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TESTING THE Q THEORY OF INVESTMENT IN THE FREQUENCY DOMAIN

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Testing the Q theory of investment in the frequency domain*

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Abstract

We revisit the empirical performance of the Q theory of investment, explicitly taking into account the frequency dependence of investment, Tobin's Q, and cash flow. The time series are decomposed into orthogonal components of different frequencies using wavelet multiresolution analysis. We find that the Q theory fits the data much better than might be expected (both in-sample and out-of-sample) when the frequency relationship between the variables is taken into account. Merging the wavelet approach and proxies for Q recently suggested in the investment literature also significantly improves the quality of short-term forecasts.

Keywords: investment, Tobin's Q, bond Q, intangible Q, intangible investment, cash flow, discrete wavelets, frequency estimation, forecast

JEL codes: C49, E22, G31

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1 Introduction

The investment equation resulting from the first-order condition of a profit-maximizing firm facing convex adjustment costs (see *e.g.* Romer, 2012, chapter 9) is of the form $IK_t = \beta_0 + \beta_Q Q_{t-1}$. It relates the investment rate (IK) – usually defined as corporate fixed investment over the replacement cost of property, plant, and equipment – to Tobin's Q – usually computed as the ratio between the firm's market value (stock price) and the cost of replacing its physical assets. The investment equation captures Tobin's original idea that "the *rate* of investment – the speed at which investors wish to increase the capital stock – should be related, if to anything, to q, the value of capital relative to its replacement cost." (see Tobin, 1969, page 21). While Lucas and Prescott (1971) provided the initial theoretical foundations for the Q theory of investment by developing a dynamic investment model with convex adjustment costs, they mentioned no explicit link to Tobin's Q. This oversight on the link between convex adjustment costs and Q theory was addressed fairly soon after by Mussa (1977) and Abel (1983).[1]

Two implications of Q-theory investment models based on convex adjustment costs are that 1) Q is a sufficient statistic for the rate of investment and 2) β_Q is inversely related to the adjustment cost parameter. Despite the attractiveness of its simplicity and sound theoretical underpinnings, empirical tests have largely rejected the Q theory. In-sample estimates of the investment equation usually show that Q has poor explanatory power, β_Q is very low (and not always statistically significant), and residuals are highly serially correlated (see *e.g.* Chirinko, 1993 and Bond and Van Reenen, 2007). Likewise, the out-of-sample empirical evidence in support of Q as a predictor of aggregate investment is rather weak, especially for long time series using US data. For instance, Oliner et al. (1995) find that, when Q is significant, it has the wrong sign, while Grullon et al. (2014) find that cash flow predicts future investment much better than Q.[2]

Because of these unsatisfactory (both in-sample and out-of-sample) results, the common perception is that Q has rather little practical value in explaining or predicting aggregate investment. Several theoretical and empirical explanations for this poor performance have been proposed in the literature. On the theoretical side, model misspecification is the most commonly cited culprit. According to the Q theory, marginal Q is the ratio of the

[1] Over the years, researchers have extended the Q investment model to encompass more realistic features. For example, Veracierto (2002) includes irreversibilities, Gomes (2001) and Hennessy et al. (2007) introduce financing constraints, Khan and Thomas (2008) bring in non-convex adjustment costs, and Verona (2014a,b) incorporate information costs.

[2] Using a shorter span of data for the UK, Price and Schleicher (2005) find that Q has predictive power for investment. Moreover, Rapach and Wohar (2007) find that the Q investment model – when compared to other investment models like the neoclassical model (Jorgenson, 1963) or the stock price model (Barro, 1990) – produces the most accurate one-quarter-ahead forecast.

market value of an additional unit capital to its replacement cost, and investment should be an increasing function of marginal Q. However, we can only observe the ratio of market value of existing capital to its replacement value, so what we can measure from the data (albeit imprecisely) is average Q. Hayashi (1982) shows that marginal and average Q are the same for a competitive firm with constant returns to scale (both in the production function and the adjustment cost function). When these conditions are not met, Q is no longer a sufficient statistic for investment and the Q theory fails. On the empirical side, the most common explanation is that Q is poorly measured. Empirical studies often attribute poor Q measurement to factors such as bubbles or fads in the stock market, or simply because market expectations of a firm's value differ from its fundamental value (see *e.g.* Blanchard et al., 1993 and Brynjolfsson et al., 2002).

Another suggested explanation for the empirical failure is that Q may have effects on investment at different frequencies. For instance, Gallegati and Ramsey (2013a) show that the in-sample fit of the investment equation is better over the long run than in the short run, thus providing empirical evidence in support of the view that the relationship between investment and Q is frequency dependent. Thus, how the empirical tests deal with Q, *i.e.* treating it as having on only a short-run or long-run effect on investment, or as having both long-run and short-run effects on investment, could be determinative of the Q theory's success.[3]

The idea of a long-run relationship between investment and Q dates back at least to Keynes (1936), who noted: "... firms' decisions to invest are mainly based upon expectations of the prospective yield of an investment over a long term of years, with such expectations reflecting the state of long-term expectations." Engle and Foley (1975) initially suggest that most of the power in the relationship between investment and Q is found at low frequencies, and thus a long-run relationship. More recently, (Abel and Eberly, 2002, Lettau and Ludvigson, 2002, Alti, 2003, Almeida et al., 2004 and Philippon, 2009) have conjectured that not only Q, but other variables such as cash flow, may have an effect on investment at different frequencies. Indeed, the information provided by Q and cash flow could, at least to some extent, be complementary. On one hand, Q is usually linked to the value of long-term growth potential. Hence, it might perform poorly in controlling for current investment, but nevertheless do a good job in explaining long-term investment patterns. Conversely, as cash flow is usually linked to current demand and productivity, it might then explain short-term investment better than long-term investment.

[3] In addition to the frequency dependence among investment and Q (and possibly other variables), Gallegati and Ramsey (2013b) and McLean and Zhao (2014) show that the investment sensitivities to Q and cash flow are also time varying.

Motivated by these arguments, in this paper we revisit the empirical performance of the Q theory of investment by *explicitly* taking into account the frequency dependence of investment, Q, and cash flow.

First, we analyze the in-sample fit of the investment equation in the frequency domain by decomposing the time series into orthogonal components of different frequencies by means of wavelet multiresolution analysis (as in *e.g.* Kim and In, 2005, Crowley, 2007 and Xue et al., 2013). As described in section 2, this method has become popular in econometric analysis thanks to its ability to overcome certain limitations of traditional frequency domain tools (*e.g.* Fourier analysis). The method employed here allows decomposition of a time series into n orthogonal time-series components, each capturing the oscillations of the variable within a specific frequency band. The lower frequencies represent the long-term dynamics of the original variable, while the higher frequencies capture its short-term dynamics. As these n frequency-decomposed time series are orthogonal, simply summing them allows recovery of the original variable time series. Thereafter, it is straightforward to estimate (*e.g.* by OLS) the coefficients of the investment equation across different frequency bands. This method thus allows the use of wavelets tools without abandoning traditional time series analysis.

