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Monetary and Fiscal Policy Interactions in a Monetary Union with Country-size Asymmetry*

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Abstract

This paper analyses optimal discretionary non-coordinated monetary and fiscal stabilization policies in a micro-founded New-Keynesian model of a two-country monetary union with country-size asymmetry, under two policy scenarios. A balanced-budget policy scenario and a policy scenario where the presence of government debt limits the macroeconomic stabilization effort and enlarges the sources of strategic policy interactions.

Numerical results indicate that non-cooperation exacerbates the fiscal policy activism of a small country while moderating that of a large country. In the balanced-budget scenario, non-cooperation improves (reduces) welfare for a small (large) country while, in the high-debt scenario, it produces the opposite results. Cooperation dominates non-cooperation for the union as a whole.

Keywords: Monetary union; optimal fiscal and monetary policies; asymmetric countries. *JEL codes:* E52; E61; E62; E63

1 Introduction

In the European Monetary Union (EMU) a common monetary policy coexists with decentralized fiscal policies. There is the case for fiscal policy to be used as a stabilization device but, strategic interactions between non-coordinated policies may meaningfully limit such a role. Country-size asymmetry, such as that

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observed within the EMU, as well as the constraints imposed by the available sources of government financing are critical for the design of optimal policies and may crucially shape the strategic interactions between policymakers.

Small and large union-countries are diversely affected by equivalent idiosyncratic shocks; their fiscal policies cause asymmetric cross-border effects and impinge on union-wide variables with different magnitudes. Suffering to a larger extent the effects of country-specific shocks, a small country is more likely to experience a worse stabilization performance than a large one. Furthermore, as its fiscal policy spillovers are smaller, the government of a small country faces accrued incentives towards fiscal policy activism, while that of a large country faces opposite incentives. These divergent incentives may even have different stabilization consequences, whether or not policymakers are constrained by the need to control government debt. Relative to a debt-unconstrained policy scenario – where lump sum taxes always adjust to ensure balanced-budgets – a debt-constrained policy scenario limits the macroeconomic stabilization effort and, since monetary policy now has debt repercussions, enlarges the sources of strategic interactions between monetary and fiscal policies that can be differently exploited by small and large countries.

Key questions are then (i) to assess how debt constraints and different policy regimes shape the macroeconomic stabilization outcomes in a country-size asymmetric monetary union; (ii) to appraise how far these outcomes are from the optimal solution; and, (iii) to establish under which conditions alternative institutional policy arrangements are welfare-improving and supported by large countries.

In order to address these issues we use a two-country micro-founded macroeconomic model with monopolistic competition and sticky prices, in line with that developed by Beetsma and Jensen (2004, 2005). As in Leith and Wren-Lewis (2007a, 2007b), the model allows for fiscal policy to have demand and supply-side effects, by considering as fiscal policy instruments the home-biased public consumption and the tax rate, under two policy scenarios. A balanced-budget policy scenario, where the ability of fiscal policy to promote stabilization is magnified and the sources of strategic interactions between monetary and fiscal policies are minimized; and, a high-debt policy scenario, where fiscal policy stabilization gains are restricted by the need to control government debt and the risk of harmful policy-mixes is inflated by the great effectiveness of monetary policy on debt adjustment.

We assume that the monetary authority – maximizing the union-wide welfare – and the fiscal authorities – maximizing their national counterpart – engage in discretionary policy games. Optimal solutions are computed numerically using appropriate algorithms that reflect the different timing structures of the (non-cooperative) policy games: Nash, monetary leadership and fiscal leadership. We follow the methodology developed in the recent work of Kirsanova and co-authors (Blake and Kirsanova, 2009, for a closed-economy setup, and Kirsanova *et al.*, 2005, for an open-economy setup). Moreover, we examine whether the solutions obtained under these different policy games can be improved either by policy cooperation or by monetary policy delegation to a weight-conservative central bank.

