse effect in which automation could also affect the demographic structure of an economy.

From these contributions, a few relate closely to ours: Irmen (2020), Acemoglu & Restrepo (2021), Basso & Jimeno (2021), Zhang et al. (2021) and Stähler (2021). All these papers explore automation and population aging; however, they do not include endogenous fertility nor heterogeneous birth rates across skill groups. In particular, Irmen (2020) presents an extensive production side where competitive firms perform tasks to produce output, and tasks require labor and machines as inputs. Zhang et al. (2021) contribution relies on an overlapping generations model that allows for labor market frictions and skill heterogeneity. Acemoglu & Restrepo (2021) include heterogenous labor regarding the workers age, instead of their skill. Nevertheless, Basso & Jimeno (2021) have a more sophisticated production and demographic side; namely, the authors include a tractable life-cycle that allows them to investigate the implications of changes in the delay of the retirement age. Likewise, Stähler (2021) analyzes how this issue affects inequality, namely, regarding labor income, wealth, and consumption.

The challenges brought by an ageing population and automation adoption and the scarce literature on the interplay between these two topics motivate the study and the need to understand this relationship to a fuller extent. Therefore, we propose to complement the existing literature and analyze whether both drivers of population ageing, a low birth rate and high life expectancy have the same sort of impact on automation adoption.[1] Additionally, we study the implications of automation on demographic dynamics.

We solve our model numerically and use the US data for the calibration. We identify several empirical moments for the US for 1970 and beyond and match the corresponding variable's level and trend. Our results show production and technological, and demographic dynamics consistent with the US trends from 1970 to 2019. In particular, we introduce different types of child-raising transfers that are crucial to combine the growth of real wages over time and either a fall or an increase in birth rates in the model. Such dynamics is consistent with the different behavior exhibited in the data by the birth rates of low- and high-skilled households – a (clear) decrease in the former and a (slight) increase in the latter, from 2006 to 2017 (the period with available data for these variables).[2] To the best of our knowledge, ours is the first model (i) to combine the growth of real wages over time and the heterogeneous behavior of the birth rate across skill groups and (ii) to display birth rates converging towards a constant level as wages (possibly) grow unbounded, in a framework of both endogenous growth and endogenous fertility.

The endogenous structure of the birth rates allows our model to feature a demographic response as automation impacts differently high- and low skilled-wages and, hence, the households' fertility choices. At the same time, the model allows for an automation response, as changes in the demographic side shift the households' education decisions and, thereby, the

[1]The substantial decline in the birth rate and the increase in life expectancy have been the main drivers to explain this issue in developed countries. Note that because migration only accounts for 3.4% of the total world population, its impact on the age structure of large countries tends to be dwarfed by fertility and mortality dynamics (Prettner & Bloom, 2020, Chapter 6, pp. 167-8).

[2]At the same time, the US data shows a clear decline in the total birth rate for the full period from 1970 onwards.

firms' incentives to automate. We show that a technological one-off shock that accelerates the stock of robots, obtained in the model by increasing R&D labor efficiency, impacts low- and high-skilled wages differently. Consequently, the demographic side of our model is also affected, specifically, the declining trend of the low-skilled birth rate slows down as a direct reaction to the shock and then becomes more accentuated. At the same time, the (slightly) increasing trend of the high-skilled birth rate intensifies. Overall, due to the shock, the total birth rate accentuates the negative trend vis-à-vis the scenario of no shock and, thereby, population ageing accelerates.

Furthermore, we analyze the possible different effects of the drivers of ageing on automation through the effects of a positive and negative one-off shock, respectively, to the survival probability (implying a longer expected life span) and the preferences for having children (inducing lower birth rates). Our model shows that, in the short and medium run, an increase in the old-age dependency ratio, either by any of the mentioned shocks, has a positive effect on the dynamics of the stock of robots. Nevertheless, the long-run effect of population ageing on robot adoption might differ regarding the primary driver of ageing: lower birth rates or higher lifespan. More specifically, our results suggest that countries with lower birth rates as the primary driver of ageing are expected to have less robot intensity, whereas those with higher lifespans face more robot adoption.

Finally, to address the advancements of more sophisticated ways of automation, such as artificial intelligence, we elaborate a short analysis where it is assumed both low- and high-skilled workers are substitutes to automation in production (i.e., a "full-labor automation" scenario). The results show that the pace of automation is slower than in the baseline model, since, in the absense of complementarity between robots and high-skilled labor (with the latter potentially growing given households' fertility and education choices), the incentives to automation are less intense. Moreover, there is also a reversed effect on the demographic side of the economy. Given the lack of impact of automation on the wages of both low- and high-skilled workers in this context, the birth rates are roughly constant, yielding a less severe ageing problem.

This paper is organized as follows. In Section 2, we present and solve the model and analyze the key transmission mechanisms underlying our framework. Then, Section 3 presents the calibration of the model and provides the results of our numerical simulation, namely, the model's dynamics and shock responses. Section 4 shows the results of a scenario where automation can replace both types of labor in production activities. Finally, our conclusions are shown in Section 5.

2 Model

To explain the challenges brought by automation and demographic issues, we consider an analytical framework based on Baldanzi, Bucci, & Prettner (2019); Baldanzi, Prettner, & Tscheuschner (2019) and Prettner & Strulik (2020). We consider an overlapping generations model with two life periods. Individuals enter the economy as young adults and face three main decisions, regarding their consumption, education and number of children. The interplay between the choice of education and that of fertility will be crucial for the results of our model, as will be shown in Section 3.

As households decide to pursue a college degree, they lose a fraction of time available to supply labor. They save for retirement, which occurs in the second period of their life, and gain utility from consumption and the number of children. To introduce mortality in our framework, we consider that households have a certain probability of dying at the beginning of the second period of life. Then, at the end of this period, they die with certainty. The size of working population is L_t and its growth depends on households' decision regarding the number of children.

Furthermore, automation enters this economy via the production side, where both labor and robots are used as inputs. Since individuals differ in terms of ability levels, not all of them pursue higher education. Hence, our structure deals with both low- and high-skilled labor, which are affected by automation in different ways. As usual in the literature on automation, we consider that only low-skilled labor can be automated; thus, only this type of workers can be substituted by robots.

2.1 Households

In this economy, individuals obtain utility from consumption in both life periods, the number of children they have, and disutility from the effort taken obtaining a college degree. In period t, the utility function for an individual of type $j = H, L$ (i.e., high- versus low-skilled) is given by[3]

$$u_{j,t} = \log(c_{1,j,t}) + \beta\phi\log(R_{t+1}s_{j,t}) + \epsilon\log(n_{j,t}) - \mathbb{1}_{[j=H]}v(a) \tag{1}$$

where $c_{1,j,t}$ is the first period consumption of the generation born at time t, R_{t+1} is gross rental rate of capital, $s_{j,t}$ denotes savings such that $c_{2,j,t} = R_{t+1}s_{j,t}$ refers to consumption in the second period of life, $n_{j,t}$ is the number of children the household decides to have, β is the discount factor, ϕ represents the probability of surviving to the next period and ϵ denotes the utility weight of children. The component $v(a)$ represents the disutility brought by the effort of pursuing higher education where the indicator function $\mathbb{1}_{[j=H]}$ can be translated in one or zero if the individuals decide to invest in a college degree or not, respectively.[4] Note that usually, when modelling fertility and education decisions, the household is the one choosing the child's education level. In our case, we follow an alternative approach so that the individual chooses his/her own education level by deciding whether to pursue higher education or not.

The budget constraint faced by the individual is the following

$$(1 - \tau n_{j,t} - \eta_j)w_{j,t} + \mu_j n_{t,j} = c_{1,j,t} + s_{j,t}, \tag{2}$$

where we include not only the opportunity (or time) cost related with raising a child, which is given by $\tau n_{j,t}w_{j,t}$, with $w_{j,t}$ denoting real wage, but also an exogenous (net) contribution μ_j that can be interpreted as a family allowance transferred by the government to encourage childbearing[5], where $\mu_L > \mu_H$;

[3] As usual in this literature, we assume the household consists of a single individual.

[4] The details on function $v(a)$ will be provided in Section 2.1.2

[5] In order to simplify the analysis, we do not explicitly consider a government sector in the model, but we adopt the usual (underlying) assumption that the government balances its budget every period by levying the necessary amount of lump-sum taxes.

however, if $\mu_j < 0$, households support a direct net cost to childbearing. Finally, η_j represents the time cost of pursuing higher education.

2.1.1 Consumption and Fertility decisions

With a view to maximizing (1) given (2), we obtain the following first order conditions:

$$c_{1,j,t} = \frac{w_{j,t}(1-\eta_j)}{1+\beta\phi+\epsilon}, \tag{3}$$

$$s_{j,t} = \frac{\beta\phi w_{j,t}(1-\eta_j)}{1+\beta\phi+\epsilon} \tag{4}$$

$$n_{j,t} = \frac{\epsilon w_{j,t}(1-\eta_j)}{(1+\beta\phi+\epsilon)(\tau w_{t,j}-\mu_j)}. \tag{5}$$

The different types of child-raising transfers included in the model, μ_j and τ, are crucial to obtain an endogenous birth rate as shown in equation (5), that is, to have $n_{j,t}$ dependent on an endogenous variable (the wage, $w_{j,t}$). This is different from the typical results in the literature on fertility choice, where a semi-endogenous result obtains (that is, where $n_{j,t}$ depends only on a vector of structural parameters of the model; e.g., (Galor & Weil, 2000; Strulik et al., 2013; Baldanzi, Bucci, & Prettner, 2019)). Furthermore, depending on the sign of μ_j, we are able to have different impacts of wages on the birth rates, as will be further explained below.

To fully understand our framework, we elaborate, in this section, a partial equilibrium analysis that allows us to emphasize the transmission mechanism exhibited in the model. To that end, we consider, for the time being, that wages are given and analyze how fertility, consumption and savings react to a given shift in both wages and key parameters that characterize the household's optimization problem. We further assume that $\tau w_{j,t} - \mu_j > 0$, since it seems natural that child-raising expenses are not fully covered by government allowences.[6]

[6] This condition is verified under the calibration of the model carried out in Section 3.

Proposition 2.1. *Let* $\tau w_{j,t} - \mu_j > 0$.

i) When the wage, $w_{j,t}$*, increases, the birth rate,* $n_{j,t}$*, decreases if* $\mu_j > 0$ *and increases if* $\mu_j < 0$*. In any case, as* $w_{j,t} \to \infty$*,* $n_{j,t}$ *converges to a constant. Furthermore, an increase in* $w_{j,t}$ *increases consumption,* $c_{1,j,t}$*, and savings,* $s_{j,t}$*.*

ii) When the cost of raising a child, τ*, and the survival probability,* ϕ*, increase,* $n_{j,t}$ *decreases. However,* $n_{j,t}$ *increases if the (net) contribution,* μ_j*, and the households' preferences for a higher number of children,* ϵ*, increase.*

Proof. i) Taking the derivative of (5), (3) and (4) with respect to $w_{j,t}$ we have

$$\begin{aligned}
\frac{\partial n_{j,t}}{\partial w_{j,t}} &= -\frac{\epsilon(1-\eta_j)\mu_j}{(1+\beta\phi+\epsilon)(\tau w_{j,t}-\mu_j)^2} \\
\frac{\partial c_{1,j,t}}{\partial w_{j,t}} &= \frac{(1-\eta_j)}{1+\beta\phi+\epsilon} > 0, \\
\frac{\partial s_{j,t}}{\partial w_{j,t}} &= \frac{\beta\phi(1-\eta_j)}{1+\beta\phi+\epsilon} > 0.
\end{aligned}$$

The sign of $\frac{\partial n_{j,t}}{\partial w_{j,t}}$ depends on the sign of μ_j.

ii) Taking the derivative of (5) with respect to τ, ϕ, μ and ϵ we have

$$\begin{aligned}
\frac{\partial n_{j,t}}{\partial \tau} &= -\frac{\epsilon w_{t,j}^2(1-\eta_j)}{(1+\beta\phi+\epsilon)(\tau w_{j,t}-\mu_j)^2} < 0, \\
\frac{\partial n_{j,t}}{\partial \phi} &= -\frac{\beta\epsilon w_{t,j}(1-\eta_j)}{(1+\beta\phi+\epsilon)^2(\tau w_{t,j}-\mu_j)} < 0, \\
\frac{\partial n_{j,t}}{\partial \mu_j} &= \frac{\epsilon w_{t,j}(1-\eta_j)}{(1+\beta\phi+\epsilon)(\tau w_{j,t}-\mu_j)^2} > 0, \\
\frac{\partial n_{j,t}}{\partial \epsilon} &= \frac{(1+\beta\phi)w_{t,j}(1-\eta_j)}{(1+\beta\phi+\epsilon)^2(\tau w_{j,t}-\mu_j)} > 0.
\end{aligned}$$

□

We notice that, for a given wage, a higher survival probability, ϕ, by reducing the effective intertemporal discount rate and thus incentivising savings, implies a decrease in the birth rate, $n_{j,t}$ (and also in present consumption, $c_{1,j}$). However, an increase in the preferences for having children, ϵ, incentivises, for a given wage, a higher birth rate, at the expense of lower present and future consumption. This result is consistent with the literature covering demographic effects on economic growth (e.g., Barro & Becker, 1989; Prettner, 2013; Baldanzi, Bucci, & Prettner, 2019; Baldanzi, Prettner, & Tscheuschner, 2019)

Furthermore, the fact that the sign of $\frac{\partial n_{j,t}}{\partial w_{j,t}}$ depends on the sign of μ_j will play a relevent role in the calibration of the model in order to address the dynamics of the birth rate in the data (see Section 3, below). We recall that μ_j can be interpreted as a family allowence, if $\mu_j > 0$, or a net cost to childbearing, if $\mu_j < 0$. Then note that, when $\mu_j < 0$, $w_{j,t}$ affects positively $n_{j,t}$ since, in this case, the household faces a net cost per child and, hence, when the wage increases, it alleviates the impact of the net cost in the budget constraint and also offsets the opportunity cost of raising a child, which grows in proportion with the wage level. However, in the opposite scenario of $\mu_j > 0$, there exists an allowance per child that is independent of the wage level. Then, an increase in the wage level implies an increase in the opportunity cost of raising a child, which, in this case, offsets the relief brought by the wage in the budget constraint. This mechanism is particularly interesting in order to attain different results regarding the relationship between wages and the birth rate of low- and high-skilled individuals, by considering different signs for μ_L and μ_H, respectively.