Second, we evaluate the out-of-sample forecasts performance of the investment equation in the frequency domain and compare it with the traditional time series forecast. As Rua (2011) shows, wavelet multiresolution analysis can improve forecast accuracy. With this method, each frequency band is first forecast separately, and then the individual forecasts are aggregated to produce the forecast for the original time series.

In recent years, researchers have also improved the empirical fit of the investment equation through better measures and proxies for Q and investment (see *e.g.* Hall, 2001, McGrattan and Prescott, 2005, Cummins et al., 2006, Merz and Yashiv, 2007, Philippon, 2009, Peters and Taylor, 2016 and Celil and Chi, 2016).[4] For this reason, we not only test the Q theory in the frequency domain using the traditional measures of investment, Q and cash flow, but also use a proxy for Q computed from bond market data as suggested by Philippon (2009), as well as measures of Q and investment that take into account intangible capital as proposed by Peters and Taylor (2016). In particular, given the increasing importance of intangible capital in modern economies, it seems quite appropriate to test the Q theory using these non-traditional measures. In fact the US economy has gradually shifted toward service- and technology-based industries in recent decades, highlighting the role of intangible assets (*e.g.* human capital,

[4] Alternatively, researchers have estimated the investment equation using different methods. For instance, Erickson and Whited (2000) use generalized method of moments estimators to purge Q from measurement errors.

branding, intellectual property, and software).

The results of the in-sample analysis find quite good in-sample fit of the investment equation, especially in the frequency bands corresponding to traditional business cycle and long-term fluctuations. At those frequencies, both the beta coefficients and the R^2s are larger than in the traditional time domain analysis. The alternative measure of Q proposed by Philippon (2009) has a good, uniform fit across all frequencies, and the measure proposed by Peters and Taylor (2016) clearly has the best fit in the long run. Actually it best explains the low-frequency upward trend that characterizes the dynamics of intangible investment. The results of the out-of-sample analysis suggest that both alternative measures of Q are good predictors for investment, while cash flow does not predict future investment as well as either of these proxies of Q. Finally, wavelet methods are shown to outperform traditional time series methods (at least up to a horizon of two quarters), thus confirming the potential for gains in forecasting economic variables with wavelet methods.

The paper is organized as follows. In section 2, we introduce wavelets and describe how they are used to decompose and forecast a time series. The data are described in section 3, and the empirical analyses performed in section 4. Section 5 concludes.

2 Wavelet analysis of time series: decomposition and forecasting

Time domain analysis is the most widespread approach to the study of temporal properties of financial and economic time series. A smaller strand of literature focuses on frequency domain analysis. Tools such as spectral analysis and Fourier transforms are useful in identifying and quantifying the importance of various frequency components of the variable under investigation. For instance, they permit to inference of information about the length of the business or the financial cycle (see *e.g.* Strohsal et al., 2015 and Verona, 2016). Nevertheless, these methods suffer from two significant drawbacks. First, they require that the data be stationary, a condition at odds with the evolution of many economic and financial variables. Second, they provide no information regarding how the frequency content of the variable changes over time. That is, they permit analysis of how much of each frequency exists in the variable without specifying when these frequency components exist. Such frequency decomposition thus only makes sense if the importance of the various frequency components remains stable over the sample period, which again is at odds

with the behavior of financial variables and seems unlikely for most series over long sample periods.[5]

A growing strand of literature uses wavelet tools to overcome these limitations. Wavelets are a natural extension to these frequency domain tools, providing a more complete decomposition of the original time series without the above-mentioned weaknesses. Unlike in Fourier analysis, wavelets are defined over a finite support/window in the time domain, with the size of the window adjusted automatically according to the frequency of interest. In other words, a small window permits viewing of fine (*i.e.* high frequency) features of the time series, whereas by looking at the same variable through a large window shows the coarse features (*i.e.* the low frequency components). By adjusting the size of the window, one can thus capture both the time- and the frequency-varying features of the variable simultaneously. Wavelets are therefore extremely useful for time series with structural breaks or jumps, as well as with non-stationary time series.

Wavelet methods can be *discrete* or *continuous*. Discrete methods are used to extract the components of a specific variable operating within a desired frequency range, while continuous methods are those applied across all frequencies. The pioneering papers in applying wavelet methods in economics and finance – Ramsey and Lampart (1998a,b) – rely on the discrete wavelet transform (DWT). A recent application closely related with our work is Gallegati and Ramsey (2013a). They use the DWT to analyze the relationship between the standard measure of investment and two measures of Q. They do not, however, control for cash flow (as is typical in the empirical investment literature) and focus instead solely on the in-sample fit of the investment equation. Here, besides controlling for cash flow, we also 1) test the Q theory with intangible capital and investment, and 2) run an out-of-sample forecasting exercise to check whether wavelet methods improve traditional time series methods with respect to forecasting aggregate investment.

The following description of the method used in this paper – maximal overlap DWT multiresolution analysis – provides a simple overview. For greater technical detail, Percival and Walden (2000) provide a good discussion of this method.[6]

[5] See Example 1 and Figure 1 in Aguiar-Conraria and Soares (2014).

[6] See Crowley (2007) and Aguiar-Conraria and Soares (2014) for detailed descriptions of discrete and continuous wavelets, respectively, as well as for further references.

Maximal Overlap DWT MultiResolution Analysis (MODWT MRA)

Using a wavelet filter, the MODWT MRA allows decomposition of a time series $x(t)$ as

$$x(t) = \sum_{j=1}^{J} D_j(t) + S_J(t) \quad , \tag{1}$$

where the $D_j(t)$ are the wavelet details and $S_J(t)$ is the wavelet smooth. The original time series is thus decomposed into orthogonal components ($D_1(t)$ to $D_J(t)$ and $S_J(t)$) called *crystals*, each defined in the time domain and representing the fluctuation of the original time series in a specific frequency band (or time scale).[7]

Our empirical analysis considers five scales, *i.e.* $J = 5$ in (1). Each variable is thus decomposed into five wavelet details ($D_1(t)$ to $D_5(t)$) and a wavelet smooth ($S_5(t)$). Since we use quarterly data, detail levels $D_1(t)$ and $D_2(t)$ capture oscillations with a period of 0.5–1 and 1–2 years, respectively. They represent the short-run (high frequency) dynamics and likely contain most of the noise of the variable. Details levels $D_3(t)$ and $D_4(t)$ capture oscillations with a period of 2–4 years and 4–8 years, respectively, which correspond to the traditional business cycle frequencies. Finally, $D_5(t)$ captures a cycle with long duration (between 8 and 16 years), while the smooth component $S_5(t)$ captures oscillations with a period longer than 16 years (*i.e.* the series trend).[8,9]

After decomposing the variables into their orthogonal frequency components, it is possible to test for the (in-sample) frequency dependence between two (or more) variables by simply running OLS regressions using the components of the variables in each frequency band, as done by *e.g.* Ramsey and Lampart (1998a), Gallegati et al. (2011) and Gallegati and Ramsey (2013a). Instead of looking at the relationship between investment and Q "averaged" across all frequencies (as done implicitly in the time series analysis), we disentangle and examine the investment-Q

[7] Loosely speaking, the frequency is inversely related to the time scale, *i.e.* large-scale wavelets are associated with low frequencies.

