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**SOVEREIGN RATINGS AND INVESTOR BEHAVIOR**

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# Sovereign Ratings and Investor Behavior\*

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## Abstract

Our premise in this paper is that credit rating agencies account for roll-over risk, which in turn depends on credit ratings. Credit ratings influence subjective beliefs and roll-over decisions, which then influence borrower's creditworthiness and affect credit rating agencies' pronouncements. These effects are stronger when the borrower is in distress, or when credit ratings are more precise than other sources of financial information. In these cases, rating events have a disproportionate impact on investor behavior, thus making credit quality difficult to assess.

We extend our analysis to investigate the role of financial prices, the case in

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which agencies follow investor opinion, the impact of rating-contingent regulation, and the strategic behavior of credit rating agencies.

*Keywords:* Sovereign ratings, roll-over risk, creditor coordination.

*JEL Classification Codes:* D82, G18, G24.

# 1 Introduction

The aim of this paper is to understand how credit ratings affect investor sentiment, and in turn how investor sentiment influences borrowers' creditworthiness and credit rating agencies' pronouncements.

There are two basic motivations for this work. On the one hand, investor sentiment explains the sharp response of investors to downgrades for low credit quality firms (see for example Hand, Holthausen and Leftwich 1992, Hite and Warga 1997, Goh and Ederington 1999, and Dichev and Piotroski 2001), and for sovereigns in distress (see for example Aizenman, Binici and Hutchison 2013). On the other hand, if rating agencies account for investor sentiment, then these sentiments explain the excessive volatility of sovereign ratings during financial crises (see, for example, Ferri, Liu and Stiglitz 1999, and Mora 2006).

The novel contribution of this paper is to show how investor sentiment explains the above results. In a nutshell, the central argument of the paper is that agencies account for roll-over risk, which in turn depends on credit ratings. This is because ratings influence beliefs about what will happen, and beliefs about what will happen influence what does happen. If downgrades contribute to lenders' unwillingness to roll over debt claims, then the lowering of a credit score may induce default because of the inability to sell new debt. Rating agencies thus face the problem of setting independent credit ratings when their pronouncements affect the credit quality of issuers.

To present this argument, we develop a theoretical model for analyzing the relationship between ratings and sentiments. Our analysis builds on the model of debt roll-over developed in Morris and Shin (2004). Rational investors (who we also call creditors or lenders) choose whether or not to refinance a borrower with uncertain capacity to repay. The borrower's project succeeds (in our case, the sovereign government is able to repay its debt) if sufficiently many lenders roll over their loans, and fails otherwise,

with a resulting loss to investors. Lenders thus face a coordination problem. Even if the project is viable, a lender may be tempted not to roll over its loan, fearing similar decisions by other lenders. Such fears are self-fulfilling, since they may lead to the failure of the project. We examine the role of credit ratings in this setting, and we obtain the following insights based on the analysis of the model.

First, creditors pay more attention to the credit ratings of borrowers with intermediate values of the fundamentals (which we call distressed borrowers). When economic fundamentals are unambiguously strong or weak, most investors share the same perceptions about credit quality, and ratings have little impact on investor sentiment. But, for intermediate values of the fundamentals, creditors are unsure about whether others will roll over their maturing claims or not, and they will pay close attention to credit ratings so as to guess what others will do. A downgrade, however small, induces many investors to decline to roll over their claims with fear that others will do the same, thereby affecting the ability of the borrower to refinance his debt. Rating announcements thus become self-fulfilling prophecies for distressed borrowers, as investors overreact to rating news and induce considerable changes in creditworthiness.<sup>1</sup>

Second, credit ratings are volatile and inaccurate for borrowers in distress. The interaction between investors and rating agencies creates feedback effects. The rating determines the roll-over decisions, which affect the borrower's default probability. This, in turn, influences the rating. Agencies anticipate the effects of their actions on roll-over decisions, and multi-notch downgrades may occur in response to small shocks to economic fundamentals. Moreover, agencies base their analysis on noisy information about the borrower. Since the impact of ratings is large for intermediate values of the fundamentals, so is the impact of any noise in ratings that inevitably creeps in. As a result, credit ratings become hypersensitive to noise when they are more influential,

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<sup>1</sup>The problems created by the potential for overreaction to public information are well-known to government officials with high visibility. Central bankers have developed specific communication skills, knowing how their public statements may disproportionately influence financial markets.

thus supporting measures to curb the use of credit ratings for borrowers in distress and during episodes of financial turmoil.

Third, regulation should acknowledge the coordination role of credit ratings and reflect the structure of information in the various segments of financial markets. Ratings convey to investors not only information about the borrower, but also valuable information about the beliefs of other investors. As a result, investors overly rely on agencies' pronouncements when ratings are the main source of public information.<sup>2</sup> For example, much detailed information about the risk characteristics of the underlying assets is lost in the process of securitization. Such information loss is serious in view of the collateral's heterogeneity and the complexity of the design of structured debt securities. Absent such information, credit ratings become the main (and often the only) source of information, thus becoming a focal point for investors' beliefs. In this case, rating-contingent regulation should assume that a *AAA* rating in structured finance does not mean the same as a *AAA* rating for corporate or sovereign debt. Indeed, a *AAA* rated structured product is more vulnerable to investor sentiment than a *AAA* corporate or sovereign bond.<sup>3</sup>

The article's first objective is to develop a simple framework that is able to capture and build on the relationship between credit ratings and investor sentiment. In most of the paper, agencies merely function as a pass-through entity of new information. Still, one might conjecture that credit rating agencies are tempted to manipulate investor sentiment to their benefit. Accordingly, the second objective of the paper is to analyze the strategic behavior of rating agencies. We extend the basic model of credit ratings so as to incorporate a richer set of incentives and investigate the dynamic behavior of credit rating agencies. In this extended model, we are able to analyze how agencies'

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<sup>2</sup>This effect is not related with the lack of alternative sources of information, or with the costs of producing private information. A public signal with the same quality as a private signal will have more impact on investor behavior than the private signal (see, for example, Morris and Shin 2002).

<sup>3</sup>European authorities have decided that the rating scale for complex securities in the structured finance segment must be different from the rating scale for other segments, so as to ensure that investors understand the difference between both types of ratings.

incentives condition credit ratings and assess the impact on borrowers, and we obtain the following insights.

First, agencies can exploit the self-fulfilling nature of credit ratings and announce ratings that trigger market reactions which induce the desired equilibrium in environments for which multiple equilibrium would otherwise exist. The existence of a "tough-rating-agency equilibrium" and a "soft-rating-agency equilibrium" demonstrates how borrowers are vulnerable to agencies' incentives.<sup>4</sup>

Second, we show that low quality grades are more accurate than high quality grades when agencies manipulate the precision of credit ratings so as to favour their clients and preserve business relationships. With this type of incentives, agencies invest on accurate ratings when available public information is unfavorable.

Credit ratings influence sentiments as much as sentiments influence credit ratings. Although rating agencies present themselves as detached observers of financial events, they are themselves an integral part of these events because they are vehicles for the spread of opinions and sentiment. Yet, little effort has been done to understand the feedback effects linking credit ratings to market sentiment.<sup>5</sup> We see uncovering these feedback effects as a necessary first step in understanding the functioning of sovereign ratings. Filling this void is our primary goal.

The paper is organized as follows. Section 2 sets up the model of debt roll-over in which rating agencies obtain noisy information about the sovereign borrower and provide reports communicating that information. We then check the robustness of the results in a variety of setups. Section 3 incorporates a market for credit derivatives, and provides clues about the role of credit default swap prices in the design of regulation.

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<sup>4</sup>To use Manso (2013)'s expressions.

<sup>5</sup>Previous work on feedback effects in credit ratings have linked mechanically ratings to debt covenants which are triggered when the credit rating crosses some threshold, but such "rating triggers" are unusual in sovereign debt. Instead, we investigate the role of investor sentiment on feedback effects.

In Section 4 we address the case in which rating agencies follow investor opinion, and Section 5 shows that the coordination effect of credit ratings is exacerbated by market practices, laws and regulations that hardwire buy or sell decisions to rating thresholds. We discuss the strategic behavior of rating agencies in Sections 5 and 6, and Section 7 concludes.

*Relationship to the literature.* The paper is related to several disjoint bodies of literature. The importance of sentiments to explain anomalies in asset pricing is a classic in financial theory, but previous theoretical literature on credit ratings has mostly ignored the role of investor sentiment.<sup>6</sup> Still, our work is related with three notable exceptions. Carlson and Hale (2006) were the first to apply global games to study credit ratings. We extend their analysis by incorporating a number of new features, such as financial markets, investor opinion, and the hardwiring of credit ratings in standards and regulation, so as to fully understand the implications of credit ratings for financial stability. Unlike them, we also study the strategic behavior of rating agencies in the cases of unique and multiple equilibria.

Holden, Natviky and Vigierz (2012) apply the same equilibrium refinements to explain investor behavior. Additionally, they posit that reputational concerns penalize rating agencies for making inaccurate predictions and reward them for making accurate ones. They consider a payoff function which penalizes the difference between the rating and the actual default or continuation of the borrower. Since the final result of the game is either "default" or "no default", the payoff function provides incentives to announce extreme ratings. By issuing extreme ratings, agencies also tilt investor behavior in

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<sup>6</sup>To date, there has been no commonly accepted definition of investor sentiment, since the term may be used in a variety of ways depending on the context. We broadly define a model of investor sentiment as a model of how investors form beliefs. For our purposes, we consider the terms "investor sentiment" and "market sentiment" as equivalent. Researchers in modern behavioral finance have narrowed the scope of the concept, by focusing on investors' erroneous perceptions about the fundamentals. Instead, we focus on a definition of sentiment expressed in terms of rational expectations. This conservative approach to investor sentiment is appropriate for our study, as we impose as few requirements on beliefs as possible, so as to undoubtedly establish a role for sentiments.



their favor, and may improve ex post accuracy. In equilibrium, the rating agencies never issue intermediate ratings.

Their contribution illustrates how the choice of the payoff function can distort the behavior of rating agencies and have profound impact on the final results. Instead, we use a definition of accuracy which we think is more palatable, and we consider milder assumptions on the payoff function of rating agencies. In our case, agencies want their rating grades to mirror ex post default probabilities.

As with Boot, Milbourn and Schmeits (2006), we also see ratings as a coordination mechanism. In their model, agencies may want to influence their clients, and may have an incentive to make announcements that trigger market reactions which discipline borrowers. They show how credit watch procedures may be used in this vein. There are important differences with our paper: first, they posit the existence of a set of institutional investors who mechanically condition their decisions on the credit rating, whereas we take very seriously the rationality of investors and explore information imperfections in financial markets; second, we extend their analysis by incorporating a derivatives market and considering the case in which rating agencies follow investor sentiment.

Our work is also closely related with Manso (2013). He models the interaction between the credit rating agency and the borrowing firm as a game with strategic complementarity, and shows that the interaction between the borrower and the rating agency produces feedback effects. There are two key differences with our work. First, we focus on roll-over risk which is an essential feature of sovereign debt markets, whereas Manso (2013) assumes the borrower uses ratings-based performance sensitive debt (which is a very specific case since, for example, sovereign debt is seldom contingent). The second key difference to our paper is that his work is not about investor behavior. According to Manso (2013) the rating agency exercises direct control over the borrowers through debt payments which are contingent on the ratings, whereas

in our case the control comes from investors that base their roll over decisions on the credit rating and in doing so give credibility to agencies' announcements.

Two assumptions are common to both Boot, Milbourn and Schmeits (2006) and Manso (2013). First, available funding depends mechanically on credit ratings and, second, rating agencies are relevant because they help to sort out the multiple equilibrium problem which results from the interaction between them, borrowers, and investors. We revisit these assumptions, and we replicate their results. Still, in most of the paper we do not follow their approach. Instead, we apply the results in the literature of global games to pin down the unique equilibrium, so as to show that credit ratings cannot be assessed without taking into account the information structure of financial markets and the position of the borrower.

Our work is related to recent models focusing on how incentive problems of financial intermediaries reduce the quality of the information disclosed to investors (see, for example, Mathis, McAndrews and Rochet 2009, Skreta and Veldkamp 2009, Bar-Isaac and Shapiro 2011, Bolton, Freixas and Shapiro 2012, Mariano 2012, and Bar-Isaac and Shapiro 2013).<sup>7</sup> Also, the issuance of ratings based on coarse information is seen by many not only as the source of pre-crisis mispricing of asset backed securities but also as the reason for the subsequent sequence of downgrades in the subprime crisis. The issuance of uninformative ratings is highlighted by Pagano and Volpin (2012) as a major inefficiency and suggests that there is a discrepancy between the private and the social benefits of transparency in debt issues. Finally, Mählmann (2011) suggests that credit rating agencies have an incentive to rescue borrowers in order to uphold their business relations with them.

Also related with our work, Chiu and Koepl (2015) show that agencies chose the

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<sup>7</sup>Many incentive problems discussed in the literature are not regarded as central to the failure of sovereign ratings, as most of the information used in the construction of sovereign ratings is public, there is competition among agencies which produce sovereign ratings, and "ratings shopping" is not seen as a major problem in sovereign debt markets.

accuracy of their ratings so as to provide enough liquidity to the secondary market. In their model (i) agencies choose the accuracy of credit ratings ex ante whereas in our model agencies pick its ex post precision (although rating methodologies are disclosed ex ante, rating committees do have sufficient room for maneuver to select the accuracy of their announcements ex post), and (ii) dealers choose endogenously whether the debt issuer should ask for a rating or not, which is an adequate assumption for some types of corporate debt (but is not appropriate for other types of debt, such as sovereign debt).

