

# INCOME EFFECTS AND THE CYCLICALITY OF JOB SEARCH EFFORT\*

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## Abstract

The canonical matching model predicts procyclical job search effort while the evidence suggests otherwise. In this paper, we assess whether introducing income effects into a model with matching frictions leads to acyclical or countercyclical job search effort as in data. We find that income effects improve the cyclical behavior of job search effort because they make the value of leisure procyclical. But the procyclicality of the value of leisure also magnifies the procyclicality of the wage. Unless we make unsound assumptions to mute this effect on the wage, the model generates either acyclical or procyclical unemployment. Thus, our paper casts doubt on the role of income effects in generating cyclical patterns of job search effort consistent with the data.

*JEL classification:* E24; E32; J64.

*Keywords:* Matching Frictions; Job Search Effort; Income Effects.

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## 1. Introduction

The cyclical nature of job search effort is a key element in understanding unemployment dynamics. Suppose that the economy is expanding and job search effort is procyclical. Because workers are more-intensively searching for jobs, vacancies are filled faster, implying a steeper fall in unemployment than it would occur otherwise. On the contrary, if job search effort is countercyclical, it dampens the fluctuations in unemployment, preventing it from rising further in recessions.

With the availability of new data sources on the search behavior of unemployed workers, a growing empirical literature emerged trying to assess the cyclical nature of job search effort. [Shimer \(2004\)](#) uses the Current Population Survey (CPS) data and treats the number of job search methods in the CPS as a proxy for job search effort. He concludes that job search effort is countercyclical. Using the data from the American Time Use Survey (ATUS), [DeLoach and Kurt \(2013\)](#) conclude that job search effort is acyclical due to two counteracting forces. The procyclical job-finding probability contributes to procyclical job search effort. But wealth effects contribute to countercyclical search effort, neutralizing the effect of the procyclical job-finding probability. [Gomme and Lkhagvasuren \(2015\)](#) merge the CPS with the ATUS data and document that the job search effort of short-term unemployed is procyclical; yet, they also document that average search effort is slightly and insignificantly procyclical, suggesting that it is acyclical. [Mukoyama, Patterson and Şahin \(2018\)](#) extend the analysis in [Shimer](#) by combining the CPS with the ATUS data. They document that job search effort is countercyclical. [Leyva \(2018\)](#) uses the ATUS data and documents results similar to [DeLoach and Kurt](#). In [Leyva](#), two counteracting forces render search effort acyclical (or somewhat countercyclical): the effect of countercyclical value of a job offsets the effect of the procyclical job-finding probability. Importantly, all these papers (except [Shimer, 2004](#)) report that unemployed workers increased job search effort at the onset of the Great Recession, pointing to countercyclical job search effort.

Though indirectly, two other papers are also pointing to countercyclical job search effort. [Faberman and Kudlyak \(2016\)](#) find that workers tend to search less in areas with tighter labor markets, which suggests that job search effort is countercyclical. [Hornstein and Kudlyak \(2017\)](#) conclude that job search effort is countercyclical if the elasticity of the matching function with respect to vacancies is in the range of one-third to one. This range covers most of the empirically plausible range that [Petrongolo and Pissarides \(2001\)](#) report for the elasticity of the matching function.

This literature strongly links to [Shimer's \(2005\)](#) critique. [Shimer](#) shows that the canonical matching model (as the one, e.g., in Chapter 29.3 of [Ljungqvist and Sargent, 2018](#) or the discrete time version of the one in Chapter 1 of [Pissarides, 2000](#)) generates volatilities of unemployment and labor market tightness far below the ones observed in the data. Furthermore, when extended with endogenous search effort, the canonical matching model predicts procyclical search effort because the returns to search (determined by wages and the job-finding probability) are procyclical. In expansions,

procyclical effort increases the chance of workers to leave unemployment. And it also allows firms to easily fill their vacancies, motivating firms to further increase vacancies. Both effects lead to a greater fall in unemployment in expansions, exacerbating volatility relative to the canonical model. This magnification mechanism can be found in, for example, [Merz \(1995\)](#) and [Gomme and Lkhagvasuren \(2015\)](#).

Yet, the body of direct and indirect evidence on the cyclical nature of job search effort points against procyclical search effort. This raises the question: why are workers not exerting more search effort when the returns to search increase? In other words, why is job search effort not procyclical as predicted by models with matching frictions?

A number of researchers suggest that income effects and, more broadly, wealth effects are important drivers of job search effort. [Faberman and Kudlyak \(2016\)](#) find that those with the lowest returns to search tend to exert more job search effort, suggesting that income effects dominate substitution effects in job search effort. [Leyva \(2018\)](#) argues that his evidence of countercyclical value of a job may result from income effects. [DeLoach and Kurt \(2013\)](#) document that the elasticity of job search effort with respect to wealth is negative and three times, in absolute value, the elasticity of effort with respect to market tightness. [Mukoyama, Patterson and Şahin \(2018\)](#) document that search effort is negatively correlated with several measures of aggregate wealth including housing and stock prices.

In this paper, we assess whether introducing income effects into a dynamic general equilibrium model featuring matching frictions may reverse the prediction of procyclical effort. We introduce income effects by deviating from the assumption of risk-neutral household used in the canonical matching model. We employ the constant relative risk aversion utility specification and control the magnitude of income effects by controlling the households' attitude toward risk. A higher degree of risk aversion implies that the marginal utility of consumption falls at a higher rate, inducing stronger income effects. In expansions, higher income allows households to enjoy more consumption. In the presence of income effects, exerting more search effort to increase the job-finding probability becomes less desirable because of the diminishing marginal utility of consumption. As a result, the household may lower search effort to enjoy more leisure –which is valued higher in terms of goods. Both if and the extent to which search effort drops in expansions and increases in recessions depend on the magnitude of income effects.

For the remaining building blocks of the model, we draw on standard assumptions in the macro-labor literature: (i) a Cobb-Douglas matching function determines the number of matches, (ii) Nash bargaining determines wages, and (iii) workers share consumption within households (e.g., [Merz, 1995](#); [Andolfatto, 1996](#); [Pissarides, 2000](#), Chapter 3.4).

We find that the cyclical behavior of search effort depends on the magnitude of income effects. If income effects are absent or low, search effort is procyclical, at odds

with the evidence. On the contrary, if income effects are moderate or high, search effort is, respectively, acyclical or countercyclical, consistent with the evidence. Yet, income effects also alter the behavior of other variables. If income effects are moderate, both unemployment and labor market tightness are acyclical. In this case, [Shimer's \(2005\)](#) critique is acute; and neither [Hagedorn and Manovskii's \(2008\)](#) calibration strategy nor [Pissarides's \(2009\)](#) fixed-matching-costs technique can overcome [Shimer's](#) critique. If income effects are high, unemployment is procyclical and vacancies are countercyclical, at odds with conventional wisdom and data.

Concisely, in our model, income effects improve the predicted cyclical behavior of search effort but worsen that of unemployment and vacancies. [Mukoyama, Patterson and Şahin \(2018\)](#) show that countercyclical search effort reduces the volatility of unemployment by amplifying the congestion externalities of matching frictions.<sup>1</sup> In our model, this channel is also at work; however, income effects essentially propagate through the wage. Depending on the magnitude of income effects, a version of our model with *exogenous* search effort also predicts acyclical or procyclical unemployment. Because of income effects, in expansions, workers demand higher wages to forgo their leisure, severely undermining job creation. In other words, income effects make the opportunity cost of working procyclical. And [Chodorow-Reich and Karabarbounis \(2016\)](#) show that a model with procyclical opportunity cost of working (as found in data) generates lower unemployment volatility relative to a model with acyclical opportunity cost.