[8] In this paper we report the results when performing a MODWT MRA using the Daubechies least asymmetric wavelet filter of length 8 ($LA8$) with reflecting boundary conditions. This filter is widely used with quarterly data (see *e.g.* Gencay et al., 2001). As the wavelet family may influence the results, we also run the simulations using the Daubechies wavelet filter with the filter length L=8 and the Coiflet wavelet filter with the filter length L=6. Our results are robust to changes in the wavelet family. As regards the choice of J, the number of observations dictates the maximum number of frequency bands that can be used. In particular, if R is the number of observations in the in-sample period, then J has to satisfy the constraint $R \geq 2^J$.

[9] So why not use the more popular Baxter and King (1999) band-pass filter, which also permits isolation of oscillations in different frequency bands? The band-pass filter is a combination of a moving average in the time domain with a Fourier decomposition in the frequency domain, which is optimized by minimizing the distance between the Fourier transform and an ideal filter. It applies a kind of optimal Fourier filtering on a sliding window (in the time domain) with constant length similar to the "short-time" Fourier transform. Wavelet filtering, in contrast, provides better resolution in the time domain as the wavelet basis functions are time-localized (not just scale-localized), which is useful for capturing the changing volatility of the time series. Moreover, Murray (2003) notes that the band-pass filter may introduce spurious dynamic properties.

relationship in each frequency band separately.

A largely ignored, but potentially rich, area of wavelet-based research is forecasting (Rua, 2011 and Faria and Verona, 2016a,b). Rua (2011) suggests that, once the series to be forecasted and to be used to forecast are decomposed into their time-scale components, a different forecasting method could be used for each frequency band, with the overall forecast of the series obtained by adding up the individual forecasted series. The main benefit of using wavelets in forecasting is that, by decomposing the total series into its constituent time scales, it is possible to tailor specific forecasting techniques to each time scale component and thereby increase the efficiency of the forecast. In fact, Rua (2011) finds that significant predictive short-run improvements can be achieved by using the MODWT MRA (in combination with factor-augmented models). Likewise, Faria and Verona (2016a,b) show that it is possible (and even recommended in forecasting stock market returns) to keep the frequency bands that have the best predictive power and exclude the noisiest frequency bands.

3 The data[10]

We use quarterly aggregate US data over the period 1972:Q1–2007:Q2. We consider two measures of aggregate investment rate. Our standard measure, denoted IK^*, is computed as the ratio between corporate fixed private nonresidential investment in equipment and structures and their replacement cost. The alternative measure, denoted IK^T, taken from the recent proposal of Peters and Taylor (2016), considers intangible capital and investment. It is computed as the ratio between total investment (the sum of investment in physical capital and investment in intangible capital) and the total capital stock (the sum of physical and intangible capital stocks). We consider three proxies for Tobin's Q. The literature's standard Q measure, denoted Q^*, is constructed as in Hall (2001) and defined as the ratio of the value of ownership claims on the firm, less the book value of inventories, to the replacement cost of equipment and structure. The first alternative proxy, bond Q, is denoted Q^b. It is taken from the suggestion of Philippon (2009) and is the function of four variables: relative prices of corporate and government bonds, average book leverage, average idiosyncratic volatility, and the ex-ante real interest rate. The second alternative proxy, denoted Q^T, was proposed by Peters and Taylor (2016). It is computed as the ratio of total ownership claims on

[10] The data have been provided by, and are used in, Philippon (2009) and Peters and Taylor (2016).

firm value, less the book value of inventories, to the total capital stock (the sum of tangible and intangible capital). Finally, following Philippon (2009), cash flow (cf) is the ratio of corporate profits over GDP.

The time series of the variables are reported (together with NBER recessions) in the top rows in Figures 1-6. The standard measure of investment rate (Figure 1) is highly pro-cyclical and exhibits slow booms followed by quick busts. The alternative measure of investment rate (Figure 2) shows a similar cyclical behavior, but also features a slow upward trend over the sample. In addition to saw-tooth movements, the standard measure of Q (Figure 3) exhibits large and low-frequency swings, moving downward until 1982, then upward until the burst at the beginning of 2000, and finally stabilizing toward the end of the sample. The alternative measure with intangible capital (Figure 4) shows a similar behavior, as (by construction) it only differs from Q^* by a component that changed slowly over time (*i.e.* the ratio of physical to total capital). On one hand, no clear relationship seems to emerge between Q^* and IK^*; they diverge in the mid-70s/early-90s period, with Q being low both absolutely and in relation to investment (as pointed out by *e.g.* Summers, 1981 and Alpanda and Peralta-Alva, 2010), while they both sharply increase from the mid-90s onward. On the other hand, a visual inspection of the time series of bond Q (Figure 5) reveals that Q^b is much less volatile and much more correlated with IK^* than Q^*. Indeed, except in the period between 1985 and 2000, IK^* and Q^b have very similar behaviors throughout the entire sample.[11] Cash flow (Figure 6) displays a slow downward trend until 1985, followed by an upward trend until the end of the sample (even with a large drop from 1997 to 2001). Cash flow seems to lead the business cycle, as it usually peaks a few quarters before the beginning of the subsequent business cycle recession.

The remaining rows in Figures 1-6 report the MODWT MRA decomposition of each series. We report all the detail levels D_1 to D_5, as well as the long-run (and possibly non-stationary) component S_5. Overall, the frequency-decomposed time series exhibit a wide degree of variation in the type of fluctuations that are embodied and hidden in the original time series. As expected, the lower the frequency, the smoother the resulting filtered time series. Furthermore, the above-mentioned trends in the time series of some variables are captured by (and easily seen from) the smooth component S_5.

Having described the data, we move on to our empirical analysis, starting with in-sample fit of the investment equation and finishing with the out-of-sample analysis.

[11] The sample correlation between IK^* and Q^* (Q^b) is 0.29 (0.72).

4 Empirical tests of the Q theory

4.1 In-sample fit

In the baseline time-series OLS regressions, two measures of investment rate are regressed on different proxies for Q and on cash flow, measured at the end of the previous quarters:

$$IK_t^* = \beta_{0^*} + \beta_{q^*} Q_{t-1}^* + \beta_{q^b} Q_{t-1}^b + \beta_{cf} cf_{t-1} + \varepsilon_t^* \tag{2}$$

$$IK_t^T = \beta_{0^T} + \beta_{q^T} Q_{t-1}^T + \beta_{q^b}^T Q_{t-1}^b + \beta_{cf}^T cf_{t-1} + \varepsilon_t^T \ . \tag{3}$$