Our main contribution to the existing literature is twofold. First, we allow for country-size asymmetry in a non-cooperative monetary union. Second, our model also captures how optimal discretionary stabilization policies are constrained by the need to ensure debt sustainability. A number of recent papers, also using a micro-founded DSGE framework, have examined monetary and fiscal policy interactions in a monetary union. However, a significant number have analyzed the nature of optimal policy only under policy cooperation, as is the case of Beetsma and Jensen (2004) or Galí and Monacelli (2008), in a balanced-budget policy scenario, and Ferrero (2009), in a debt-policy scenario. Another important branch of this literature has considered the case of non-cooperation but only few authors have used a dynamic model.[1] In this spirit and closer to our paper, van Aarle *et al.* (2002) and Beetsma and Jensen (2005), for instance, analyze non-cooperative monetary and fiscal policies under Nash, while Kirsanova *et al.* (2005) examine the case of monetary leadership.[2]

Furthermore, to our knowledge, country-asymmetry is rarely addressed in the literature on policy interactions in a monetary union. Canzoneri *et al.* (2005), using a theoretical model calibrated to represent the Euro area, found that a common monetary policy favours macroeconomic stabilization of the larger countries. However, their results are obtained under non-optimal policies.[3] At the empirical level and for a broad sample of countries, Furceri and Karras (2007, 2008) found that small countries have higher business cycle volatility and the findings of Furceri and Ribeiro (2009) suggest that smaller countries have more volatile government consumption.

In line with these results, our numerical simulations confirm that a small country performs a more active fiscal policy than a large one and that non-cooperation reinforces this discrepancy. In a debt-constrained setting and following a shock, time-consistency requires policy instruments to stabilize debt at its pre-shock level (debt stabilization bias, under discretion). For sufficiently high levels of public debt, monetary policy complements fiscal policy on debt adjustment at the union level and small and large countries rely differently on monetary policy to adjust domestic debts. Thus, under non-cooperation, fiscal policy is further (less) active towards debt stabilization for the small (large) country. As a consequence, in the high-debt scenario and relative to policy cooperation, non-cooperation reduces (improves) welfare for a small (large) country and amplifies the asymmetric distribution of the stabilization burden across countries, with negative welfare consequences for the union as a whole. Mone-

[1] The traditional literature has studied this question using static models (see Beetsma and Debrun, 2004, for a thorough review of this literature). Static models allow for analytical solutions but cannot conveniently incorporate expectations nor can they be used to analyze appropriately the role of public debt in policy interactions. However, some authors, like Chari and Kehoe (2007), introduce dynamics in a tractable way through a two-period model where public debt is set strategically.

[2] In contrast with the generality of this literature, Forlati (2009) examines optimal non-coordinated (Nash) monetary and fiscal policies with fully micro-founded welfare criteria.

[3] Ferrero (2009) also allows for country-size asymmetry in his model calibration of the EMU, but he doesn't explicitly examine this issue, which is of minor relevance under cooperation.

tary conservatism has proved to be a fruitful device to improve welfare for the small country and for the union under fiscal leadership, where the large country can benefit from its larger strategic position vis-à-vis the central bank.

This paper is organized as follows. In Section 2 we develop the setup for policy analysis. In Section 3 we perform policy analysis related with dynamic responses and welfare evaluation under different policy regimes. Finally, in Section 4 we present concluding remarks and suggest extensions for future work.

2 Setup for Policy Analysis

The model developed by Beetsma and Jensen (2004, 2005) is extended to capture country-size asymmetry, to allow for a more generic case of cross-country consumption elasticity and to include different fiscal policy scenarios.

The monetary union is modelled as a closed area with two countries, H (Home) and F (Foreign), populated by a continuum of agents $\in [0,1]$. The relative dimension of country i ($i = H, F$) is $n_i \in (0,1)$, with $n_H + n_F = 1$. While subject to idiosyncratic shocks, the countries are assumed to have identical economic structures and each one is characterized by two private sectors - households and firms -, one fiscal authority, and is subject to a common monetary policy.