We subject our findings to numerous robustness checks. Two stand out. First, if we depart from Nash bargaining and assume *fully* rigid wages (i.e., constant wages), we close the channel through which income effects influence wages. In this case, income effects imply strongly countercyclical unemployment and countercyclical search effort, consistent with the direct evidence in [Shimer \(2004\)](#) and [Mukoyama, Patterson and Şahin \(2018\)](#) and the indirect evidence in [Faberman and Kudlyak \(2016\)](#) and [Hornstein and Kudlyak \(2017\)](#). But if we slightly deviate from full wage rigidity, the model's predictions are similar to those we obtain under Nash bargaining. Given that the wages of the newly hired workers are strongly procyclical in data ([Pissarides, 2009](#)), assuming constant wages is not plausible. Second, if we assume that the opportunity cost of working only includes unemployment benefits but not leisure, the model delivers reasonable responses of unemployment and search effort. In this case, income effects do not play directly a role in the wage formation but still directly affect job search effort. Yet, this scenario requires an implausible replacement rate (greater than 85%) and an excessively high risk aversion. In summary, accounting for the income effects in the model does not generate the cyclical patterns observed in the data unless we make unsound assumptions. Thus, our results cast doubts on the role of income effects in generating the cyclical patterns of job search effort observed in the data.

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<sup>1</sup>[Mukoyama, Patterson and Şahin \(2018\)](#) build a model with a generalized matching function and endogenous job search effort. They show that if the elasticity of job search effort with respect to labor market tightness is negative, the volatilities drop.

In the next section, we present our model. The model builds on assumptions standard in the macro-labor literature and features matching frictions and endogenous search effort. In Section 3., we present our calibration strategy. In Section 4., using the results of our numerical simulations, we evaluate the roles of income effects and of endogenous search effort. In the same section, we study the robustness of our results to alternative calibrations and specifications of the model. In Section 5., we offer some concluding remarks.

## 2. The Model

Time is discrete and denoted by  $t$ . There is a representative household and a representative firm. The representative household consists of employed and unemployed workers. Unemployed workers exert effort in searching for jobs. When matched with a vacant job, unemployed workers start working in the following period. Employed workers produce, bargain wages with the firm, but do not search on-the-job.<sup>2</sup> Each period, a constant fraction of jobs, denoted by  $\delta$ , are exogenously destroyed. The law of motion of employment is

$$n_{t+1} = (1 - \delta)n_t + m_t, \quad (1)$$

where  $n_t$  is the employment rate and  $m_t$  is new matches. Vacancies,  $v_t$ , and total search effort,  $\bar{e}_t u_t$ , determine new matches via a matching function

$$m_t = \sigma v_t^\eta (\bar{e}_t u_t)^{1-\eta}, \quad (2)$$

where  $\sigma$  is a scale parameter and  $0 < \eta < 1$  is the elasticity of the matching function with respect to vacancies. Total search effort is the product of the average job search effort,  $\bar{e}_t$ , and the unemployment rate,  $u_t \equiv 1 - n_t$ .

We define the labor market tightness as the vacancy-unemployment ratio

$$\theta_t \equiv \frac{v_t}{u_t}. \quad (3)$$

We write the job-finding probability per unit of search effort as

$$f(\bar{e}_t, \theta_t) = \sigma \left( \frac{\theta_t}{\bar{e}_t} \right)^\eta, \quad (4)$$

and the job-filling probability as

$$\mu(\bar{e}_t, \theta_t) = \sigma \left( \frac{\theta_t}{\bar{e}_t} \right)^{\eta-1}. \quad (5)$$

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<sup>2</sup>We abstract from on-the-job search because we focus on the search effort of unemployed workers.

## 2.1. The Household

The household derives utility from consumption and disutility from exerting effort in job search and from working in firms.<sup>3</sup> It pools consumption and discounts future utility by  $0 < \beta < 1$ . In maximizing the lifetime utility, the household takes the employment rate, the labor market tightness, the average job search effort, the dividend income,  $d_t$ , and the wage,  $w_t$ , as given and chooses the paths of consumption,  $c_t$ , and job search effort,  $e_t$ . The optimization problem is

$$V_t = \max_{\{c_t, e_t\}} \left[ \frac{c_t^{1-\gamma} - 1}{1-\gamma} - \frac{\psi}{\zeta} e_t^\zeta (1 - n_t) - \chi n_t + \beta E_t V_{t+1} \right],$$

subject to

$$\begin{aligned} c_t &\leq w_t n_t + d_t, \\ n_{t+1} &\leq (1 - \delta) n_t + e_t f(\bar{e}_t, \theta_t) (1 - n_t). \end{aligned}$$

$V_t$  is the lifetime utility of the household. The parameters  $\psi$  and  $\zeta > 1$  measure the scale and the convexity of the disutility from search effort, respectively, and  $\chi$  scales the disutility from working. The parameter  $\gamma \geq 0$  is the coefficient of relative risk aversion and measures the curvature of the utility of consumption.

The first-order condition for search effort is

$$\psi e_t^{\zeta-1} = \beta f(\bar{e}_t, \theta_t) E_t V_{n,t+1}, \quad (6)$$

where  $V_{n,t}$  is the value of an additional employed worker to the household. It is given by

$$V_{n,t} = w_t c_t^{-\gamma} + \frac{\psi}{\zeta} e_t^\zeta - \chi + (1 - \delta - e_t f(\bar{e}_t, \theta_t)) \beta E_t V_{n,t+1}. \quad (7)$$

Eq. (6) equates the marginal disutility from exerting effort to the expected discounted value of an additional employed worker to the household multiplied by the probability the worker finds a job. The value of an additional employed worker to the household is the sum of the continuation value of  $V_{n,t}$  and three components: wage income of a worker valued at the marginal utility, the increase in the utility (because the worker stops searching), and the decrease in the utility (because the worker starts working).

If  $\gamma > 0$ , we deviate from the risk-neutrality assumption and introduce income effects into the model. In this case, income affects both the level and the marginal utility

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<sup>3</sup>We follow [Andolfatto \(1996\)](#) and [Merz \(1995\)](#) by assuming that working generates disutility because of forgone leisure. An equally valid assumption could be that workers also forgo unemployment benefits when they start working. We consider this case in our robustness checks in Section 4.3. Furthermore, we assume that exerting search effort also generates disutility. If we assume that search effort deprives workers of their income rather than their leisure, Nash bargaining implies that search effort is proportional to labor market tightness. In this case, income effects do not play a direct role in search effort.

of consumption. In good times, due to the decreasing marginal utility of consumption, *ceteris paribus*, the household increases leisure by decreasing search effort. Thus, income effects render a negative correlation between consumption and effort possible.