As regards the analysis in the frequency domain, after decomposing the variables into their different frequency components, the following regressions are estimated by OLS:

$$IK^* \left[D_j\right]_t = \beta_{0^*}^{D_j} + \beta_{q^*}^{D_j} Q^* \left[D_j\right]_{t-1} + \beta_{q^b}^{D_j} Q^b \left[D_j\right]_{t-1} + \beta_{cf}^{D_j} cf \left[D_j\right]_{t-1} + \varepsilon_t^{*D} \tag{4}$$

$$IK^* \left[S_5\right]_t = \beta_{0^*}^{S_j} + \beta_{q^*}^{S_j} Q^* \left[S_5\right]_{t-1} + \beta_{q^b}^{S_j} Q^b \left[S_5\right]_{t-1} + \beta_{cf}^{S_j} cf \left[S_5\right]_{t-1} + \varepsilon_t^{*S} \tag{5}$$

and

$$IK^T \left[D_j\right]_t = \beta_{0^*}^{D_j^T} + \beta_{q^T}^{D_j} Q^T \left[D_j\right]_{t-1} + \beta_{q^b}^{D_j^T} Q^b \left[D_j\right]_{t-1} + \beta_{cf}^{D_j^T} cf \left[D_j\right]_{t-1} + \varepsilon_t^{TD} \tag{6}$$

$$IK^T \left[S_5\right]_t = \beta_{0^*}^{S_j^T} + \beta_{q^T}^{S_j} Q^T \left[S_5\right]_{t-1} + \beta_{q^b}^{S_j^T} Q^b \left[S_5\right]_{t-1} + \beta_{cf}^{S_j^T} cf \left[S_5\right]_{t-1} + \varepsilon_t^{TS} \ , \tag{7}$$

where $[D_j]$, $j = 1, 2, 3, 4, 5$, represent the components of the variables at each time-scale level j and $[S_5]$ the long-run components.

The time-series estimation results of equation (2) are reported in Panel A of Table 1. The first three columns report the results for the bivariate regressions, while the remaining columns report the results of the multivariate regressions. In all the regressions, the standard errors control for autocorrelation in the error terms up to 12 quarters (as in *e.g.* Peters and Taylor, 2016).

As expected, β_{Q^*} is very low and not significant, and Q^* alone only explains 4 % of the variations of investment. The fit of the investment equation improves significantly by replacing equity Q with bond Q. β_{Q^b} is over 30 times

larger than β_{Q^*} and accounts for 47 % of variations in the investment rate. As pointed out by Philippon (2009), the higher elasticity of investment is an encouraging result, because it implies that adjustment costs are an order of magnitude smaller than with the standard measure of Q. By significantly reducing the implied adjustment costs, bond Q thus mitigates the problem of low elasticity that has long puzzled empirical investment researchers. Like Q^*, cash flow is not significant at the aggregate level. Looking at the time-series results overall, bond Q appears to be a better proxy for investment opportunities than Q^* or cash flow.

Panel B of Table 1 presents the estimated regression results in the frequency domain (Equations 4 and 5). The results show that the relationship between investment and Q^* varies widely across frequencies as regards the degree of fit and the significance of the estimated coefficients. The adjusted R^2 increases from nearly 0 in the very short run (*i.e.* in high frequency bands D_1 and D_2) to 0.56 in the frequency band D_5. The size of β_{Q^*} is low (and not significant) at high frequencies (D_1 to D_3), while in the bands D_4 and D_5 it is statistically significant and much larger than in the time domain. In particular, in the frequency band D_5 the regression displays the largest estimated value for β_{Q^*} (0.012), which is approximately seven times larger than the coefficient in the time domain. Unlike Q^*, Q^b displays a uniform behavior across frequencies. At the detail levels D_3 to D_5, the adjusted R^2s are quite high, ranging from 0.59 to 0.83. As to the size and significance of β_{Q^b}, except for the very short and very long run (D_1 and S_5), the estimated coefficients are statistically significant, with the medium- and long-run (levels D_4 and D_5) coefficients even larger than in the time domain. The results in the frequency domain bolster Philippon's explanation of the good performance of bond Q , whereby Q^b is successful because it effectively captures the effect of Q on investment at different frequencies. With regard to cash flow, the beta coefficients are significant only in the frequency bands corresponding to traditional business cycle frequencies (D_3 and D_4). Moreover, cash flow alone at most explains 29 % of variations of investment (in the 2–4 year frequency band).

Looking at the multivariate regressions results, including Q^* or cash flow in a regression with Q^b does not notably improve the fit of the equation (compared to the bivariate regression with Q^b) or the significance of the beta coefficients in either the time or frequency domains. Interestingly, when used together, Q^* and cash flow are not significant in the time domain, but they are in the frequency bands corresponding to medium-to-long run fluctuations (D_4 and D_5). In these frequency bands, Q^* and cash flow jointly explain over 85 % of investment fluctuations, with cash flow significant at shorter horizons.

The time-series OLS estimation results of Equation (3), which considers the alternative measures of investment and Q that include intangible capital, are quite different from the results with standard measures. As Panel A of Table 2 shows, Q^T and cash flow are significant, while Q^b is not in either the bivariate or the multivariate regressions. As already shown by Peters and Taylor (2016), Q^T seems to be an excellent predictor for total investment, as it explains 61 % of the volatility of total investment. This R^2 is even higher than the 47 % R^2 obtained by regressing the standard measure of investment on bond Q, and is probably the best in-sample results obtained so far in any empirical investment studies (even though the total investment-Q^T sensitivity is not as high as the investment-Q^b sensitivity). Interestingly, and unlike most findings in the literature (see *e.g.* Philippon, 2009), Q^T drives out cash flow from the regression, as cash flow loses its significance and explanatory power once Q^T and cash flow are both included in the regression.

Peters and Taylor's argument behind the good fit of Q theory when using intangible capital and investment is that these measures are better able to explain the low-frequency trends in Q and investment. This argument is strengthened by our estimation results in the frequency domain (equations 6 and 7), which are reported in Panel B of Table 2. In fact, in the bivariate regressions with Q^T, the R^2s are 0.69 and 0.75 in the frequency bands that capture the lowest frequency movements of the series (D_5 and S_5, respectively). While not significant in the time domain, Q^b alone nevertheless explains a large share of total investment fluctuations at business cycle frequencies (D_3 and D_4), and about 50 % of the volatility in long cycles (8–16 years). Moreover, Q^b further improves the fit of the investment equation when used in combination with Q^T. Except for the very short run (frequency bands D_1 and D_2), the R^2s of the multivariate regressions of total investment with both two measures of Q (Q^T and Q^b) are very high (and the coefficients usually significant), ranging from 0.67 (in the frequency band 2-4 years) to 0.81 (at frequencies more than 16 years), reaching an astonishing 90 % in the frequency band D_5 (fluctuations of 8–16 years). Similar to our time-series results, Q^T drives out cash flow in the long run.

Overall, in the frequency bands corresponding to business cycle and long-term fluctuations, the in-sample fit of the investment equation is remarkably good. Moreover, both the beta coefficients and the R^2s are larger than in the time domain. The alternative measure of Q proposed by Philippon (2009) has a good and uniform fit across all frequencies, while the measure of Q proposed by Peters and Taylor (2016) clearly has the best fit over the long run. When comparing the traditional and the alternative variables in the very long run (*i.e.* in the frequency band

S_5), the beta coefficients are never significant using the traditional variables, but usually are with the alternative variables.