To the best of our knowledge, ours is the first model that compatibilizes growth of real wages over time and either a fall or an increase in birth rates, with the latter converging towards a constant level as the wage grows unbounded, in a framework of both endogenous growth and endogenous fertility.

2.1.2 Education decision

The education decision in our model depends on the effort associated with tertiary education, $v(a)$, and which works as a desutility factor in the individual's utility function (1). To this end, we follow the framework of Prettner & Strulik (2020) and consider that desutility is negatively related to the individual's level of ability, that is $v(a) = \theta \log \big(\psi/(a - a_{min})\big)$, with $a \geq a_{min}$, where a is the ability of an individual, a_{min} the minimum ability required to pursue a college degree, ψ and θ are parameters used for calibration. Note that ψ must be sufficiently large, that is $\psi > a - a_{min}$, so that $v(a) > 0$; otherwise, $v(a) < 0$, which means individuals would obtain direct utility from pursuing higher education. To reach the realistic environment in which there is always some portion of workers without higher education, it is defined that $v(a) = \infty$ for $a < a_{min}$.

Nevertheless, for those with enough ability, there is also a threshold, $\bar{a}$, above which an individual decides to invest in education. The indifference condition affecting the decision of an individual to invest or not invest in education is given by $u_{H,t} = u_{L,t}$, which is satisfied for an individual with ability $\bar{a}$. Replacing (3), (4), (5) and considering $\eta_H = \eta$ and $\eta_L = 0$ for simplification, we obtain

$$v(\bar{a}) = (1 + \beta\phi + \epsilon) \log \left[\frac{w_{H,t}(1-\eta)}{w_{L,t}}\right] + \epsilon \log \left[\frac{\tau w_{L,t} - \mu_L}{\tau w_{H,t} - \mu_H}\right]. \tag{6}$$

We then use function $v(a)$ to get the ability threshold

$$\bar{a}_t = \psi \left[\left(\frac{w_{H,t}(1-\eta)}{w_{L,t}}\right)^{-\frac{1+\beta\phi+\epsilon}{\theta}} \cdot \left(\frac{\tau w_{L,t} - \mu_L}{\tau w_{H,t} - \mu_H}\right)^{-\frac{\epsilon}{\theta}}\right] + a_{min}. \tag{7}$$

Comparing with the threshold in Prettner & Strulik (2020), here $\bar{a}$ depends on demographic factors controlled by ϕ, μ_j, τ and ϵ. In particular, there is a new term (the second term inside the square brackets) that confronts net cost per child for individuals of type L and that for individuals of type H, i.e., the (time) cost related with raising a child minus the family (net) allowance transferred by the government to encourage childbearing, $\tau w_j - \mu_j$. This term arises because the fertility-related cost competes for resources in the individual's resource constraint with consumption and education activities. In turn, the first term inside the square brackets confronts the direct net benefits from education for individuals of type H versus type L.

For $a > \bar{a}$ individuals choose to obtain higher education, that is, to become high-skilled, whereas for levels of $a < \bar{a}$, they remain low-skilled. We then use a cumulative distribution function of ability, denoted by $F(a)$, to define the fraction of high- and low-skilled labor in the economy, so that

$$L_{H,t} = (1 - F(\bar{a}_t))L_t \tag{8}$$

and

$$L_{L,t} = F(\bar{a}_t)L_t, \tag{9}$$

respectively.

Moreover, to complete the ability threshold analysis, we repeat the partial-equilibrium exercise presented in Section 2.1.1. We assume that $\tau w_{j,t} - \mu_j > 0$ (as in Proposition 2.1).

Proposition 2.2. *Let* $\tau w_{j,t} - \mu_j > 0$.

i) If $\tau w_{j,t} - \mu_j > \epsilon\mu_j/(1+\beta\phi)$, *then an increase in the low (respectively, high)-skilled wage,* $w_{L,t}$ $(w_{H,t})$, *implies an increase (decrease) in the ability threshold,* $\bar{a}$, *which then implies an increase in the measure of low (high)-skilled labor,* L_L (L_H). *Otherwise, an increase in* $w_{L,t}$ *and* $w_{H,t}$ *implies a decrease in* L_L *and* L_H, *respectively.*

ii) When (net) allowences for low-skilled families, μ_L, *increase,* $\bar{a}$ *also increases. Neverthless, when (net) allowences for high-skilled families,* μ_H, *increase,* $\bar{a}$ *decreases.*

iii) When the survival probability, ϕ, *increases,* $\bar{a}$ *decreases if* $w_{H,t}(1-\eta)/w_{L,t} > 1$. *Otherwise, an increase in* ϕ *decreases* $\bar{a}$.

iv) When preferences for having children, ϵ, *increase,* $\bar{a}$ *decreases as long as* $w_{H,t}(1-\eta)/w_{L,t} > (\tau w_{L,t} - \mu_L)/(\tau w_{H,t} - \mu_H)$. *Otherwise, an increase in* ϵ *decreases* $\bar{a}$.

v) When childcaring expenses, τ, *increase,* $\bar{a}$ *decreases as long as* $w_{H,t}/w_{L,t} < (\tau w_{H,t} - \mu_H)/(\tau w_{L,t} - \mu_L)$. *Otherwise, an increase in* τ *increases* $\bar{a}$.

Proof. Taking the derivatives of $\bar{a}$ with respect to wages, family allowences and childcaring expenses, respectively, we have

i)

$$\frac{\partial \bar{a}}{\partial w_{H,t}} = (\bar{a} - a_{min})\left[\left(\frac{\epsilon}{\theta}\right)\frac{\tau}{\tau w_{H,t} - \mu_H} - \left(\frac{1+\beta\phi+\epsilon}{\theta}\right)\frac{1}{w_{H,t}}\right] \tag{10}$$

and

$$\frac{\partial \bar{a}}{\partial w_{L,t}} = (\bar{a} - a_{min}) \left[\left(\frac{1 + \beta\phi + \epsilon}{\theta} \right) \frac{1}{w_{L,t}} - \left(\frac{\epsilon}{\theta} \right) \frac{\tau}{\tau w_{L,t} - \mu_L} \right] \tag{11}$$

The sign of (10) and (11) is negative and positive, respectively, provided that $\tau w_{j,t} - \mu_j > \epsilon\mu_j/(1 + \beta\phi)$.

ii)

$$\frac{\partial \bar{a}}{\partial \mu_H} = -\frac{\psi\epsilon}{\theta} \left(\frac{w_{H,t}(1-\eta)}{w_{L,t}} \right)^{-\frac{1+\beta\phi+\epsilon}{\theta}} \left(\frac{\tau w_{L,t} - \mu_L}{\tau w_{H,t} - \mu_H} \right)^{\frac{-\epsilon-\theta}{\theta}} \frac{\tau w_{L,t} - \mu_L}{(\tau w_{H,t} - \mu_H)^2} < 0 \tag{12}$$

and

$$\frac{\partial \bar{a}}{\partial \mu_L} = \frac{\psi\epsilon}{\theta} \left(\frac{w_{H,t}(1-\eta)}{w_{L,t}} \right)^{-\frac{1+\beta\phi+\epsilon}{\theta}} \left(\frac{\tau w_{L,t} - \mu_L}{\tau w_{H,t} - \mu_H} \right)^{\frac{-\epsilon-\theta}{\theta}} \frac{1}{\tau w_{H,t} - \mu_H} > 0 \tag{13}$$

iii)

$$\frac{\partial \bar{a}}{\partial \phi} = -\frac{\beta}{\theta}(\bar{a} - a_{min}) \log \left(\frac{w_{H,t}(1-\eta)}{w_{L,t}} \right) \tag{14}$$

Under the condition $w_{H,t}(1-\eta)/w_{L,t} > 1$, the sign of (14) is negative. This condition means that there is a skill premium even when one considers the wage ratio adjusted by the time cost of higher education, $(1-\eta)$.

iv)

$$\frac{\partial \bar{a}}{\partial \epsilon} = -\frac{1}{\theta}(\bar{a} - a_{min}) \left[\log \left(\frac{w_{H,t}(1-\eta)}{w_{L,t}} \right) + \log \left(\frac{\tau w_{L,t} - \mu_L}{\tau w_{H,t} - \mu_H} \right) \right] \tag{15}$$

If $\frac{w_{H,t}(1-\eta)}{w_{L,t}} > \frac{\tau w_{L,t} - \mu_L}{\tau w_{H,t} - \mu_H}$, the sign of (15) is negative.

v)

$$\frac{\partial \bar{a}}{\partial \tau} = -\frac{\psi\epsilon}{\theta} \left(\frac{w_{H,t}(1-\eta)}{w_{L,t}} \right)^{-\frac{1+\beta\phi+\epsilon}{\theta}} \left(\frac{\tau w_{L,t} - \mu_L}{\tau w_{H,t} - \mu_H} \right)^{\frac{-\epsilon-\theta}{\theta}} \frac{w_{L,t}(\tau w_{H,t} - \mu_H) - w_{H,t}(\tau w_{L,t} - \mu_L)}{(\tau w_{H,t} - \mu_H)^2} \tag{16}$$

The sign of (16) is negative if $w_{H,t}/w_{L,t} < (\tau w_{H,t} - \mu_H)/(\tau w_{L,t} - \mu_L)$.

We notice that the conditions indicated in parts (i), (iii), (iv), and (v) of the proof are all verified under the calibration of the model presented in Section 3 (see Table 1, below). □

It is noteworthy that the impact of w_j on $\bar{a}$ works through the first term inside the square brackets in equation (7), meaning that the direct net benefits from education in terms of wage outweigh the wage effect on net cost per child. Because of this, the impact of μ_j (which alliviates the cost per child) on $\bar{a}$ turns out to have the same sign as the impact of w_j.

Furthermore, given the calibration to be provided in Section 3, an increase in ϕ has a negative impact on $\bar{a}$, implying that when individuals face an expected higher lifespan, they have more incentives to become high skilled. Moreover, given the negative impact of ϵ on $\bar{a}$, we conclude that when the preferences for having children are stronger, individuals also have more incentives to become high skilled.

Note that these relationships will be crucial for the new results of the model relating fertility choice, (exogenous) mortality and human capital choice, on one hand, and automation, on the other (see Section 3).

2.2 Demographic dynamics

In this economy, population growth is endogenous. We define the total birth rate as

$$n_t = \frac{n_{H,t}L_{H,t} + n_{L,t}L_{L,t}}{L_t}, \tag{17}$$

where $n_{H,t}$ and $n_{L,t}$ are the birth rates chosen by the households belonging to each skill group as shown in equation (5) and $L_{H,t}$ and $L_{L,t}$ are the measures of high- and low-skilled workers in the economy, as determined by equations (8) and (9), respectively. Keep in mind that children from both types of households may become either low- or high-skilled workers, as determined by the level of $\bar{a}$. Population dynamics is, thus, given by

$$L_{t+1} = n_t L_t. \tag{18}$$

Note that, from equations (8), (9) and (18), the growth rates of L_H and of L_L, respectively, $g_{L_{H,t}}$ and $g_{L_{L,t}}$, are functions of n_t and $F(\bar{a}_t)$.