## 2.2. The Firm

The firm produces a homogeneous good with the technology

$$y_t = z_t n_t, \quad (8)$$

where  $y_t$  is output and  $z_t$  is the productivity. The firm pays  $\kappa$  per vacancy. It takes the employment rate, the job-filling probability, and the wage as given and chooses the number of vacancies to maximize its value,  $J_t$ . The optimization problem is

$$J_t = \max_{v_t} (z_t n_t - w_t n_t - \kappa v_t + \beta \mathbb{E}_t [(c_{t+1}/c_t)^{-\gamma} J_{t+1}]),$$

subject to

$$n_{t+1} = (1 - \delta)n_t + \mu(\bar{e}_t, \theta_t)v_t.$$

The future value of the firm is discounted by the term  $\beta (c_{t+1}/c_t)^{-\gamma}$  because the household owns the firm. The first order condition for vacancies is

$$\beta \mu(\bar{e}_t, \theta_t) \mathbb{E}_t [(c_{t+1}/c_t)^{-\gamma} J_{n,t+1}] = \kappa, \quad (9)$$

where  $J_n$  is the value of an additional worker to the firm. It is given by

$$J_{n,t} = z_t - w_t + (1 - \delta)\beta \mathbb{E}_t [(c_{t+1}/c_t)^{-\gamma} J_{n,t+1}]. \quad (10)$$

Eq. (9) states that firms open vacancies until the increase in the continuation value of the firm equals the cost of opening vacancies. The value of an additional worker to the firm is the sum of output per worker net of the wage and the continuation value of  $J_{n,t}$ .

## 2.3. The Wage

Workers and firms bargain over wages such that the bargained wage maximizes the Nash product. Namely,

$$w_t = \arg \max \left( V_{n,t}/c_t^{-\gamma} \right)^\phi J_{n,t}^{1-\phi}, \quad (11)$$

where the parameter  $0 < \phi < 1$  measures the worker's bargaining power and we convert the value of an additional employed worker to the household,  $V_{n,t}$ , to goods terms by dividing it by the marginal utility of consumption,  $c_t^{-\gamma}$ . The equilibrium wage is

$$w_t = \phi \left( z_t + e_t f(\bar{e}_t, \theta_t) \frac{\kappa}{\mu(\bar{e}_t, \theta_t)} \right) + (1 - \phi) \left( \chi - \frac{\psi}{\zeta} e_t^\zeta \right) \frac{1}{c_t^{-\gamma}}. \quad (12)$$

Thus, the wage increases with (i) the productivity of the match; (ii) the hiring costs because by accepting the match, the worker is providing savings for the firm; and (iii) the net disutility of working measured in goods terms.

## 2.4. Equilibrium

In equilibrium, each worker chooses the same job search effort and, thus,  $e_t = \bar{e}_t$ . To close the model, we write the resource constraint

$$y_t = c_t + \kappa v_t, \quad (13)$$

which states that output equals the sum of consumption and the costs of job creation.

## 3. Calibration

Table 1 summarizes our calibration choices. We calibrate the model to monthly data. In the benchmark calibration, we target an annual discount rate of 4.91%, implying that  $\beta = 0.996$ . Drawing on [Shimer's \(2012\)](#) measurement, we set the employment exit probability,  $\delta$ , to 3.6%. We fix  $\zeta = 2$ , implying a quadratic disutility for job search effort ([Gomme and Lkhagvasuren, 2015](#); [Yashiv, 2000](#)). Drawing on [Petrngolo and Pissarides \(2001\)](#), we set  $\eta = 0.6$ .<sup>4</sup> We also set  $\phi = 0.6$ .

Table 1: Benchmark Calibration

Discount factor:	$\beta = 0.996$
Rate of job destruction:	$\delta = 0.036$
Matching function elasticity:	$\eta = 0.6$
Workers' bargaining power:	$\phi = 0.6$
Convexity of search effort disutility:	$\zeta = 2$
Relative risk aversion:	$\gamma \in [0, 4]$
Autocorrelation of productivity:	0.98
Standard deviation of productivity shock:	0.005

Because the value of  $\gamma$  is central to our analysis, we assume a variety of values ranging from zero to four.<sup>5</sup> For each value of  $\gamma$ , we pin down four parameters  $\chi$ ,  $\psi$ ,  $\sigma$ , and  $\kappa$  using four steady-state targets. By dropping the time subscript  $t$  from variables to denote the steady-state values, Table 2 summarizes the four steady-state targets. Namely, we target (i) an unemployment rate of 5.7%, (ii) a search effort value of 1, (iii) a tightness value of 0.72 ([Pissarides, 2009](#)), and (iv) a ratio of total hiring costs to output of 1%

<sup>4</sup>Eqs. (4) and (A.3) imply that for a constant marginal utility of consumption, the elasticity of the job-finding rate with respect to tightness is  $\eta(1 - 1/\zeta)$ . As  $\zeta$  is set at 2,  $\eta = 0.6$  implies that the targeted elasticity is within the range [Petrngolo and Pissarides](#) report.

<sup>5</sup>In one experiment in Section 4.3.4., we increase the upper bound of  $\gamma$  to 20.



(Christiano, Eichenbaum and Trabandt, 2016).<sup>6</sup>

Table 2: Imposed Steady-State

Employment:	$n = 0.943$
Labor market tightness:	$\theta = 0.72$
Job search effort per worker:	$e = 1$
Total matching costs:	$\kappa v = 0.01y$

We study shocks to aggregate productivity, measured by  $z_t$ . For that, we assume that the aggregate productivity follows a first-order autoregressive process with a correlation coefficient of 0.98 and a standard deviation of the innovations of 0.005 (Shimer, 2010).

## 4. Results

Throughout this section, we use the log-linear version of the model to study how income effects alter the responses of key labor market variables to a positive productivity shock. We focus on the volatility and cyclicity of labor market tightness, unemployment, and job search effort. As measures of volatility and cyclicity, we use the magnitude and the sign of (cumulative) responses.

### 4.1. The Role of Income Effects

Figure 1 plots the responses of labor market tightness, unemployment, and job search effort to a positive productivity shock for three different levels of risk aversion,  $\gamma$ . First, as in the canonical matching model, we assume linear utility ( $\gamma = 0$ ) and, thus, ignore income effects. In this case, the model predicts procyclical search effort, opposing the evidence reported in DeLoach and Kurt (2013), Leyva (2018), Mukoyama, Patterson and Şahin (2018), and Shimer (2004).<sup>7</sup> The model also predicts procyclical tightness and countercyclical unemployment.

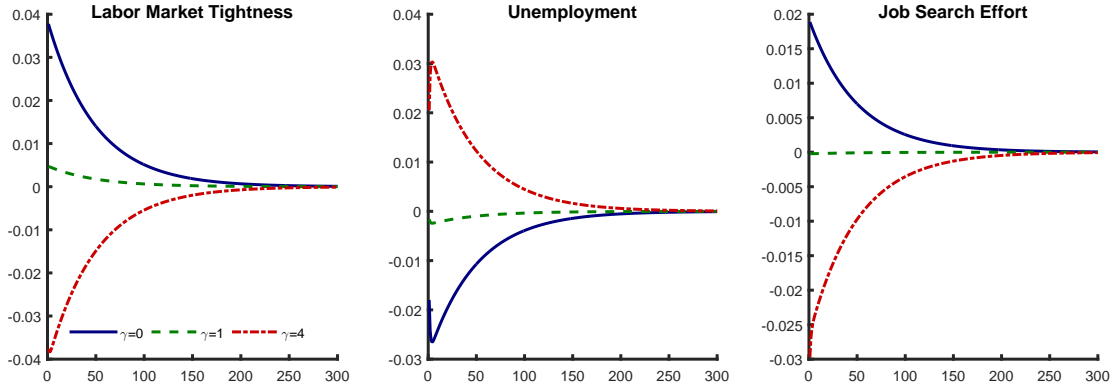
In the model, after log-linearization, search effort is proportional to the difference between labor market tightness and the marginal utility of consumption:

$$\zeta \hat{e}_t = \hat{\theta}_t - \gamma \hat{c}_t, \quad (14)$$

<sup>6</sup>Our calibration implies that in steady state working gives higher disutility than searching, i.e.,  $\chi > \psi e^\xi / \zeta$ .