4.2 Out-of-sample forecasts performance

We now evaluate the out-of-sample (OOS) predictability performance of the investment equation. We use a recursive scheme to generate out-of-sample forecasts for the investment rate. We first divide the total sample of T=142 observations into in-sample and out-of-sample portions, where the in-sample observations span the first R=102 observations. The out-of-sample period thus starts in 1997:Q3. This is an interesting period as starts around the middle of the longest US economic expansion, and includes the investment bust that led to the 2001 recession.[12]

In the time domain, the predictive regression model is

$$IK_{t+h} = \alpha + \gamma \mathbf{X}_{t-1+h} + \varepsilon_{t+h} \ , \tag{8}$$

where $\mathbf{X}$ are the regressors and h is the forecast horizon.

The one-step-ahead OOS forecast of the investment rate is given by $\hat{IK}_{t+1} = \hat{\alpha} + \hat{\gamma}\mathbf{X}_t$, where $\hat{\alpha}$ and $\hat{\gamma}$ are the OLS estimates of α and γ, respectively. Similarly, the two-step-ahead OOS forecast is given by $\hat{IK}_{t+2} = \hat{\alpha} + \hat{\gamma}E_t\mathbf{X}_{t+1}$. So, for $h = 2$ we also need to forecast the independent variable, as we would need to compute *e.g.* E_tQ_{t+1}. To do so, in each iteration, we fit an AR(1) process for *e.g.* Q_t: $Q_t = \hat{\delta}_0 + \hat{\delta}_1 Q_{t-1} + \varepsilon_t$, so that $E_tQ_{t+1} = \hat{\delta}_0 + \hat{\delta}_1 Q_t$.[13]

To make the forecast in the frequency domain, we follow the method proposed by Rua (2011). Thus, we first fit a model like (8) to each frequency band component of the wavelet decomposition of the variables. Next, to obtain the forecast of investment rate as a whole, we sum the individual forecasts. As the components are orthogonal, the "information" we are using is equivalent to the "information" in the original time series. Moreover, we recompute the crystals at each iteration of the out-of-sample forecast process. As we only use only past and current information in making the forecast, our method does not suffer from look-ahead bias.[14]

[12] We run the same exercise using R=94 (so as to have a longer out-of-sample period) and obtain similar results.

[13] We find similar results assuming that the independent variable follows a random walk, *i.e.* $E_tQ_{t+1} = Q_t$.

[14] As Faria and Verona (2016a,b) show, there may be forecasting gains from ignoring the noisiest frequency bands. In this application, we only find marginal forecasting gains by ignoring the highest frequency bands (D_1 and D_2).

Tables 3 and 4 present the mean squared forecast error (MSFE) for each of the forecasting methodology (time series versus wavelets) relative to the benchmark model, which we consider to be the forecast made in the time domain using Q^* (Q^T) as the only predictor for IK^* (IK^T). Entries in bold indicate a value lower than one (which means that the proposed forecasting model is better than the benchmark), while entries in italics denote the best-performing model for each forecast horizon. We report the results for two forecasting horizons, h=1 and 2 (Panels A and B, respectively).

As regards the traditional variables (Table 3), we emphasize three results, which hold regardless of the forecast horizon. First, it is easy to outperform the benchmark model. It is evidenced by the large number of entries in bold. Second, when comparing the forecasting performance of the different (combination of) potential predictors, Q^b is always the best predictor in the time domain, and Q^b and Q^* together are always the best predictors in the frequency domain. Third, when comparing the forecasting performance of the two techniques (time series versus wavelets), the wavelet method is usually preferable, especially when the in-sample fit of the equation is modest.

As regards the alternative variables (Table 4), we also highlight three results. First, and unlike the traditional variable case (Table 3), only two forecasting models outperform the benchmark, thus suggesting that Q^T is an excellent out-of-sample predictor of total investment. Second, when comparing the forecasting performance of the different (combination of) potential predictors, Q^T together with cash flow (Q^b) is the best predictor in the time (frequency) domain. Third, and like the traditional variable case, when comparing the forecasting performance of the two different techniques (time versus frequency domain), it seems that the frequency domain forecast is preferable whenever the in-sample fit of the equation is poor.

Finally, regardless of the variables used (traditional or alternative) or the forecasting horizon considered (h=1 or 2), the best result (*i.e.* the lowest MSFE) is always obtained when forecasting using wavelets *and* both proxies for Q (Q^* and Q^b for investment, and Q^T and Q^b for total investment). To check whether the differences in the forecasting performance (in terms of the MSFE) between the best forecasting model and the other models are statistically significant, we apply the Harvey et al. (1997) modified version of the Diebold and Mariano (1995) test. Tables 5 and 6 report the results of the test. Overall, for both forecast horizons, the differences between the models are usually statistically significant.[15]

[15] We also investigate longer forecast horizons (*i.e.* $h \geq 3$). Overall, the forecasting gains of using the wavelet approach are less expressive and the significance decreases as the forecast horizon increases.

So, the results of the out-of-sample analysis suggest that Q^b is a good predictor for the standard measure of investment and Q^T a good predictor for the alternative measure of investment that includes intangible capital. Moreover, cash flow does not seem to predict future investment better than these two proxies of Q. Finally, at least for horizons up to two quarters, wavelet forecasting methods outperform traditional time-series forecasting methods.

5 Conclusions

Despite the empirical investment literature "full of disappointments" (Caballero et al., 1995, p.1), we have analyzed the investment equation from the Q theory of investment from a new perspective – the frequency domain. In particular, we used wavelets tools to analyze the in-sample fit and out-of-sample forecasting performance of the investment equation. Our findings suggest that the Q theory fits the data much better than suggested by earlier studies (both in-sample and out-of-sample) once the frequency relationship between Q and investment is taken into account. In other words, failing to take into account the frequency dependence among these variables is another reason for the well-known poor empirical performance of the Q theory. Moreover, we find that merging the wavelet approach and the proxies for Q that have been recently suggested in the literature significantly enhances the predictive power of the investment equation.

