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REACHING AN OPTIMAL MARK-UP BID THROUGH THE VALUATION OF THE OPTION TO SIGN THE CONTRACT BY THE SUCCESSFUL BIDDER

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Reaching an Optimal Mark-Up Bid through the Valuation of the Option to Sign the Contract by the Successful Bidder*

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Abstract

This paper aims to establish a support decision model by which an optimal mark-up (profit margin) in the context of a bidding process is reached through the valuation of the option to sign the contract assuming the contractor is chosen to perform the project. The price included in the bid proposal remains unchanged from the moment the offer is sealed until the contractor has the right - but not the obligation - to sign the contract, whereas construction costs vary stochastically throughout the period. Contractors should only sign the contract if the construction costs, at that moment, are lower than the price previously defined. We evaluate the option using an adapted version of the Margrabe (1978) exchange option formula and we also assign a probability of winning the bid for each profit margin using a function that respects the inverse relationship between these two variables. We conclude that to the higher value of the option - weighted by the probability of winning the contract - corresponds the optimal mark-up bid. Finally, we consider the existence of penalty costs which makes the model more efficient in explaining what actually takes place in some legal environments; we then conclude that the option to sign the contract and, therefore, the optimal mark-up bid are affected by their existence.

JEL classification codes: G31; D81

Keywords: optimal bidding; real options; construction projects; price determination

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1 Introduction

In this paper we aim to reach an optimal profit margin in the context of a bidding contracting process using a real options approach. The model herein proposed focuses on optimizing the contractor´s price through the valuation of the option to perform the project. When a contractor presents a bid to the client and assuming that the probability of winning the bid is greater than zero, the option to sign the contract - and subsequently executing the job - does have value, as clearly established in the option pricing theory. The motivation behind our work is supported by the presence of uncertainty since the estimated costs of delivering the project will vary from the moment the price is fixed until the preferred bidder has to decide whether to sign the contract or not. Even though the price remains unchanged during this period, the uncertainty in construction costs will lead to changes in the expected profit margin until the contract is signed and the parties are legally bounded. [1]

As far as the present research is concerned, contractors are firms operating in the construction industry whose business consists of executing a set of tasks previously defined by the client. The amount of tasks to be performed constitute a project, job or work. A significant amount of projects in the construction industry are assigned through what is known as tender or bidding processes (Christodoulou, 2010; Drew et al., 2001), being this the most popular form of price determination (Liu and Ling, 2005; Li and Love, 1999). A bidding process consists of a number of contractors competing to perform a particular job by submitting sealed proposals until a certain date previously defined by the client. The usual format of a bidding process is based on the rule that - all other things being equal - the contract will be awarded to the competitor which submitted the lowest bid (Chapman et al., 2000), *i.e.*, the lowest price. Bearing this in mind, it is easy to conclude that the client´s decision is very straightforward but the contractor´s decision on what price to bid is way more difficult to reach, being probably one of the most difficult decisions management has to face during the bid preparation (Li and Love, 1999).

The construction industry is known for featuring strong levels of price competitiveness (Chao and Liu, 2007; Mochtar and Arditi, 2001; Ngai et al., 2002) and the competitive pressures are probably more intense than in any other industry (Drew and Skitmore, 1997; Skitmore, 2002), which often leads contractors to lower their profit margins in order to produce a more competitive bid. Thus, it is not rare to see the winning bid include a near zero-profit margin (Chao and Liu, 2007). Moreover, under-pricing in the context of competitive bidding is a common phenomenon, namely explained by the need for work and penetration strategies (Drew and Skitmore, 1997; Fayek, 1998; Yiu and Tam, 2006), even tough bidding below cost does not necessarily guarantee a successful result to the bidder (Tenah and Coulter, 1999).

Contractors realize that bidding low when facing strong competition increases the chance of being chosen to perform the work but they are also aware of the opposite: if the price included in their

[1] It should be mentioned that this risk cannot be hedged since the contractor does not know when the bidding process will end.

proposal is higher, the likelihood of winning the bid will definitively be lower. This inverse relationship between the level of the profit margin (commonly known in the construction management literature as the "mark-up bid") and the probability of winning the bid is an accepted fact both in the construction industry and within the research community (see, for example, Christodoulou, 2010; Kim and Reinschmidt, 2006; Tenah and Coulter, 1999; Walllwork, 1999).

Competitive bidding has been a subject of research since the important papers of Friedman (1956) and Gates (1967) set the standards for future discussion. Both models proposed a probabilistic approach to determine the most appropriate mark-up value for a given contract and were supported by the definition of a relationship between the mark-up level and the probability of winning the bid. For that purpose, the two authors assumed the existence of previous bidding data - leading to the definition of the bidding patterns of potential competitors - and their models are supported by two fundamental parameters: the estimated cost of completing the project and the expected number of bidders. Gates had the merit to extend the model built by Friedman and turned it into a general strategic model, with general applicability, setting the foundations for what is now commonly known as *Tendering Theory* (Runeson and Skitmore, 1999). All attempts to establish a relationship between the probability of winning the contract and the level of the profit margin were based on previous bidding data – in line with the mentioned pioneer models. Carr (1982) proposed a model similar to Friedman's but differing in the partitioning of underlying variables: Friedman (1956) used a single independent variable, a composite bid-to-cost ratio, whereas Carr (1982) crafted his model around two distributions: one that standardizes the estimated cost of the analyzing bidder to that of all competitor bids, and another that standardizes the bids of an individual competitor against that of the analyzing bidder´s estimated costs. More recently, Skitmore and Pemberton (1994) presented a multivariate approach by assuming that an individual bidder is not restricted to data for bids in which he or she has participated, as in the case of Friedman and Gates' models, both based on bi-variate approaches. Instead, the bidder is able to incorporate data for all auctions in which competitors and potential competitors have participated, regardless of the individual bidder´s participation. This methodology had the merit of increasing the amount of data available for estimating the model parameters. An optimal mark-up value was then reached against known competitors, as well as other types of strategic mark-ups.