<sup>7</sup>It also opposes the indirect evidence of countercyclical job search effort in Faberman and Kudlyak (2016) and Hornstein and Kudlyak (2017). And it opposes the evidence suggesting acyclical average job search effort in Gomme and Lkhagvasuren (2015).

Figure 1: Impulse Response Functions



*Note:* The figure plots the impulse response functions. The horizontal axis measures time in months. The vertical axis measures the logarithmic/percentage deviation from the steady state. The impulse is a one standard deviation increase in productivity. Solid lines represent the responses in the case of  $\gamma = 0$ , the dashed lines represent the responses in the case of  $\gamma = 1$ , and the dot-dashed lines represent the responses in the case of  $\gamma = 4$ .

where a hat denotes the log-deviation from the steady state.<sup>8</sup> Therefore, in the absence of income effects,  $\gamma = 0$ , procyclical labor market tightness entails procyclical search effort.

Next, as is standard in many macroeconomics models, we assume log-utility ( $\gamma = 1$ ) and, thus, introduce somewhat moderate income effects. In this case, the model predicts acyclical search effort. The reason for acyclical search effort is particularly congruent with the evidence in DeLoach and Kurt (2013). In DeLoach and Kurt, job search effort is acyclical due to two counteracting forces: procyclical job-finding probability and wealth effects. In the model, a positive productivity shock increases labor income and consumption thanks to higher wages and employment. Assuming moderate income effects ( $\gamma = 1$ ), the fall in the marginal utility of consumption neutralizes the effect of higher job-finding probability, which generates acyclical job search effort (see Eq. 14). Yet, the model also predicts acyclical labor market tightness and unemployment. Shimer (2005) shows that the volatilities of unemployment and labor market tightness in matching models fall short of their empirical counterparts. Income effects exacerbate this volatility problem.

To see the consequences of strong income effects, we then consider an arbitrarily large coefficient of relative risk aversion,  $\gamma = 4$ . In this case, the model has an aberrant behavior. It predicts countercyclical search effort, in line with the direct evidence in Shimer (2004) and Mukoyama, Patterson and Şahin (2018) and with the indirect evidence in Faberman and Kudlyak (2016) and Hornstein and Kudlyak (2017). But it also

<sup>8</sup>See Appendix A for the derivation of Eq. (14).

predicts procyclical unemployment and countercyclical tightness, at odds with the evidence and conventional wisdom. To see the reason for procyclical unemployment and countercyclical tightness note that the equilibrium wage,

$$\hat{w}_t = \frac{\phi}{w} \left( z\hat{z}_t + \frac{(\zeta - 1)\kappa\theta}{\zeta} \hat{\theta}_t \right) + \gamma \frac{(1 - \phi)\chi c^\gamma}{w} \hat{c}_t, \quad (15)$$

is increasing in income effects.<sup>9</sup> A positive productivity shock shifts wages upwards, thereby increasing labor income and consumption. If  $\gamma = 0$ , the increase in consumption has no effect on wages. If  $\gamma > 0$ , however, the marginal utility of consumption decreases with the level of consumption. Accordingly, the worker demands higher wages to trade leisure for consumption. In the case of strong income effects ( $\gamma = 4$ ), the upward pressure in wages is so high that firms decrease vacancies. It follows that the labor market tightness becomes countercyclical and unemployment procyclical.

The magnitude of income effects determines the behavior of the labor market. To illustrate this claim, in Figure 2, we plot the impact of  $\gamma$  on the 8-year-cumulative responses of  $\theta_t$ ,  $u_t$ , and  $e_t$  to a positive productivity shock. Low income effects (low  $\gamma$ ) imply higher volatility, but also procyclical search effort. A small interval of  $\gamma$  implies qualitatively-reasonable but also dull responses. High income effects (high  $\gamma$ ) imply countercyclical job search effort, but also qualitatively-unreasonable responses of unemployment and tightness. Concisely, in our model, income effects bring the predicted cyclical behavior of search effort closer to data; but income effects also worsen the predicted cyclical behavior of unemployment and vacancies.

## 4.2. The Role of Search Effort

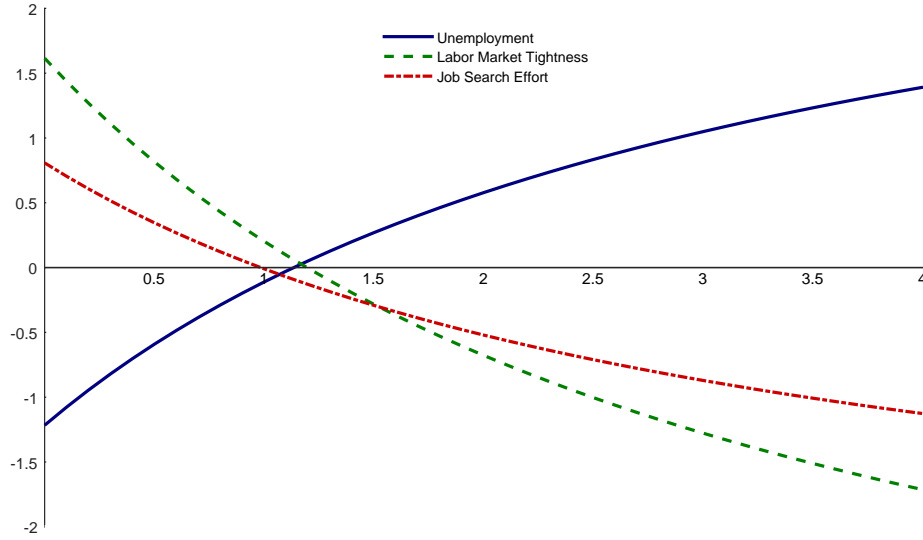
Because models with matching frictions do not usually feature endogenous search effort, we now consider a variant of the model with exogenous search effort.<sup>10</sup> Figure 3 plots the 8-year-cumulative responses of unemployment and labor market tightness as functions of  $\gamma$  in the models with endogenous and exogenous search effort. As before, the impulse is a positive productivity shock. The responses in both models coincide in the neighborhood of  $\gamma = 1$ , i.e., in models assuming log-utility, search effort plays an insignificant role. In the case of linear utility, the response of unemployment nearly triples and the response of labor market tightness doubles by endogenizing search effort. As  $\gamma$  increases, unemployment and tightness first become less cyclical and then become qualitatively-inconsistent with data. Consequently, the finding in Gomme and Lkhagvasuren (2015) that endogenous search effort amplifies the responses of labor market variables only applies to the case of weak income effects.

Figure 3 highlights another important finding. The counterfactual results – that un-

<sup>9</sup>Eq. (15) is in log-linear form. Recall that variables without a time subscript denote values in the steady state.

<sup>10</sup>Appendix B1. outlines the model with exogenous search effort.

Figure 2: The Role of Income Effects



*Note:* The horizontal axis measures the degree of income effects,  $\gamma \in [0, 4]$ . The vertical axis measures the 8-year cumulative responses to a one standard deviation increase in productivity. The solid line represents unemployment; the dashed line represents labor market tightness; and the dot-dashed line represents job search effort.

employment (tightness) is procyclical (countercyclical)– are not induced by endogenous search effort but by income effects. The reason is that the wage still positively depends on income effects even if search effort is exogenous.<sup>11</sup> Therefore, if  $\gamma$  is large enough, firms open less vacancies when productivity rises. Endogenous search effort just slightly reduces the threshold value of  $\gamma$  that generates procyclical unemployment and countercyclical labor market tightness.