None of the above-mentioned papers directly addresses the excessive responsiveness of investors to credit ratings. Through its emphasis on this issue, our paper is reminiscent of the wide body of literature linking the structure of information to investor coordination. Our article emphasizes the distinction between private information and the different types of public information. In that, it is particularly related to Morris and Shin (2002), Angeletos and Werning (2006) and Dasgupta (2007) who show that investors overreact to public information. We extend their analysis by incorporating credit ratings as an endogenous source of public information.

## 2 The basic model with exogenous information

The model builds on Morris and Shin (2004). A sovereign government has an outstanding amount of one period debt that equals 1 and is about to mature. The government can and is willing to repay an exogenous share  $\theta$  of this debt— $\theta$  may be interpreted as available cash—while the remaining amount of debt  $(1 - \theta)$  needs to be rolled over. Government debt is held by a continuum of short term creditors indexed by  $i$  and uniformly distributed over the  $[0, 1]$  interval. Each short term creditor individually decides whether or not to roll over his unit of debt, and we define  $a_i \in \{0, 1\}$  as individual investment. Let  $A = \int_0^1 a_i di$  denote the aggregate level of investment.

We introduce strategic complementarity by assuming that the individual return to

investment depends on the aggregate level of investment. Accordingly, investors have utility

$$u(a_i, A, \theta, \psi) = \begin{cases} R & \text{when } a_i = 1 \text{ and } \psi + A + \theta \geq 1 \\ R - \Delta & \text{when } a_i = 1 \text{ and } \psi + A + \theta < 1 \\ 1 & \text{when } a_i = 0 \end{cases}$$

where  $R$  and  $\Delta$  are constants with  $0 < R - 1 < \Delta \leq R$ . Parameter  $\psi$  measures the amount of exogenous funding that the government can guarantee. We give the following interpretation to the payoff function. Provided the mass of investors  $A$  is large enough, the government is able to fulfil its promise and repays  $R$ ; otherwise, the country is forced to default on its debt and repays  $R - \Delta$  (where  $\Delta$  measures Loss Given Default, LGD). Alternative investment opportunities yield zero interest.

For simplicity, we assume the borrower's finances are viable in the long run as long as  $\theta + \psi \geq 0$ . If  $\theta + \psi < 0$ , the country is not solvent. When  $\theta + \psi \geq 1$ , the debtor country faces no liquidity problems because the sovereign has enough funds to meet the maturing debt. In the intermediate range  $0 \leq \theta + \psi < 1$ , the fate of the country lies in the hands of its short term creditors, and it will default if it is unable to convince creditors to roll over their claims.

Parameter  $\psi$  is an indicator of how easy it is to coordinate investors. Factors determining this parameter are:

- Financial assistance from international agencies—for instance, obtained through stabilization programmes by the International Monetary Fund (IMF) or the European Stability Mechanism.
- The proportion of long term debt to total debt. It is possible to redefine the outstanding amount of one period debt, and let it be equal to  $1 - \psi$ . According to this interpretation,  $\psi$  represents the proportion of long term debt. Long term creditors are passive, as they are summoned only when the debtor country

defaults.

- Exogenous shocks in the preferences of investors, which affect the demand for sovereign bonds.

**Information and transparency.** The  $\theta$  stands for the underlying economic fundamental. The fundamental  $\theta \in \mathbb{R}$  is not known at the time the lending decisions are made, and short term creditors have heterogeneous beliefs about  $\theta$ . The fundamental  $\theta$  is drawn from an improper uniform prior over the real line. A sufficient statistic  $z$  summarizes the public information such that  $z = \theta + \sigma_z \varepsilon$ , where  $\sigma_z > 0$  and  $\varepsilon$  is standard normal, independent of  $\theta$  and common across agents. The private information of short term creditor  $i$  is summarized by a sufficient statistic  $x_i = \theta + \sigma_x \xi_i$ , where  $\sigma_x > 0$  and  $\xi_i$  is standard normal, independent of  $\theta$  and independent and identically distributed across agents.

The information structure is parametrized by the standard deviations  $\sigma_x$  and  $\sigma_z$  or, equivalently, by  $\alpha_x = \sigma_x^{-2}$  and  $\alpha_z = \sigma_z^{-2}$ , the precision of private and public information. The information structure and the values of parameters  $\alpha_x$  and  $\alpha_z$  are common knowledge. Private signals introduce heterogeneity in market expectations about the economic fundamental, and may be understood as heterogeneity in the reading and interpretation of available information.

Short term creditors face two types of uncertainty—fundamental uncertainty and strategic uncertainty. Fundamental uncertainty refers to uncertainty concerning the value of  $\theta$ . The posterior belief of agent  $i$  about  $\theta$  is normal with mean  $E[\theta|x_i, z] = \frac{\alpha_x}{\alpha_x + \alpha_z} x_i + \frac{\alpha_z}{\alpha_x + \alpha_z} z$  and variance  $Var[\theta|x_i, z] = \frac{1}{\alpha_x + \alpha_z}$ . Strategic uncertainty refers to the uncertainty concerning the actions of other creditors. Strategic uncertainty implies that each investor must form beliefs about the decisions of others, since the aggregate level of investment  $A$  determines the final payoffs.

## 2.1 Equilibrium

A strategy for short term creditor  $i$  is a decision rule which defines an action conditional on the observed signal  $x_i$ . Throughout the paper, we restrict attention to equilibria in which short term creditors use *switching strategies*, in which case creditor  $i$  rolls over the loan whenever the private signal  $x_i$  is higher than some given threshold level  $x^*(z)$ . Later, we comment on why this is the only possible class of strategies in equilibrium.

Since  $x_i = \theta + \sigma_x \xi_i$ , aggregate investment is increasing in the fundamental  $\theta$ . As a result, there is a threshold for the economic fundamental  $\theta^*(z)$  below which there is default because an insufficient number of short term creditors chooses to roll over their loans. When  $\theta \geq \theta^*(z)$  there is no default, because a sufficient mass of short term creditors receives private signals above  $x^*(z)$ .

Let  $p_i$  be the probability that agent  $i$  attributes to  $\theta \geq \theta^*(z)$ . An agent finds it optimal to invest, when the expected return from roll-over is larger than the payoff from the alternative, that is, when  $p_i R + (1 - p_i)(R - \Delta) \geq 1$ .

We restrict  $\sigma_z^2 \sqrt{2\pi} > \sigma_x$ . This suffices for the equilibrium to be unique and amounts to saying that public information cannot be too precise, otherwise there are multiple equilibria. Let  $\Phi$  and  $\phi$  denote, respectively, the standard normal cumulative distribution function and the standard normal density distribution function.

**Proposition 1** *If  $\sigma_z^2 \sqrt{2\pi} > \sigma_x$ , there exists a unique equilibrium, and  $\theta^*(z)$  is implicitly determined by  $\theta^* = \Phi(\sigma_x [\sqrt{\alpha_x + \alpha_z} \Phi^{-1}(1 - \frac{R-1}{\Delta}) + \alpha_z(\theta^* - z)]) - \psi$ . The default threshold  $\theta^*$  is decreasing in the return  $R$ , increasing in the LGD  $\Delta$ , decreasing in the statistic for public information  $z$ , and decreasing in exogenous funding  $\psi$ .*

Equilibrium is described by a threshold  $x^*(z)$  for the switching strategy of short term creditors. The threshold  $x^*(z)$  defines a critical value  $\theta^*(z)$  for the fundamental

$\theta$ , below which the debtor country defaults. Since there is a one-to-one relationship between the thresholds  $x^*(z)$  and  $\theta^*(z)$ , it suffices to define the value of the cutoff for the fundamental. Both thresholds decrease with  $z$ , since favorable public information has a positive effect on investment decisions.

The previous proposition accounts for a number of stylized facts. First, a high promised return  $R$  attracts short term creditors and reduces the probability of default while a high LGD has the opposite effect. Second, favorable public information, measured by  $z$ , improves confidence regarding the capability of the sovereign to fulfil its obligations and favours debt roll-over. Since  $z = \theta + \sigma_z \varepsilon$ , a stronger fundamental eases access to credit, which is a desirable property for any model of defaultable debt. Third, exogenous and long term funding, measured by  $\psi$ , reduce the incidence of failure.

We have considered a solution in switching strategies. Morris and Shin (2004) show that condition  $\sigma_z^2 \sqrt{2\pi} \geq \sigma_x$  is sufficient for there to be a unique dominance-solvable equilibrium and, therefore, is sufficient for uniqueness of equilibrium in any class of strategies.

**Efficiency.** Since  $A \leq 1$ , the borrower is unable to roll over its short term debt whenever  $\psi + \theta < 0$ . In this sense, the sovereign is deemed insolvent when  $\theta < -\psi$ . Since the equilibrium default threshold  $\theta^*$  lies above the insolvency threshold  $-\psi$ , the borrower is unable to roll over its debt for  $\theta \in [-\psi, \theta^*)$ . In this case, a solvent borrower is forced into failure.

It is instructive to contrast a single creditor decision problem with the current setup. If there were one single creditor (with one unit of funds), he would provide the funds as long as  $\theta \geq -\psi$ . The country would not default, as long as the fundamental  $\theta$  is above the insolvency threshold  $-\psi$ .

But dispersed creditors face a coordination problem. Since they roll over their loans if and only if they believe their peers will do the same, an insufficient number of lenders

rolls over when  $\theta \in [-\psi, \theta^*)$ . For values of the fundamental in this interval, liquidation is inefficient but it is forced on the sovereign. We interpret the value of the threshold  $\theta^*$  as a measure of the inefficiency due to coordination failure.

## 2.2 Model with a credit rating agency

In this section, we add a public signal which represents the credit rating. Credit rating agencies (CRAs) base their analysis on public information and on confidential information which borrowers agree to share with them. We abstract away from any conflict of interest between the rating agency and investors or creditors. More specifically, we posit that the credit rating agency's objective is to set an accurate rating based on available information. An accurate rating informs investors about the probability of default of the borrower.<sup>8</sup>

**Setup.** We assume that there is one single representative rating agency which privately observes a signal  $\rho = \theta + \sigma_\rho \varepsilon_\rho$ , where  $\sigma_\rho > 0$  and  $\varepsilon_\rho$  is standard normal, independent of  $\theta, \varepsilon$ , and  $\xi_i$ . Signal  $\rho$  represents *new* available information not previously accessible to investors; variable  $\varepsilon_\rho$  is associated with errors in the agency signal. These errors may be interpreted as mistakes resulting from the process of collecting information and have the potential to mislead investors.

The objective of the CRA is to set an accurate rating based on available information. Let  $p_{CRA}$  be the probability that the sovereign is able to repay its debt, conditional on  $z$  and  $\rho$ —the information available to the CRA. The CRA maps the probability  $p_{CRA}$

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<sup>8</sup>Although CRAs do not explicitly quantify their rating scales, they do provide ex-post summaries of default rates by rating category. Moreover, they reveal their target default probabilities and loss rates for some types of debt (for example, Moody's uses idealized default rates by rating category based on historical default rates). CRAs do not try to maintain constant default rates for given rating category (as this would imply en masse adjustments in response to changes in cyclical conditions), but it is commonly understood that CRAs implicitly control the relationship between rating grades and ex-post default statistics, and use this relationship as the main indicator of their success. Ratings are often used as though they map into specific credit-risk metrics, including the Basel Accord standardized approach. The European Central Bank's collateral framework has explicitly mapped credit ratings into one year default probabilities (ECB, 2008).



into rating grades. A rating on sovereign debt  $\hat{\rho}$  is accurate if  $\hat{\rho} = p_{CRA}$ .

The individual roll-over decision is a function of  $x, z$  and  $\hat{\rho}$ , and aggregate investment is a function of  $\theta, z$  and  $\hat{\rho}$ . We define an equilibrium as follows.

**Definition 1** *An equilibrium is a rating function  $\hat{\rho}(z, \rho)$ , and individual strategies for investment in public debt  $a(x, z, \hat{\rho})$ , which satisfy*

$$\begin{aligned}\hat{\rho}(z, \rho) &\in \arg \min_{\hat{\rho} \in [0,1]} \hat{\rho} - p_{CRA} \\ a(x, z, \hat{\rho}) &\in \arg \max_{a \in \{0,1\}} E[u(a, A(\theta, z, \hat{\rho}), \theta, \psi) | x, z, \hat{\rho}] \\ A(\theta, z, \hat{\rho}) &= E[a(x, z, \hat{\rho}) | \theta, z, \hat{\rho}]\end{aligned}$$

and the sovereign government defaults if and only if  $\psi + A(\theta, z, \hat{\rho}) + \theta < 1$ .

The above conditions define a perfect Bayesian equilibrium with  $\hat{\rho} = p_{CRA}$ . In the base model we consider the simplest case in which the precision of the agency's signal  $\alpha_\rho = \sigma_\rho^{-2}$  is fixed (see below the case in which  $\alpha_\rho$  is endogenous).

**Equilibrium.** We focus on strategies in which there is a one-to-one mapping between  $\rho$  and  $\hat{\rho}$ , so that these variables have identical informational content (see below the discussion on the case in which the relationship may not be unique). Given the public signal  $z$ , it is indifferent which one is announced in equilibrium. Rational investors recognize how the agency forms the credit rating, and extract the *new* (and relevant) information produced by the CRA. Still, while  $z$  and  $\rho$  are exogenous variables, the rating  $\hat{\rho}$  is determined endogenously.