Past research seems to suggest that it would be difficult to establish a link - with general applicability - between the mark-up level and the probability of winning the bid. Contractors may recur to previous bidding data and assume that bidders are likely to bid as they have done in the past in order to shape the relationship that best describes their specific situation. However, as Fayek (1998) stated, past bidding information is not always available and – even if it is – we believe that past bidding experiences may not always be useful since the circumstances surrounding every bidding process differ from all the others. Moreover, often contractors do not possess information about which competitors will prepare and present a bid proposal, which is generally the case in public contracting processes.

Several studies suggest that decisions regarding the definition of the mark-up level were mainly

supported using subjective judgment, gut feeling and heuristics (Hartono and Yap, 2011), hence acknowledging the fact that managers actually do have a perception in real-world situations as to how the mark-up level will affect the probability of winning the contract. Our experience confirms such: even though, in general, managers do not support their mark-up bid decisions using some sort of mathematical expression linking the price and the probability of winning, they are aware that higher mark-ups will lead to lower chances of getting the job and do have a perception of how their decision regarding the definition of the mark-up bid will affect the probability of being successful. Bearing this in mind, we decided to support our model in a mathematical expression linking the mark-up level with the probability of winning the bid that (i) respects the generally accepted inverse relationship between these two variables; (ii) allows for flexibility and thus can be adapted to accommodate each contractors´ circumstances surrounding a particular bidding process. The formulation for the probability of winning that we propose will allow contractors to explicitly shape their previous perception, a determinant aspect for reaching the optimal mark-up bid, as we will demonstrate.

Most of the recent contributions to the optimal mark-up bid debate have been concerned with the selection of factors construction managers should take into account when deciding what price to bid (Christodoulou, 2010). Research by authors such as Drew et al. (2001), Drew and Skitmore (1992) and Shash (1993) stated that different bidders apply different mark-up policies which may be variable or fixed. These authors list a long set of factors aiming to explain the rationale behind mark-up bidding decision making: (1) amount of work in hand; (2) number and size of bids in hand; (3) availability of staff, including architects and other supervising officers; (4) profitability; (5) contract conditions; (6) site conditions; (7) construction methods and programme; (8) market conditions and (9) identity of other bidders, to name the ones they considered to be the most prominent. In general terms, factors are grouped in different categories and we sympathize with the 5 categories defined by Dulaimi and Shan (2002): (1) project characteristics; (2) project documentation; (3) contractor characteristics; (4) bidding situation and (5) economic environment. Following this line of thought, innovative research on the subject has been embracing more sophisticated methodologies. The paper by Li and Love (1999) managed to combine rule-based expert systems with Artificial Neural Networks (ANN) in the context of mark-up bid estimation, following previous research conducted by Li (1996), Moselhi et al. (1991), amongst others. In fact, the most recent and innovative models use ANN (as in Christodoulou, 2010 and Liu and Ling, 2005) or Goal Programming Techniques (Tan et al., 2008), where those determinants (or attributes) provide the ground where models are built upon, thus recognizing the crucial importance of possessing a strong knowledge of the factors influencing the contractors´ bid mark-up decision for the purpose of identifying the optimal mark-up level (Dulaimi and Shan, 2002).

Even though some work has been developed in the construction management field applying the real options approach (see, for example, Espinoza, 2011; Mattar and Cheah, 2006; Ng et al., 2004; Ng and Chin, 2004; Tseng et al., 2009; Yiu and Tam, 2006), there seems to be a lack of research contributing to the optimal mark-up debate using this methodology, motivating us to build up a

model embracing the real options approach and aiming to reach the mark-up level that maximizes the project expected value. This is achieved by evaluating the option to sign the contract and weighting the value of the option by the probability of winning the bid. According to our model, contractors should mark-up their bids with the amount that corresponds to the higher value of the option to sign the contract, weighted by the probability of winning the bid. Under the real options approach, this is the right perspective to follow: to the higher value of the option (weighted by the probability of winning the contract) will correspond a certain level of the profit margin, this being the optimal mark-up bid. The remainder of this paper unfolds as follows. In Section 2, each of the model´s components is described and the model´s numerical solution is presented. In Section 3, a numerical example is given and a sensitivity analysis to the parameters that shape the probability of winning the bid for each mark-up value is performed, in order to evaluate the impact in the optimal mark-up ratio. In Section 4, we consider the existence of penalty costs if the successful bidder decides to not sign the contract, adapting the model accordingly and presenting the new results based on the previous numerical example. Finally, in Section 5, conclusions are given.

2 The Model

2.1 Theoretical Background

The model herein presented has a prescriptive nature as it aims to equip contractors with a tool to address the mark-up bid decision making process by recognizing the existence of uncertainty in one of the key value drivers of the project: its construction costs. Opposed to describing "how a decision is made", our model attempts to alert contractors to "how the mark-up decision should be made", recognizing the real options approach as an effective methodology in addressing the optimal mark-up debate. Uncertainty is present throughout the bidding process and construction managers should be aware of the fact that construction costs will most likely vary from the moment the price is fixed until the preferred bidder has to decide whether to perform the project or not, as he or she has the right - but not the obligation - to sign the contract. Thus, the expected profit will also change during the same period and the optimal decision may be not to sign the contract if the variation in the construction costs lead to a negative profit margin.