Total population in period t is then given by $L_t + \phi L_{t-1}$ where L_t are individuals born in t (the young) and ϕL_{t-1} are the surviving individuals on their second period of life in t (the old). Hence, we define the old-age dependency ratio in our model as[7]

$$OAR_t = \frac{\phi L_{t-1}}{L_t} = \frac{\phi}{n_t}. \tag{19}$$

2.3 Firms

The production side of the economy is based on Prettner & Strulik (2020). The final-good production sector uses both low- and high-skilled labor as inputs combined with machines in the form of automation, i.e., robots. Here, machines only replace labor performed by low-skilled workers, whereas they are complemented by high-skilled labor. The final good is produced according to the function

$$Y_t = L_{H,Y,t}^{1-\alpha}\Big(L_{L,t}^{\alpha} + \sum_{i=1}^{A_t} x_{i,t}^{\alpha}\Big), \tag{20}$$

where: $L_{H,Y,t}$ is high-skilled labor used as an input in final-good production; $L_{L,t}$ is low-skilled labor; $x_{i,t}$ is the quantity of machines (robots) of variety i; A_t represents the level of technology advancement, i.e., the measure of available varieties of machines at time t; and $\alpha \in (0,1)$ denotes the elasticity of output with respect to low-skilled labor and to machines. Under production function (20), automation (i.e., an increase in A_t) encapsulates a 'share effect', as usually considered in the literature: automation implies replacing low-skilled labor, thereby decreasing the share of the latter in the production process.[8]

Let us denote $p_{i,t}$ as the price of a unit of a machine of variety i and $w_{H,Y,t}$ and $w_{L,t}$ the wages of each type of human labor (respectively, high- and low-skilled). Maximizing the profits of this sector, we obtain the following factor

[7]See Irmen (2020) for a similar use of this concept in the context of an OLG model.

[8]In fact, it can be shown automation translates into a reduction in the labor share in the production sector, with the latter converging asymptotically to $1-\alpha$ (see Prettner & Strulik, 2020).

prices

$$w_{H,Y,t} = (1-\alpha)L_{H,Y,t}^{-\alpha}\Big(L_{L,t}^{\alpha} + \sum_{i=1}^{A_t} x_{i,t}^{\alpha}\Big), \tag{21}$$

$$w_{L,t} = \alpha\left(\frac{L_{H,Y,t}}{L_{L,t}}\right)^{1-\alpha}, \tag{22}$$

$$p_{i,t} = \alpha L_{H,Y,t}^{1-\alpha} x_{i,t}^{\alpha-1}. \tag{23}$$

We see from these results that, under equation (20), there is an impact of changes in A_t on the factor prices which is different from the one in the benchmark literature (see, e.g. Romer (1990); C. I. Jones (1995)). To obtain such result, it is essential that we introduce a substitution relationship between low-skilled labor and machines. With this in mind, we choose the framework presented in equation (20), which implies that low-skilled labor and machines are not, nevertheless, strictly perfect substitutes and, therefore, $w_{L,t}$ is independent of A_t. Note that we could have assumed perfect substitution so that we would get a result in which A_t would affect negatively the marginal productivity of $L_{L,t}$ and, thereby, $w_{L,t}$.[9] This result is apparently not consistent with the empirical literature (see, for instance, Acemoglu (2002); Acemoglu & Autor (2011)).

The machine-producing sector uses traditional physical capital as input to produce machines (robots). The production function is $x_{i,t} = K_{i,t}$, where $K_{i,t}$ is the amount of physical capital employed by each machine producer.[10] They use a blueprint (patent) from the R&D sector as fixed input. These firms operate under monopolistic competition, their profits are given by $\pi_{i,t} = p_{i,t}x_{i,t} - R_t x_{i,t}$, and their production is subject to the demand by the final-good sector given by equation (40). Profit maximization yields

$$p_{i,t} \equiv p_t = \frac{R_t}{\alpha} \tag{24}$$

[9] In this case, we would have $Y_t = L_{H,Y,t}^{1-\alpha}\Big(L_{L,t} + \sum_{i=1}^{A_t} x_{i,t}\Big)^{\alpha}$.

[10] For simplicity and without any loss of generality, we assume that physical capital depreciates fully within one period (i.e., one generation).

In equilibrium, each firm charges the same price p_t, and produces the same amount $x_{i,t} \equiv x_t$. Each firm's profit is, thus,

$$\pi_{i,t} \equiv \pi_t = \alpha(1-\alpha)x_t^{\alpha} L_{H,Y,t}^{1-\alpha}. \tag{25}$$

Finally, the R&D sector uses high-skilled labor as input to produce blueprints for new varieties of robots. Following C. I. Jones (1995), the production function in this sector is given by

$$A_{t+1} - A_t = \bar{\delta}_t L_{H,A,t}, \tag{26}$$

where $L_{H,A,t}$ represents the scientists recruited from the pool of high-skilled workers, $\bar{\delta}_t = \frac{\delta A_t^{\gamma}}{L_{H,A,t}^{1-\lambda}}$ denotes the productivity level of scientists that depends on the efficiency parameter δ, on the strength of intertemporal knowledge spillovers given by $\gamma \in (0,1]$ and the congestion or duplication effects represented by $1-\lambda$, with $\lambda \in [0,1]$. The component $\bar{\delta}$ is external to each individual R&D firm.

Firms in this sector sell their blueprints at price $p_{A,t}$ and pay the wages of scientists given by $w_{H,A,t}$. Hence, their profits are given by $p_{A,t}(A_{t+1} - A_t) - w_{H,A,t}L_{H,A,t}$. Profit maximization and free entry imply the optimality condition

$$w_{H,A,t} = \bar{\delta}_t p_{A,t}, \tag{27}$$

where $p_{A,t} = \pi_t$.

2.4 Equilibrium

Combining demand from the final-good and the machines-producing sector, we obtain the demand for robots x_t. Inserting (23) in (24) we get

$$x_{i,t} \equiv x_t = L_{H,Y,t} \left(\frac{\alpha^2}{R_t} \right)^{\frac{1}{1-\alpha}}. \tag{28}$$

Aggregating, the final-good production function is given by

$$Y_t = L_{H,Y,t}^{1-\alpha}\bigg(L_{L,t}^{\alpha} + A_t x_t^{\alpha}\bigg). \qquad (29)$$

Then, capital market clearance requires that $K_t = \sum_{i=1}^{A_t} K_{i,t} = A_t x_t$. Inserting equation (28) in the latter, we find the endogenous interest rate $R_t = \alpha^2 k_t^{\alpha-1}$ where $k_t \equiv \frac{K_t}{A_t L_{H,Y,t}}$. Moreover, under equilibrium in the final-good market, we have $K_{t+1} = L_t s_t$. Considering the two types of households in the economy, then $K_{t+1} = L_{L,t}s_{L,t} + L_{H,t}s_{H,t}$. It results from here that

$$k_{t+1} \equiv K_{t+1}/(A_{t+1}L_{H,Y,t+1}) = (L_{L,t}s_{L,t} + L_{H,t}s_{H,t})/(A_{t+1}L_{H,Y,t+1}). \qquad (30)$$

Replacing (4) in (30) for high- and low-skilled optimal savings, we obtain the equation for capital dynamics as

$$k_{t+1} = \frac{\beta\phi}{(1+\beta\phi+\epsilon)A_{t+1}L_{H,Y,t+1}}\bigg\{\alpha L_{L,t}^{\alpha}L_{H,Y,t}^{1-\alpha} + (1-\eta)(1-\alpha)L_{H,t}\big[\big(L_{L,t}/L_{H,Y,t}\big)^{\alpha} + A_t k_t^{\alpha}\big]\bigg\}. \qquad (31)$$

Hence, the system of difference equations (26) and (31) characterize the model dynamics.

Finally, we guarantee labor market equilibrium through $L_{H,t} = L_{H,Y,t} + L_{H,A,t}$. In equilibrium, wages of high-skilled workers in the final-good and R&D sector are equalized, $w_{H,A,t} = w_{H,Y,t}$. Replacing (25) in (27), we have

$$\alpha\delta x_{i,t}^{\alpha} A_{t-1}^{\gamma} L_{H,Y,t} L_{H,A,t}^{\lambda-1} = L_{L,t}^{\alpha} + A_t x_{i,t}^{\alpha}.$$

Rewriting and inserting (28), we get the implicit function

$$G(\cdot) \equiv \alpha\delta A_{t-1}^{\gamma}(L_{H,t} - L_{H,A,t})L_{H,A,t}^{\lambda-1} - k_t^{-\alpha}\left[\frac{(L_t - L_{H,t})}{(L_{H,t} - L_{H,A,t})}\right]^{\alpha} - A_t = 0 \qquad (32)$$

which, together with equation (8), yields $L_{H,A,t}$ for given L_t, A_t and k_t. Recall that the time paths of the latter three variables is determined by the difference equations (18), (26), and (31), jointly with the initial conditions L_0, A_0 and k_0.

2.5 Balanced Growth Path

We now characterize the Balanced-Growth Path (BGP) of this model, where g_z will denote generically $(z_{t+1} - z_t)/z_t$. A BGP equilibrium is a (long-run) equilibrium path along which: i) all variables grow at a constant rate, namely, $g_{Y/L_t} = g^*_{Y/L}$, $g_{K/L_t} = g^*_{K/L}$, $g_{A_t} = g^*_A$, and $g_{L_t} = g^*_L$, where $g_{L_t} = n_t - 1$ and $g^*_L = n^* - 1$, and ii) the sectoral shares of labor, $L_{H,Y,t}/L_t$, $L_{H,A,t}/L_t$ and $L_{L,t}/L_t$ are constant.

Proposition 2.3. *The model exhibits an asymptotic BGP where* $g^*_{Y/L} = g^*_{K/L} = g^*_A = (n^*)^{\left(\frac{\lambda}{1-\gamma}\right)} - 1$.

Proof. First, replace equation (28) in (29), to get output per worker

$$\frac{Y_t}{L_t} = \frac{L_{H,Y,t}^{1-\alpha}\left(L_{L,t}^{\alpha} + A_t L_{H,Y,t}^{\alpha} k_t^{\alpha}\right)}{L_t}. \tag{33}$$

Assuming that k_t and the sectoral shares of labor are constant, then Y_t/L_t grows proportionally with $[(L_{L,t}/[L_{H,Y,t}k_t])^{\alpha} + A_t]$, where the first term of the expression is constant. In turn, this implies that, asymtotically, Y_t/L_t will grow proportionally with A_t, that is $g^*_{Y/L} = g^*_A$.

On the other hand, from the final-good market equilibrium condition, we have $K_{t+1}/L_t = s_t$. Using equations (4), (21) and (22), and again assuming the sectoral shares of labor are constant, we see that K_{t+1}/L_t grows proportionally with a term ($constant + A_t$). Then, this implies that, asymtotically, K_{t+1}/L_t grows proportionally with A_t, similarly to Y_t/L_t, meaning also that k_t is constant asymtotically.

Furthermore, from equation (26), we reach the growth rate of A_t, which is given by

$$g_{A_t} \equiv \frac{A_{t+1} - A_t}{A_t} = \delta A_t^{\gamma-1} L_{H,A,t}^{\lambda}. \tag{34}$$

It is then straightforward to show that we have a constant growth rate of technology when

$$g_A^* = \left(1 + g_{L_{H,A}}^*\right)^{\frac{\lambda}{1-\gamma}} - 1. \tag{35}$$

where $g_{L_{H,A}}^*$ is the constant growth rate of $L_{H,A,t}$.