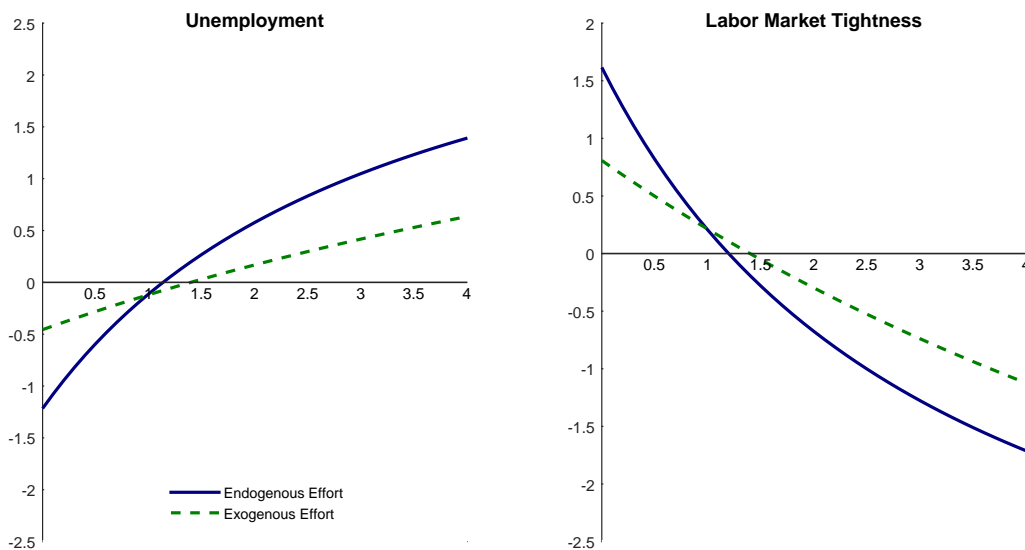
### 4.3. Robustness Checks

We conduct several experiments to assess the robustness of our results. Recall that search effort is acyclical only if Shimer’s (2005) critique is rather acute. For this reason, we start by experimenting with solutions shown in the literature that help to overcome Shimer’s critique. In particular, we consider fixed matching costs a la Pissarides (2009), the alternative calibration proposal of Hagedorn and Manovskii (2008), and wage rigidity.<sup>12</sup> Then, we study the impact of introducing unemployment benefits and capital into the model. As a summary statistic, we use the 8-year cumulative responses of  $\theta_t$ ,  $u_t$ , and  $e_t$  to a positive productivity shock. Table 3 summarizes the results of each experiment for three levels of relative risk aversion:  $\gamma = \{0, 1, 4\}$ .

<sup>11</sup>In the case of exogenous search effort, the wage is given by  $\hat{w}_t = \phi [z\hat{z}_t + \kappa\theta\hat{\theta}_t] / w + \gamma(1 - \phi)\chi c^\gamma \hat{c}_t / w$ .

<sup>12</sup>See Ljungqvist and Sargent (2017) for a detailed analysis of these and other reconfigurations.

Figure 3: Endogenous vs Exogenous Search Effort



Note: The horizontal axis measures the degree of income effects,  $\gamma \in [0, 4]$ . The vertical axis measures the 8-year cumulative responses to a one standard deviation increase in productivity. The solid (dashed) lines represent the responses in the model with endogenous (exogenous) search effort.

Table 3: Sensitivity Analysis

	$\gamma = 0$			$\gamma = 1$			$\gamma = 4$		
	$\theta$	$u$	$e$	$\theta$	$u$	$e$	$\theta$	$u$	$e$
Benchmark	1.62	-1.22	0.81	0.20	-0.11	-0.01	-1.71	1.39	-1.13
$v\mu(e, \theta)H = 0.0033y$	1.77	-1.31	0.82	0.23	-0.13	-0.01	-1.77	1.41	-1.11
$v\mu(e, \theta)H = 0.005y$	1.86	-1.36	0.83	0.25	-0.14	-0.01	-1.80	1.43	-1.09
$\phi = 0.05$	27.12	-20.42	13.56	0.09	-0.03	-0.06	-3.36	2.58	-1.82
$\kappa v = 0.002y$	8.08	-6.08	4.04	0.12	-0.05	-0.05	-2.93	2.26	-1.62
$\rho = 0.99$	2.06	-1.52	0.93	0.57	-0.35	0.06	-1.47	1.26	-1.13
$\rho = 1$	33.60	-19.64	1.76	30.45	-16.36	-2.22	25.05	-10.75	-9.02
$b = 0.4$	1.62	-1.22	0.81	0.78	-0.54	0.27	-0.71	0.66	-0.71
Model with Capital	1.62	-1.22	0.81	0.73	-0.51	0.25	-0.43	0.44	-0.53

Note: The table gives the 8-year cumulative responses of  $\theta_t$ ,  $u_t$ , and  $e_t$  to a one standard deviation increase in productivity. The panels only differ in the degree of risk aversion,  $\gamma = \{0, 1, 4\}$ . The first row presents the results under the benchmark calibration. The second and third rows present the results for the case of [Pissarides's](#) fixed matching costs: if  $H = 0.09$ , fixed matching costs represent 33% of total hiring costs; and if  $H = 0.14$ , fixed matching costs represent 50% of total hiring costs. The fourth and fifth rows present the results for the case of [Hagedorn and Manovskii's](#) calibration strategy:  $\phi = 0.05$  implies a very low value of workers' bargaining power; and  $\kappa v = 0.002y$  implies an augmented value of leisure. The fifth and sixth rows present the results with wage rigidity. The seventh row presents the results with a replacement rate of unemployment benefits slightly above 40%. The last row presents the results of the model with capital.

### 4.3.1. Fixed Matching Costs

Pissarides (2009) shows that the addition of fixed matching costs raises the elasticity of labor market tightness. We embed this idea into our model by rewriting the value of the firm,  $J_t$ , as

$$J_t = \max_{v_t} \left( z_t n_t - w_t n_t - (\kappa + \mu(\bar{e}_t, \theta_t)H)v_t + \beta E_t \left[ (c_{t+1}/c_t)^{-\gamma} J_{t+1} \right] \right),$$

in which we assume that firms pay  $\kappa$  per vacancy and  $H$  per match.<sup>13</sup> Following Pissarides, to accommodate  $H$ , we adjust  $\kappa$  such that we keep the steady-state value of hiring costs,  $v(\kappa + \mu(e, \theta)H)$ , unchanged.<sup>14</sup> With lower  $\kappa$  and higher  $H$ , hiring costs are less responsive to market conditions, thus inducing firms to open more vacancies when productivity rises. We experiment with two values of  $H$ . Consistent with Pissarides, we find that fixed matching costs enhance the volatilities of labor market variables in the model. Nevertheless, in the model with fixed matching costs we obtain acyclical or countercyclical job search effort together with the severely deteriorated cyclical behavior of unemployment and labor market tightness, as in the baseline model.

### 4.3.2. Hagedorn and Manovskii's Calibration Strategy

In the next two experiments, we focus on the workers' bargaining power and on the value of non-market activity. Hagedorn and Manovskii (2008) show that decreasing workers' bargaining power and increasing the value of non-market activities generates volatile labor market variables. By decreasing workers' bargaining power,  $\phi$ , the wage becomes less sensitive to changes in productivity and labor market tightness, rendering labor costs also less sensitive and increasing the incentives to open vacancies. In our model, the value of non-market activities indirectly corresponds to the disutility of working,  $\chi$ .<sup>15</sup> By increasing  $\chi$ , the equilibrium value of an additional worker to the household,  $V_n$ , falls. Because of Nash bargaining, a lower  $V_n$  implies a lower  $J_n$  (the equilibrium value of an additional worker to the firm). Therefore, small changes in productivity have large effects on  $J_n$ , increasing the incentives to open vacancies.