For the sake of simplicity we assume that the rating agency publicly announces its private signal  $\rho$ , and short term creditors update their common prior accordingly. The new prior about  $\theta$  is normal with mean  $E[\theta | z, \rho] = \frac{\alpha_z}{\alpha_z + \alpha_\rho} z + \frac{\alpha_\rho}{\alpha_z + \alpha_\rho} \rho$  and precision  $\alpha_z + \alpha_\rho$ . Short term creditors combine this information with their private signals  $x_i$ . We

focus on the case in which there is a unique equilibrium, i.e.  $\sigma_z^2 \sigma_\rho^2 \sqrt{2\pi} > \sigma_x (\sigma_z^2 + \sigma_\rho^2)$ .

**Proposition 2** *If  $\sigma_z^2 \sigma_\rho^2 \sqrt{2\pi} > \sigma_x (\sigma_z^2 + \sigma_\rho^2)$ , there exists a unique equilibrium, and  $\theta^*(z, \rho)$  is implicitly determined by  $\theta^* = \Phi(\sigma_x [\sqrt{\alpha_x + \alpha_z + \alpha_\rho} \Phi^{-1}(1 - \frac{R-1}{\Delta}) + (\alpha_z + \alpha_\rho) (\theta^* - \frac{\alpha_z}{\alpha_z + \alpha_\rho} z - \frac{\alpha_\rho}{\alpha_z + \alpha_\rho} \rho)]) - \psi$ . The default threshold  $\theta^*$  is decreasing in the signal  $\rho$ .*

The threshold  $\theta^*(z, \rho)$  decreases with the signal  $\rho$  disclosed by the CRA, but creditors are unable to identify the source of favorable information—a strong fundamental  $\theta$  or a mistake in the information being disclosed (i.e. a positive shock  $\varepsilon_\rho$ ).

**Rating actions.** The rating on sovereign debt equals the probability that the actual value of the fundamental  $\theta$  lies above the threshold  $\theta^*$  conditional on  $z$  and  $\rho$ ,

$$\hat{\rho}(z, \rho) = \Phi\left(\left(\frac{\alpha_z}{\alpha_z + \alpha_\rho} z + \frac{\alpha_\rho}{\alpha_z + \alpha_\rho} \rho - \theta^*\right) \sqrt{\alpha_z + \alpha_\rho}\right). \quad (1)$$

The agency is not merely disclosing its private signal; it is rendering an opinion on creditworthiness as it anticipates the chain of events which will unfold (and lead to a shift in the threshold  $\theta^*$ ) following the issue of the rating. The response of credit ratings to a shift in the fundamental  $\theta$  equals

$$\frac{\partial \hat{\rho}}{\partial \theta} = \sqrt{\alpha_z + \alpha_\rho} \phi(\Phi^{-1}(\hat{\rho})) + \frac{\phi(\Phi^{-1}(\hat{\rho})) \phi(\Phi^{-1}(\theta^* + \psi)) \sigma_x (\alpha_z + \alpha_\rho)^{\frac{3}{2}}}{1 - \phi(\Phi^{-1}(\theta^* + \psi)) \sigma_x (\alpha_z + \alpha_\rho)} > 0 \quad (2)$$

where we use the result  $\partial z / \partial \theta = \partial \rho / \partial \theta = 1$ . The term  $\sqrt{\alpha_z + \alpha_\rho} \phi(\Phi^{-1}(\hat{\rho}))$  expresses the conventional intuition that better fundamentals reduce the probability of default. The term  $\frac{\phi(\Phi^{-1}(\hat{\rho})) \phi(\Phi^{-1}(\theta^* + \psi)) \sigma_x (\alpha_z + \alpha_\rho)^{\frac{3}{2}}}{1 - \phi(\Phi^{-1}(\theta^* + \psi)) \sigma_x (\alpha_z + \alpha_\rho)}$  is the novel feature. It arises because better fundamentals are correlated with favorable information, and auspicious information favours coordination, thereby reducing the default threshold  $\theta^*$ . We call this term the *coordination effect* of credit ratings.

## 2.3 Empirical implications

In this section, we use the specification of our model as a basis for our main empirical predictions. Because credit ratings coordinate agent's expectations, there are *crisis zones* where ratings (*i*) have a big influence on sentiments and credit quality, and (*ii*) are inaccurate. In this subsection, we assume  $R = 1 + \Delta/2$  in order to keep the analysis tractable enough to investigate the relationship between ratings and sentiments.

**Excess sensitivity of the default threshold.** The absolute value of the derivative  $\frac{\partial \theta^*}{\partial \rho}$  (computed in Proposition 2) measures the sensitivity of the default threshold to variations in the *new* information disclosed by the CRA. One can show that this sensitivity is larger for  $E[\theta|z, \rho] = \theta^*$ , which means that the response of the default threshold  $\theta^*$  is maximum when public information reveals that the fundamental is near this threshold. This is intuitive, as in this case most investors are uncertain about whether others will roll over their claims or not, and they pay close attention to public information in order to guess what others will do. A small downgrade is enough to induce many investors to withhold short term credit with fear that others will do the same, thereby making the default threshold very sensitive to small adjustments in the credit rating. The response of the default threshold  $\theta^*$  to public information  $z$ , and to exogenous funding  $\psi$ , is also larger when  $E[\theta|z, \rho] = \theta^*$ .

The default threshold is not affected by the credit rating when the fundamental  $\theta$  is very strong or weak, as  $\lim_{E[\theta|z, \rho] \rightarrow +\infty} \frac{\partial \theta^*}{\partial \rho} = \lim_{E[\theta|z, \rho] \rightarrow -\infty} \frac{\partial \theta^*}{\partial \rho} = 0$ . This result makes clear that credit ratings have little impact when the fundamental is very strong or very weak. In these cases, the willingness to invest will not shift following an upgrade or a downgrade.<sup>9</sup>

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<sup>9</sup>Two examples come to mind: the US and Greece. In 2011, Standard & Poor's downgraded US treasury bonds from AAA to AA+. Many observers were surprised to see yields on US bonds actually decline when this happened. This could be because US fundamentals were perceived as unambiguously strong, so that the downgrade had virtually no feedback effect. On the other end of the spectrum is Greece. At this point, market participants are well aware of Greece's poor fundamentals, and there is little risk that another downgrade would generate feedback effects.

A strand of empirical research uses event studies to measure the impact of credit ratings on capital markets. Most studies use data on credit risk sensitive securities, such as stock and bond prices, and more recently credit default swap (CDS) spreads, to gauge the impact of credit rating events. The predictions of our model are largely confirmed by Hand, Holthausen and Leftwich (1992), Hite and Warga (1997), Goh and Ederington (1999), and Dichev and Piotroski (2001). Using data on bond and stock prices, they find that the magnitude of downgrading effects increases substantially as their samples move from investment grade to noninvestment grade firms.

The same results hold for upgrades. Finnerty, Miller and Chen (2013) document a significant impact on CDS spreads from corporate upgrade announcements. They document larger impact on CDS spreads from credit rating changes that cross over the dividing line that separates investment grade and noninvestment grade. Using data on sovereign bonds of emerging markets, Ismailescu and Kazemi (2010) find that CDS spreads of noninvestment grade entities react strongly to credit rating upgrades. Norden and Weber (2004) investigate a sample of investment grade bonds and report no significant reaction to positive credit rating events, but acknowledge that this lack of significance may be due to the composition of their sample.

Aizenman, Binici and Hutchison (2013) study the effect of sovereign credit rating change announcements on the CDS spreads of European sovereign bonds in the context of the recent crisis. They report insensitive CDS spreads at the top of the rating scale (above *A*), and a high sensitivity for middle ratings (with the highest sensitivity at the *BB+* level), which broadly confirms our predictions.<sup>10</sup>

Still, the above results require a careful interpretation, since this type of excess sensitivity may follow from regulation and standards preventing (institutional) investors from holding bonds below investment grade. More on this below.

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<sup>10</sup>Surprisingly, Aizenman, Binici and Hutchison (2013) report high sensitivity of CDS spreads at the very low end of the rating scale.

**Excess sensitivity of the credit rating.** The derivative  $\frac{\partial \hat{p}}{\partial \theta}$  (obtained in expression 2) measures the sensitivity of the credit rating to variations in the fundamental  $\theta$ . The credit rating is more sensitive when public information signals that the fundamental is near the default threshold. In this case, investors pay close attention to the information received so as to guess if others will roll over or not. A small deterioration of the fundamental entails (slightly) negative information, which increases the proportion of creditors who decline to roll over, thereby raising significantly the likelihood of default. The CRA anticipates this chain of events, and incorporates these effects in its pronouncements, making the rating sensitive to apparently innocuous shifts in the fundamental.

A strand of the empirical research uncovers the determinants of sovereign ratings, using linear estimation methods and ordered response models. Sovereign debt rating notations are modelled as a function of explanatory variables like per capita income, growth, inflation, fiscal balance, external balance, short term debt, economic development, default history, and bond yield spreads. A number of studies analyze the 1997 Asian crisis. For example, Ferri, Liu and Stiglitz (1999) show that the drop in actual ratings was sharper than the predictions of a model of ratings based on economic fundamentals, suggesting that rating downgrades were larger than what economic fundamentals would justify; similar anomalies have been documented by the International Monetary Fund (1999). Mora (2006) suggests that assigned ratings react to market sentiment (as proxied by bond spreads) and overreact during crises.

The credit rating is very stable when the fundamental  $\theta$  is very strong or weak, because  $\lim_{E[\theta|z,\rho] \rightarrow +\infty} \frac{\partial \hat{p}}{\partial \theta} = \lim_{E[\theta|z,\rho] \rightarrow -\infty} \frac{\partial \hat{p}}{\partial \theta} = 0$ . Hence, the model predicts that sharp downgrades should be less frequent for strong fundamentals. Indeed, transition matrices among rating grades confirm that over the long term, higher rating grades are more stable than the intermediate rating categories (see, for example, International Monetary Fund 2010, or Standard & Poor's 2011).

As  $E[\theta|z, \rho]$  approaches  $\theta^*$ , the behavior of credit ratings and creditors changes considerably when compared with their behavior in normal times (as suggested by Aizenman, Binici and Hutchison 2013). This feature is particularly important for estimating models of sovereign ratings, which often lack data for borrowers in distress. When most of the observations are from periods of financial stability, models of ratings are likely to underestimate the role of market sentiment. As a result, these models are likely to obtain overreaction of credit ratings in financial crisis, when market sentiment becomes important. This would explain the discrepancy between predicted ratings and assigned ratings, as in Mora (2006).

## 2.4 Distressed borrowers

In this subsection, we examine the role of credit ratings for values of the fundamental  $\theta$  in the neighborhood of the default threshold  $\theta^*$ , so as to study the case of distressed borrowers.

Specifically, we evaluate the sensitivity of the default threshold to  $\varepsilon_\rho$ , which we interpret as mistakes made by the CRA, and we perform the analysis for values of the fundamental near the threshold  $\theta^*$ . There is default if and only if  $\theta < \theta^*(z, \rho)$ , where  $z = \theta + \sigma_z \varepsilon$  and  $\rho = \theta + \sigma_\rho \varepsilon_\rho$ . Since function  $\theta^*(z, \rho)$  is continuously decreasing in both arguments, the sovereign defaults if and only if  $\theta < \hat{\theta}(\varepsilon, \varepsilon_\rho)$ , where  $\hat{\theta}(\varepsilon, \varepsilon_\rho)$  is the unique solution to

$$\begin{aligned} \hat{\theta} = & \Phi \left( \sigma_x \left[ \sqrt{\alpha_x + \alpha_z + \alpha_\rho} \Phi^{-1} \left( 1 - \frac{R-1}{\Delta} \right) + \right. \right. \\ & \left. \left. + (\alpha_z + \alpha_\rho) \left( \hat{\theta} - \frac{\alpha_z}{\alpha_z + \alpha_\rho} (\hat{\theta} + \sigma_z \varepsilon) - \frac{\alpha_\rho}{\alpha_z + \alpha_\rho} (\hat{\theta} + \sigma_\rho \varepsilon_\rho) \right) \right] \right) - \psi. \end{aligned}$$

or, equivalently,  $\hat{\theta} = \Phi \left( \sigma_x \left[ \sqrt{\alpha_x + \alpha_z + \alpha_\rho} \Phi^{-1} \left( 1 - \frac{R-1}{\Delta} \right) - \sqrt{\alpha_z} \varepsilon - \sqrt{\alpha_\rho} \varepsilon_\rho \right] \right) - \psi$ . Since

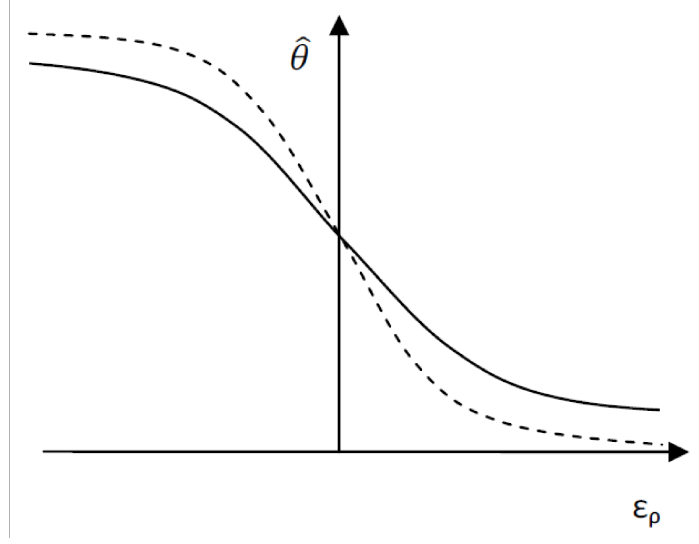


Figure 1: Default threshold as a function of  $\varepsilon_\rho$ . The dashed line corresponds to higher ratio  $\frac{\alpha_\rho}{\alpha_x}$ .