It remains to be shown that $L_{H,A,t}/L_{H,t}$ and $L_{H,t}/L_t$ are indeed constant in the asymptotic BGP and, thus, $g_{L_{H,A}}^* = g_{L_H}^* = g_L^* = n^* - 1$ and also that $n = n^*$ is attained. □

Lemma 2.4. *The model obtains an asymptotic BGP with constant sectoral shares of labor, when $\bar{a}_t \to a_{min}$ and there is a constant population growth rate, $n_t = n^*$.*

Proof. Recalling equation (8), we see that $L_{H,t}$ grows at a constant rate, $g_{L_H}^* = g_L^* = n^* - 1$, when both $F(\bar{a}_t)$ and n_t are constant.[11] Then, the sectoral share $L_{H,t}/L_t$ (and, of course, $L_{L,t}/L_t$) is constant. In turn, the latter, together with equation (32), garantees that the sectoral shares of labor $L_{H,Y,t}/L_{H,t}$ and $L_{H,A,t}/L_{H,t}$ are (asymptotically) constant. Divinding the left-hand side of (32) by A_{t-1} and using equation (34), yields

$$\alpha g_{A_t} \left(L_{H,t} - L_{H,A,t}\right) L_{H,A,t}^{-1} - k_t^{-\alpha} \left(\frac{L_t - L_{H,t}}{L_{H,t} - L_{H,A,t}}\right)^{\alpha} \frac{1}{A_{t-1}} - \left(g_{A_t} + 1\right) = 0$$

Then, evaluating this equation at the asymptotic BGP, so that $g_{A_t} = g_A^*$ and $A_{t-1} \to \infty$, and considering again a constant k_t and constant sectoral shares of labor also asymptotically, we get

$$\alpha g_A^* \left(L_{H,t} - L_{H,A,t}\right) L_{H,A,t}^{-1} - \left(g_A^* + 1\right) = 0 \Leftrightarrow$$

$$\Leftrightarrow \left(\frac{L_{H,Y}}{L_{H,A}}\right)^* = \frac{1}{\alpha}\left(1 + \frac{1}{g_A^*}\right).$$

The condition that $F(\bar{a}_t)$ is contant is satisfied with $\bar{a}_t \to a_{min}$, which requires $w_{H,t}/w_{L,t} \to +\infty$ under $1 + \beta\phi > 0$ (see equation 7). Furthermore,

[11] Note that, what is relevant here is n_t and not the birth rate of the high-skilled, $n_{H,t}$, because the pool of workers from which high-skilled labor is drawn from, for a given $\bar{a}$, is L, which grows at the rate n_t over time; see equation (18).

by replacing (8) and (9) in equation (17), we have

$$\begin{aligned} n_t \equiv \frac{n_{H,t}L_{H,t} + n_{L,t}L_{L,t}}{L_t} &= \frac{n_{H,t}(1 - F(\bar{a}_t))L_t + n_{L,t}F(\bar{a}_t)L_t}{L_t} = \\ &= n_{H,t} + (n_{L,t} - n_{H,t})F(\bar{a}_t). \end{aligned}$$

Hence, $n_{H,t}$, $n_{L,t}$ and $F(\bar{a}_t)$ must be constant so that $n_t = n^*$. Constant $n_{H,t}$ and $n_{L,t}$ are obtained, from equation (5), either with constant $w_{j,t}$ or with $w_{j,t} \to +\infty$ since

$$\lim_{w_{j,t}\to\infty} n_{j,t} = \frac{\epsilon(1 - \eta_j)}{(1 + \beta\phi + \epsilon)\tau}.$$

Finally, we show that, indeed, $w_{L,t}$ will be constant asymptotically, whereas $w_{H,t} \to +\infty$, which in turn guarantees that $w_{H,t}/w_{L,t} \to +\infty$ (as required for $\bar{a}_t \to a_{min}$). By equation (22), it is immediate to see that $w_{L,t}$ is constant when the sectoral shares of labor are constant. On the other hand, by equation (21), jointly with (20), we see that $w_{H,t}$ grows at the same rate as Y_t/L_t also under constant sectoral shares of labor, i.e., grows asymptotically with A_t. □

3 Results

3.1 Calibration and numerical results

We solve our model numerically using equations (18), (26), (31), (32) and (8) and calibrate it to fit a number of empirical moments for the US for 1970 and beyond. The values are depicted in Table 1.

The values for α, ψ, θ, a_{min} and L_0 are based on Prettner & Strulik (2020).[12] We then calibrate ϕ to match the average survival probability to the age of 65 of about 0.7 in 1970 – which we compute using World Bank (2019b) data on both female and male population – and define β so that alongside with the values for ϕ, ϵ and η, in equation (4), we get a saving rate

[12]The parameter α is set at a value higher than in the standard literature because, as in Prettner & Strulik (2020), we target the price-elasticity of robot demand in the data. According to the authors' computations, this implies a value of α of about 0.8.

Literature-based parameters		
α	0.78	Elasticity of output with respect to labor
ψ	23	Intercept parameter of $v(a)$
θ	0.38	Slope parameter of $v(a)$
a_{min}	100	Minimum ability required to pursue a college degree
L_0	1000	Initial level of population
Data-based parameters		
β	0.52	Discount factor
ϕ	0.7	Survival probability
η	0.05	Opportunity cost of higher education
δ	0.12	R&D labor efficiency
λ	0.55	Congestion or duplication effects
γ	0.6	Intertemporal knowlegde spillovers
ϵ	0.3	Utility weight of children
μ_H	-0.01	High-skilled family (net) allowences
μ_L	0.1	Low-skilled family (net) allowences
τ	0.31	Cost per child
A_0	2	Initial level of technology
k_0	1	Initial level of capital per capita

Table 1: Baseline calibration of the model. See text for details.

of 0.21, the average observed in the US between the 70s of the 20th century and the last decade of this century (Sequeira et al., 2018).

Additionally, we define the values of the parameters δ, λ, γ, η, A_0 and k_0 to match the following targets regarding empirical trends for the US: a skill premium (high-skill/low-skill wage ratio) in 1970 of 1.6, followed by an upward trend specially after 1981 (Autor, 2010); an annual TFP growth rate of 1.5% in 1970, with an upward trend up to the mid 1990's and a slight downward trend onwards (Feenstra et al., 2015); an R&D share of 0.4% in 1970, followed by a gradual upward trend – which we compute using the number of Full-Time-Equivalent (FTE) R&D scientists and engineers in R&D-performing companies from the National Science Foundation (NSF, 2019) and the total labor force from the Penn World Tables (Feenstra et al., 2015); a college share (graduates) of 20% in 1970, also followed by a discernible rising trend – which we derive using data on years of schooling completed by people that are 25 years old or more (Bureau US Census, 2019); a GDP growth rate of about 3% in 1970, followed by a slight downard trend (World Bank, 2020c); and, finally, a stock of robots of about 43.5 thousand in 1993 (the first year with available data), followed by a significant upward trend (International Federation of Robotics, 2020).

Finally, we choose the values for ϵ, μ_H, μ_L and τ to fit the following demographic targets: an old age dependency ratio of 16.25% in 1970, followed by an upward trend (World Bank, 2020a), and three targets regarding the evolution of the birth rate in the US, i.e., the total birth rate and the birth rates by skill group. With recent data from the US Census Population (Bureau US Census, 2018) and the National Center for Health Statistics (Martin et al., 2017), we are able to compute distinct birth rates across skill groups for the period from 2006 to 2017. To this end, we use data on women with one birth in the past year broken down by educational attainment and combine this with the total number of births in the same year to obtain the total number of births by education level. Then, we divide the number of births by population for each education level to obtain the respetive birth rate (see Figure 1). This exercise shows that the well-known decrease in birth rates recently observed in the US is mainly due to the decrease in low-skilled birth rates. Interestingly, we can see a slight increase in high-skilled birth rates after 2010. As a reference for the calibration of the model, we look at the dynamics of the birth rate by skill group, as depicted by Figure 1. For the total birth rate, however, we consider the behavior over the period 1970-2018.

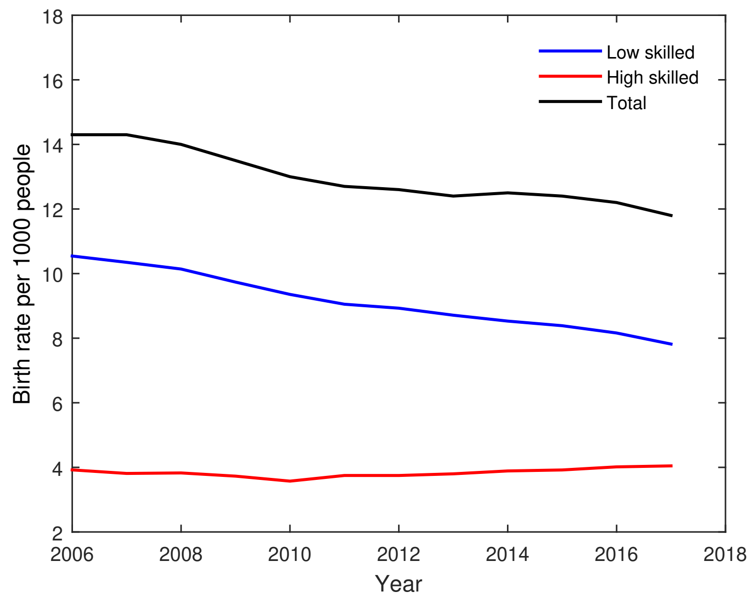

Figure 1: Birth rates across skill group from 2006 to 2017. The low- and high-skilled birth rates result from our own calculations using data from the US Census Population on women with a child in the past year detailed by educational attainment and on the number of births. See data sources in the text.

We compute our simulations using the calibration shown in Table 1 and the cumulative distribution function of a standard normal distribution to specify the ability distribution $F(\bar{a}_t)$. Figures 2 and 3 show the transitional dynamics of our model given the initial conditions outside the BGP regarding the main variables versus the data.[13] Blue lines represent the simulation results and dashed red lines the data.[14]

[13]Figure 3 does not include the data series corresponding to the variables L_L birth rate and L_H birth rate (already depicted in Figure 1) because they pertain to a very short time period and thus would not be perceptible given the time scale of the simulation exercise.

[14]Our model provides not enough degrees of freedom to calibrate the level of some variables with the available parameters; however, the focus of our paper is on the dynamics of the variables. Given this, we use an ad hoc procedure to adjust the scale of the simulated series, namely for the TFP growth, R&D share and stock of robots.

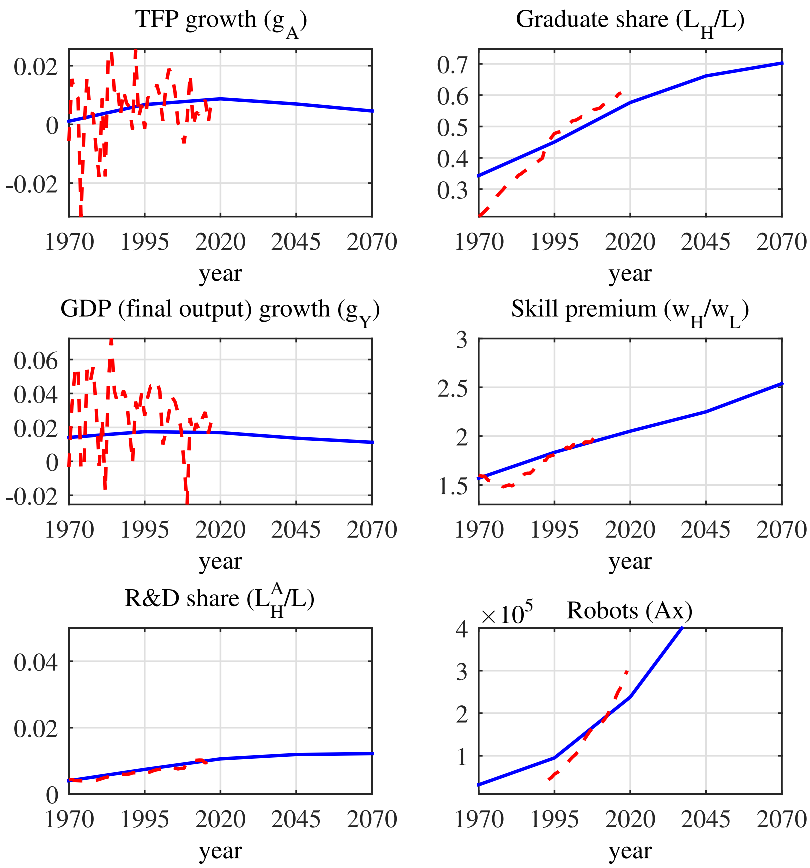

Figure 2: Dynamics of technological and production variables given initial conditions at 1970. Blue lines represent simulation results and red dashed lines the data. See data sources in the text.

Overall, our model generates dynamics consistent with the US trends for the key technological and production variables. Figure 2 shows that the TFP growth path in the model captures the acceleration in the data during the last quarter of the 20th century, followed by a nearly constant trend at the beginning of the 21st century (the model shows a downward trend slightly later than what is observable in recent data). Regarding the ratio of high-skilled workers to the labor force (graduate share), the model replicates the positive trend exhibited in the data, although with a slight overshooting of the data levels during the first years of the simulation. Also, the behavior of the share of R&D workers, the stock of robots in production and the skill premium in the model follows from close the time series in the data.