<sup>13</sup>Accordingly, the free-entry condition, Eq. (9), is replaced by

$$\beta E_t \left[ (c_{t+1}/c_t)^{-\gamma} J_{n,t+1} \right] \mu(\bar{e}_t, \theta_t) = \kappa + \mu(\bar{e}_t, \theta_t)H;$$

the wage equation, Eq. (11), is replaced by

$$w_t = \phi \left[ z_t + e_t f(\bar{e}_t, \theta_t) \left( \frac{\kappa}{\mu(\bar{e}_t, \theta_t)} + H \right) \right] + (1 - \phi) \left( \chi - \frac{\psi}{\zeta} e_t^\zeta \right) \frac{1}{c_t^{-\gamma}},$$

where we assume, as Pissarides (2009), that  $H$  is sunk at the time of bargaining; and finally the resource constraint, Eq. (13), is replaced by

$$y_t = c_t + (\kappa + \mu(\bar{e}_t, \theta_t)H)v_t.$$

<sup>14</sup>In the way indicated, the steady-state values of labor market tightness and the wage remain the same.

<sup>15</sup>In our calibration, we use  $\chi$  to target relative hiring costs,  $\kappa v/y$ . A lower weight implies a higher  $\chi$ .

We find that the logic above only applies to the case of small income effects.<sup>16</sup> Interestingly, in the case of log-utility,  $\gamma = 1$ , [Hagedorn and Manovskii](#)'s proposal decreases the volatilities of unemployment and labor market tightness even further. And if income effects are stronger, unemployment is even more procyclical than in the benchmark. The reason is that by reducing  $\phi$  and increasing  $\chi$ , income effects has a higher weight on wages (recall that the last term on the right hand side of the wage equation, Eq. 15, is  $\gamma(1 - \phi)\chi c^\gamma \hat{c}_t/w$ ). If  $\gamma = 0$ , income effects vanish and, thus, the calibration of [Hagedorn and Manovskii](#) generates highly volatile and countercyclical unemployment. But if  $\gamma > 0$ , their calibration increases the weight of income effects in the wage dynamics. Therefore, wages tend to increase more and reduce job creation.

### 4.3.3. Wage Rigidity

We have shown that income effects play a determinant role in the evolution of wages. If income effects are high, workers demand higher wages to exchange their leisure for additional consumption, thereby undermining the value of job creation. Yet, this follows from Nash bargaining. In this section, we deviate from this assumption and instead assume rigid wages. In particular, as in [Shimer \(2010\)](#), wages are a weighted average of lagged wages and the wage obtained through Nash bargaining,  $w_t^*$ :

$$w_t = \rho w_{t-1} + (1 - \rho)w_t^*, \quad (16)$$

where  $0 < \rho \leq 1$  governs the degree of rigidity.<sup>17</sup> Fully rigid wages amplify the volatility of unemployment and labor market tightness (as in, for example, [Shimer, 2005](#) and [Hall, 2005](#)). We also find that if  $\gamma = 1$  or higher, job search effort is countercyclical without compromising the qualitative responses of unemployment and tightness.<sup>18</sup> Thus, if wages are fully rigid, the model is in line with the direct evidence in [Shimer \(2004\)](#) and [Mukoyama, Patterson and Şahin \(2018\)](#) and with the indirect evidence in [Faberman and Kudlyak \(2016\)](#) and [Hornstein and Kudlyak \(2017\)](#).

The high volatility of unemployment and vacancies follows from the increase in incentives to open vacancies. If wages are constant, following an increase in productivity, firms do not have to share the increased surplus with workers. The natural response

<sup>16</sup>[Chodorow-Reich and Karabarbounis \(2016\)](#) show that the calibration strategy of [Hagedorn and Manovskii \(2008\)](#) only improves the predictions of the canonical matching model because the opportunity cost of working is constant in the model. In our model, income effects imply that the opportunity cost of working is procyclical, consistent with the evidence documented by [Chodorow-Reich and Karabarbounis](#).

<sup>17</sup>In this case, the wage equation, Eq. (11), is replaced by

$$w_t^* = \phi \left( z_t + e_t f(\bar{e}_t, \theta_t) \frac{\kappa}{\mu(\bar{e}_t, \theta_t)} \right) + (1 - \phi) \left( \chi - \frac{\psi}{v} e_t^v \right) \frac{1}{c_t^{-\gamma}} + (1 - \delta_n - e_t f(\bar{e}_t, \theta_t)) \beta E_t [(c_{t+1}/c_t)^{-\gamma} (w_{t+1}^* - w_{t+1})].$$

<sup>18</sup>Job search effort is procyclical if  $\gamma < 0.41$ .

of firms is, thus, to open more vacancies than under flexible wages, leading to lower unemployment. To understand the dynamics of search effort under wage rigidity, recall that in equilibrium the marginal disutility of search effort equals the expected discounted value of an additional employed worker to the household multiplied by the probability the worker finds a job (Eq. 6). The job-finding probability is highly procyclical. But, in good times, the value of an additional worker to the household tends to decrease because the same wage evaluated in utility terms falls. If income effects are strong enough, the second effect dominates the first.

Hence, rigid wages allows the model with income effects to be reconciled with the some of the evidence. The drawback is that the model requires wages to be constant ( $\rho = 1$ ). For example, even if  $\rho = 0.99$ , the results are not much different from the benchmark. As shown in the literature surveyed by Pissarides (2009), the wage of new hires –those that determine unemployment fluctuations in the model– changes almost one to one with productivity, whereas  $\rho = 1$  implies that the wage of new hires is constant.

#### 4.3.4. Unemployment Benefits

In our benchmark model, we assume that the opportunity cost of working is only forgone leisure (which is in utility terms). Now we assume that, as in Nakajima (2012) and Chodorow-Reich and Karabarbounis (2016), the opportunity cost of working also includes forgone unemployment benefits (which are in goods terms). To see why this can be relevant for our results, let  $\tau_t$  denote lump-sum taxes and  $b$  denote unemployment benefits.<sup>19</sup> The households' budget constraint is then

$$c_t \leq w_t n_t + d_t + b(1 - n_t) - \tau_t,$$

implying that the wage is given by

$$w_t = \phi \left( z_t + e_t f(\bar{e}_t, \theta_t) \frac{\kappa}{\mu(\bar{e}_t, \theta_t)} \right) + (1 - \phi)b + (1 - \phi) \left( \chi - \frac{\psi}{\zeta} e_t^\xi \right) \frac{1}{c_t^{-\gamma}}. \quad (17)$$

Unemployment benefits,  $b$ , reduce the required value of  $\chi$  that pins down relative hiring costs in our calibration ( $\kappa v/y = 0.01$  in our benchmark), which, in turn, reduces the relevance of income effects for the dynamics of wages (the last term in Eq. 15). By reducing wage fluctuations relative to the benchmark, introducing unemployment benefits into the model increases the incentives for job creation. At the same time, the equation driving search effort (Eq. 14) remains unchanged. Thus, unemployment benefits may reconcile the model with the evidence.