$\hat{\theta}$  is the default threshold written as a function of noise  $\varepsilon$  and  $\varepsilon_\rho$ , one can compute

$$\frac{\partial \hat{\theta}}{\partial \varepsilon_\rho} = -\phi\left(\Phi^{-1}\left(\hat{\theta} + \psi\right)\right) \frac{\sigma_x}{\sigma_\rho} < 0.$$

For given  $\varepsilon$ , function  $\hat{\theta}(\varepsilon, \varepsilon_\rho)$  satisfies the single crossing property with respect to the ratio  $\frac{\sigma_x}{\sigma_\rho}$ . Figure 1 depicts the default threshold as a function of the information divulged by the CRA (as measured by  $\varepsilon_\rho$ ), with the dashed line corresponding to a higher precision  $\alpha_\rho$  than the solid one.

The curvature of the relation is a key observation of the paper, and has important implications regarding the role of credit ratings. If the relation were linear, investors would assign constant weights to private and public information. Since the relative accuracy of the credit rating is constant, the curvature shows that short term creditors give more importance to the credit rating when the fundamental is near the default threshold, thus showing that these creditors tend to take CRA's pronouncements too seriously.

Near the default threshold, the issuer is vulnerable to information released by the CRA because the probability of default is largely determined by the beliefs of market participants, and credit ratings signal *and* influence what market participants are doing and thinking. Noisy (but on average accurate) private information about the fundamentals of the borrowers is not so valuable; short term creditors place more weight on public information, which leads to the default threshold becoming more dependent on ratings.

Figure 1 shows that the curvature of the relation becomes more pronounced when the credit rating is relatively more precise. This is because the credit rating provides considerably more information about the roll-over decision of others (than the private signal), so that individuals place more weight on the credit rating. For example, Cornaggia, Cornaggia and Israelsen (2015) document stronger impact of municipal credit ratings among more opaque issuers. They suggest that the reliance on credit ratings is greatest among opaque issuers for which investors lack alternative sources of reliable information.

When the fundamental is near the default threshold  $\theta^*$ , the relationship between the credit rating and the information released by the CRA (measured by  $\varepsilon_\rho$ ) may be written as  $\hat{\rho}(\varepsilon, \varepsilon_\rho) = \Phi\left(\sqrt{\frac{\alpha_z}{\alpha_z + \alpha_\rho}}\varepsilon + \sqrt{\frac{\alpha_\rho}{\alpha_z + \alpha_\rho}}\varepsilon_\rho\right)$ . For given  $\varepsilon$ , function  $\hat{\rho}(\varepsilon, \varepsilon_\rho)$  satisfies the single crossing property with respect to the ratio  $\frac{\alpha_\rho}{\alpha_z}$ . Figure 2 plots the credit rating as a function of  $\varepsilon_\rho$ , with the dashed line corresponding to a higher ratio  $\frac{\alpha_\rho}{\alpha_z}$  than the solid one. The curvature of the relationship shows that credit ratings unduly influence roll-over decisions near the default threshold, thereby affecting the ability of the borrower to refinance its debt. Agencies anticipate (and incorporate) the strong impact of their pronouncements, thus making credit ratings very sensitive to new information. The curvature becomes more pronounced when credit ratings are more accurate than other sources of public information, that is when the ratio  $\frac{\alpha_\rho}{\alpha_z}$  is large.

**Policy implications.** As can be seen from Figures 1 and 2, the effects of varia-



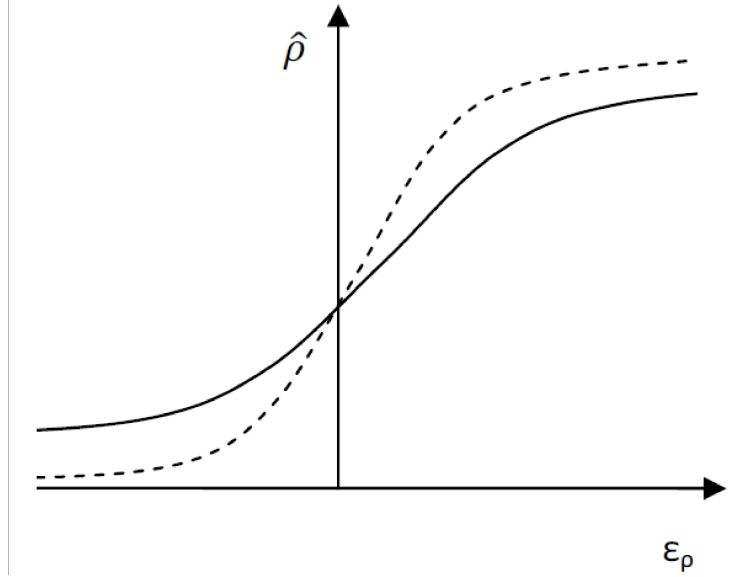


Figure 2: Credit rating as a function of  $\varepsilon_\rho$ . The dashed line corresponds to higher ratio  $\frac{\alpha_\rho}{\alpha_z}$ .

tions in noise  $\varepsilon_\rho$  depend on the relative precision of the information contained in the credit rating, and we conclude that credit ratings should reflect the specific features of each market. During the subprime crisis, CRAs are generally seen to have performed well in the corporate bond market. It is for complex securities in the structured finance segment that ratings performance has come under severe criticism (see, for example, the recommendations made by the Issing Committee 2009). Ratings of complex securities are very sensitive to the information disclosed by CRAs because there are few other sources of information available (thus increasing the *relative* precision of the information contained in credit ratings). These features support the decision by European authorities that CRAs should discriminate between ratings for structured finance products and ratings for other financial obligations.<sup>11</sup>

The effect of exogenous finance  $\psi$  on the default threshold is higher when  $E[\theta|z, \rho]$

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<sup>11</sup>Moreover, agencies issue nearly identical forecasts for simple assets, but for complex assets ratings often reveal large assessment differences which create incentives to shop for the best rating.

is near  $\theta^*$ , lending support to the hypothesis of catalytic finance by Morris and Shin (2006) and Corsetti, Guimarães and Roubini (2006).<sup>12</sup> Liquidity provision by an official institution like the IMF can work to prevent a destructive run by moving the default threshold  $\theta^*$  downwards. Yet, the excess sensitivity of the default threshold  $\theta^*$  to  $\varepsilon_\rho$  may offset the positive influence of catalytic finance. Mistakes in credit ratings may easily counteract the effects of financial assistance, suggesting that regulators should be able to temporarily suspend the use of sovereign ratings for a country undergoing an international bailout programme. This is what the European Central Bank (ECB) has implicitly done by adapting its collateral framework during the crisis to accept lower-rated assets as collateral.<sup>13</sup>

### 3 Financial markets

There is evidence that CDS positions influence firms' credit risk and credit ratings. Spikes in CDS prices have effects on the value of the firm, its likelihood of default and credit rating (see Subrahmanyam, Tang and Wang 2014, and Hortaçsu, Matvos, Syverson and Venkataraman 2013). Moreover, Afonso, Furceri and Gomes (2012) find that CDS spreads anticipate sovereign rating announcements in a 1 – 2 week window, and Norden and Weber (2004), Hull, Predescu and White (2004), and Finnerty, Miller and Chen (2013) confirm that CDS markets anticipate corporate rating downgrades.

To investigate the role of prices, we draw on the analysis of Angeletos and Werning (2006), and we introduce a financial market where agents trade a derivative security prior to playing the coordination game. Because the return on the derivative depends on

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<sup>12</sup>The idea behind catalytic finance is that the provision of official assistance to a sovereign with limited access to financial markets stimulates the private sector creditors into rolling over short term loans, thereby alleviating the funding crisis faced by the debtor country.

<sup>13</sup>For crisis countries, the Eurosystem changed its collateral framework a number of times to allow government bonds to become acceptable as collateral again. For example, when Greek government debt was falling below the *BBB-* threshold required to be acceptable as collateral, the ECB changed the eligibility criteria and the valuation haircuts applied on collateral.

the underlying fundamental, the equilibrium price conveys information that is valuable in the coordination game.

**Setup.** As before, the fundamental  $\theta$  is drawn from an improper uniform distribution over the real line, and each agent  $i$  receives the exogenous private signal  $x_i = \theta + \sigma_x \xi_i$ . For tractability reasons we separate the investment coordination game from the derivatives market. Agents can be seen as interacting in segmented markets and in two separate stages.

The first stage happens in the derivatives market: agents trade a risky asset with return  $\theta$  at a price  $p$ . We adopt the CARA-normal framework introduced by Grossman and Stiglitz (1976). The utility of agent  $i$  is  $v(w_i) = -e^{-\gamma w_i}$  for  $\gamma > 0$ , where  $w_i = w_0 + (\theta - p)k_i$  is the final wealth,  $w_0$  is the initial endowment, and  $k_i$  is investment in the asset. The supply of the asset is uncertain and not observed, given by  $K^S(\varepsilon_s) = \sigma_s \varepsilon_s$ , with  $\sigma_s > 0$ , and  $\varepsilon_s$  is standard normal, independent of  $\theta$  and  $\xi_i$ . This formulation means that the derivative security exists in zero net supply plus some noise—parametrized by  $\sigma_s$ —which prevents a fully revealing equilibrium.<sup>14</sup>

The second stage is essentially the same as the model with credit rating agency in the previous section: short term creditors decide whether to invest or not; the sovereign government defaults if and only if  $\psi + A + \theta < 1$  and the payoff from this stage is  $u(a_i, A, \theta, \psi)$ . There is an important difference in the second stage with respect to the model in Section 2.2. Short term creditors observe the price that cleared the financial market, and the endogenous price  $p$  replaces the exogenous public signal  $z$ . The eventual default by the sovereign, the derivative's return and the payoffs from both stages are realized at the end of stage 2.

Individual demand of the derivative security is a function of  $x$  and  $p$ , the realizations of the private and public signals, and the corresponding aggregate is a function of  $\theta$  and

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<sup>14</sup>Indeed, Stanton and Wallace (2011) suggest that volatility in CDS spreads is often driven by the demand and supply of credit protection, and not by the underlying risk itself.

$p$ . The individual roll-over decision is a function of  $x, \rho$  and  $p$ , and aggregate investment is a function of  $\theta, \rho$  and  $p$ . We define an equilibrium as follows.

**Definition 2** *An equilibrium is a price function,  $P(\theta, \varepsilon_s)$ , and individual strategies for investment in the derivative and in public debt,  $k(x, p)$  and  $a(x, \rho, p)$ , which satisfy:*

$$\begin{aligned} k(x, p) &\in \arg \max_{k \in \mathbb{R}} E[v(w_0 + (\theta - p)k) | x, p] \\ K(\theta, p) &= E[k(x, p) | \theta, p] \\ K^S(\varepsilon_s) &= K(\theta, P(\theta, \varepsilon_s)) \\ a(x, \rho, p) &\in \arg \max_{a \in \{0, 1\}} E[u(a, A(\theta, \rho, p), \theta, \psi) | x, \rho, p] \\ A(\theta, \rho, p) &= E[a(x, p) | \theta, \rho, p] \end{aligned}$$

and the sovereign government defaults if and only if  $\psi + A(\theta, \rho, P(\theta, \varepsilon_s)) + \theta < 1$ .

The above conditions define a rational expectations equilibrium for the first stage, and a perfect Bayesian equilibrium for stage 2. For the sake of simplicity we have already imposed that the CRA discloses  $\rho$  (and  $\hat{\rho} = p_{CRA}$ ).

**Equilibrium.** In the first stage, we look for a linear price function. Observing the price realization then is equivalent to observing a normally distributed signal with some precision  $\alpha_p = \sigma_p^{-2} \geq 0$ . The posterior of  $\theta$  conditional on  $x$  and  $p$  is normally distributed with mean  $\frac{\alpha_x}{\alpha_x + \alpha_p}x + \frac{\alpha_p}{\alpha_x + \alpha_p}p$  and precision  $\alpha_x + \alpha_p$ . Individual asset demand is  $k(x, p) = \frac{\alpha_x}{\gamma}(x - p)$  and aggregate demand is  $K(\theta, p) = \frac{\alpha_x}{\gamma}(\theta - p)$ . Market clearing implies  $P(\theta, \varepsilon_s) = \theta - \gamma\sigma_s\sigma_x^2\varepsilon_s$ , which verifies the initial guess with  $\sigma_p = \gamma\sigma_s\sigma_x^2$ . This result highlights the informative role of prices because the precision of public information improves with private information.

The second stage is equivalent to the benchmark model in the previous section, with the price  $p$  playing the role of the public signal. Replace  $\sigma_z$  with  $\sigma_p$ , and the

uniqueness condition becomes  $\gamma^2 \sigma_s^2 \sigma_x^3 \sigma_\rho^2 \sqrt{2\pi} > \gamma^2 \sigma_s^2 \sigma_x^4 + \sigma_\rho^2$ .

**Proposition 3** *If  $\gamma^2 \sigma_s^2 \sigma_x^3 \sigma_\rho^2 \sqrt{2\pi} > \gamma^2 \sigma_s^2 \sigma_x^4 + \sigma_\rho^2$ , there exists a unique equilibrium, and  $\theta^*(p, \rho)$  is implicitly determined by  $\theta^* = \Phi(\sigma_x [\sqrt{\alpha_x + \alpha_p + \alpha_\rho} \Phi^{-1}(1 - \frac{R-1}{\Delta}) + (\alpha_p + \alpha_\rho)(\theta^* - \frac{\alpha_p}{\alpha_p + \alpha_\rho} p - \frac{\alpha_\rho}{\alpha_p + \alpha_\rho} \rho)]) - \psi$ . The default threshold  $\theta^*$  is decreasing in the price  $p$ .*

The model behaves in a similar way to the model with CRA in Section 2.2, with prices replacing the public signal  $z$ ; high prices signal a strong fundamental and favor debt roll-over.