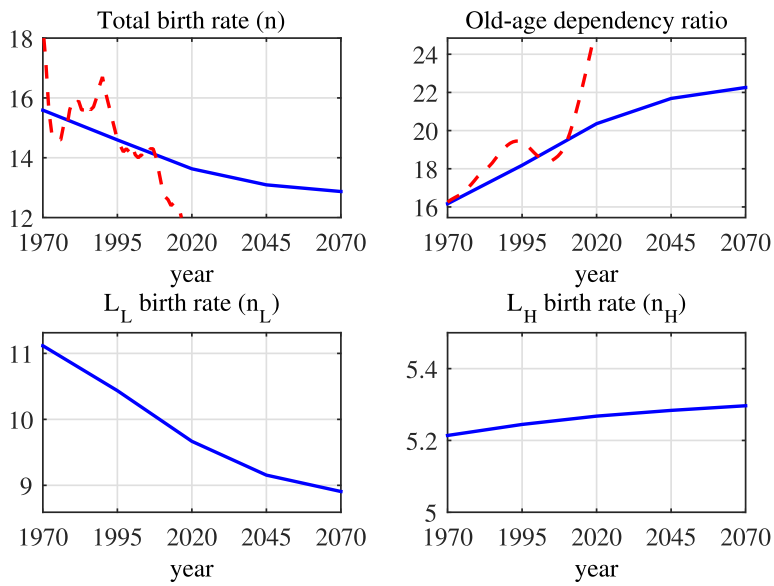

Figure 3: Dynamics of demographic variables given initial conditions at 1970. Blue lines represent simulation results and red dashed lines the data. See data sources in the text.

The demographic dynamics of our model are shown in Figure 3. Our model captures the downward trend of the total birth rate in the US, although this decrease is somewhat smaller than in the data after 2008. Similarly, the old-age dependency ratio in the model captures the upward trend in the data, but exhibiting a milder ageing trajectory than that empirically observable also after 2008. Indeed, our model is not able to capture the intensity of ageing in its full empirical extent because of the referred to smoother decrease in the total birth rate (recall equation (19)), but specially due to the fact the model considers a constant death rate, $1 - \phi$ (whereas, in the data, the death rate has fallen, generating a positive impact on life expectancy). The remaining panels show a downward movement for the low-skilled birth rate and an upward movement for that of the high skilled, which qualitatively matches the dynamics in the data, as depicted by Figure 1. The results show that the decrease in the total birth rate is primarily explained by the low-skilled birth rate, whereas the behaviour of the high-skilled birth rate shows a very mild rising trend.

Overall, we underline that, differently from the existing literature, our model is able to compatibilize real wages growing over time and either falling or increasing birth rates under a framework of both endogenous growth and endogenous fertility. We accomplish this by allowing for family (net) allowences transferred by the government, μ_j, in the household's budget constraint (2), and which may be either positive or negative. As shown in Proposition 2.1, the sign of μ_j determines the sign of the correlation between birth rates and wages over time.

3.2 Exogenous shock

As already emphasized, the structure of the model laid out in Section 2 allows for the study of interactions between automation, demography, and growth in a general-equilibrium framework. To explore the mechanisms underlying those interactions, we now analyze the effects of exogenous one-off shocks to both the technological and the demographic side to this economy, namely, to δ, ϕ and ϵ, while keeping the values of the remaining parameters unchanged.

The ongoing discussion in the literature on whether automation affects demography or vice versa (see, e.g., Prettner & Bloom (2020)) motivates this type of study. Furthermore, the choice of these specific shocks is suggested by the following US empirical events. First, the increase in the operational stock of robots of about 426% from 1995 to 2019 (International Federation of Robotics, 2020), which we replicate by raising the R&D-efficiency parameter, δ, by 61%.[15] Second, the notable decrease in birth rates of about 20% from

[15]Notice that, under this exercise, we consider a shift in δ alone in order to capture the dynamics of the stock of robots in the data from 1995 on, while the remaining parameters are kept unchaged in their baseline values of Table 1. In contrast, in the baseline calibration, the value of δ was determined simultaneously with the value of a number of other parameters in order to capture several moments in the data (usually, from 1970 on) besides the dynamics of the stock of robots. The same logic applies to the exercises pertaining to ϕ and to ϵ.

1995 to 2019 (World Bank, 2020b), which we introduce in the model by reducing the utility weight of children, ϵ, by 11.6%. Finally, the average probability to survive to the age of 65 in 2018 (World Bank, 2019b), which we match by raising the surviving-probability parameter, ϕ, by 10 p.p.

We present the results of this exercise in the following subsections. Herein, blue lines in the figures represent the dynamics of the model in the baseline case (no shocks) and green dashed lines the simulated response of key macroeconomic variables to a given one-off shock as of 1995.

3.2.1 Technological shock

Figure 4 shows the transitional-dynamics effects of an increase in δ of 61% on key technological and production variables and compares them with the transition path under the baseline scenario (no shock). The impact on these variables reveals that the acceleration of the stock of robots is paralleled by an increase in GDP growth over transition vis-à-vis the time path in the baseline scenario. This positive impact is explained by the increase in the TFP growth rate, resulting directly from the increase in R&D labor productivity induced by the increase in δ. As its productivity increases, the share of high-skilled labor allocated to R&D accelerates and, therefore, the share of this type of labor in final-good production weakens. Nevertheless, as the former starts to decrease, the latter increases, reaching a level above the time path in the baseline scenario. Furthermore, the positive trend of the high-skilled wage intensifies due to the direct effect of the increase in R&D (high-skilled) labor productivity (recall equation (27)). In parallel, the increase in δ has a short- and medium-run negative impact on the upward trend of the low-skilled wage, followed by a positive effect towards the asymptotic BGP. This behavior occurs due to the impact of the share of high-skilled labor in final-good production (recall equation (22)). Consequently, there is a first stage under which the skill premium raises due to a negative and positive response of, respectively, the low- and high-skilled wage trends. Afterward, despite the acceleration in the low-skilled wage, the skill premium continues to grow, given the more significant increase in the high-skilled wage. Moreover, the behaviour already described of both wages decreases the ability threshold, $\bar{a}$, and, hence, increases the share of graduates in the economy (see Proposition 2.2). Our results also show that the boost of the stock of robots is due first to the direct rise of blueprints because of the increased R&D efficiency and then to the increase in the high-skilled labor allocated to final-good production (as this increases the demand for robots).

Additionally, we look at the long-run (BGP) effects of this shock (not shown in Figure 4). Our model shows that a one-off increase in the labor efficiency of R&D has permanent effects in the skill-premium and the stock of robots (i.e., the shock sets their upward time path permanently above the time path under no shock) explained by, respectively, the (permanent) acceleration of the high-skilled wage, directly affected by δ, and the increase in the stock of blueprints. However, the TFP and GDP growth rates and the sectoral shares of labor all follow a hump-shaped transition path towards the unaltered asymptotic BGP. As explained in Section 2.5, in the asymptotic BGP, the ability threshold converges to the value of a_{min}; hence, whatever the shock, the share of high-skilled labor stays unaltered. Moreover, TFP and GDP growth rates are also unchanged vis-à-vis the BGP with no shock due to the unaffected total birth rate (as will be further explained below). In turn, the latter implies that the sectoral shares of high-skilled labor converge to the same BGP level as in the baseline scenario (see Proposition 2.3 and Lemma 2.4).

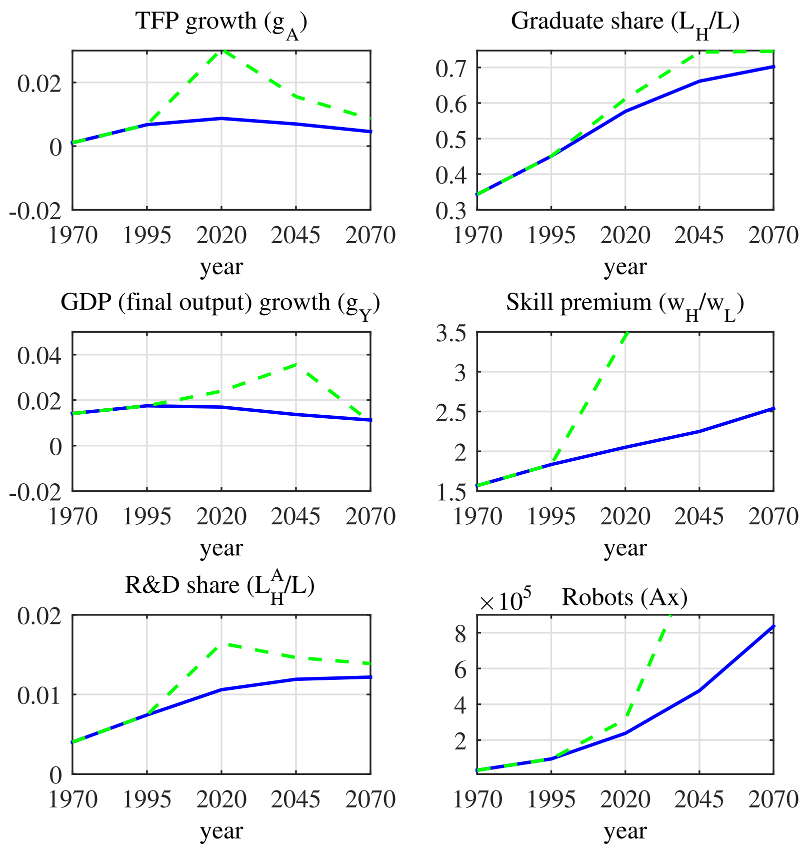

Figure 4: Response of technological and production variables to an exogenous one-off increase of 61% in δ in 1995. Blue lines represent the baseline scenario ($\delta = 0.12$) and green dashed lines the shock response ($\delta = 0.1932$).

In turn, Figure 5 shows the implication of the increase in δ, leading to an acceleration of the stock of robots, on demography. The recent discussion of the interplay between automation and demography has brought the issue of whether robots can affect the demographic dynamics and age structure of an economy (Prettner & Bloom, 2020). We find, in our model, that robots and demography interact by means of the acceleration of wages. As we mentioned above, the one-off shift in δ originates an acceleration of high-skilled wages, whereas the upward trend of low-skilled wages first weakens and then is followed by a boost. Consequently, there is an intensification of the increase in the high-skilled birth rate, while the low-skilled birth rate first attenuates the fall and then intensifies it. Overall, these movements translate into a more intense fall in the total birth rate than in the baseline scenario. In turn, the latter leads to an acceleration of the old-age dependency ratio.

However, the analysis of the BGP effects indicates that the change in δ has no long-run impact on the demographic side of the model. The low- and high-skilled birth rates are unaltered vis-à-vis the BGP with no shock since the low-skilled wage also remains unaltered while, in the case of high-skilled labor, as the wage tends to infinity, the birth rate converges to the same BGP level than in the scenario with no shock (recall Lemma 2.4). Hence, with the sectoral shares of labor and the skill-type birth rates unaffected, there is also no permanent effect on the total birth rate. As a consequence, the old-age dependency ratio also returns to the pre-shock level in the long-run equilibrium.

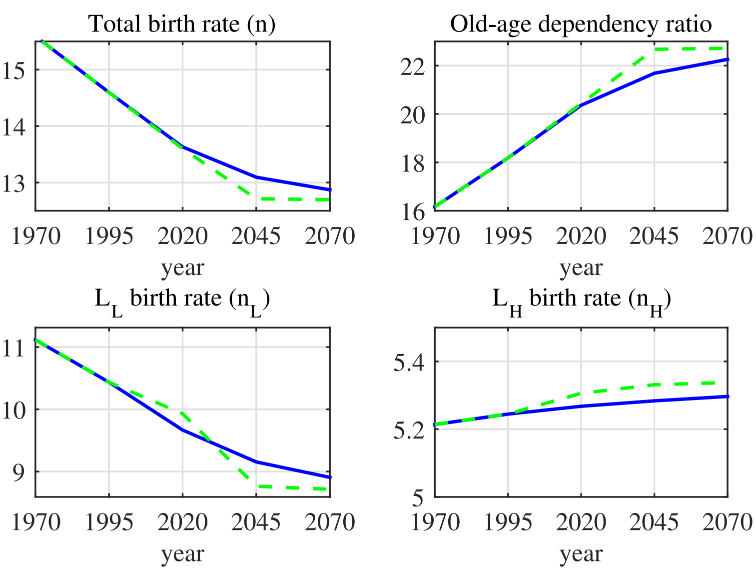

Figure 5: Response of demographic variables to an exogenous one-off increase of 61% in δ in 1995. Blue lines represent the baseline scenario ($\delta = 0.12$) and green dashed lines the shock response ($\delta = 0.1932$).

3.2.2 Demographic shocks

We now analyze the effect of demographic shocks versus the baseline scenario (no shocks). Figures 6 and 8 display the effect of a one-off increase of 10 pp. in ϕ to match the share of people that survives to age 65 in 2018. Figures 7 and 9 show the impact of a one-off decrease of 11.6% in ϵ to replicate the decrease of about 20% in the birth rate from 1995 to 2019.