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		IK^*								
		Q^*	Q^b	cf	Q^*	Q^b	Q^*	cf	Q^b	cf
Panel A: time domain										
	β	0.0018	0.0613	0.1021	0.0005	0.0604	0.0015	0.0530	0.0612	0.0051
Time series	t-stat	0.71	6.82	0.69	0.32	6.59	0.52	0.33	5.84	0.05
	$\bar{R}^2$	0.04	0.47	0.02		0.47		0.04		0.47
Panel B: frequency domain										
	β	0.0000	0.0065	0.0139	0.0001	0.0065	0.0000	0.0138	0.0066	-0.0049
D_1	t-stat	0.05	1.86	0.39	0.10	1.86	0.00	0.38	1.89	-0.14
$(6m - 1y)$	$\bar{R}^2$	0.00	0.04	0.00		0.04		0.00		0.04
	β	0.0007	0.0141	0.1052	-0.0005	0.0144	0.0008	0.1061	0.0125	0.0474
D_2	t-stat	0.61	2.36	1.69	-0.29	2.25	0.65	1.73	2.32	1.14
$(1y - 2y)$	$\bar{R}^2$	0.00	0.16	0.06		0.17		0.07		0.18
	β	0.0050	0.0558	0.5160	-0.0028	0.0590	0.0003	0.5112	0.0530	0.0653
D_3	t-stat	1.67	5.87	3.04	-0.99	5.07	0.07	2.26	5.42	0.65
$(2y - 4y)$	$\bar{R}^2$	0.05	0.66	0.29		0.68		0.29		0.67
	β	0.0104	0.0745	0.3294	0.0056	0.0697	0.0093	0.2491	0.0821	-0.1883
D_4	t-stat	4.47	16.37	3.60	9.00	25.12	3.50	2.29	13.51	-1.87
$(4y - 8y)$	$\bar{R}^2$	0.20	0.83	0.10		0.89		0.26		0.86
	β	0.0119	0.0756	0.1812	0.0088	0.0574	0.0135	0.2964	0.0864	-0.1313
D_5	t-stat	7.44	5.42	1.67	9.22	7.46	11.68	5.61	4.53	-1.13
$(8y - 16y)$	$\bar{R}^2$	0.56	0.59	0.08		0.86		0.76		0.62
	β	-0.0005	0.0286	-0.0254	-0.0008	0.0318	-0.0007	0.0292	0.0315	0.0359
S_5	t-stat	-0.40	1.31	-0.18	-0.74	1.57	-0.36	0.13	1.41	0.30
$(> 16y)$	$\bar{R}^2$	0.01	0.08	0.00		0.11		0.01		0.09

Table 1: Investment regressions, 1972:Q1-2007:Q2, traditional variables

Notes. The wavelet decomposition analysis has been performed by applying the maximal overlap discrete wavelet transform (MODWT) and using the Daubechies least asymmetric (LA) wavelet filter of length 8 with reflecting boundary conditions. Constant terms are omitted. The standard errors control for autocorrelation in the error terms up to 12 quarters.

		IK^T								
		Q^T	Q^b	cf	Q^T	Q^b	Q^T	cf	Q^b	cf
Panel A: time domain										
	β	0.0172	0.0554	0.5457	0.0162	0.0312	0.0158	0.1927	0.0440	0.4760
Time series	t-stat	6.77	1.73	5.09	6.51	1.71	5.30	1.59	1.53	3.40
	$\bar{R}^2$	0.61	0.14	0.21		0.66		0.64		0.29
Panel B: frequency domain										
	β	0.0026	0.0001	0.0032	0.0026	0.0003	0.0026	-0.0067	0.0000	0.0031
D_1	t-stat	0.86	0.02	0.03	0.87	0.07	0.90	-0.07	0.01	0.03
$(6m - 1y)$	$\bar{R}^2$	0.01	0.00	0.00		0.01		0.01		0.00
	β	0.0033	0.0111	0.1575	0.0022	0.0100	0.0034	0.1600	0.0071	0.1246
D_2	t-stat	1.09	1.28	2.06	0.64	1.06	1.08	2.15	0.93	2.12
$(1y - 2y)$	$\bar{R}^2$	0.02	0.04	0.06		0.05		0.08		0.07
	β	0.0074	0.0688	0.6862	-0.0052	0.0733	-0.0012	0.7008	0.0619	0.1598
D_3	t-stat	1.26	5.50	3.54	-1.01	4.83	-0.16	2.62	4.94	1.43
$(2y - 4y)$	$\bar{R}^2$	0.04	0.65	0.33		0.67		0.33		0.67
	β	0.0155	0.0760	0.3768	0.0090	0.0702	0.0137	0.2784	0.0815	-0.1372
D_4	t-stat	4.55	13.32	3.89	6.56	19.60	3.45	2.57	15.60	-1.32
$(4y - 8y)$	$\bar{R}^2$	0.23	0.79	0.12		0.87		0.30		0.81
	β	0.0188	0.0738	0.0427	0.0152	0.0501	0.0198	0.1634	0.1002	-0.3195
D_5	t-stat	10.75	4.81	0.41	15.13	6.53	12.67	3.51	5.02	-3.13
$(8y - 16y)$	$\bar{R}^2$	0.69	0.49	0.00		0.90		0.75		0.64
	β	0.0177	-0.0023	0.9953	0.0188	-0.0648	0.0155	0.2144	0.0928	1.1760
S_5	t-stat	7.23	-0.03	6.24	8.67	-2.94	4.17	0.78	2.29	5.05
$(> 16y)$	$\bar{R}^2$	0.75	0.00	0.49		0.81		0.77		0.58

Table 2: Investment regressions, 1972:Q1-2007:Q2, alternative variables

Notes. The wavelet decomposition analysis has been performed by applying the maximal overlap discrete wavelet transform (MODWT) and using the Daubechies least asymmetric (LA) wavelet filter of length 8 with reflecting boundary conditions. Constant terms are omitted. The standard errors control for autocorrelation in the error terms up to 12 quarters.

	IK^*					
	Q^*	Q^b	cf	(Q^*, Q^b)	(Q^*, cf)	(Q^b, cf)
	Panel A: h=1					
time series	1.00	**0.31**	1.15	**0.54**	1.44	**0.36**
wavelets	**0.31**	**0.62**	**0.90**	***0.22***	**0.42**	**0.41**
	Panel B: h=2					
time series	1.00	**0.32**	1.03	**0.61**	1.40	**0.35**
wavelets	**0.37**	**0.57**	**0.82**	***0.23***	**0.42**	**0.37**

Table 3: Mean squared forecast error (relative to the benchmark model), h-step-ahead forecasts, traditional variables
Notes. Each entry of the table corresponds to the ratio between the MSFE of each forecasting model and the MSFE of the benchmark model (*i.e.*, the time series estimation using Q^*). Bold entries indicate a value lower than one, *i.e.* the model beats the benchmark. Entries in italics denote the best-performing model for each forecast horizon.

	IK^T					
	Q^T	Q^b	cf	(Q^T, Q^b)	(Q^T, cf)	(Q^b, cf)
	Panel A: h=1					
time series	1.00	5.33	3.85	1.65	**0.88**	4.32
wavelets	1.04	2.80	3.02	***0.48***	1.24	2.20
	Panel B: h=2					
time series	1.00	5.05	3.81	1.59	**0.91**	4.20
wavelets	1.16	2.87	2.99	***0.56***	1.32	2.33

Table 4: Mean squared forecast error (relative to the benchmark model), h-step-ahead forecasts, alternative variables
Notes. Each entry of the table corresponds to the ratio between the MSFE of each forecasting model and the MSFE of the benchmark model (*i.e.* the time series estimation using Q^T). Bold entries indicate a value lower than one, *i.e.* the model beats the benchmark. Entries in italics denote the best-performing model for each forecast horizon.