**Credit ratings.** The CRA incorporates the information embedded in the financial price into the credit rating, so that

$$\hat{p}(p, \rho) = \Phi\left(\left(\frac{\alpha_p}{\alpha_p + \alpha_\rho} p + \frac{\alpha_\rho}{\alpha_p + \alpha_\rho} \rho - \theta^*\right) \sqrt{\alpha_p + \alpha_\rho}\right). \quad (3)$$

It follows that  $\frac{\partial \hat{p}}{\partial \varepsilon_s} < 0$ . A positive supply shock reduces the price of the derivative which the CRA interprets as a deterioration in credit quality. Noise in financial markets has two effects on the rating of sovereign debt. First, there is a direct effect because low prices signal low credit quality. Second, there is the coordination effect working through the default threshold, because low prices impair coordination among dispersed creditors.

**Distressed borrowers.** Near the default threshold, the sensitivity of credit ratings to noise in prices increases with the relative precision of the information contained in prices (the computations are available in the appendix). This result calls for a reassessment of the current trend towards more transparent derivatives markets as regulators move over-the-counter (OTC) contracts to exchanges (and clearing houses) as this move may have unintended consequences. By making prices more widely available to buyers of OTC derivatives, regulators will raise the coordination role of prices,

thereby creating new challenges for the credit rating industry.<sup>15</sup>

Our model points out two opposite forces in sovereign debt markets. On the one hand, CRAs will pay less attention to bond yields when financial markets are more volatile (since public information embedded in prices is less precise). On the other hand, agencies may adjust their ratings as a result of financial shocks which are uncorrelated with the country's fundamentals.

## 4 Following investor opinion

Many claim that CRAs are as much following investor opinion as leading it (see, for example, Mora 2006). In this section, we consider the case in which CRAs do not disclose information which borrowers agree to share with them, but instead issue ratings based on anticipating what short term creditors will do. Formally, we examine situations in which the credit rating originates within the coordination game itself. More specifically, the CRA divulges a public signal about the aggregate level of investment in sovereign debt.

**Setup.** The model is identical to the basic model whose equilibrium is characterized in Section 2.1, except that the public signal  $z$  is replaced with  $y = \Phi^{-1}(1 - A) + \sigma_y \varepsilon_y$ , where  $\varepsilon_y$  is standard normal and independent of  $\theta$  and  $\xi_i$  (see Dasgupta 2007, for details on this specification). We focus on strategies in which there is a one-to-one mapping between the signal  $y$  and the credit rating. For simplicity, we assume that the CRA discloses  $y$ .

We posit that the CRA can condition the rating on the noisy indicator  $y$  of contemporaneous aggregate behavior. This should not be taken literally, as it is not fully consistent from a game theoretical perspective; rather, we think this assumption cap-

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<sup>15</sup>Vives (2014) shows how introducing a derivatives market may backfire, aggravating financial fragility.

tures in a simple way the idea that rating agencies issue and revise ratings based on information about investor behavior. Still, our specification extends to a simple dynamic model with a perfect Bayesian equilibrium in which the CRA does not have information about contemporaneous actions of investors. An example illustrating this sequential variant is available from the authors on request.<sup>16</sup>

We define an equilibrium as follows.

**Definition 3** *An equilibrium consists of an endogenous signal  $y$ , an individual investment strategy  $a(x, y)$  and aggregate investment  $A(\theta, y)$ , such that:*

$$\begin{aligned} a(x, y) &\in \arg \max_{a \in \{0,1\}} E[u(a, A(\theta, y), \theta, \psi) | x, y] \\ A(\theta, y) &= E[a(x, p) | \theta, y] \\ y &= \Phi^{-1}(1 - A(\theta, y)) + \sigma_y \varepsilon_y \end{aligned}$$

*and the sovereign government defaults if and only if  $\psi + A(\theta, y) + \theta < 1$ .*

Our equilibrium definition is a hybrid of rational expectations and perfect Bayesian equilibrium concepts, as in the model with financial markets in Section 3. In an equilibrium with switching strategies an agent invests if and only if  $x \geq x^*(y)$ , and the sovereign defaults if and only if  $\theta \leq \theta^*(y)$ . Hence, an equilibrium is identified with functions  $x^*(y)$ ,  $\theta^*(y)$  and  $y = Y(\theta, \varepsilon_y)$ . The uniqueness condition becomes  $\sigma_y^2 \sigma_x \sqrt{2\pi} > 1$ .

**Proposition 4** *If  $\sigma_y^2 \sigma_x \sqrt{2\pi} > 1$ , there exists a unique equilibrium, and  $\theta^*(y)$  is deter-*

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<sup>16</sup>Investors receive their private signals before the credit rating is released, and the CRA samples a group of investors with the objective of learning about their private signals (for example, analysts at CRAs may call a representative sample of fund managers). The CRA then issues a rating based on this data, which serves as a noisy indicator of aggregate behavior. Investors incorporate the rating into their set of available information, and take their roll-over decisions accordingly. The case with simultaneous moves studied here is equivalent to this sequential variant, since the CRA fully anticipates investors' reaction to the credit rating. Moreover, the case with simultaneous moves is approached in the sequential variant, as the size of the sample of investors converges to zero.

mined by

$$\theta^* = \Phi \left( \frac{1}{\sigma_x} \left( \frac{1}{\sqrt{\alpha_x + \frac{1}{(\sigma_x \sigma_y)^2}}} \Phi^{-1} \left( 1 - \frac{R-1}{\Delta} \right) + \frac{\frac{1}{(\sigma_x \sigma_y)^2}}{\alpha_x + \frac{1}{(\sigma_x \sigma_y)^2}} \sigma_x y \right) \right) - \psi.$$

Just as in the benchmark model of Section 2.2, equilibrium depends on the credit rating. There is a fundamental difference, though, as the *new* information provided by the CRA is now endogenous.

**Financial markets.** As in the Section 3, a financial market opens in the first stage and reveals the price of the derivative. In this setup, it remains to characterize the public signal  $y$  released by the CRA. Signal  $y$  about the aggregate level of investment comprises two types of information. First, it includes information embedded in market prices, because prices signal aggregate behavior. Were this the only type of information disclosed by the CRA, and  $y = \Phi^{-1}(1 - A) + \frac{\sigma_p}{\sigma_x} \varepsilon_s$ ; this type of information would be redundant. The second type of information contained in signal  $y$  is *new* information, independent of prices. Signal  $y$  is informative as long as the noise in  $y$ —parametrized by  $\sigma_y \varepsilon_y$ —is less than the noise in prices. Maintaining the restriction  $\sigma_y^2 \sigma_x \sqrt{2\pi} > 1$ , we show that a model with direct signals on the actions of investors is equivalent to a model with exogenous signals.

**Proposition 5** (*Equivalence Principle*) *In a market-based financial system, the equilibrium obtained when the CRA follows investor opinion is equivalent to the equilibrium obtained when the CRA discloses a private signal on the economic fundamental.*

It follows that the results regarding the noise in prices and ratings hold with this form of herding. The key issue is whether CRAs improve the precision of the information provided by financial markets, or if they just reproduce the information already contained in prices.



## 5 Hardwiring

One key concern is whether downgrades destabilize financial markets because ratings are embedded in many regulations and private contracts (Opp, Opp and Harris 2013, analyze the distortion in ratings caused by regulatory requirements like rating-contingent regulation which favor highly rated securities). Prudential regulations typically allow for less capital or liquidity to be held against highly rated securities. Central banks use ratings to determine which assets can serve as collateral in their money market operations. Suitability standards, which discipline fund managers by restricting investments to assets with certain risk characteristics, are often based on rating thresholds. Credit ratings are used as triggers for collateral calls in margin agreements in financing transactions. In these ways, ratings drive institutional demand and market liquidity.

The preceding sections contain descriptions drawn from individuals' optimal behavior. This section considers the mechanical use of credit ratings in investment decisions. To do this we consider an additional set of creditors who use credit ratings as *buy-sell triggers*, and we reinterpret  $\psi$  as the amount of debt roll-over by those investors who use this type of triggers.

**Setup.** The model is identical to the model whose equilibrium is characterized in Section 3, except that the debt roll-over  $\psi$  is replaced with  $\tilde{\psi} = \Phi\left(\rho + \frac{\alpha_p}{\alpha_\rho}p\right)\bar{\psi}$  with  $\bar{\psi}$  denoting the total amount of debt in the hands of those investors who use credit ratings as buy-sell triggers. Following the same steps as in Section 3, we obtain the following result.

**Proposition 6** (*Hardwiring*) *If  $\gamma^2\sigma_s^2\sigma_x^3\sigma_\rho^2\sqrt{2\pi} > \gamma^2\sigma_s^2\sigma_x^4 + \sigma_\rho^2$ , there exists a unique equilibrium, and  $\theta^*(p, \rho)$  is implicitly determined by  $\theta^* = -\Phi\left(\rho + \frac{\alpha_p}{\alpha_\rho}p\right)\bar{\psi} +$*

$$\Phi\left(\sigma_x\left[\sqrt{\alpha_x + \alpha_p + \alpha_\rho}\Phi^{-1}\left(1 - \frac{R-1}{\Delta}\right) + (\alpha_p + \alpha_\rho)\left(\theta^* - \frac{\alpha_p}{\alpha_p + \alpha_\rho}p - \frac{\alpha_\rho}{\alpha_p + \alpha_\rho}\rho\right)\right]\right).$$

It could be argued that market practices, laws and regulations that hardwire buy or sell decisions to rating thresholds make the amount of debt held by institutional investors  $\tilde{\psi}$  vary with the credit rating  $\hat{\rho}$ , and not with  $\rho$  or  $p$ . In the current setup, both representations are equivalent because  $\rho$  and  $\hat{\rho}$  have the same informational content once the price  $p$  is known. We represent the relationship between  $\tilde{\psi}$  and  $\hat{\rho}$  as a function  $\hat{\psi}(\hat{\rho})$ . The degree of hardwiring of investment decisions to credit ratings is measured by  $\hat{\psi}'(\hat{\rho}) = \frac{\partial[\Phi(\rho + \frac{\alpha_p}{\alpha_\rho} p)\tilde{\psi}]/\partial p}{\partial \hat{\rho}/\partial p} = \frac{\partial[\Phi(\rho + \frac{\alpha_p}{\alpha_\rho} p)\tilde{\psi}]/\partial \rho}{\partial \hat{\rho}/\partial \rho} > 0$ . Lower ratings reduce the amount of debt in the hands of those investors who use ratings as buy-sell triggers, which is a desirable property for any model of hardwiring.

Repeating the comparative-statics analysis performed earlier, reveals that hardwiring amplifies the response of the credit rating and the default threshold to shocks in prices ( $\varepsilon_s$ ), and to shocks in the information disclosed by the CRA ( $\varepsilon_\rho$ )—the results are available in the appendix. The coordination effect of credit ratings is exacerbated by market practices, laws and regulations that hardwire buy or sell decisions to rating thresholds. This hardwiring is a cause of herding in market behavior because regulations and rules effectively require or motivate large numbers of market participants to act in similar fashion and, as a result, it contributes significantly to market reliance on ratings, reinforcing their role as focal points. Omitting references to ratings in regulation and reducing their use as buy-sell triggers mitigates the effect of shocks and would thus stabilize financial markets. The same results hold for regulations and rules that rely on market-based indicators like financial prices.<sup>17</sup>

**Multiple equilibria.** When  $\gamma^2 \sigma_s^2 \sigma_x^3 \sigma_\rho^2 \sqrt{2\pi} < \gamma^2 \sigma_s^2 \sigma_x^4 + \sigma_\rho^2$  the interaction between the credit rating  $\hat{\rho}$  and the default threshold  $\theta^*$  creates multiple equilibria as the relative precision of public information is too large. In this case, the same public signals  $\rho$  and  $p$  may yield different default thresholds  $\theta^*$  which means that the same public information

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<sup>17</sup>Some have suggested replacing credit ratings with CDS premia and credit spreads, but these too may increase the coordination motive and bring instability to financial markets. In June 2007, CDS prices for the banking sector were at a record low level and, in 2009, sovereign debt prices for countries like Ireland, Greece, Portugal, Italy and Spain were almost the same as for Germany.

may yield different probabilities of default and, consequently, different credit ratings.

We assume that the CRA discloses two pieces of information to borrowers. First, the CRA discloses information about the underlying economic fundamental by announcing the signal  $\rho$ . Second, it discloses information about roll-over risk through the disclosure of the rating  $\hat{\rho}$ . In practice, agencies accompany their announcements with written reports in which they provide information about fundamental economic variables and detail the risk exposure of the borrower. Moreover, the regular interaction between rating agencies and borrowers provides valuable information to financial markets.

The existence of multiple equilibria gives a powerful role to CRAs as they are able to select the equilibrium. Let us sketch the argument behind this result. The default threshold  $\theta^*$  satisfies

$$\theta^* + \hat{\psi}(\hat{\rho}) = \Phi \left( \sigma_x \left[ \sqrt{\alpha_x + \alpha_p + \alpha_\rho} \Phi^{-1} \left( 1 - \frac{R-1}{\Delta} \right) + (\alpha_p + \alpha_\rho) \left( \theta^* - \frac{\alpha_p}{\alpha_p + \alpha_\rho} p - \frac{\alpha_\rho}{\alpha_p + \alpha_\rho} \rho \right) \right] \right). \quad (4)$$

The critical value  $\theta^*$  is obtained as the intersection of a cumulative normal distribution function with a line with slope 1 and intercept  $\hat{\psi}(\hat{\rho})$ . Figure 3(a) plots two solid straight lines with slope 1 and intercepts  $\hat{\psi}(\hat{\rho}_1)$  and  $\hat{\psi}(\hat{\rho}_2)$  as well as the cumulative normal distribution function, and illustrates how equilibrium is obtained.