Our model is related to the *Tendering Theory*, which is a theory of price determination and differs from the well-known *Game Theory*. *Game Theory* is based on the analysis of problems involving the interactions of rational agents and requires that all bidders consider their strategies and select the most appropriate strategy assuming that all other competitors do the same (Runeson and Skitmore, 1999). Thus, *Game Theory* does not apply to situations where one agent (bidder) adopts his preferred strategy without any attempts from other bidders to modify their strategies in response or, as stated by Fudenberg and Tirole (1989), *Game Theory* only applies when there is a "conscious conflict", meaning that the outcome depends not only on one bidder´s action and chance but on

the actions of the other bidders. This goes against what we can observe in reality since contractors prepare and present their bids without engaging in any kind of interaction with other competitors.

2.2 Assumptions

In our model we assume that (i) each bidder is non-cooperative (hence deciding what price to include in its proposal in isolation); (ii) each bidder prepares his or her proposal simultaneously with the other competitors; (iii) each bidder presents a single-sealed bid proposal to the client; (iv) each bidder has access to the available information concerning the project in hands and all documentation to support the cost estimation and the final bid decision, in line with all other potential bidders; (v) the chosen bidder will only decide if he or she is going to perform the project at the moment the contract needs to be signed and not before that date.

Our model is thus based on the existence of a single-sealed bid process where there is no interaction or contact of any kind with other bid participants. We will further assume that bidders have no information about the number of competitors until the actual results are available to all contractors. Moreover, the outcome of our model is independent of the number of bidders competing with the contractor when he or she has to decide which mark-up value to include in the bid proposal. In the basic scenario, we assume that there are no penalty costs if the successful bidder decides not to sign the contract and, hence, not perform the project. [2]

2.3 Model Description

The model aims to determine the optimal mark-up bid to be included in the contractor´s proposal. As we will see, the optimal bid depends upon two different components: (i) the value of the option to sign the contract, which will be modeled as a contingent claim, adapting the exchange option model proposed by Margrabe (1978); (ii) since this option is only available to the successful bidder, we must weight the value of the option by the probability of winning the bid.

It is easy to see that there is a positive relationship between the price included in the bid proposal and the value of the option to sign the contract. However, the higher the price the lower will be the probability of winning, as we previously mentioned. The optimal price derives from the solution of a maximization problem. We will begin by explaining the two components separately.

2.3.1 The Option to Sign the Contract

The Margrabe (1978) exchange option model builds on the famous Black and Scholes (1973) model, used to evaluate a typical european *call option* considering the existence of only one

[2] This assumption will be relaxed in Section 4.

stochastic variable: the price of the "underlying asset", whereas the Margrabe (1978) formula incorporates two "underlying assets", being the formula´s outcome the value of an european *call option* to exchange one asset for another. We adapt the Margrabe (1978) exchange option model to accomodate the fact that only the exercise price is uncertain, *i.e.*, the construction costs. Thus, only K, the construction costs follow a stochastic process known as Geometric Brownian Motion, with the following diffusion process:[3]

$$dK = \alpha K dt + \sigma K dz \tag{1}$$

where α is the drift parameter, dt is the time interval, σ is the standard deviation (volatility parameter) and dz is the increment of a standard Wiener process. The Margrabe (1978) formula (F) becomes:

$$F(P,K) = PN(d_1) - KN(d_2) \tag{2}$$

being (d_1) and (d_2):

$$d_1 = \frac{ln(P/K) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}} \tag{3}$$

$$d_2 = d_1 - (\sigma\sqrt{T-t}) \tag{4}$$

$N(d_1)$ and $N(d_2)$ are the probability density functions for the values resulting from expressions (d_1) and (d_2), respectively and:

- σ^2 is the variance (volatility), which, in our model, equals σ_K^2. [4]
- $T-t$ is the time between the moment the bid proposal is closed and the moment the contract needs to be signed.

2.3.2 The Probability of Winning the Bid

Based on our previous considerations, we propose an inverse relationship linking the mark-up ratio, (P/K) and the probability of winning the bid, W given by the following expression:

[3] As, in our case, the price remains unchanged from the moment the bid proposal is closed until the contract is signed, then $dP = 0$.

[4] Since $\sigma^2 = \sigma_P^2 - 2\sigma_P\sigma_K\rho_{PK} + \sigma_K^2$, where ρ_{PK} is the correlation coefficient between the price, P and the construction costs, K; since P remains unchanged, then σ_P^2 equals zero and so does $2\sigma_P\sigma_K\rho_{PK}$.