We compare first the implications regarding the technology and production side of the model, depicted in Figures 6 and 7. The increase in ϕ has a negative effect on the ability threshold, thus positively affecting the share of high-skilled labor (graduate share). However, a decrease in ϵ has the opposite effect on the ability threshold, and hence, it decreases the share of qualified labor; nevertheless, in the first period after this shock, the share of high-skilled labor slightly accelerates due to the endogenous effect of wages on the ability threshold (recall Proposition 2.2). Regarding the share of R&D labor, both shocks induce a (slight) oscillatory behavior in which first there is an acceleration and then there is a downward movement, reaching a level below that of the baseline scenario. Moreover, high-skilled labor allocated to final-good production intensifies under the two types of shocks despite the first response of the R&D labor share. Note that during the first period after the shock, total high-skilled labor is increasing (even when its share is receding). Following this, there is a reallocation of high-skilled workers between the two sectors. Finally, reflecting the reallocation of labor described above, the TFP growth rate reacts to an increase in ϕ by accelerating in the first and second period and then slowing down vis-à-vis the baseline scenario. In the case of a decrease in ϵ, the TFP growth rate (slightly) accelerates in the first period and begins to slow down right afterward, thus displaying a faster adjustment than under the shift in ϕ. The GDP growth rate presents a similar qualitative response to that of the TFP growth rate, with the acceleration induced by the shock remaining longer in the case of an increase in ϕ.

Furthermore, importantly, our model shows that the effects of a shock inducing population ageing on the skill premium and the stock of robots may differ when ageing arises from a decrease in the birth rate (inducted by a decrease in ϵ) or an increase in the individual's expected lifespan (increase in ϕ). The increasing trend of the skill premium slows down in the first scenario due to the less intense upward movement in the high-skilled wage, whereas it becomes more accentuated in the second one given the acceleration of this wage. The different response of the stock of robots explains the different response of high-skilled wage (recall equation (21)). Note that, from the combination of the response of the TFP growth rate and the sectoral shares of labor, it results that this stock tends to increase below the baseline time path in the case of a one-off decrease in ϵ and increases above the baseline under a one-off increase in ϕ. However, our results also show that during the first period after the shock in ϵ, the stock of robots also slightly accelerates.[16] Therefore, even though both drivers of an ageing population can affect robot adoption positively in the short and medium run, the result can be different in the long run, as further explained below.

The long-run analysis reveals for both shocks that the TFP and GDP growth rates face a permanently lower level than that of the baseline scenario, explained by the permanent decrease in the birth rate (as shown below). As already mentioned, the share of high-skilled labor remains unaffected asymptotically due to $\bar{a}_t \rightarrow a_{min}$, whereas the R&D share attains a permanently lower level than in the baseline case in parallel with the referred to decrease in the TFP growth rate. Hence, high-skilled labor used in the final-good sector increases (see Lemma 2.4).

Additionally, the stock of robots shows a permanent lower (respectively, higher) level than that in the baseline case in the scenario of a one-off decrease (increase) in ϵ (ϕ). The same occurs to the skill premium. Hence, the long-run effect of population ageing indicates that its impact on the skill premium and robot adoption differs regarding the primary driver of ageing: a lower total birth rate or a longer expected lifespan.

[16] Although the behaviour of the high-skilled wage follows that of the stock of robots during the first period after the decrease in ϵ, afterwards the upward trend of this wage weakens while the stock of robots accelerates. This discrepancy is due to the negative effect of the decrease in the low-skilled labor on the high-skilled wage (recall equation 21).

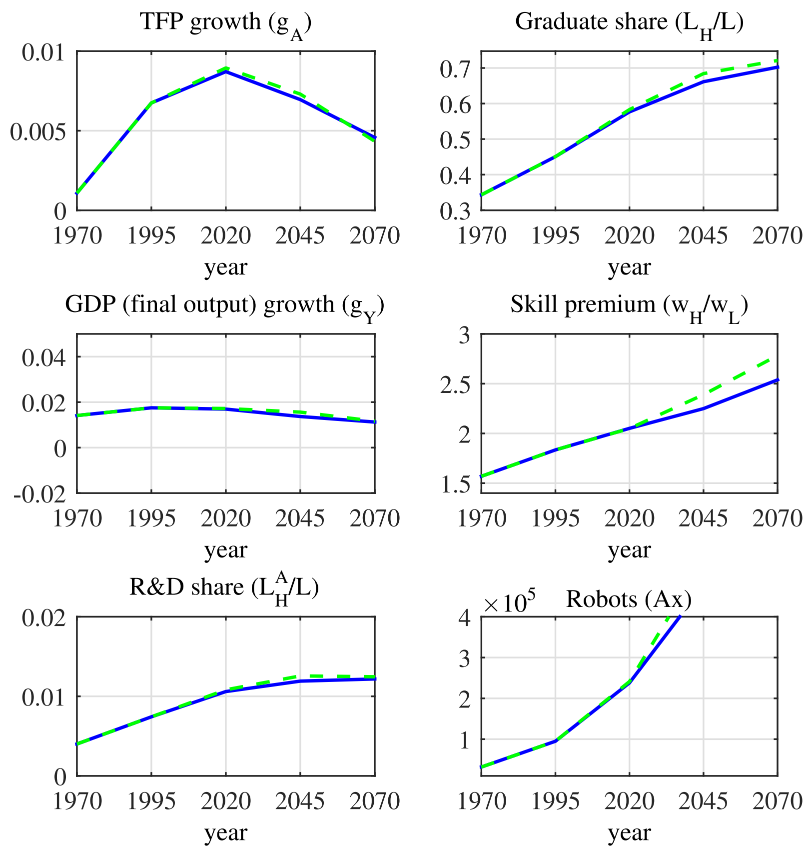

Figure 6: Response of technological and production variables to an exogenous one-off increase of 10 p.p. in ϕ in 1995. Blue lines represent the baseline scenario ($\phi = 0.7$) and green dashed lines the shock response ($\phi = 0.8$).

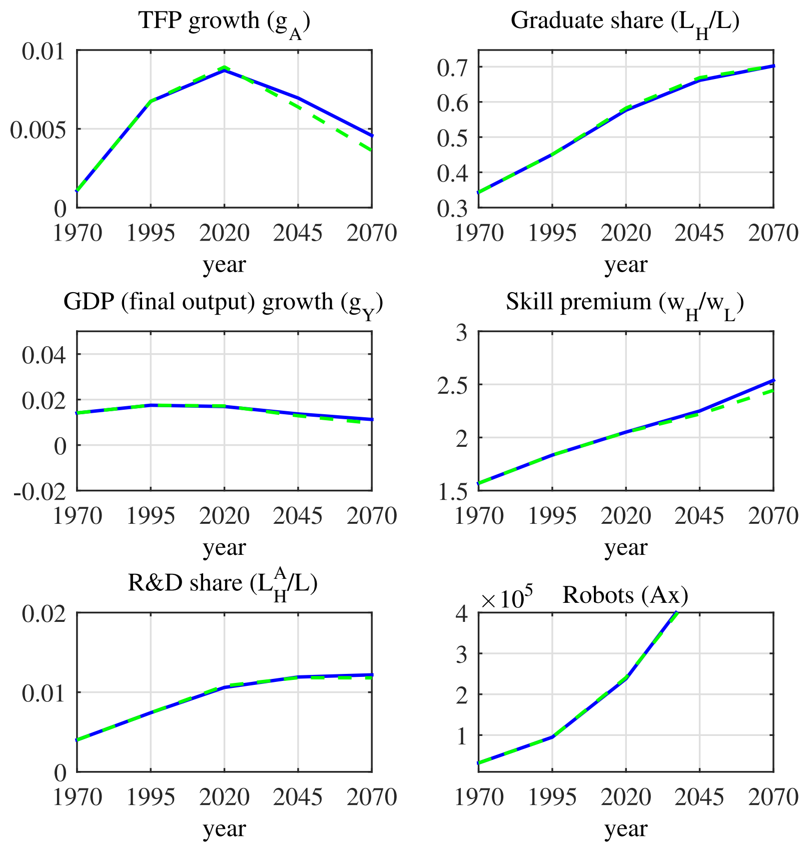

Figure 7: Response of technological and production variables to an exogenous one-off decrease of 11.6% in ϵ in 1995. Blue lines represent the baseline scenario ($\epsilon = 0.3$) and green dashed lines the shock response ($\epsilon = 0.265$).

The impact of these shocks on the demography side is straightforward since ϕ and ϵ directly affect consumption and fertility decisions. An increase in the survival probability and a decrease in the preference for having children decreases the birth rates, intensifying the downward trajectory vis-à-vis the baseline case of no shock (see Proposition 2.1). Facing a longer lifespan due to a higher ϕ, individuals choose to save more, which implies a lower consumption level and a lower birth rate for both types of labor in the first period of life (in the case of the high-skilled, the short- and medium-run response to the shock implies that the birth rate detaches from the upward trajectory observed in the baseline scenario). This is a typical result in the literature of demography and economics; see, for instance, Prettner (2013); Baldanzi, Prettner, & Tscheuschner (2019). In this case, the old-age dependency ratio intensifies its increase due to the increase in ϕ and the (endogenous) decrease in the total birth rate. On the other hand, as ϵ represents the utility weight of having children, a decrease in this parameter intensifies the decrease in both birth rates (see Proposition 2.1). Here, the old-age dependency ratio increases only due to the (endogenous) fall in the (total) birth rate.

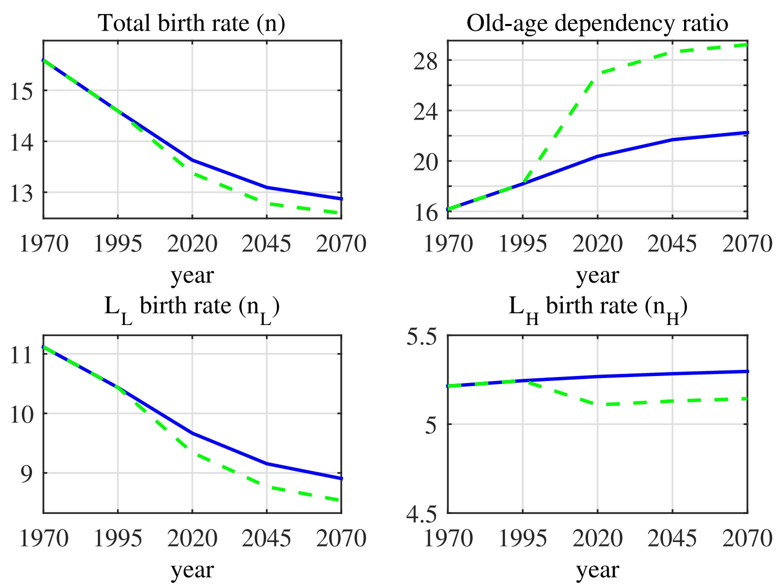

Figure 8: Response of demographic variables to an exogenous one-off increase of 10 p.p. in ϕ in 1995. Blue lines represent the baseline scenario ($\phi = 0.7$) and green dashed lines the shock response ($\phi = 0.8$).

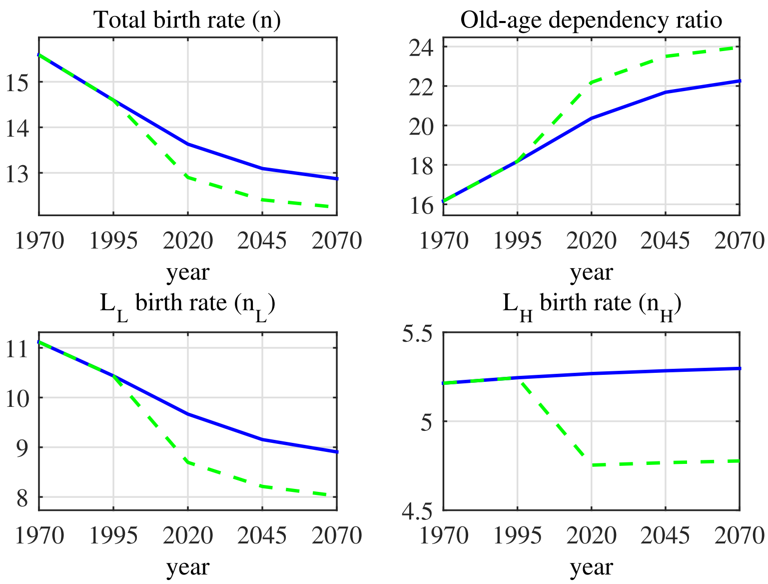

Figure 9: Response of demographic variables to an exogenous one-off decrease of 11.6% in ϵ in 1995. Blue lines represent the baseline scenario ($\epsilon = 0.3$) and green dashed lines the shock response ($\epsilon = 0.265$).