The credit rating  $\hat{\rho}$  corresponds to the default probability that the actual value of the fundamental  $\theta$  lies above the threshold  $\theta^*$ , as defined in expression (3). The amount of debt that investors roll over as a result of buy-sell triggers equals

$$\tilde{\psi} = \hat{\psi} \left( \Phi \left( \left( \frac{\alpha_p}{\alpha_p + \alpha_\rho} p + \frac{\alpha_\rho}{\alpha_p + \alpha_\rho} \rho - \theta^* \right) \sqrt{\alpha_p + \alpha_\rho} \right) \right). \quad (5)$$

Figure 3(b) plots this relationship between  $\tilde{\psi}$  and  $\theta^*$ . The equilibrium value of the default threshold  $\theta^*$  is obtained as the joint solution to equations (4) and (5).

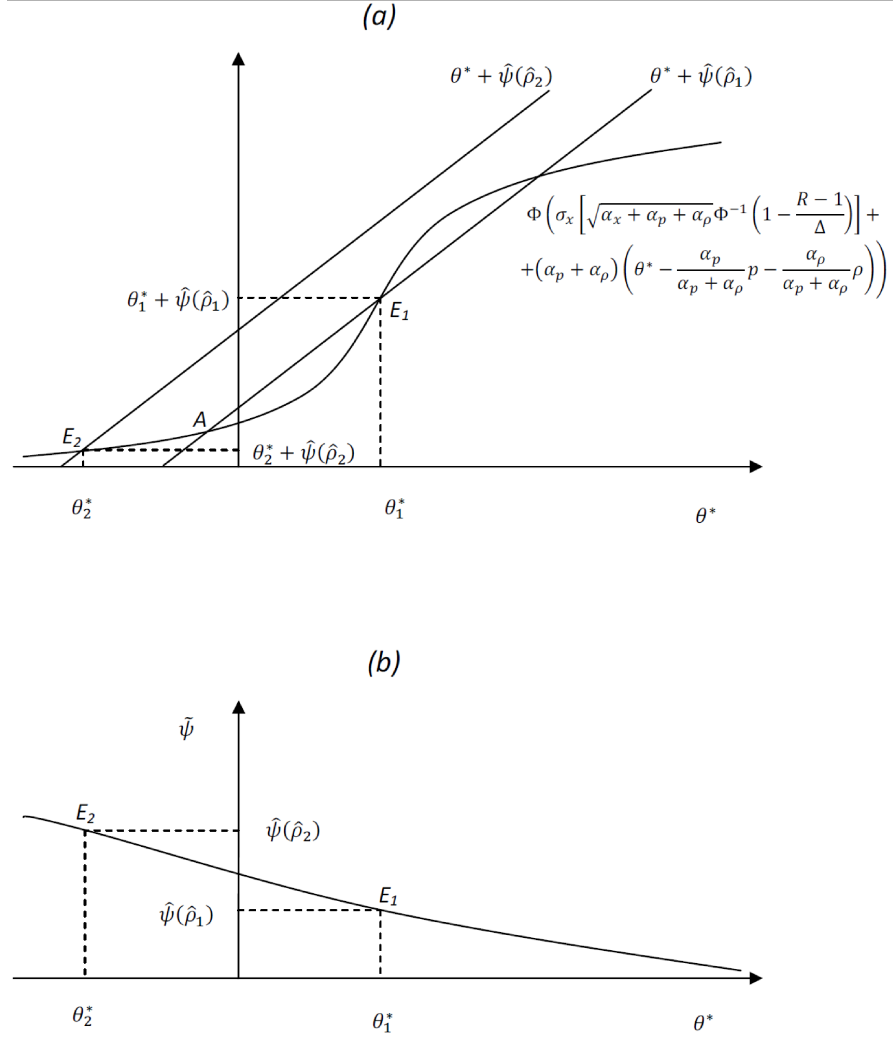


Figure 3: Hardwiring and multiple equilibria. (a) illustrates how the default threshold  $\theta^*$  is determined; both solid straight lines have slope 1, the upper left line has intercept  $\hat{\psi}(\hat{\rho}_2)$  and the lower right line has intercept  $\hat{\psi}(\hat{\rho}_1)$ . (b) represents the relationship between the amount of debt roll-over  $\tilde{\psi}$  and the threshold  $\theta^*$ .

Figure 3(a) and (b) illustrates a case in which there are multiple joint solutions to equations (4) and (5). From Figure 3(a) one can see that equilibrium  $E_1$  features a high default threshold  $\theta_1^*$ , which implies a large default probability and a low rating  $\hat{\rho}_1$ . From Figure 3(b), one can verify that this credit rating implies debt roll-over as a result of triggers equal to  $\hat{\psi}(\hat{\rho}_1)$ —which is consistent with the straight line intersecting the curved line at equilibrium  $E_1$  in Figure 3(a).<sup>18</sup> Equilibrium  $E_2$  features  $\theta_2^*$  as the default threshold, implies a lower default probability, a credit rating  $\hat{\rho}_2 > \hat{\rho}_1$  and, as a result,  $\hat{\psi}(\hat{\rho}_2) > \hat{\psi}(\hat{\rho}_1)$ .

The game has multiple equilibria as there is more than one solution to the credit rating. To see this, consider two possible announcements for the same values of  $p$  and  $\rho$ .

- Tough-rating-agency equilibrium. In equilibrium  $E_1$  the CRA announces a low rating  $\hat{\rho}_1$ , which induces low debt-roll over by those investors who use buy-sell triggers. Other creditors anticipate the reaction by those investors who use triggers, and they too refrain from investing. These reactions increase the likelihood of default, thus vindicating the opinion of the CRA.
- Soft-rating-agency equilibrium. In equilibrium  $E_2$  the CRA announces a high rating  $\hat{\rho}_2$ , which triggers market reactions that induce high roll-over by all creditors. The corresponding reduction in the probability of default confirms the rating.

As with Boot, Milbourn and Schmeits (2006), the credit ratings help fix the desired equilibrium when there are multiple equilibria. The CRA is able to select which equilibrium will be played when it selects the credit rating  $\hat{\rho}$  and determines the amount of roll-over  $\tilde{\psi} = \hat{\psi}(\hat{\rho})$ . By assigning a favorable rating  $\hat{\rho}_2$ , the agency coordinates investors into the equilibrium with a lower default threshold.

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<sup>18</sup>Point  $A$  does not represent an equilibrium since it would imply a lower default threshold and a better rating than  $\hat{\rho}_1$ , which would shift the straight line upwards.

The final outcome depends on the announcement of the CRA. In this setting:

- Credit ratings can serve as focal points and help to mitigate the multiple equilibria problem. The soft-rating-agency equilibrium purports credit ratings as an insurance mechanism against uncoordinated jumps to the bad equilibrium.
- Borrowers are vulnerable to agencies' opinions. The CRA can pick the equilibrium which suits better its incentives. If the CRA wishes to preserve business relationships, it will pick the soft-rating-agency equilibrium.
- Investors give credibility to the credit rating by basing their (mechanical) roll-over decisions on the agency's pronouncement.
- Credit ratings are not biased since they are confirmed ex post.

## 6 Strategic behavior

Up to this point we have assumed that CRAs maximize the accuracy of their ratings, and do not account for the effect of their opinions on their business. We now consider that CRAs have an incentive to *rescue* borrowers in order to preserve their business relationships with them. We perform the analysis within the setting of Section 2.2.

**Setup.** So far we have assumed that the precision of the private signal  $\alpha_\rho$  is constant. We now endogenize  $\alpha_\rho$ , and allow the CRA to choose the precision of its signal  $\rho$  so as to minimize the likelihood of default of the borrower (or, equivalently, issue the highest rating possible). The sequence of decisions can be summarized as follows.

1. Public information  $z$  is disclosed;  $z$  is common knowledge.

2. The CRA chooses the precision  $\alpha_\rho$  of its signal. In equilibrium, rational investors figure out the value of  $\alpha_\rho$ .
3. The CRA obtains a private signal  $\rho$ , and discloses the credit rating  $\hat{\rho}$ . There is a one-to-one relationship between  $\rho$  and  $\hat{\rho}$ . The credit rating is common knowledge among all.
4. Each investor  $i$  observes the realization of his private signal  $x_i$ .
5. Based on the information  $z$ ,  $\hat{\rho}$ , and his private signal  $x_i$ , each investor individually decides whether or not to invest his unit of debt.

The three final stages of the extensive form game are similar to the benchmark model of Section 2.2, and we employ the results in Proposition 2 to obtain the critical threshold  $\theta^*$  at which sovereign default is triggered. The solution for  $\theta^*$  can then be used in evaluating the decision of the CRA in deciding the precision of the credit rating.

Given the signals  $z$  and  $\rho$ , the default probability of the borrower is equal to  $\Phi\left(\left(\theta^* - \frac{\alpha_z}{\alpha_z + \alpha_\rho}z - \frac{\alpha_\rho}{\alpha_z + \alpha_\rho}\rho\right)\sqrt{\alpha_z + \alpha_\rho}\right)$ . The CRA chooses the precision  $\alpha_\rho$  so as to minimize the expected probability of default, conditional on the available public information  $z$ . Formally, the agency solves

$$\begin{aligned} \min_{\alpha_\rho \in \left[0, \frac{\sigma_z^2 \sqrt{2\pi} - \sigma_x}{\sigma_z^2 \sigma_x}\right)} \quad & E \left[ \Phi \left( \left( \theta^* - \frac{\alpha_z}{\alpha_z + \alpha_\rho}z - \frac{\alpha_\rho}{\alpha_z + \alpha_\rho}\rho \right) \sqrt{\alpha_z + \alpha_\rho} \right) | z \right] \\ \text{s.t.} \quad & \hat{\rho} = p_{CRA} \end{aligned} .$$

The CRA chooses the value of  $\alpha_\rho$  without knowing the actual values taken by  $\rho$  or  $\theta^*$ . The distribution of  $\theta^*$  has no simple analytical expression, and we resort to numerical solutions.

**Numerical results.** The CRA chooses imprecise ratings when public information is favorable, and invests on precise ratings when public information is unfavorable. The

intuition for this result is as follows:

- When available public information is negative, announcing a precise rating may reduce the negative effects of public information and improve the position of the borrower. For short term creditors  $E[\theta|z, \rho] = \frac{\alpha_z}{\alpha_z + \alpha_\rho}z + \frac{\alpha_\rho}{\alpha_z + \alpha_\rho}\rho$ , so that accurate ratings reduce the weight given to  $z$  and raise the weight placed on  $\rho$ . Choosing an accurate rating is beneficial for two reasons. First, it mitigates the negative effects of available public information as investors reduce the weight given to  $z$ . Second, the CRA may receive a good signal and issue a high rating, which will reduce the default probability of a borrower in a delicate position (since  $\rho$  is noisy, a low rating is also possible and will worsen the already delicate position of the borrower). Figure 4(a) plots the relationship between the  $\alpha_\rho$  and the expected probability of default, conditional on  $z = 0.1$  (which we see as unfavorable public information). Since it wishes to minimize the default probability of the borrower, the CRA will choose to set the rating as accurately as possible.<sup>19</sup>
- When the public signal  $z$  is large, issuing precise ratings is likely to reduce the good effect of public information. With precise ratings, individuals reduce the weight given to favorable public information and place more weight on the credit rating and on any noise that inevitably creeps in. Figure 4(b) depicts the relationship between the  $\alpha_\rho$  and the expected probability of default for  $z = 1$  (which we see as favorable public information), and shows that the CRA wishes to set the precision  $\alpha_\rho$  as low as possible.

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<sup>19</sup>In practice, though, collecting information is costly, which sets limits to the accuracy of the information being collected.



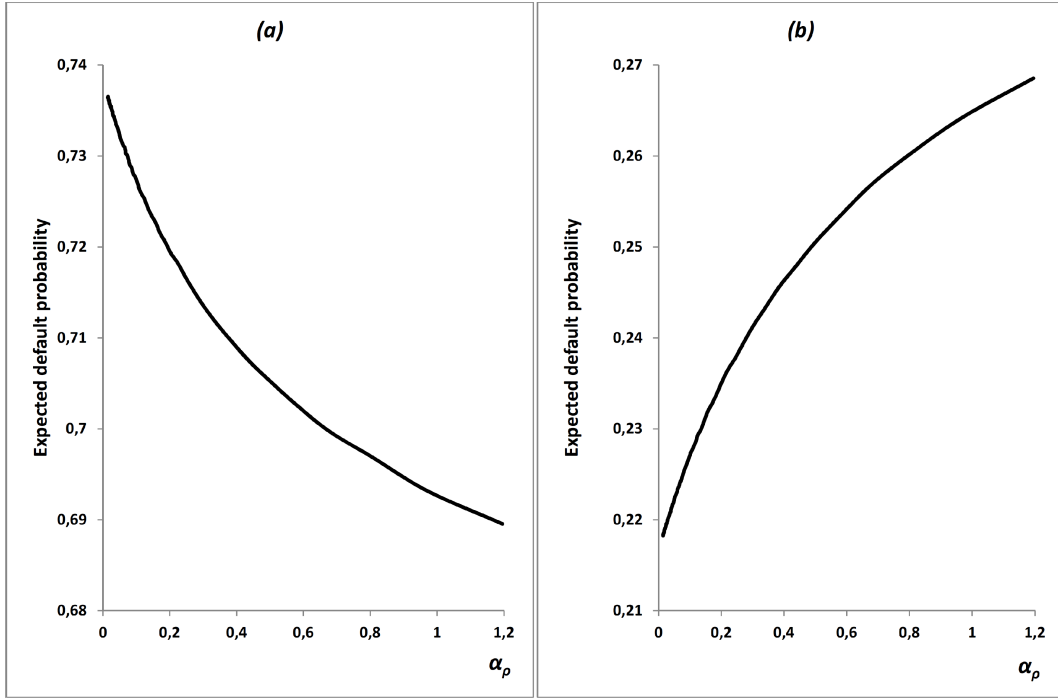


Figure 4: The relationship between the precision  $\alpha_\rho$  and the expected default probability of the borrower when  $\alpha_z = \alpha_x = 1, R = 1.2, \Delta = 0.4, \psi = 0$ . In (a)  $z = 0.1$  and in (b)  $z = 1$ .