	IK^*					
	Q^*	Q^b	cf	(Q^*, Q^b)	(Q^*, cf)	(Q^b, cf)
	Panel A: h=1					
time series	4.46***	1.39*	5.14***	2.40***	6.41***	1.60**
wavelets	1.38*	2.78***	4.03***	—	1.89***	1.82***
	Panel B: h=2					
time series	4.31***	1.38	4.44***	2.63***	6.01***	1.52*
wavelets	1.60*	2.44***	3.54***	—	1.80***	1.60***

Table 5: Harvey et al. (1997) test, h-step-ahead forecasts, traditional variables

Notes. Asterisks denote rejection of the null hypothesis of equal forecast accuracy at 10 % (*), 5 % (**) and 1 % (***) significance level, computed using the Harvey et al. (1997) modified version of the Diebold and Mariano (1995) test.

	IK^T					
	Q^T	Q^b	cf	(Q^T, Q^b)	(Q^T, cf)	(Q^b, cf)
	Panel A: h=1					
time series	2.09***	11.12***	8.02***	3.44***	1.83***	9.01***
wavelets	2.17***	5.83***	6.31***	—	2.58***	4.59***
	Panel B: h=2					
time series	1.80*	9.08***	6.85***	2.85***	1.64	7.56***
wavelets	2.09***	5.16***	5.38***	—	2.38**	4.19***

Table 6: Harvey et al. (1997) test, h-step-ahead forecasts, alternative variables

Notes. Asterisks denote rejection of the null hypothesis of equal forecast accuracy at 10 % (*), 5 % (**) and 1 % (***) significance level, computed using the Harvey et al. (1997) modified version of the Diebold and Mariano (1995) test.

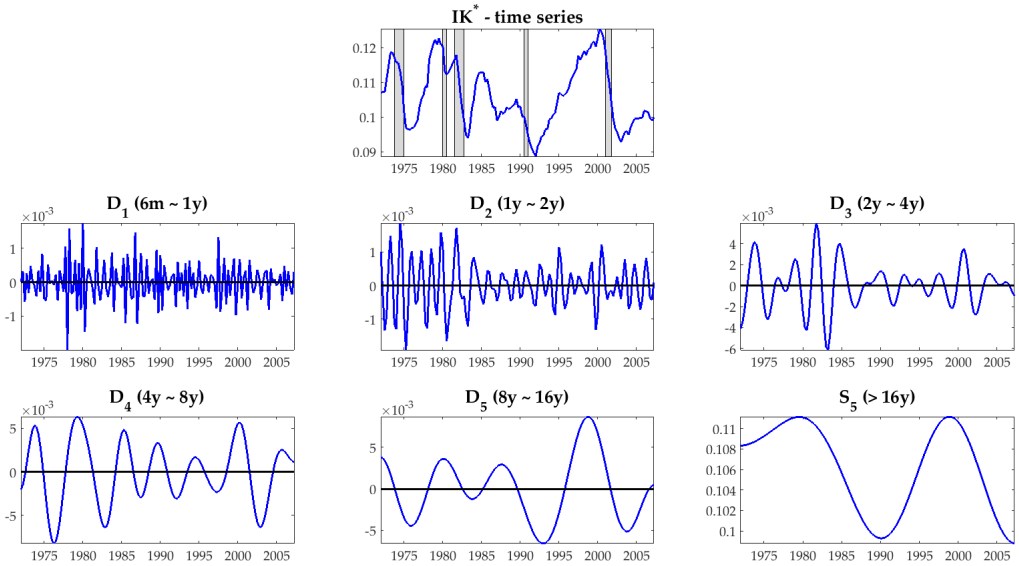

Figure 1: IK^*: time series (top row; grey bars: NBER recessions) and MODWT decomposition (middle and bottom rows)

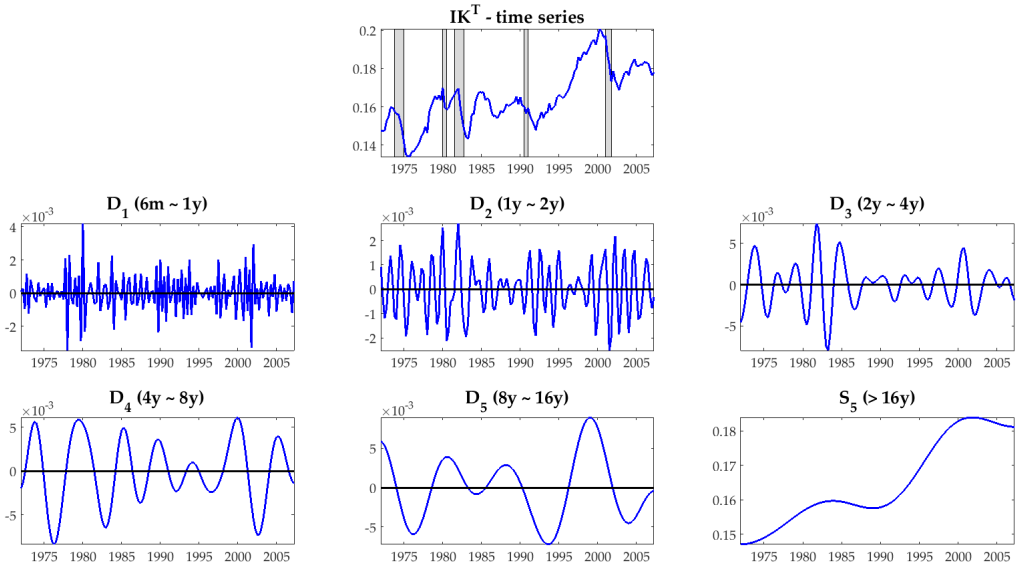

Figure 2: IK^T: time series (top row; grey bars: NBER recessions) and MODWT decomposition (middle and bottom rows)

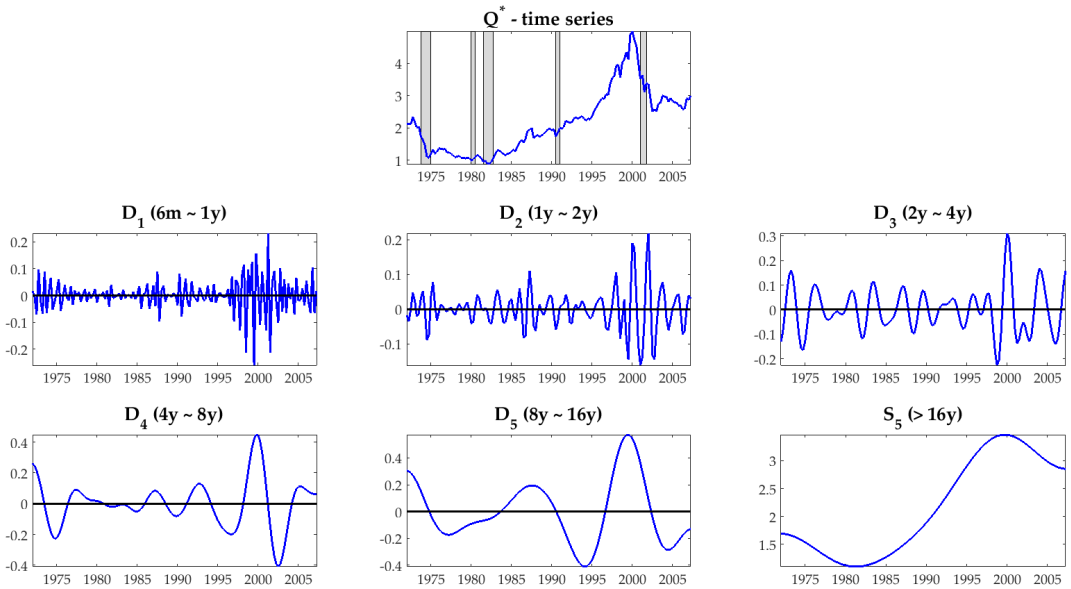

Figure 3: Q^*: time series (top row; grey bars: NBER recessions) and MODWT decomposition (middle and bottom rows)