We conduct two experiments with unemployment benefits. In the first one, we assume that  $b = 0.4$ , implying a replacement rate of unemployment benefits just slightly

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<sup>19</sup>We assume that the government runs a balanced budget every period, implying  $\tau_t = b_t(1 - n_t)$ .



above 40% (which is close to the one in [Shimer, 2005](#)). In the case of  $\gamma = 1$ , assuming that  $b = 0.4$  qualitatively changes the behavior of search effort: effort rises when productivity rises, at odds with the evidence. In other cases, the models with  $b = 0$  (benchmark) and  $b = 0.4$  behave similarly. Assuming that  $b = 0.4$  only increases the thresholds after which search effort becomes countercyclical and after which unemployment becomes procyclical.

In our second experiment, we consider the extreme case that the opportunity cost of working only includes forgone unemployment benefits. In this case,  $\chi = 0$  and we pin down the value of  $b$  to target relative hiring costs. Thus, income effects do not play any direct role for wage dynamics. We present the implications of this experiment in [Figure 4](#), where we plot the impact of  $\gamma$  on the 8-year-cumulative responses of  $\theta_t$ ,  $u_t$ , and  $e_t$  to a positive productivity shock. We find that the range of values that generates acyclical or countercyclical search effort without compromising the cyclical behavior of unemployment is extremely large: search effort becomes countercyclical if  $\gamma > 6.2$  and unemployment only becomes procyclical if  $\gamma > 39.9$ . Furthermore, the 8-year-cumulative response of labor market tightness is almost unresponsive to the degree of risk aversion.<sup>20</sup> Thus, if  $\chi = 0$ , the model's predictions are much more in line with the evidence. The problem, however, is that this only occurs if  $\gamma$  is very high (around nine) and the replacement rate of unemployment benefits exceeds 86%, much higher than its empirical counterpart.

#### 4.3.5. The Model with Capital

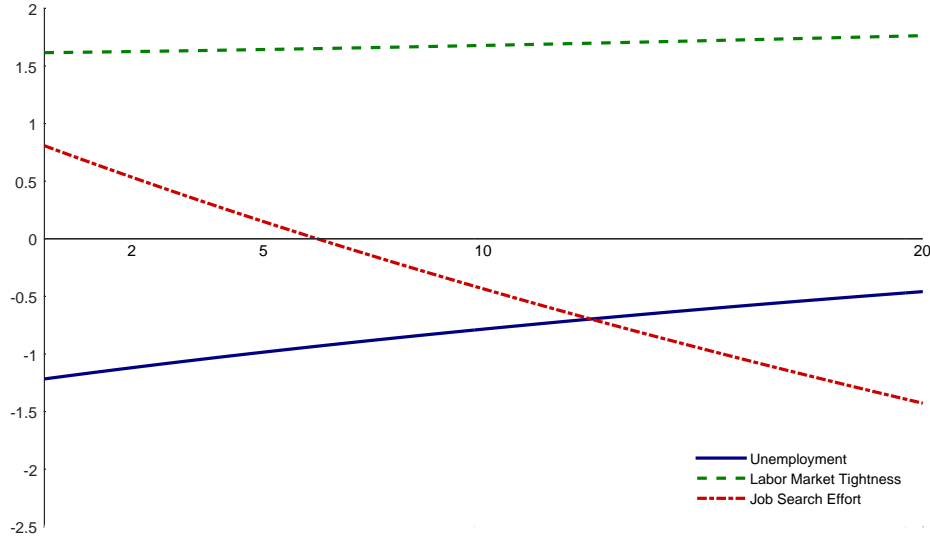
In our last experiment, to make our model closer to the standard real business cycle model, we introduce capital.<sup>21</sup> The results resemble those of the model with unemployment benefits assuming  $b = 0.4$ . Introducing capital only increases the thresholds after which search effort becomes countercyclical and after which unemployment becomes procyclical. In expansions households increase their capital stock by delaying consumption. This, in turn, reduces the change in the marginal utility of consumption (relative to the benchmark) and, consequently, the relevance of income effects both for the wage and job search effort.<sup>22</sup>

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<sup>20</sup>In this extreme case of  $\chi = 0$ , the calibration strategy of [Hagedorn and Manovskii \(2008\)](#) magnifies the volatility of unemployment and labor market tightness. As explained in [Section 4.3.2.](#), decreasing  $\phi$  or the relative hiring costs  $\kappa v/y$  increases the relevance of income effects for the wage dynamics. But, if  $\chi = 0$ , this channel is muted.

<sup>21</sup>[Appendix B2.](#) outlines the model with capital.

<sup>22</sup>Using the model with capital, we have also investigated the results for fixed matching costs, calibration strategy *a la* [Hagedorn and Manovskii \(2008\)](#), rigid wages, and unemployment benefits. The main conclusions relative to the model without capital remain unchanged and for the sake of brevity, we do not report those results.

Figure 4: The Case  $\chi = 0$ 

*Note:* The horizontal axis measures the degree of income effects,  $\gamma \in [0, 20]$ . The vertical axis measures the 8-year cumulative responses to a one standard deviation increase in productivity. The solid line represents unemployment; the dashed line represents labor market tightness; and the dot-dashed line represents job search effort.

## 5. Concluding Remarks

In this paper, we study the role of income effects in a general equilibrium model extended with matching frictions. In particular, we assess whether the model generates cyclical patterns consistent with data: acyclical or countercyclical search effort and, at the same time, strongly countercyclical unemployment and strongly procyclical labor market tightness.

We find that income effects dictate the behavior of the model. In the baseline model, depending on the magnitude of income effects, one of three possibilities emerge. If income effects are absent or low, search effort is procyclical, which agrees with the evidence for search effort of the short-term unemployed in [Gomme and Lkhagvasuren \(2015\)](#) but disagrees with the evidence for average search effort in the same study and in the other empirical studies. If income effects are moderate, search effort is acyclical, which concurs with the direct evidence in [DeLoach and Kurt \(2013\)](#) and [Leyva \(2018\)](#). But, in this case, [Shimer's \(2005\)](#) critique is acute in the model because the volatilities of the other labor market variables are extremely low. If income effects are high, search effort is countercyclical, which concurs with the direct evidence in [Shimer \(2004\)](#) and [Mukoyama, Patterson and Şahin \(2018\)](#) and the indirect evidence in [Faberman and Kudlyak \(2016\)](#) and [Hornstein and Kudlyak \(2017\)](#). But, high income effects render unemployment procyclical and labor market tightness countercyclical, a result

at odds with conventional wisdom and data.

In our model, income effects can be broadly interpreted as wealth effects because household's wealth is mainly the present discounted value of future income. This allows us to induce a broader implication from our results: as long as both exerting search effort and working reduce leisure, income and (more generally) wealth effects worsen the predictions of the model. [Chodorow-Reich and Karabarbounis \(2016\)](#) reach a similar conclusion. After documenting that the opportunity cost of working is procyclical in US data, they show that a model with procyclical opportunity cost of working generates lower unemployment volatility relative to a model with acyclical opportunity cost. Thus, income and wealth effects may make job search effort acyclical or countercyclical as suggested by most of the empirical literature. But in the presence of income and wealth effects, the opportunity cost of working becomes procyclical, which renders the wage more procyclical and job creation less procyclical or even countercyclical.