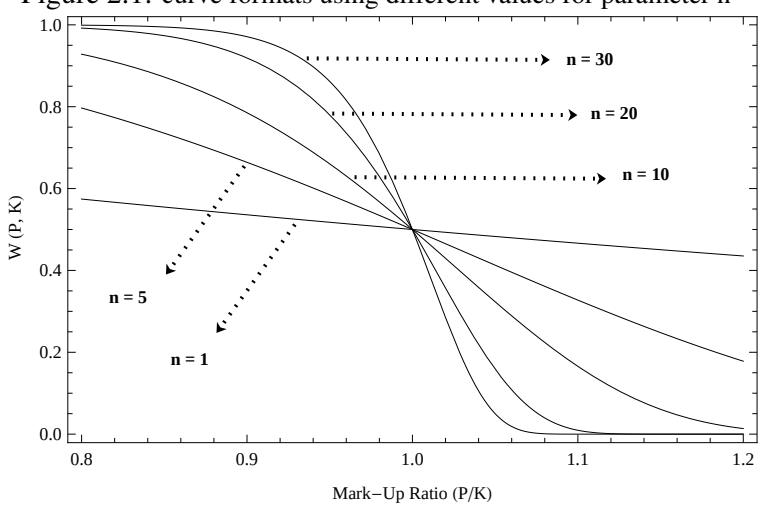
$$W(P,K) \quad = \quad e^{-b(P/K)^n} \tag{5}$$

where $W(P,K)$ is the probability of winning the bid, and n and b are parameters which should be used to calibrate the expression linking the mark-up level and the probability of winning the contract in order to best reflect each contractor´s specific circumstances. We will graphically show how each of them affect the configuration of equation (5).

Parameter n

Parameter n is responsible for shaping expression (4) in terms of its the concavity and convexity. Assuming parameter b equals $ln(1/0.5)$, the figure below shows the impact caused by parameter n in the expression´s curve format, for a set of five different values.

Figure 2.1: curve formats using different values for parameter n

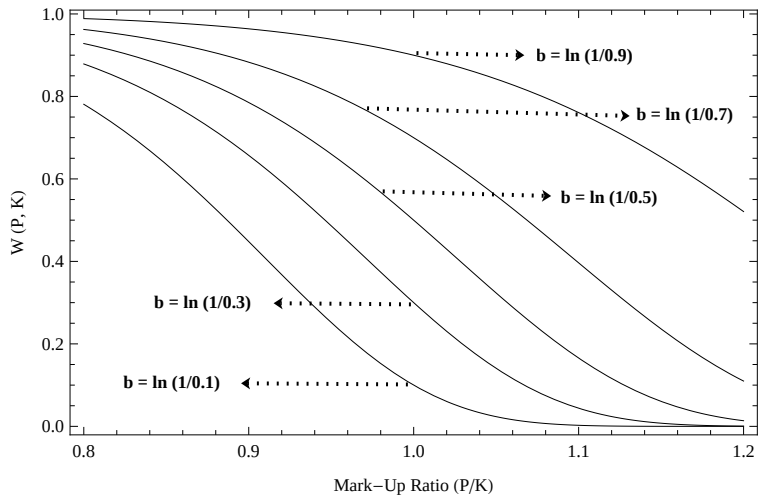


The figure above shows that the curve becomes more pronounced as n assumes higher values. In its concave region, changes in the mark-up ratio cause increasing variations in the probability of winning, whereas in the convex region variations in the mark-up ratio lead to decreasing variations in the probability of winning, and hence these effects assume more importance as the curve stretches out as a consequence of greater values of n. For lower values of n, the relationship between the mark-up ratio and the probability of winning becomes (almost) linear as the function shrunks towards the center, gradually showing both weaker concavity and convexity and thus traducing lower sensitivity to changes in the mark-up ratio. In a real-world situation, managers should calibrate this parameter to establish the existence and the pace at which these two unproportional effects take place .

Parameter b

Assuming parameter n equals 10, the following figure shows the impact on the curve´s format caused by five different values for parameter b.

Figure 2.2: curve formats using different values for parameter b



This parameter enables contractors to calibrate the functional relationship between the mark-up ratio and and the probability of wining the contract by setting the probability of winning when the price includes a zero profit margin (or, which is the same, when the mark-up ratio equals 1). The figure above shows that the greater the probability of winning the contract with a zero-profit margin (curves are designed for 10%, 30%, 50% ,70% and 90% probability with a zero-profit margin) the more shifted up and to the right the curve is . Also, the greater this probability is the less pronounced is the convexity region, reflecting the fact that variations in the mark-up ratio for values situated in this area will cause smaller decreasing impacts in the probability of winning the contract. On the contrary, as the curve shifts up and to the right, the curve´s concave region becomes more pronounced and variations in the mark-up ratio located in this area will lead to greater increasing variations in the probability of winning the bid.

2.3.3 The Optimal Price

The optimal price will be the one that maximizes the value of the option to invest weighted by the probability of winning the bid. Thus, the model´s outcome is the solution for the following maximization problem:

$$V(P,K) \quad = \max_{P} \quad \{[PN(d_1) - KN(d_2)][W(P,K)]\} \tag{6}$$

which means that the option value for each mark-up will be given by the outcome of the adapted Margrabe (1978) formula weighted by the probability of winning the bid. To the higher value

of the option weighted by the probability of being awarded the contract will correspond a specific price (P) and, therefore, a specific mark-up value (M) and the correspondent mark-up ratio (P/K). This will be the optimal price (P^*), the optimal mark-up value (M^*) and the optimal mark-up ratio (P/K^*), as we illustrate in the following numerical example.

3 Numerical Example

3.1 Basic Case

The next table includes information about the inputs used in our numerical example.