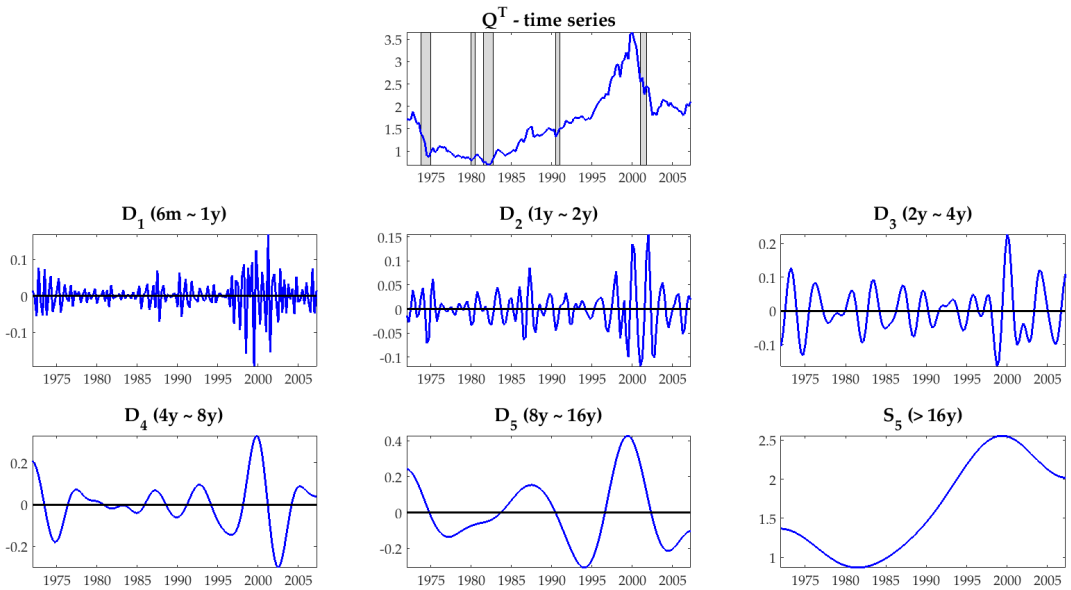

Figure 4: Q^T: time series (top row; grey bars: NBER recessions) and MODWT decomposition (middle and bottom rows)

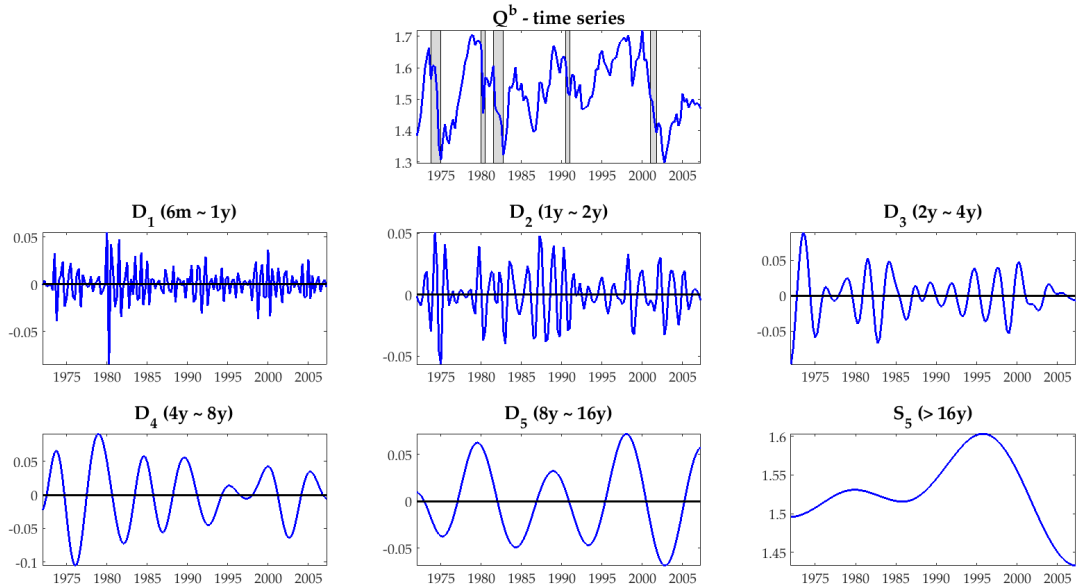

Figure 5: Q^b: time series (top row; grey bars: NBER recessions) and MODWT decomposition (middle and bottom rows)

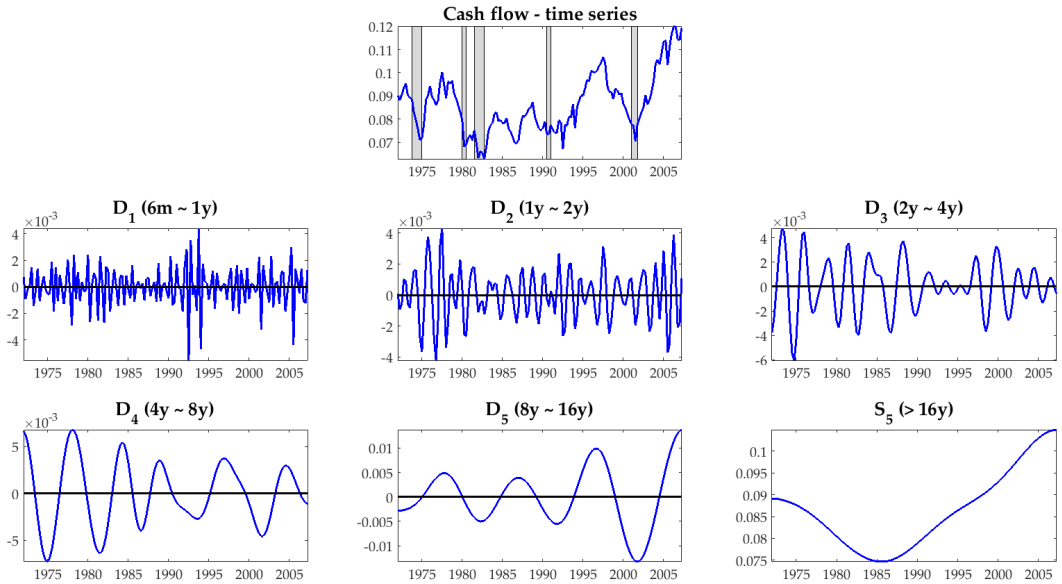

Figure 6: cash flow: time series (top row; grey bars: NBER recessions) and MODWT decomposition (middle and bottom rows)