To start, we address the optimization problem of households and firms, living at country H (equivalent to that at F). The next step is to describe the policy environment which includes the presentation of the policy instruments, the equilibrium conditions and the policy objectives. The remainder of this section characterizes the policy games and presents the benchmark calibration.

2.1 Households

The j-household seeks to maximize the following lifetime utility (U_0^j).

$$
\begin{aligned}
U_0^j &= E_0 \sum_{t=0}^{\infty} \beta^t \left[u\left(C_t^j, \overline{C}_t^H\right) + V\left(G_t^H\right) - v\left(L_t^j\right)\right] \qquad (1) \\
& \text{where} \\
& u\left(C_t^j, \overline{C}_t^H\right) = \tfrac{\sigma}{\sigma-1}\left(C_t^j\right)^{\frac{\sigma-1}{\sigma}}\left(\overline{C}_t^H\right)^{\frac{1}{\sigma}} \\
& V\left(G_t^H\right) = \delta \tfrac{\psi}{\psi-1}\left(G_t^H\right)^{\frac{\psi-1}{\psi}} \\
& v\left(L_t^j\right) = \tfrac{d}{1+\eta}\left(L_t^j\right)^{1+\eta}
\end{aligned}
$$

with C_t^j, G_t^i and L_t^j denoting, respectively, private consumption, *per capita* public consumption on domestically produced goods and hours of work. $\overline{C}^i$ is an exogenous disturbance which affects the demand for consumption goods and

C^j is a real consumption Dixit-Stiglitz index defined as

$$C^j \equiv \left[n_H^{\frac{1}{\rho}} \left(C_H^j \right)^{\frac{\rho-1}{\rho}} + n_F^{\frac{1}{\rho}} \left(C_F^j \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \tag{2}$$

where ρ the elasticity of substitution between H and F consumption baskets, C_H^j and C_F^j are consumption sub-indexes of the continuum of differentiated goods produced, respectively, in country H and F

$$C_{H,t}^j \equiv \left[\left(\frac{1}{n_H} \right)^{\frac{1}{\theta}} \int_0^{n_H} c_t^j (h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}} ; \ C_{F,t}^j \equiv \left[\left(\frac{1}{n_F} \right)^{\frac{1}{\theta}} \int_{n_H}^{1} c_t^j (f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}} \tag{3}$$

and θ is the elasticity of substitution between goods produced in each country.

Maximization of (1) is subject to a sequence of budget constraints of the form

$$P_t C_t^j + E_t \left(Q_{t,t+1} D_{t+1}^j \right) = W_t (j) L_t^j + \int_0^{n_H} \Pi_t^j (k) \, dk - P_t T_t^H + D_t^j \tag{4}$$

where P is the consumption-based price index defined below, $W(j)$ is the nominal wage rate of labour of type j, $\Pi^j(k)$ is the share of profits of domestic firm k going to household j in country H and T^H is a *per capita* lump sum tax. Household j has access to a complete set of state-contingent securities that span all possible states of nature and are traded across the union. D_{t+1}^j denotes the nominal payoff of a portfolio of state-contingent securities, purchased by the j-household at date t, while $Q_{t,t+1}$ is the stochastic discount factor for one-period ahead nominal payoffs, common across countries.

Assuming no trade barriers and given the structure of preferences, purchasing power parity holds, and the underlying consumption-based price index (P_t) is defined as

$$P_t \equiv \left[n_H P_{H,t}^{1-\rho} + n_F P_{F,t}^{1-\rho} \right]^{\frac{1}{1-\rho}} , \tag{5}$$

while the country-specific price indexes P_H and P_F are given by

$$P_{H,t} \equiv \left[\frac{1}{n_H} \int_0^{n_H} p_t (h)^{1-\theta} dh \right]^{\frac{1}{1-\theta}} ; \ P_{F,t} \equiv \left[\frac{1}{n_F} \int_{n_H}^{1} p_t (f)^{1-\theta} df \right]^{\frac{1}{1-\theta}} \tag{6}$$

where $p(h)$ and $p(f)$ are the prices of typical goods h and f produced in H and F, respectively.