To sum up, one of this study's main findings regarding the impact of demographic shocks is that when the old-age dependency ratio increases, either by a decrease in ϵ or an increase in ϕ, the short- and medium-run effect on the demand for robots is positive. The sign of the relationship between ageing and automation in our model is consistent with recent literature, e.g. Acemoglu & Restrepo (2017, 2021); Basso & Jimeno (2021); Zhang et al. (2021); yet, their results refer to the long-run impact of ageing. In turn, in our model, the long-run effect on the dynamics of the stock of robots differs depending on whether population ageing is induced by a lower total birth rate or a longer expected lifespan. In particular, the mechanisms in our model suggest that countries experiencing a fall in birth rates do not have many incentives to adopt automation, at least in the long run. Our findings contrast with those of Irmen (2020), first regarding the short- and medium-run effect of ageing, since, in that paper, there is a negative impact on automation, and also regarding that paper's result that both forces of ageing have the same positive impact on automation in the long-run.

Additionally, our model's interaction between human capital and fertility decision complements the standard literature on child quantity versus quality. We obtain a different result for the effect of ϵ on human capital and the effect of wages on the fertility decision. If households have strong preferences to have children (higher ϵ), the ability threshold decreases – which implies an increase in the share of high-skilled labor in the economy – whereas, in the standard literature, it decreases due to the trade-off between child quantity and quality (e.g. Barro & Becker, 1989; Prettner, 2013; Baldanzi, Prettner, & Tscheuschner, 2019; Baldanzi, Bucci, & Prettner, 2019). In our case, the decision on whether an individual becomes high or low skilled is her own and not her parents'; therefore, our model represents an economy in which households who wish to have more children tend to become high skilled, hence, receive a higher wage level. At the same time, given the calibration of our model, we obtain a positive relation between the wage and fertility decision for high-skilled labor and a negative one for low-skilled labor, which seems to be the case in light of the recent data depicted in Figure 1.

4 Extension: full-labor automation

The view that only low-skilled labor can be automated is expected to be outdated by the rise of a more sophisticated technology, e.g., artificial intelligence (AI). Up until now, the majority of automated work has been replacing positions occupied by low-skilled labor. However, the development of AI alongside big data and machine learning increased the pace of automation by enabling high-skilled positions to become also a job for robots. Examples of such occupations include accounting, mortgage origination, management consulting, financial planning, paralegals, and various medical specialties, including radiology, general practice, or even surgery (Acemoglu & Restrepo, 2018b). By allowing machines to adapt to new tasks, AI should permanently facilitate machines catching up with labor in producing each task (Guimarães & Gil, 2019).

With this in mind, we now consider a production function modified so that automation can replace labor performed by both low- and high-skilled workers in production. Aggregate production occurs according to the function

$$Y_t = L_{Y,t}^{1-\alpha} + \sum_{i=1}^{A_t} x_{i,t}^{\alpha}, \tag{36}$$

where $L_{Y,t} = (L_{H,Y,t}^{\sigma} L_{L,t}^{1-\sigma})$ represents the human labor in the production process, with high-skilled labor denoted by $L_{H,Y,t}$ and low-skilled labor by $L_{L,t}$ and where $\sigma \in (0,1)$ controls for the elasticity of substitution between high- and low-skilled labor. As before, $x_{i,t}$ is the quantity of machines (robots) of variety i, A_t represents the measure of available varieties of machines at time t, and $\alpha \in (0,1)$ denotes the elasticity of output with respect to automation. We show in the appendix the details on the derivations regarding this version of the model.[17,18]

To fully understand how the interplay of demography and automation changes under the future scenario of machines replacing both types of labor, we repeat our simulation exercise, focusing on the period of 2020 onwards. Note that the new model is not able to capture the trends in the data up until now, in constrast to the model introduced in Section 2. This suggests that today's context is still not characterized by a state where robots replace high-skilled labor. Nevertheless, as we expect advancements in technology development and automation adoption, it seems reasonable to investigate such a possible future scenario.

Figures 10 and 11 show the dynamics for the baseline model (blue lines)

[17] Notice that the production function in (36) implies that labor and automation are not, nevertheless, strictly perfect substitutes. Therefore, as shown in the appendix, wages $w_{L,t}$ and $w_{H,t}$, are independent of A_t (instead of falling with A_t). In this context, automation induces a fall in the labor share in the production sector towards zero, but only asymptotically, as A_t grows relative to $L_{Y,t}$ without bounds.

[18] The extreme specification for the aggregate production function in (36) implies decreasing returns to scale. The factor income distribution problem is well defined if, say, we also consider a fixed overhead in (36), measured in terms of final output and increasing with A_t and L_t (because Y_t grows with A_t and L_t in the long run). In that case, in each period t, the marginal cost function corresponding to (36) is increasing in the quantity of output, while the average cost function is U-shaped. This implies that there is an optimal scale of production in the final-good sector, where the average cost is minimized. At this scale and with factors priced at their marginal products, total factor income will equal output, Y_t, net of the fixed overhead.

versus the model with the modified production function (black dashed lines) as of 2020. We use the same calibration as in Table 1, except for δ, ϵ and ϕ, which we change for the values used in the shock exercises in Section 3.2 in both models, and α and (the new parameter) σ, for the model with full-labor automation, so that $\alpha = 0.65$ and $\sigma = 0.25$. The results show that as high-skilled labor is replaced by automation, the graduate share flattens, triggered by the unresponsiveness of high-skilled wage to the dynamics of A_t. In fact, the high-skilled wage decreases due to the rise of high-skilled labor in the context of a positive birth rate. Since a similar dynamics occurs as regards the low-skilled wage, the skill premium is almost unchanged in the model with full-labor automation, in contrast with the steep increase in the baseline model. Likewise, the R&D share also flattens. In addition, the TFP growth rate exhibits a smooth slowdown vis-à-vis the baseline model; however, this is not extended to the GDP growth rate since the more favorable behavior of the total birth rate (as explained below) offsets the behavior of the TFP growth rate. Finally, the stock of robots exhibits a slower upward trend than that of the baseline scenario. Under full-labor automation, the demand for robots is not boosted by the high-skilled labor in the final goods sector since this type of labor is no longer complementary to robots.

Regarding the demographic dynamics, the decreasing trend exhibited by low- and high-skilled wages results in birth rates with a soft but opposite behavior to that of the baseline model: the high-skilled birth rate (slightly) increases, whereas the low-skilled birth rate (slightly) decreases under the model with full-labor automation. In turn, these imply a different behavior of the total birth rate, here characterized by a nearly constant trend, resulting in the flattening of the old-age dependency ratio.

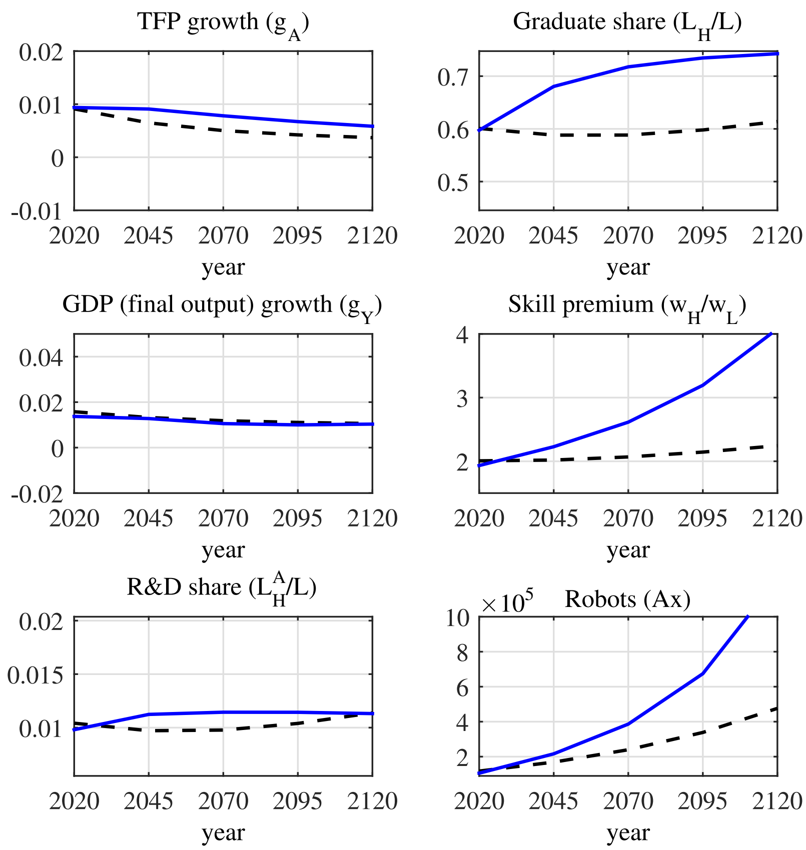

Figure 10: Dynamics of technological and production variables given initial conditions at 2020. Blue lines represent the baseline model and black dashed lines the simulation results from the model with full-labor automation.

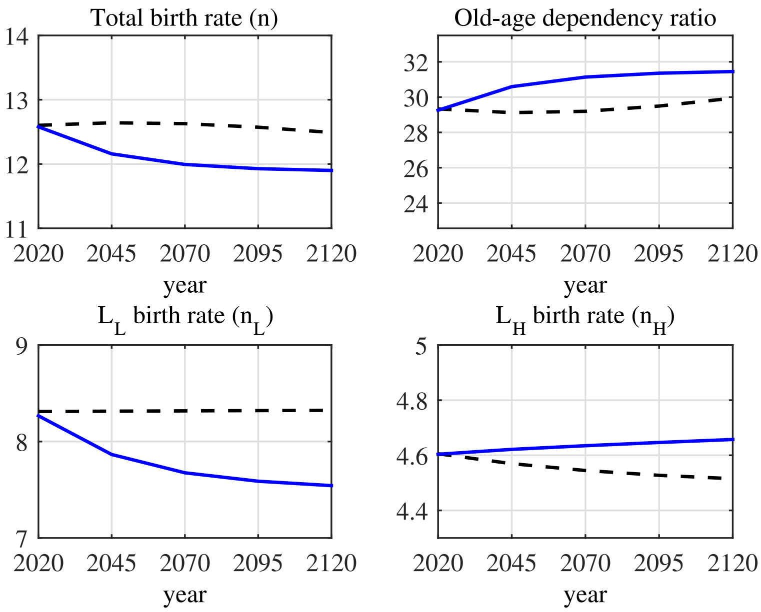

Figure 11: Dynamics of demographic variables given initial conditions at 2020. Blue lines represent the baseline model and black dashed lines the simulation results from the model with full-labor automation.

To sum up, the scenario of full-labor automation in production presented in this section seems to constraint the dynamics of the baseline model. In particular, this occurs for the future dynamics (after 2020) of the skill premium, the stock of robots, and the TFP growth rate. But it also weakens the interaction between technology and demography exhibited in the baseline model, since the benefits of the complementarity between high-skilled labor and robots no longer accrue, in spite of the context of potential demographic growth, and hence, of growth of the high-skilled labor.

5 Conclusion

Recent studies have found that population ageing and automation adoption are linked since an ageing workforce can incentivize firms to become more technological dependent (Acemoglu & Restrepo, 2021; Irmen, 2020; Basso &

Jimeno, 2021; Zhang et al., 2021; Stähler, 2021). Our paper extends this literature by focusing on the interaction between automation and demography and the effects of different drivers of ageing, i.e., a low birth rate or high life expectancy. With this in mind, we built an R&D-based model extended to include automation in the production function, as well as endogenous education and a demographic structure in the household sector by means of the introduction of fertility choice and a survival probability from young to old age.

Our model reveals consistent dynamics with the US trends from 1970 to 2019. Specifically, on the demographic side of the model, we can combine the growth of real wages over time, and the heterogeneous behavior of the birth rate across skill groups — a dynamics consistent with recent data regarding the behaviour of low- and high-skilled birth rates.

To explore the mechanisms underlying our model, we run a number of exercises considering one-off shocks to key technological and demographic parameters. Our model reveals that an increase in the R&D labor efficiency – which increases automation intensity – induces a temporary boost to TFP and GDP growth rates and to the graduate share and a permanent (long-run) increase in the skill premium. Yet, it has a temporary negative effect on the total birth rate, increasing the old-age dependency ratio.