Our paper calls for further research to understand the cyclicity of job search effort and to amend the matching model. We summarize this call in two questions. First, if effort is not procyclical, why are workers not increasing search effort when the returns to search are higher? One salient explanation is income and wealth effects ([DeLoach and Kurt, 2013](#); [Faberman and Kudlyak, 2016](#)). But, in the model, income effects mainly magnify the wage response, which, in turn, severely deteriorates the cyclical behavior of unemployment and tightness. Thus, our paper casts doubt on the validity of income and wealth effects as an explanation for acyclical or countercyclical search effort in a general equilibrium context. Second, in matching models, how can we simultaneously overcome [Shimer's](#) critique and reverse the prediction of procyclical search effort? We find that leading reconfigurations of matching models that generate high unemployment volatility fail once we account for income effects. We only find two reconfigurations that improve the predictions of the model. Both work by reducing the role of income effects on wages but, unfortunately, both are implausible: one is to assume a fully rigid wage; the other is to assume excessively high risk aversion together with excessively high replacement rate of unemployment benefits. Thus, adding income effects to the model does not generate the cyclical patterns we observe in data unless we make unsound assumptions. We hope that our paper stimulates further research in this direction.

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## A Derivation of the Effort Equation

Below we derive Eq. (14) that we presented in Section 4. The Nash bargaining problem set in Eq. (11) implies

$$V_{n,t} = \frac{\phi}{1-\phi} J_{n,t} c_t^{-\gamma}. \quad (\text{A.1})$$

First, we lead this equation one period and take period  $t$  expectation. Then, we use the result to substitute out  $E_t V_{n,t+1}$  from the first order condition for effort ( $\psi e_t^{\zeta-1} = \beta f(\bar{e}_t, \theta_t) E_t V_{n,t+1}$ ). This gets

$$\psi e_t^{\zeta-1} = \beta f(\bar{e}_t, \theta_t) \frac{\phi}{1-\phi} E_t \left[ J_{n,t+1} c_{t+1}^{-\gamma} \right]. \quad (\text{A.2})$$

Then, using the free-entry condition ( $\beta E_t [J_{n,t+1} c_{t+1}^{-\gamma}] = \kappa c_t^{-\gamma} / \mu(\bar{e}_t, \theta_t)$ ) in Eq. (A.2) implies that

$$\psi e_t^{\zeta-1} = f(\bar{e}_t, \theta_t) \frac{\phi}{1-\phi} \frac{\kappa c_t^{-\gamma}}{\mu(\bar{e}_t, \theta_t)}.$$

This further simplifies to

$$\psi e_t^{\zeta} = \theta_t \frac{\phi}{1-\phi} \kappa c_t^{-\gamma}, \quad (\text{A.3})$$

because  $f(\bar{e}_t, \theta_t) = \frac{\theta_t}{\bar{e}_t} \mu(\bar{e}_t, \theta_t)$  and  $\bar{e}_t = e_t$ . Log-linearizing Eq. (A.3) yields Eq. (14) in the main text.

## B Two Variants of the Model

In this appendix, we outline two variants of the baseline model in Section 2., the model with exogenous search effort and the model with capital.

### B1. Variant with Exogenous Search Effort

To generate Figure 3, we employ a variant of the model with exogenous search effort. Assuming exogenous effort changes the matching function, the job-finding probability, and the job-filling probability:

$$\begin{aligned} m_t &= \sigma v_t^\eta (u_t)^{1-\eta}, \\ f(\bar{e}_t, \theta_t) &\equiv f(\theta_t) = \sigma (\theta_t)^\eta, \\ \mu(\bar{e}_t, \theta_t) &\equiv \mu(\theta_t) = \sigma (\theta_t)^{\eta-1}. \end{aligned}$$

Assuming exogenous effort also changes the optimization problem of the household:

$$V_t = \max_{\{c_t\}} \left[ \frac{c_t^{1-\gamma} - 1}{1-\gamma} - \chi n_t + \beta E_t V_{t+1} \right],$$

subject to

$$n_{t+1} \leq (1 - \delta)n_t + f(\theta_t)(1 - n_t).$$

This change implies that Eq. (6) is dropped from the model and that the value of an additional employed worker for the household is

$$V_{n,t} = w_t c_t^{-\gamma} - \chi + (1 - \delta - f(\theta_t))\beta E_t V_{n,t+1}.$$

Finally the wage evolves according to

$$w_t = \phi(z_t + \theta_t \kappa) + (1 - \phi) \frac{\chi}{c_t^{-\gamma}}.$$

## B2. Variant with Capital

The introduction of capital,  $k_t$ , into the model requires rewriting the household's maximization problem, the firm's maximization problem, and the resource constraint.

### B2.1. Household's Maximization Problem

In the model with capital, the household maximizes lifetime utility taking the employment rate, the job-filling probability, the wage, and the rental rate of capital,  $r_t$ , as given and choosing the paths of consumption, capital, and job search effort. The optimization problem is

$$V_t = \max_{\{c_t, k_{t+1}, e_t\}} \left[ \frac{c_t^{1-\gamma} - 1}{1-\gamma} - \frac{\psi}{\zeta} e_t^\zeta (1 - n_t) - \chi n_t + \beta E_t V_{t+1} \right],$$

subject to

$$\begin{aligned} c_t + k_{t+1} &\leq w_t n_t + k_t(1 - \delta_k) + k_t r_t, \\ n_{t+1} &\leq (1 - \delta)n_t + e_t f(\bar{e}_t, \theta_t)(1 - n_t), \end{aligned}$$

where  $\delta_k$  is the rate of depreciation of capital. The first-order condition for effort (Eq. 6) and the value of an additional employed worker to the household (Eq. 7) are unchanged. The first-order condition for capital is

$$c_t^{-\gamma} = \beta E_t \left[ c_{t+1}^{-\gamma} (1 - \delta_k + r_{t+1}) \right],$$

which equates cost and benefit, in utility terms, of increasing capital stock by a unit.

## B2.2. Firm's Maximization Problem

We assume a Cobb-Douglas production function:

$$y_t = z_t k_t^\alpha n_t^{1-\alpha},$$

where  $\alpha$  is the elasticity of output with respect to capital. The firm takes the employment rate, the job-filling probability, the wage, and the rental rate of capital as given. To maximize its value, the firm chooses the number of vacancies to open and the amount of capital to rent. The optimization problem is

$$J_t = \max_{v_t, k_{t+1}} (z_t k_t^\alpha n_t^{1-\alpha} - r_t k_t - w_t n_t - \kappa v_t + \beta E_t [(c_{t+1}/c_t)^{-\gamma} J_{t+1}]),$$

subject to

$$n_{t+1} = (1 - \delta)n_t + \mu(\bar{e}_t, \theta_t)v_t.$$

The free-entry condition (Eq. 9) is unchanged and the value of an additional worker to the firm is given by

$$J_{n,t} = (1 - \alpha) \frac{y_t}{n_t} - w_t + (1 - \delta)\beta E_t [(c_{t+1}/c_t)^{-\gamma} J_{n,t+1}].$$

The first-order condition for capital implies that the marginal cost of capital equals its marginal product:

$$r_t = \alpha \frac{y_t}{k_t}.$$

## B2.3. The Wage

Wage bargaining in the model with capital implies that the wage is given by

$$w_t = \phi \left( (1 - \alpha) \frac{y_t}{n_t} + e_t f(\bar{e}_t, \theta_t) \frac{\kappa}{\mu(\bar{e}_t, \theta_t)} \right) + (1 - \phi) \left( \chi - \frac{\psi}{\zeta} e_t^\zeta \right) \frac{1}{c_t^{-\gamma}}.$$

## B2.4. Resource Constraint

Adding capital implies that the resource constraint is

$$y_t = c_t + k_{t+1} - (1 - \delta_k)k_t + \kappa v_t.$$

In our experiments, we set  $\delta_k = 1/120$  and  $\alpha = 0.33$ .