The problem of the representative household can be split into an intertemporal and an intratemporal problem. In regards to the household's intratemporal problem, it requires choosing the allocation of a given level of expenditure across the differentiated goods to maximize the consumption index, C^j. Plugging into the appropriate output aggregators the resulting individual demands and the optimal government spending allocation across domestically produced goods, we obtain the national aggregate demands, Y^H and Y^F,

$$Y_t^H = \left(\frac{P_{H,t}}{P_t}\right)^{-\rho} C_t^W + G_t^H \tag{7H}$$

$$Y_t^F = \left(\frac{P_{F,t}}{P_t}\right)^{-\rho} C_t^W + G_t^F \tag{7F}$$

where the union-wide consumption, C^W, is defined as $C^W \equiv \int_0^1 C^j dj$, and

$$\left(\frac{P_H}{P}\right)^{\rho-1} = n_{_H} + n_{_F} T^{1-\rho} \ ; \ \left(\frac{P_F}{P}\right)^{\rho-1} = n_{_H} T^{\rho-1} + n_{_F} \tag{8}$$

The variable T stands for the terms-of-trade, defined as the relative price of the F-bundle of goods in terms of the H-bundle of goods ($T \equiv P_F/P_H$). According to (8), changes in the terms-of-trade imply a larger response in a country's aggregate demand the smaller the size of the country, *i.e.*, the larger the degree of openness.

As for the household's intertemporal problem, the household chooses the set of processes $\left\{C_t^j, L_t^j; D_{t+1}^j\right\}_{t=0}^{\infty}$, taking as given all the other processes and the initial wealth, as to maximize the intertemporal utility function (1) subject to (4). Solution for this problem yields the familiar Euler equation

$$u_c\left(C_t^j, \overline{C}_t^H\right) = \beta\left(1+i_t\right) E_t\left\{\left(\frac{P_t}{P_{t+1}}\right) u_c\left(C_{t+1}^j, \overline{C}_{t+1}^H\right)\right\}, \tag{9}$$

where $1+i_t = \frac{1}{E_t Q_{t,t+1}}$ is the gross risk-free nominal interest rate. Moreover, assuming that the initial state-contingent distribution of nominal bonds is such that the life-time budget constraints of all households are identical, the risk-sharing condition implies that

$$u_c\left(C_t^H, \overline{C}_t^H\right) = u_c\left(C_t^F, \overline{C}_t^F\right) \tag{10}$$

Finally, the labour supply decision determines that the real wage for labour type j is given by

$$\frac{W_t(j)}{P_t} = \mu_{w,t}^H * \frac{v_L\left(L_t^j\right)}{u_c\left(C_t^j, \overline{C}_t^H\right)} \tag{11}$$

where $\mu_{w,t}^H \geqslant 1$ is an exogenous H-specific wage markup that is used as a device to introduce the possibility of "pure cost-push shocks" that affects the equilibrium price behaviour but does not change the efficient output, as in Benigno and Woodford (2004, 2005).

2.2 Firms

There are a continuum of firms in country H and in country F. The production function for the differentiated consumption good y, indexed by $h \in [0, n_{H})$ in country H and by $f \in [n_{H}, 1]$ in country F, is described, for $y(h)$, by

$$y_t(h) = a_t^H L_t(h) \tag{12}$$

where a_t^H is an exogenous H-specific technology shock, common to all H-firms, and $L_t(h)$ is the firm-specific labour input offered by a continuum of H-households, indexed in the unit interval. In a symmetric equilibrium, the work effort chosen by the household $\left(L_t^h\right)$ equals the aggregate labour input $(L_t(h))$.