Concerning the impact of demography on robot adoption, we show that, in the short and medium run, an increase in population ageing, either by an increase in the survival probability or by a decrease in the preferences to have children, has a positive effect on the dynamics of the stock of robots. Nevertheless, in the long run, robot adoption accelerates with the increase in the survival probability, whereas it slows down with the decrease in the preference for having children. This result can explain the differences in the rate of robot adoption faced by countries with severe population ageing. For instance, China is one of the countries with the most aged population; yet, it is not one of the top countries with higher robot density – the number of robots per 10.000 workers in an industry International Federation of Robots (2021). Our model provides a possible explanation for this since China's birth rates are meager. Certainly, longevity has also contributed to worsening its ageing problem; however, China's life expectancy at birth still remains below that of countries such as the US (World Bank, 2019a) and, simultaneously, China's one-child policy has been implemented since the 70s, being revised only in 2016 to two children, and recently, allowing as many as three children.

Furthermore, an increase in the survival probability positively impacts the TFP and GDP growth rates in the short and medium run, whereas these variables face a permanently lower level in the long run. The graduate share faces a temporary acceleration, while the skill premium permanently accelerates. The response of these variables to a decrease in the preferences to have children is similar; nevertheless, the skill premium displays an opposite permanent effect.

Finally, we extend our analysis to include a full-labor automation scenario. Taking as a reference the benchmark model, where machines can only replace low-skilled labor, we find that automation intensity slows down when robots can also substitute high-skilled labor in production. The key factor is that, in spite of a growing high-skilled labor (reflecting the households' fertility and education choices), the latter no longer stimulates the demand for robots under full-labor automation. On the other hand, since, in this case, wages of both types of labor remain relatively flattened because they do not respond directly to automation, the birth rates have also a smoother dynamics, which leads to a less severe behavior of the old-age dependency ratio. As an add-on, this version of our model also helps make it clear, in retrospective, the relevance of the complementarity between high-skilled labor and robots for the study of the rich interplay between population dynamics, human capital and technology observed in the last decades.

Although our full-labor automation exercise constrains the dynamics exhibited in the baseline model, this result may be overturned, namely for the stock of robots and the TFP growth rate, if we included automation in the R&D sector in the context of advanced AI (as in, e.g., Basso & Jimeno, 2021). The performance of the full-labor automation model relative to the baseline model would also be overturned in case the high-skilled population was expected to stagnate in the future (which would require $a \rightarrow a_{min}$ and null birth rates in the model) say as the consequence of (further) shocks to the households' preferences for children or the (net) cost of childbearing.

References

Abeliansky, A., Algur, E., Bloom, D. E., & Prettner, K. (2020). *The Future of Work: Challenges for Job Creation Due to Global Demographic Change and Automation* (Tech. Rep.). Retrieved from `https://ideas.repec.org/p/iza/izadps/dp12962.html`

Abeliansky, A., & Prettner, K. (2017). Automation and demographic change. *Available at SSRN 2959977*.

Acemoglu, D. (2002). Technical change, inequality, and the labor market. *Journal of Economic Literature*, *40*(1), 7–72.

Acemoglu, D., & Autor, D. (2011). Skills, tasks and technologies: Implications for employment and earnings. In *Handbook of Labor Economics* (Vol. 4, pp. 1043–1171). Elsevier.

Acemoglu, D., & Restrepo, P. (2017). Secular Stagnation? The Effect of Aging on Economic Growth in the Age of Automation. *American Economic Review*, *107*(5), 174–179.

Acemoglu, D., & Restrepo, P. (2018a). Artificial Intelligence, Automation and Work. *NBER working paper [24196]*.

Acemoglu, D., & Restrepo, P. (2018b). Low-Skill and High-Skill Automation. *Journal of Human Capital*, *12*(2), 204–232.

Acemoglu, D., & Restrepo, P. (2018c). The race between man and machine: Implications of technology for growth, factor shares, and employment. *American Economic Review*, *108*(6), 1488–1542.

Acemoglu, D., & Restrepo, P. (2021). Demographics and Automation. *The Review of Economic Studies*. (rdab031)

Autor, D. (2010). US labor market challenges over the longer term. *Federal Reserve Board of Governors*.

Baldanzi, A., Bucci, A., & Prettner, K. (2019). Children's health, human capital accumulation, and r&d-based economic growth. *Macroeconomic Dynamics*, 1–18.

Baldanzi, A., Prettner, K., & Tscheuschner, P. (2019). Longevity-induced vertical innovation and the tradeoff between life and growth. *Journal of Population Economics*, *32*(4), 1293-1313.

Barro, R. J., & Becker, G. S. (1989). Fertility choice in a model of economic growth. *Econometrica*, *57*(2), 481–501.

Basso, H. S., & Jimeno, J. F. (2021). From secular stagnation to robocalypse? Implications of demographic and technological changes. *Journal of Monetary Economics*, *117*(C), 833-847.

Brynjolfsson, E., & McAfee, A. (2014). *The second machine age: Work, progress, and prosperity in a time of brilliant technologies.* WW Norton & Company.

Bureau US Census. (2018). *Historical Time Series Tables - Women who had a Birth in the Past Year and Their Percentage in the Labor Force: ACS, 2006-2017.* Retrieved 2021-04-11, from https://www.census.gov/data/tables/time-series/demo/fertility/his-cps.html

Bureau US Census. (2019). *Educational Attainment in the United States.* Retrieved from https://www.census.gov/data/tables/2019/demo/educational-attainment/cps-detailed-tables.htmlhttp://www.census.gov/prod/2012pubs/p20-566.pdf

Cervellati, M., & Sunde, U. (2005). Human Capital Formation, Life Expectancy, and the Process of Development. *American Economic Review*, *95*(5), 1653–1672.

David, H. (2015). Why are there still so many jobs? the history and future of workplace automation. *Journal of Economic Perspectives*, *29*(3), 3–30.

Feenstra, R. C., Inklaar, R., & Timmer, M. P. (2015). The next generation of the penn world table. *American Economic Review*, *105*(10), 3150-82.

Galor, O., & Weil, D. N. (2000). Population, technology, and growth: From malthusian stagnation to the demographic transition and beyond. *American Economic Review*, *90*(4), 806-828.

Gehringer, A., & Prettner, K. (2019). Longevity and technological change. *Macroeconomic Dynamics*, *23*(4), 1471–1503.

Guimarães, L., & Gil, P. (2019). Looking ahead at the effects of automation in an economy with matching frictions [MPRA Paper]. (96238). Retrieved from `https://ideas.repec.org/p/pra/mprapa/96238.html`

Hansen, A. H. (1939). Economic Progress and Declining Population Growth. *American Economic Review*, *29*(1), 1–15.

International Federation of Robotics. (2020). *Operational Stock IFR data.*

International Federation of Robots. (2021). *Robot Race: The World´s Top 10 automated countries - International Federation of Robotics.* Retrieved from `https://ifr.org/ifr-press-releases/news/robot-race-the-worlds-top-10-automated-countries`

Irmen, A. (2020). Automation, factor shares and growth in the era of population aging. *CREA Discussion Paper Series*, 20-15.

Jones, B. F. (2010). Age and Great Invention. *Review of Economics and Statistics*, *92*(1), 1–14.

Jones, C. I. (1995). R&D-based models of economic growth. *Journal of Political Economy*, *103*(4), 759–784.

Leitner, S., & Stehrer, R. (2019). *The Automatisation Challenge Meets the Demographic Challenge: In Need of Higher Productivity Growth* (European Economy - Discussion Papers 2015 - No. 117). Directorate General Economic and Financial Affairs (DG ECFIN), European Commission.

Maestas, N., Mullen, K. J., & Powell, D. (2016). The Effect of Population Aging on Economic Growth, the Labor Force and Productivity. *NBER Working Papers [22452]*.

Martin, J., Hamilton, B., & Osterman, M. (2017). *Births in the United States.* Retrieved 2021-04-01, from https://www.cdc.gov/nchs/products/databriefs/db318.htm

Nerlich, C., & Schroth, J. (2018). The economic impact of population ageing and pension reforms. *Economic Bulletin Articles*, *2*. Retrieved from https://ideas.repec.org/a/ecb/ecbart/201800022.html

NSF. (2019). *Full-Time-Equivalent (FTE) R&D scientists and engineers in R&D-performing companies from the National Science Foundation.*

Prettner, K. (2013). Population aging and endogenous economic growth. *Journal of Population Economics*, *26*(2), 811–834.

Prettner, K. (2019). A note on the implications of automation for economic growth and the labor share. *Macroeconomic Dynamics*, *23*(3), 1294–1301.

Prettner, K., & Bloom, D. E. (2020). *Automation and its macroeconomic consequences: Theory, evidence, and social impacts.* Academic Press.

Prettner, K., & Strulik, H. (2020). Innovation, automation, and inequality: Policy challenges in the race against the machine. *Journal of Monetary Economics*, *116*, 249–265.

Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy*, *98*(5), S71–S102.

Sequeira, T. N., Gil, P. M., & Afonso, O. (2018). Endogenous growth and entropy. *Journal of Economic Behavior & Organization*, *154*(C), 100-120.

Stähler, N. (2021). The impact of aging and automation on the macroeconomy and inequality. *Journal of Macroeconomics*, *67*, 103278.

Strulik, H., Prettner, K., & Prskawetz, A. (2013). The past and future of knowledge-based growth. *Journal of Economic Growth*, *18*(4), 411–437.

World Bank. (2019a). *Life expectancy at birth, total (years) - China, United States - Data.* Retrieved 2021-06-19, from https://data.worldbank.org/indicator/SP.DYN.LE00.IN?locations=CN-US

World Bank. (2019b). *Survival to age 65, male (% of cohort) — Data.* Retrieved 2021-04-01, from https://data.worldbank.org/indicator/SP.DYN.TO65.MA.ZS?end=2018{&}locations=US{&}start=1968

World Bank. (2020a). *Age dependency ratio, old (% of working-age population) - United States — Data.* Retrieved 2021-04-01, from https://data.worldbank.org/indicator/SP.POP.DPND.OL?locations=US

World Bank. (2020b). *Birth rate, crude (per 1,000 people) - United States — Data.* Retrieved 2021-04-01, from https://data.worldbank.org/indicator/SP.DYN.TO65.MA.ZS?end=2018{&}locations=US{&}start=1968

World Bank. (2020c). *GDP growth (annual %) - United States.* Retrieved 2021-04-01, from https://data.worldbank.org/indicator/NY.GDP.MKTP.KD.ZG?locations=US

Zhang, X., Palivos, T., & Liu, X. (2021). Aging and automation in economies with search frictions. *Journal of Population Economics*, forthcoming.

Appendix

We use now a different final-good production function so that both low- and high-skilled labor can be replaced by robots. Aggregate product is produced according to the function

$$Y_t = (L_{H,Y,t}^{\sigma} L_{L,t}^{1-\sigma})^{1-\alpha} + \sum_{i=1}^{A_t} x_{i,t}^{\alpha}, \tag{37}$$

Maximizing the profits of this sector, we reach the following factor prices

$$\begin{aligned} w_{H,Y,t} &= \sigma(1-\alpha)\frac{\left(L_{H,Y,t}^{\sigma} L_{L,t}^{1-\sigma}\right)^{(1-\alpha)}}{L_{H,Y,t}}, & (38)\\ w_{L,t} &= (1-\sigma)(1-\alpha)\frac{\left(L_{H,Y,t}^{\sigma} L_{L,t}^{1-\sigma}\right)^{(1-\alpha)}}{L_{L,t}}, & (39)\\ p_{i,t} &= \alpha x_{i,t}^{\alpha-1}. & (40) \end{aligned}$$

The R&D and machine-producing sector are still characterized as in the main text. However, the latter faces now a different demand by the final-good sector given by equation (40). Given this, a monopolistic firm's profit is

$$\pi_{i,t} \equiv \pi_t = \alpha(1-\alpha)x_t^{\alpha} \tag{41}$$

The demand for robots x_t is now given by:

$$x_{i,t} \equiv x_t = \left(\frac{\alpha^2}{R_t}\right)^{\frac{1}{1-\alpha}}. \tag{42}$$

Aggregating, the final-good production function is given by $Y_t = L_{Y,t}^{1-\alpha} + A_t x_t^{\alpha}$. Consumption and capital dynamics are still the same as in the main text.

We face now a new implicit equation to obtain $L_{H,A,t}$ and $L_{H,Y,t}$.

$$G(\cdot) \equiv \frac{\alpha\delta}{\sigma} A_t^{\gamma}(L_{H,t}-L_{H,A,t})L_{H,A,t}^{\lambda-1}(k_t L_t)^{\alpha} - \left(L_{H,t}-L_{H,A,t}\right)^{\sigma(1-\alpha)}\left(L_t-L_{H,t}\right)^{(1-\sigma)(1-\alpha)} = 0 \tag{43}$$