Table 3.1: inputs - description and values

input	description	value
K	construction costs	USD 50,000,000
σ	standard deviation	25%
$T-t$	time from the moment the price is established until the contract is awarded	0.5 (years)
n	parameter for calibrating the relationship between P/K and W	10
b	parameter for calibrating the relationship between P/K and W	ln(1/0.5)

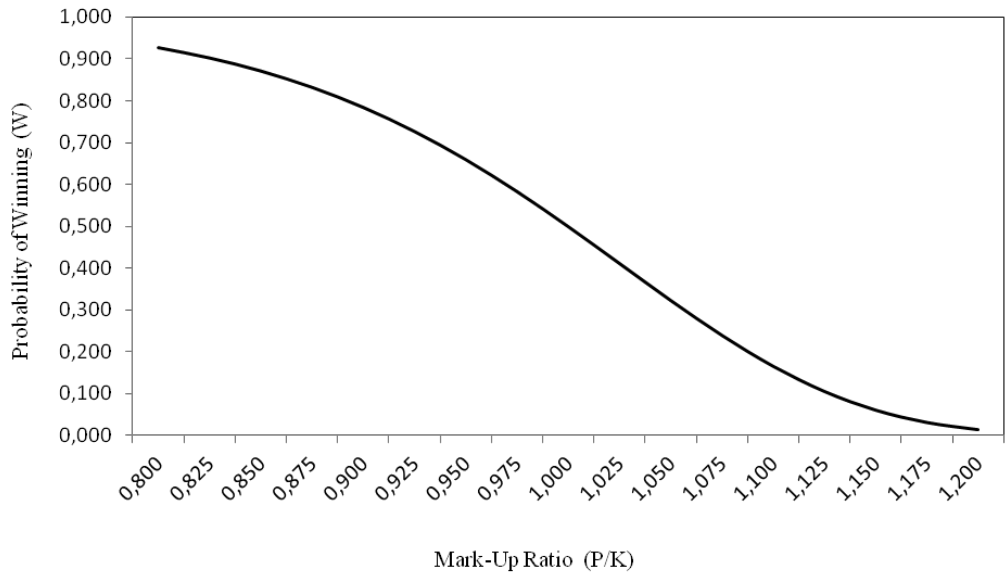
Thus, the relationship between the mark-up ratio and the probability of winning the bid is given by the following equation:

$$W(P,K) \quad = \quad e^{-ln(1/0.5)(P/K)^{10}} \tag{7}$$

meaning we set b equal to $ln(1/0.5)$, hence assuming that there is exactly a 50% probability of winning the bid if the contractor establishes a zero-profit margin, *i.e.*, $P/K = 1$, and also that parameter n equals 10.

The figure below shows the curve format for equation (7):

Figure 3.1: relationship between the mark-up ratio and the probability of winning



This S-Shaped curve respects the above mentioned inverse relationship between the mark-up ratio and the probability of winning the contract and typically comprises two different regions: a concave region until the mark-up ratio equals, approximately 1.027, corresponding to a probability of winning the contract of 40.66% and a convex region onwards. In its concave region, changes in the mark-up ratio lead to increasing variations in the probability of winning the bid, whereas in the convex region changes in the mark-up ratio will lead to decreasing variations in the probability of winning the contract.

The next table includes results for a set of different prices, P and the correspondent mark-up values, M and mark-up ratios, P/K.

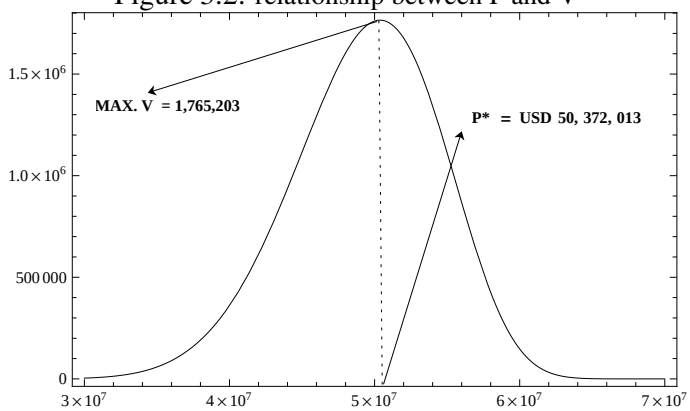
Table 3.2: results for different prices and mark-up levels

P (USD)	P/K	M (USD)	F (USD)	W (%)	V (USD)
40,000,000	0.800	-10,000,000	388,726	92.83	360,845
45,000,000	0.900	-5,000,000	1,420,579	78.53	1,115,585
50,000,000	1.000	0,000,000	3,521,599	50.00	1,760,799
50,372,013	**1.0074**	**372,013**	**3,723,805**	**47.40**	**1,765,203**
55,000,000	1.100	5,000,000	6,720,607	16.57	1,113,305
60,000,000	1.200	10,000,000	10,757,755	1.37	147,171

The results above demonstrate that the higher the value of the "underlying asset" the higher the value of F (the outcome of the adapted Margrabe formula increases in response to higher mark-

up levels, as the option pricing theory states) and the lower the value of W, since this probability decreases as the margin assumes greater values. As a result, V increases until reaches its maximum value: USD $1,765,203$. To this maximum value of the option to sign the contract corresponds the optimal mark-up value (M^*), *i.e.*, approximately USD $372,013$ and, equivalently, an optimal mark-up ratio, $(P/K)^*$ of 1.0074. Thus, the optimal price is USD $50,372,013$. Graphically, we have:

Figure 3.2: relationship between P and V



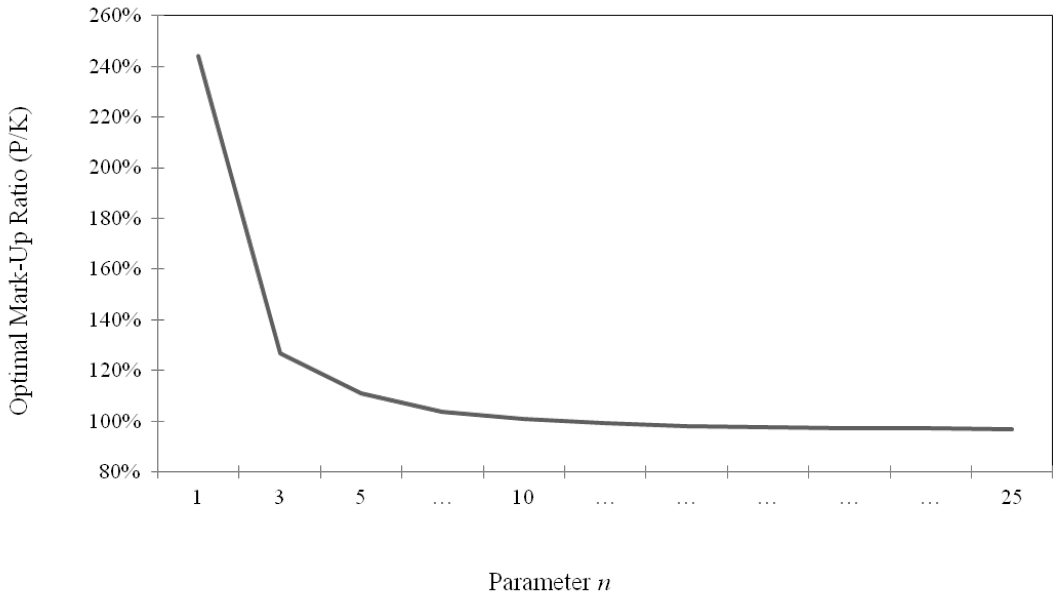
Thus, the function increases until it reaches the maximum value for V, corresponding to an optimal price of P^*. From this point onwards, the function decreases and the option value tends to zero as the margin goes to infinity.

3.2 Sensitivity Analysis

3.2.1 Parameter n

The figure below shows the impact caused by variations in parameter n, ranging from 1 to 25 and setting b equal to $ln(1/0.5)$, *i.e.,* assuming a 50% probability of winning the bid if the contractor marks up the bid with a zero-profit margin.

Figure 3.3: sensitivity analysis - parameter n



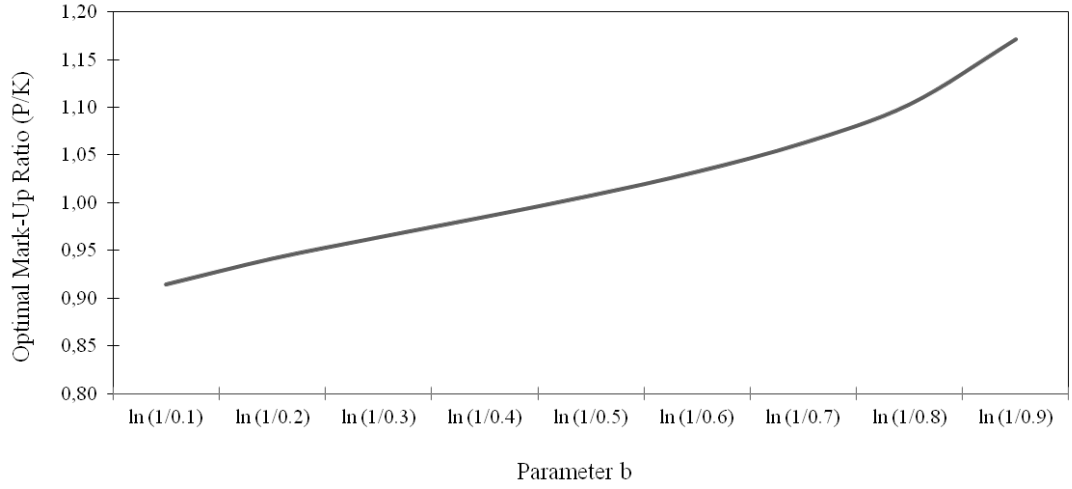
The figure shows that, the greater the value assumed by parameter n, the lower the optimal mark-up ratio due to a decrease in the option value, V which decreases as a consequence of two opposite variations: a decrease in F and an increase in W, being the former effect stronger than the later.

However, the figure also reveals that our model is much more sensitive to variations in parameter n when it assumes lower values. We can see that until $n = 5$, the impact is very strong but it becomes less important onwards (until n equals 10). For values greater than 10, the impact turns out to be almost negligible.

3.2.2 Parameter b

Next figure shows the results of the sensitivity analysis for parameter b, starting in a 10% probability of winning the contract for a zero-profit margin up to 90% probability of winning with the same zero-profit margin and fixing parameter n to 10.

Figure 3.4: sensitivity analysis - parameter b



The analysis entails that the higher the probability of winning the contract with a zero-profit margin, the higher the option value and consequently, the higher the optimal mark-up ratio (P/K^*). The option value (V) grows as a result of both an increase in F and W which explains the increase in the optimal mark-up bid. Results lead us to conclude that the model´s outcome is highly sensitive to variations in this parameter, ranging from an optimal mark-up ratio of 0.91 when b equals ln (1/0.1) to 1.17 when b equals ln (1/0.9).