## 7 Conclusion

With external ratings acting as a reference for investors' beliefs, ratings inform and at the same time influence the probability of default of borrowers who must roll over their debts. As CRAs take into account the feedback effects on credit quality, credit ratings become volatile and prone to cause financial instability. We have described the sharp response of credit ratings and default probabilities to prices and ratings in periods of market stress.

It is one thing to identify the weaknesses of credit ratings, quite another to find solutions and alternative standards that are clearly better. Part of the solution, it seems, is to mitigate the coordination motive and the feedback effects. The less embedded into deal-documentation ratings are, the lower the impact of credit ratings on investment decisions and the less focal credit ratings will be.<sup>20</sup> Ending the contribution of credit ratings to financial instability calls for the reduction of references to credit ratings in regulation and rules. Some industry leaders back these moves. Deven Sharma (2010), former president of Standard & Poor's, insisted "we support removing investor rating requirements and believe the market—not government mandates—should decide the value of our work".

But, in many areas, there are no alternative measures of creditworthiness that are as transparent and simple to use, that allow for consistent implementation across banks and markets, and that effectively differentiate risk as credit ratings. Several alternative approaches that remove references to ratings entirely have been considered in the debate, but no satisfactory substitutes have yet been identified. For example, the use of market-based indicators, such as CDS premia and credit spreads, may also

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<sup>20</sup>The concentration of the industry in a reduced number of big (and focal) rating agencies might have aggravated these problems in the past. Moreover, the conditions imposed by regulators for designating a CRA as an external credit assessment institution often require that the market already places substantial weight on the judgment of a rating agency. By giving the market a role in selecting rating agencies, regulators exacerbate the focal role of CRAs.

increase the coordination motive and bring instability to financial markets. Even if the scope for hardwiring were reduced, it is likely that credit ratings would retain a significant influence in financial markets. Small and less-sophisticated investors that do not have the economies of scale to do their own credit analyses will continue to rely extensively on credit ratings, and it is plausible that many institutional investors would be reluctant to do their own credit assessments and would continue to rely on ratings, even if these were pulled from regulations. This makes the use of credit ratings so pervasive that market participants cannot ignore them, even if they do not consider them reliable.

Credit ratings do their job well outside the crisis zones, and there are no simple alternatives to their *certification role*. For as long as ratings retain their widespread influence throughout the financial system, there seems to be a sound economic rationale for regulating CRAs. Rather than prohibiting their use, we argue in favor of a flexible use of credit ratings for regulatory purposes, and this paper provides an economic rationale behind some of the reforms being sought by regulators worldwide. Encouraging accurate and prudent smoothing rules would reduce the volatility of credit ratings in the crisis zones, and would stabilize financial markets by forcing agents to rely more on their own private information. Also, a liquidity provision by official institutions is far more effective in the crisis zones, where credit ratings are not reliable and can precipitate default. Hence, the CRAs' communication strategies should be monitored closely in periods of market stress. Michael Barnier, the former European internal market and services commissioner, argued in favor of such restrictions "It is not the thermometer that causes the fever," he said, "but the thermometer has to work properly to ensure you do not exaggerate the fever."

## References

- [1] Afonso, A., D. Furceri, P. Gomes, 2012. Sovereign Credit Ratings and Financial Market Linkages: Applications to European Data. *Journal of International Money and Finance* 31, 606-638.
- [2] Aizenman, J., M. Binici, M. Hutchison, 2013. Credit Ratings and the Pricing of Sovereign Debt during the Euro Crisis. *Oxford Review of Economic Policy* 29, 582-609.
- [3] Angeletos, G., I. Werning, 2006. Crises and Prices: Information Aggregation, Multiplicity, and Volatility. *American Economic Review* 96, 1720-1736.
- [4] Bar-Isaac, H., J. Shapiro, 2011. Credit Rating Accuracy and Analyst Incentives. *American Economic Review* 101, 120-124.
- [5] Bar-Isaac, H., J. Shapiro, 2013. Ratings Quality over the Business Cycle. *Journal of Financial Economics* 108, 62-78.
- [6] Bolton, P., X. Freixas, J. Shapiro, 2012. The Credit Ratings Game. *Journal of Finance* 67, 85-111.
- [7] Boot, A., T. Milbourn, A. Schmeits, 2006. Credit Ratings as Coordination Mechanisms. *Review of Financial Studies* 19, 81-118.
- [8] Carlson, M., G. Hale, 2006. Rating Agencies and Sovereign Debt Rollover. *Topics in Macroeconomics* 6, article 8.
- [9] Chiu, J., T. Koeppl, 2015. Livin' on the Edge with Ratings: Liquidity, Efficiency and Stability. *Queen's Economics Department Working Paper* 1335.
- [10] Cornaggia, J., K. Cornaggia, R. Israelsen, 2015. Credit Ratings and the Cost of Municipal Financing. Unpublished paper available via SSRN.

- [11] Corsetti, G., B. Guimarães, N. Roubini, 2006. International Lending of Last Resort and Moral Hazard: A Model of IMF's catalytic finance. *Journal of Monetary Economics*, 53(3), 441-471.
- [12] Dasgupta, A., 2007. Coordination and Delay in Global Games. *Journal of Economic Theory* 134(1), 195-225.
- [13] Dichev, I., J. Piotroski, 2001. The Long-run Stock Returns Following Bond Ratings Changes. *Journal of Finance* 56, 173-203.
- [14] European Central Bank (ECB), 2008. Technical Specifications for the Temporary Expansion of the Central Framework. ECB Press Release, 17 October 2008.
- [15] Ferri, G., L. Liu, J. Stiglitz, 1999. The Procyclical Role of Rating Agencies: Evidence from the East Asian Crisis. *Economic Notes* 28(3), 335-55.
- [16] Finnerty, J., C. Miller, R. Chen, 2013. The Impact of Credit Rating Announcements on Credit Default Swap Spreads. *Journal of Banking and Finance* 37, 2011-2030.
- [17] Goh, J., L. Ederington, 1999. Cross Sectional Variation in the Stock Market Reaction to Bond Rating Changes. *Quarterly Review of Economics and Finance* 29, 101-112.
- [18] Grossman, S., J. Stiglitz, 1976. Information and Competitive Price Systems. *American Economic Review* 66, 246-53.
- [19] Hand, J., R. Holthausen, R. Leftwich, 1992. The Effect of Bond Rating Agency Announcements on Bond and Stock Prices. *Journal of Finance* 47, 733-752.
- [20] Hite, G., A. Warga, 1997. The Effect of Bond-Rating Changes on Bond Price Performance. *Financial Analysts Journal*, 35-51.
- [21] Holden, S., G. J. Natvik, A. Vigier, 2012. An Equilibrium Model of Credit Rating Agencies. Norges Bank Research Working Paper 2012|23.

- [22] Hortaçsu, A., G. Matvos, C. Syverson, S. Venkataraman, 2013. Indirect Costs of Financial Distress in Durable Goods Industries: The Case of Auto Manufacturers. *Review of Financial Studies* 26, 1248-1290.
- [23] Hull, J., M. Predescu, A. White, 2004. The Relationship Between Credit Default Swap Spreads, Bond Yields, and Credit Rating Announcements. *Journal of Banking and Finance* 28, 2789-2811.
- [24] International Monetary Fund, 1999. International Capital Markets Report. July 1999.
- [25] International Monetary Fund, 2010. Sovereigns, Funding, and Systemic Liquidity. *Global Financial Stability Report*, October 2010.
- [26] Ismailescu, I., H. Kazemi, 2010. The Reaction of Emerging Market Credit Default Swap Spreads to Sovereign Credit Rating Changes. *Journal of Banking and Finance* 34, 2861-2873.
- [27] James, J., P. Lawler, 2011. Optimal Policy Intervention and the Social Value of Public Information. *American Economic Review* 101, 1561-1574.
- [28] Issing Committee, 2009. New Financial Order Recommendations by the Issing Committee. Available via the Internet.
- [29] Mählmann, T., 2011. Is There a Relationship Benefit in Credit Ratings? *Review of Finance* 15, 475-510.
- [30] Manso, G., 2013. Feedback Effects of Credit Ratings. *Journal of Financial Economics* 109, 535-548.
- [31] Mariano, B., 2012. Market Power and Reputational Concerns in the Ratings Industry. *Journal of Banking and Finance* 36, 1616-1626.

- [32] Mathis, J., J. McAndrews, J.-C. Rochet, 2009. Rating the Raters: Are Reputation Concerns Powerful Enough to Discipline Rating Agencies? *Journal of Monetary Economics* 56, 657-674.
- [33] Mora, N., 2006. Sovereign Credit Ratings: Guilty Beyond Reasonable Doubt? *Journal of Banking & Finance* 30, 2041-2062.
- [34] Morris, S., H. Shin, 2006. Catalytic Finance: When Does It Work? *Journal of International Economics* 70, 161-177.
- [35] Morris, S., H. Shin, 2004. Coordination Risk and The Price of Debt. *European Economic Review* 48, 133-153.
- [36] Morris, S., H. Shin, 2002. Social Value of Public Information. *American Economic Review* 92, 1521-1534.
- [37] Norden, L. and M. Weber, 2004. Informational Efficiency of Credit Default Swap and Stock Markets: The Impact of Credit Rating Announcements. *Journal of Banking and Finance* 28, 2813-2843.
- [38] Opp, C., M. Opp, M. Harris, 2013. Rating Agencies in the Face of Regulation. *Journal of Financial Economics* 108, 46-61.
- [39] Pagano, M., P. Volpin, 2012. Securitization, Transparency and Liquidity. *Review of Financial Studies* 25, 2417-2453.
- [40] Sharma, D., 2010. Why Rating Requirements Don't Make Sense. *Wall Street Journal*, available via the Internet.
- [41] Skreta, V., L. Veldkamp. 2009. Ratings Shopping and Asset Complexity: A Theory of Ratings Inflation. *Journal of Monetary Economics* 56, 678-695.
- [42] Standard & Poor's, 2011. Default, Transition, and Recovery: 2010 Annual Global Corporate Default Study And Rating Transitions. Standard & Poor's.

- [43] Stanton, R., N. Wallace, 2011. The Bear's Lair: Index Credit Default Swaps and the Subprime Mortgage Crisis. *Review of Financial Studies* 24, 3250-3280.
- [44] Subrahmanyam, M., D. Tang, S. Wang, 2014. Does the Tail Wag the Dog? The Effect of Credit Default Swaps on Credit Risk. *Review of Financial Studies* 27, 2927-2960.
- [45] Vives, X., 2014. Strategic Complementarity, Fragility, and Regulation. *Review of Financial Studies* 27, 3547-3592.