4 Considering the Existence of Penalty Costs

In section 2 we did not consider the existence of penalty costs. Yet, in some legal environments, the contractor may have to pay a legal compensation if he or she decides to not exercise the option to sign the contract and, hence, not perform the project. According to Halpin and Senior (2011), in the United States contractors are free to withdraw their bids without incurring in any penalties if that happens prior to the ending of the bidding period. However, if a contractor decides to withdraw the bid after that moment - and assuming that he or she is the chosen bidder - a penalty equal to the difference between the second best proposal and the chosen bid is legally imposed, even if the contract has not been signed yet. According to Halpin and Senior (2011), "this may occur in the event that the selected bidder realizes that he or she has underbid the project and that pursuing the work will result in a financial loss" (p.44). In these circumstances, the client will exercise the legal right of receiving the difference between the two prices. If, for instance, a contractor included a price of USD 5,000,000 in the bid and refuses to enter into contract and the next low bid is USD 5,100,000, the client is damaged in the amount of USD 100,000. Hence a legal compensation equal to the difference between these two values is due, *i.e.*, USD 100,000.

For the sake of convenience but also because - from the contractor´s perspective - the expected

penalty costs can be seen as a percentage of the construction costs, we will assume that g, the amount of the penalty costs, is expressed as a percentage of K, the construction costs. Thus, the payoff (at maturity) of the option to sign the contract under these conditions will be:

$$Max[P-K;-gK] \tag{8}$$

Expression (8) entails that the contractor will choose to pay the legal penalty, gK if this cost is smaller than the financial loss given by the difference between P and K (the profit generated by undertaking the project). On the contrary, if this difference is smaller than the amount of the legal compensation, gK, than the contractor will prefer to sign the contract and execute the job. Taking these new conditions into account, we again adapted the Margrabe (1978) formula. The equation below includes two important changes when compared with equation (2):

1) The exercise price is now equal to "$K-gK$" . This is due to the fact that, if the contractor decides to not perform the project, he or she incurs in a penalty cost of the amount "gK". On the contrary, if the project is undertaken, the contractor will invest the amount "K". Thus, seen in incremental terms, "$K-gK$" will gives us the exercise price if the contractor exercises the option to perform the project in the presence of such costs.

2) $N(d_2)$ is the risk-adjusted probability that the option will be exercised at maturity (Nielsen, 1992) in the original Black and Scholes (1973) formula or, in other words, the probability that the option will finish "in-the-money" in a risk-neutral world (Smith, 1976). This also holds the same meaning in the Margrabe (1978) model. Hence, $[1-N(d_2)]$ will express the probability that the option will not be exercised at the maturity as it will not finish "in-the-money". In these circumstances the contractor will prefer to incur in the legal cost, gK and not execute the project. Thus, the adapted Margrabe formula (F_g) becomes:

$$Fg = [PN(d_1)-(K-gK)N(d_2)]-gK[(1-N(d_2)] \tag{9}$$

where:

$$d1 = \frac{ln[P/(K-gK)]+\frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}} \tag{10}$$

and

$$d2 = d1-(\sigma\sqrt{T-t}) \tag{11}$$

equation (5) still holds; thus the model´s outcome is the solution for the maximization problem expressed below:

$$Vg \quad = \max_{P_g} \quad \{[PN(d_1) - (K - gK)N(d_2) - gK((1 - N(d_2))][W(P,K)]\} \tag{12}$$

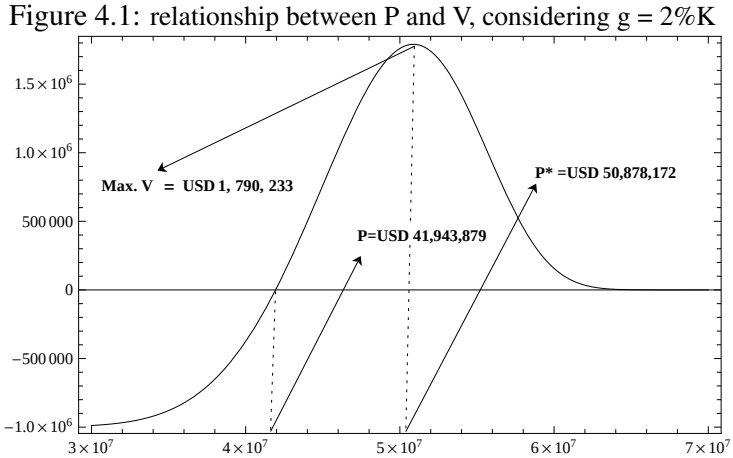
Using the same inputs included in table 3.1, the next table includes results for a set of different prices, P, the correspondent mark-up values, M and mark-up ratios, P/K and considering that the penalty costs, g account for 2% of the construction costs.

Table 4.1: results for different prices and mark-up levels, with g = 2%K

P (USD)	P/K	M (USD)	F_g (USD)	W (%)	V_g (USD)
40,000,000	0.800	-10,000,000	-405,117	92.83	-376,060
45,000,000	0.900	-5,000,000	970,174	78.53	761,881
50,000,000	1.000	0,000,000	3,519,395	50.00	1,759,698
50,878,172	**1.018**	**878,172**	**4,084,993**	**43.82**	**1,790,233**
55,000,000	1.100	5,000,000	7,128,665	16.57	1,180,902
60,000,000	1.200	10,000,000	1,145,375	1.37	156,693