## A Mathematical Appendix

### A.1 Proof of Proposition 1

Follows from Morris and Shin (2004). An agent invests if and only if  $x \geq x^*$ , where  $x^*$  solves  $p_i R + (1 - p_i)(R - \Delta) = 1$  with

$$p_i = 1 - \Phi \left( \left( \theta^* - \left[ \frac{\alpha_x}{\alpha_x + \alpha_z} x^* + \frac{\alpha_z}{\alpha_x + \alpha_z} z \right] \right) \sqrt{\alpha_x + \alpha_z} \right).$$

The mass of investors equals  $A = 1 - \Phi(\sqrt{\alpha_x}(x^*(z) - \theta))$ . The sovereign defaults if and only if  $\theta < \theta^*$ , where  $\theta^*$  solves  $\psi + A + \theta^* = 1$ , or equivalently  $\theta^* = \Phi(\sqrt{\alpha_x}(x^* - \theta^*)) - \psi$ . Solving for  $\theta^*$  yields  $\theta^*(z)$ . The condition for uniqueness ensures  $\frac{\partial \theta^*}{\partial R} < 0$ ,  $\frac{\partial \theta^*}{\partial \Delta} > 0$ ,  $\frac{\partial \theta^*}{\partial z} < 0$  and  $\frac{\partial \theta^*}{\partial \psi} < -1$ . ■

### A.2 Proof of Proposition 2

The proof follows the same steps as the proof of Proposition 1, with the posterior belief of agent  $i$  about  $\theta$  being normal with mean  $E[\theta|x_i, z] = \frac{\alpha_x}{\alpha_x + \alpha_z + \alpha_\rho} x_i + \frac{\alpha_z}{\alpha_x + \alpha_z + \alpha_\rho} z + \frac{\alpha_\rho}{\alpha_x + \alpha_z + \alpha_\rho} \rho$  and variance  $Var[\theta|x_i, z] = \frac{1}{\alpha_x + \alpha_z + \alpha_\rho}$ . The uniqueness condition ensures  $\frac{\partial \theta^*}{\partial \rho} = -\frac{\sigma_x \alpha_\rho \phi(\Phi^{-1}(\theta^* + \psi))}{1 - \sigma_x(\alpha_z + \alpha_\rho)\phi(\Phi^{-1}(\theta^* + \psi))} < 0$ . ■

### A.3 Proof of Proposition 3

The proof for the second stage follows the same steps as the proof of Proposition 1. The uniqueness condition ensures a unique equilibrium in both stages and  $\frac{\partial \theta^*}{\partial p} < 0$ . ■

#### A.4 Sensitivity of credit ratings to noise in prices for distressed borrowers

We perform the analysis for values of  $\theta$  near  $\theta^*$ . There is default if and only if  $\theta < \theta^*(p, \rho)$ , where  $p = \theta - \sigma_p \varepsilon_s$  and  $\rho = \theta + \sigma_\rho \varepsilon_\rho$ . Hence, the sovereign defaults if and only if  $\theta < \hat{\theta}(\varepsilon_s, \varepsilon_\rho)$ , where  $\hat{\theta}(\varepsilon_s, \varepsilon_\rho)$  is the unique solution to  $\hat{\theta} = \Phi(\sigma_x [\sqrt{\alpha_x + \alpha_p + \alpha_\rho} \Phi^{-1}(1 - \frac{R-1}{\Delta}) + \sqrt{\alpha_p} \varepsilon_s - \sqrt{\alpha_\rho} \varepsilon_\rho]) - \psi$ . Function  $\hat{\theta}(\varepsilon_s, \varepsilon_\rho)$  satisfies the single crossing property, because

$$\begin{aligned} \frac{\partial \hat{\theta}}{\partial \varepsilon_\rho} &= -\phi\left(\Phi^{-1}\left(\hat{\theta} + \psi\right)\right) \frac{\sigma_x}{\sigma_\rho} < 0, \\ \frac{\partial \hat{\theta}}{\partial \varepsilon_s} &= \phi\left(\Phi^{-1}\left(\hat{\theta} + \psi\right)\right) \frac{\sigma_x}{\sigma_p} = \phi\left(\Phi^{-1}\left(\hat{\theta} + \psi\right)\right) \frac{1}{\gamma \sigma_s \sigma_x} > 0. \end{aligned}$$

A reduction in the standard deviation  $\sigma_s$  or  $\sigma_\rho$  increases the sensitivity of the equilibrium outcomes to the exogenous shocks  $\varepsilon_s$  and  $\varepsilon_\rho$ . Figure 5 depicts function  $\hat{\theta}(\varepsilon_s, \varepsilon_\rho)$ , with the dashed line corresponding to a lower standard deviation  $\sigma_s$  than the solid one.

#### A.5 Proof of Proposition 4

In an equilibrium with switching strategies the mass of investors equals  $A(y, \theta) = 1 - \Phi\left(\frac{1}{\sigma_x}(x^*(y) - \theta)\right)$ . Using the definition of public signal,  $y = \Phi^{-1}(1 - A) + \sigma_y \varepsilon_y = \frac{1}{\sigma_x}(x^*(y) - \theta) + \sigma_y \varepsilon_y$ , and, thus,  $x^*(y) - \sigma_x y = \theta - \sigma_x \sigma_y \varepsilon_y$ . This expression can be seen as a function that relates  $y$  and  $z \equiv \theta - \sigma_x \sigma_y \varepsilon_y$ . Given  $y$ , agents are able to infer  $z$ , where  $\sigma_z = \sigma_x \sigma_y$ . An agent invests if and only if  $x \geq x^*(y)$ , where  $x^*(y)$  solves  $R - \Phi\left(\sqrt{\alpha_x + \alpha_z}\left[\theta^* - \left(\frac{\alpha_x}{\alpha_x + \alpha_z}x^* + \frac{\alpha_z}{\alpha_x + \alpha_z}z\right)\right]\right)\Delta = 1$ . This expression can be rewritten as

$$x^* = \theta^* + \frac{1}{\sqrt{\alpha_x + \alpha_z}} \Phi^{-1}\left(1 - \frac{R-1}{\Delta}\right) + \frac{\alpha_z}{\alpha_x + \alpha_z} \sigma_x y. \quad (6)$$

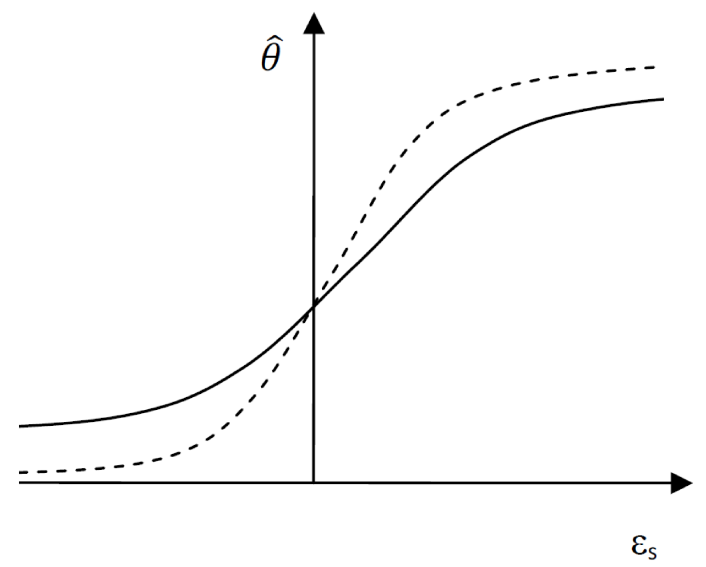


Figure 5: Default threshold  $\hat{\theta}$  as a function of  $\varepsilon_s$ . The dashed line corresponds to a lower  $\sigma_s$ .

The sovereign defaults if and only if  $\theta < \theta^*(y)$ , where  $\theta^*(y)$  solves  $\psi + A(\theta, y) + \theta = 1$ , or, equivalently,

$$\theta^* = \Phi\left(\frac{x^* - \theta^*}{\sigma_x}\right) - \psi. \quad (7)$$

Substituting (6) into (7), we get  $\theta^* = \Phi\left(\frac{1}{\sigma_x}\left(\frac{1}{\sqrt{\alpha_x + \alpha_z}}\Phi^{-1}\left(1 - \frac{R-1}{\Delta}\right) + \frac{\alpha_z}{\alpha_x + \alpha_z}\sigma_x y\right)\right) - \psi$ . This expression yields a unique solution  $\theta^*(y)$ , and substituting this solution into (6) we obtain the unique solution  $x^*(y) = \Phi\left(\frac{1}{\sigma_x}\left(\frac{1}{\sqrt{\alpha_x + \alpha_z}}\Phi^{-1}\left(1 - \frac{R-1}{\Delta}\right) + \frac{\alpha_z}{\alpha_x + \alpha_z}\sigma_x y\right)\right) - \psi + \frac{1}{\sqrt{\alpha_x + \alpha_z}}\Phi^{-1}\left(1 - \frac{R-1}{\Delta}\right) + \frac{\alpha_z}{\alpha_x + \alpha_z}\sigma_x y$ .

Finally, we confirm that  $F(y) = x^*(y) - \sigma_x y$  is indeed a function. Substituting  $x^*(y)$  yields  $F(y) = \Phi\left(\frac{1}{\sigma_x}\left(\frac{1}{\sqrt{\alpha_x + \alpha_z}}\Phi^{-1}\left(1 - \frac{R-1}{\Delta}\right) + \frac{\alpha_z}{\alpha_x + \alpha_z}\sigma_x y\right)\right) - \psi + \frac{1}{\sqrt{\alpha_x + \alpha_z}}\Phi^{-1}\left(1 - \frac{R-1}{\Delta}\right) + \left(\frac{\alpha_z}{\alpha_x + \alpha_z} - 1\right)\sigma_x y$ . Compute the sign of  $F'(y)$ , and it is easy to show that  $F(y)$  is monotonic. ■

## A.6 Proof of Proposition 5

As shown in the proof of Proposition 4, given  $y$ , agents are able to infer the public signal  $z \equiv \theta - \sigma_x \sigma_y \varepsilon_y = \theta - \sqrt{\frac{1}{\alpha_p + \alpha_{\tilde{\rho}}}} \varepsilon_y$ . Variable  $z$  incorporates information about  $p$  and the new information produced by the CRA. Given that  $z$  and  $p$  are normally distributed, the new information can be represented by a signal  $\tilde{\rho} = \theta + \sigma_{\tilde{\rho}} \varepsilon_{\tilde{\rho}}$ , where  $\varepsilon_{\tilde{\rho}}$  is standard normal, independent of  $\theta, \varepsilon_s$  and  $\xi_i$ , and with  $\sigma_{\tilde{\rho}}^2 = \frac{1}{\alpha_{\tilde{\rho}}}$ . Hence,  $z = \frac{\alpha_{\tilde{\rho}}}{\alpha_p + \alpha_{\tilde{\rho}}} \tilde{\rho} + \frac{\alpha_p}{\alpha_p + \alpha_{\tilde{\rho}}} p$  and, after the first stage, the new prior has precision  $\alpha_p + \alpha_{\tilde{\rho}}$ . The investment problem is equivalent to the second stage of the model of Section 3. Hence, Proposition 3 holds with  $\tilde{\rho}$  replacing  $\rho$ . The uniqueness condition becomes  $\gamma^2 \sigma_s^2 \sigma_x^3 \sqrt{2\pi} > 1 + \alpha_{\tilde{\rho}} \gamma^2 \sigma_s^2 \sigma_x^4$ , which is equivalent to  $\sigma_y^2 \sigma_x \sqrt{2\pi} > 1$ . ■

## A.7 Comparative statics with hardwiring

The impact of noise on credit ratings is measured by the following derivatives:

$$\begin{aligned} \frac{\partial \hat{\rho}}{\partial \varepsilon_s} &= -\phi(\Phi^{-1}(\hat{\rho})) \left( \sqrt{\frac{\alpha_p}{\alpha_p + \alpha_{\rho}}} + \frac{\sqrt{\alpha_p} \phi(\Phi^{-1}(\theta^* + \tilde{\psi})) \sigma_x \sqrt{\alpha_p + \alpha_{\rho}}}{1 - \phi(\Phi^{-1}(\theta^* + \tilde{\psi})) \sigma_x (\alpha_p + \alpha_{\rho})} + \right. \\ &\quad \left. + \frac{\phi\left(\rho + \frac{\alpha_p}{\alpha_{\rho}} p\right) \frac{\sqrt{\alpha_p}}{\alpha_{\rho}} \tilde{\psi} \sqrt{\alpha_p + \alpha_{\rho}}}{1 - \phi(\Phi^{-1}(\theta^* + \tilde{\psi})) \sigma_x (\alpha_p + \alpha_{\rho})} \right) < 0, \\ \frac{\partial \hat{\rho}}{\partial \varepsilon_{\rho}} &= \phi(\Phi^{-1}(\hat{\rho})) \left( \sqrt{\frac{\alpha_{\rho}}{\alpha_p + \alpha_{\rho}}} + \frac{\sqrt{\alpha_{\rho}} \phi(\Phi^{-1}(\theta^* + \tilde{\psi})) \sigma_x \sqrt{\alpha_p + \alpha_{\rho}}}{1 - \phi(\Phi^{-1}(\theta^* + \tilde{\psi})) \sigma_x (\alpha_p + \alpha_{\rho})} + \right. \\ &\quad \left. + \frac{\phi\left(\rho + \frac{\alpha_p}{\alpha_{\rho}} p\right) \tilde{\psi} \sqrt{\frac{\alpha_p + \alpha_{\rho}}{\alpha_{\rho}}}}{1 - \phi(\Phi^{-1}(\theta^* + \tilde{\psi})) \sigma_x (\alpha_p + \alpha_{\rho})} \right) > 0. \end{aligned}$$

Let  $\tilde{\psi} < \frac{\alpha_{\rho} \sqrt{2\pi}}{\alpha_p + \alpha_{\rho}}$ , which means that the amount of debt that is rolled over using credit ratings is (sufficiently) small. Near the default threshold, the impact of noise on the

default threshold is given by:

$$\begin{aligned}\frac{\partial \hat{\theta}}{\partial \varepsilon_\rho} &= -\frac{\phi\left(\Phi^{-1}\left(\hat{\theta} + \tilde{\psi}\right)\right) \frac{\sigma_x}{\sigma_\rho} + \phi\left(\rho + \frac{\alpha_p}{\alpha_\rho} p\right) \bar{\psi} \sigma_\rho}{1 - \phi\left(\rho + \frac{\alpha_p}{\alpha_\rho} p\right) \bar{\psi} \left(1 + \frac{\alpha_p}{\alpha_\rho}\right)} < 0 \\ \frac{\partial \hat{\theta}}{\partial \varepsilon_s} &= \frac{\phi\left(\Phi^{-1}\left(\hat{\theta} + \tilde{\psi}\right)\right) \frac{\sigma_x}{\sigma_p} + \phi\left(\rho + \frac{\alpha_p}{\alpha_\rho} p\right) \frac{\alpha_p \bar{\psi} \sigma_p}{\alpha_\rho}}{1 - \phi\left(\rho + \frac{\alpha_p}{\alpha_\rho} p\right) \bar{\psi} \left(1 + \frac{\alpha_p}{\alpha_\rho}\right)} > 0.\end{aligned}$$

Comparing these derivatives with those in Section 3, shows that the coordination effect of credit ratings is exacerbated by market practices, laws and regulations that hardwire buy or sell decisions to rating thresholds.