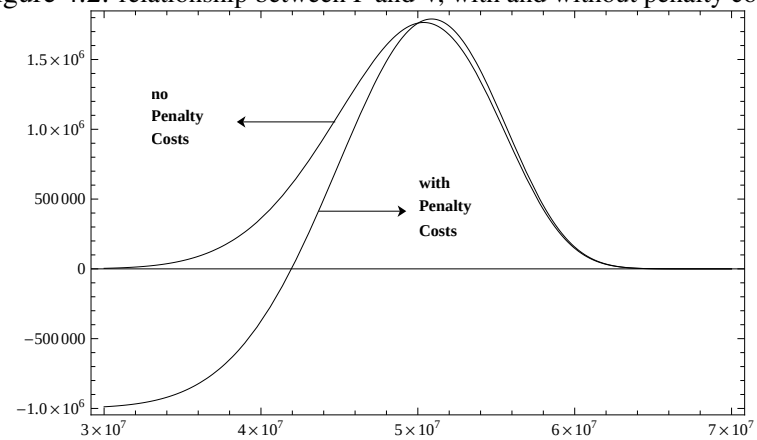
We must stress that the value of the option to sign the contract, V_g assumes negative values for low levels of P, in line with the interpretation of expression (8) and what figure 4.1 reflects. V_g becomes positive for price levels above USD $41,943,879$ and reaches its maximum when the price equals USD $50,878,172$. This is the optimal price in the presence of penalty costs, a slightly greater value than in the basic scenario, where penalty costs were not considered.

Figure 4.1: relationship between P and V, considering g = 2%K



The figure below compares the relationship between the price, P and the value of the option to sign the contract, V for both scenarios.

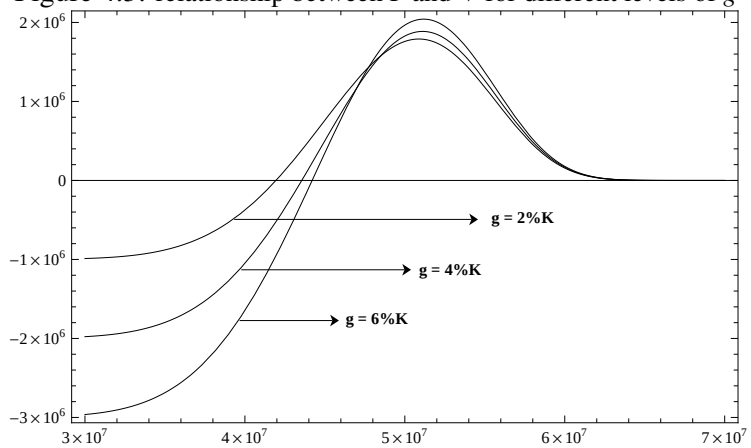
Figure 4.2: relationship between P and V, with and without penalty costs



In the ascending part of the two curves, the value of the option is greater in the absence of penalty costs for a specific price value. This difference becomes less significant as the price increases, until the maximum value for the option is reached, in both scenarios. In their descending part, the curves feature very similar configurations, with the value of the option - in the presence of penalty costs - assuming a slightly greater value for a specific price level, until both curves converge when higher price levels are reached.

Figure 4.3 shows that the value of the option to invest increases as g assumes higher levels. Consequently, the positive difference between the optimal price in the presence of penalty costs and the optimal price when these costs are not taken into account becomes greater.

Figure 4.3: relationship between P and V for different levels of g



5 CONCLUSIONS

The model herein presented aims to alert construction managers to the importance of considering uncertainty in project valuation through its impact in the behavior of construction costs within a real options framework. We identified an option available to the preferred contractor - the option to sign the contract - and the flexibility of performing the project or not does have value, as clearly established in the option pricing theory. The option to perform the project constitutes a real option and, when there are no penalty costs involved, it should only be exercised if the construction costs - at the time the contract needs to be signed - are lower than the price included in the bid proposal; however, when penalty costs are present, contractors should only exercise the option if the difference between the price and the construction costs - at that moment - is greater than the amount of this legal compensation.

Through the valuation of this option and assigning a probability of being awarded the contract for each mark-up level, the model determines that to the higher value of the option to execute the job - weighted by the probability of winning the contract - corresponds the optimal mark-up bid. According to our approach, to this optimal mark-up bid corresponds the optimal price contractors should include in their proposals.

The numerical example and the subsequent sensitivity analysis clearly showed that the model´s outcome strongly depends on the nature of the relationship between the mark-up level and the probability of winning the bid. Our model includes an expression linking these two variables which contractors may use since it respects the inverse relationship between the the mark-up value and the probability of winning the contract. This inverse relationship is a common accepted fact and contractors may use our expression and perform an adequate calibration by manipulating its parameters with the purpose of shaping their perception as how the mark-up level affects the probability of winning the bid.

Based on the input values considered in the numerical example, we concluded that, in the presence of penalty costs, the optimal price is higher since the option to execute the project as greater maximum value when contractors face the decision of eventually paying a legal compensation, compared with the optimal price in the basic scenario, where penalty costs were not considered. As the penalty costs assume higher values, the difference between the optimal price when penalty costs are included in the analysis and the optimal price when these type of costs is not present becomes greater. However, if different calibrations for parameters b and n are established, the optimal price in the absence of penalty costs may be higher than the optimal price when penalty costs are considered. This fact reinforces our previous argument that the optimal price is highly sensitive to the mathematical expression linking the mark-up level with the probability of winning the contract.

Our model, supported on the valuation of the option to perform the project, constitutes an innovative form of addressing the optimal mark-up bid debate and we hope it will be acknowledged as an effective contribution to this research subject.

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