Firms are assumed to set prices on a staggered basis, as in Calvo (1983). Each period, a randomly selected fraction of firms at H $\left(1-\alpha^H\right)$ have the opportunity to change their prices, independently of the time that has elapsed since the last price-resetting, while the remaining firms keep the prices of the previous period. If it has the chance to reset prices in period t, an optimizing h-firm will set $p_t^o(h)$ in order to maximize the expected future profits, subject to the demand for its product and the production technology. The first order condition for this optimizing wage-taker firm can be expressed as

$$\left(\frac{p_t^o(h)}{P_{H,t}}\right)^{1+\theta\eta} = \frac{\frac{\theta}{\theta-1} E_t \sum_{s=t}^{\infty} \left(\alpha^H \beta\right)^{s-t} \mu_{w,s}^H \left(1-\zeta^H\right) v_y\left(Y_s^H; a_s^H\right) \left(\frac{P_{H,s}}{P_{H,t}}\right)^{\theta(1+\eta)} Y_s^H}{E_t \sum_{s=t}^{\infty} \left(\alpha^H \beta\right)^{s-t} \left(1-\tau_s^H\right) u_c\left(C_s^H, \overline{C}_s^H\right) \left(\frac{P_{H,s}}{P_{H,t}}\right)^{\theta-1} \left(\frac{P_{H,s}}{P_s}\right) Y_s^H} \tag{13}$$

where $p_t^o(h)$ still applies at s, τ_s^H is a proportional tax rate on sales with the non-zero steady-state level $\overline{\tau}^H$, and ζ^H is an employment subsidy fully financed by lump sum taxes that, removing average monopolistic and tax rate distortions, ensures the efficiency of the steady-state output level.[4] The price index P_H evolves according to the law of motion

$$P_{H,t}^{1-\theta} = \alpha^H P_{H,t-1}^{1-\theta} + \left(1-\alpha^H\right) p_t^o(h)^{1-\theta} \tag{14}$$

2.3 Policy Environment

In this section, we describe the instruments and constraints for the monetary and fiscal policies and present a set of meaningful objective functions facing the policy authorities. These policy functions have a twofold purpose: (i) to enable the derivation of optimal discretionary policy rules across several regimes of monetary and fiscal policies interactions and (ii) to assess the welfare impacts of the different policy regimes.

[4] Following Leith and Wren-Lewis (2007a, 2007b), we use this employment subsidy as a device to eliminate linear terms in the social welfare function without losing the possibility of using the sales tax rates as fiscal policy instruments.

2.3.1 Policy instruments and constraints

The monetary authority sets a common nominal interest rate, i_t, for the union. As for fiscal policy, we assume two alternative policy scenarios. In a first set-up, lump-sum taxes $\left(T^H\right)$ are adjusted to fully finance, in each period, an employment subsidy $\left(\zeta^H\right)$ and the instruments used for stabilization purposes – the home-biased government spending $\left(G^H\right)$ and the sales tax rate $\left(\tau^H\right)$.[5] Here, fiscal policy is balanced-budget and Ricardian equivalence holds. In a second scenario, lump-sum taxes only adjust to fully accommodate the employment subsidy and the government inter-temporal solvency condition appears as an additional binding constraint to the set of possible equilibrium paths of the endogenous variables. Stabilization fiscal policy instruments are the same as in the first scenario - G^H and τ^H - and, thus, fiscal policy encompasses demand and supply-side effects. The budget constraints for the fiscal authorities can be written as

$$B_t^H = (1+i_{t-1}) B_{t-1}^H + P_{H,t} G_t^H - \tau_t^H P_{H,t} Y_t^H \tag{15H}$$
$$B_t^F = (1+i_{t-1}) B_{t-1}^F + P_{F,t} G_t^F - \tau_t^F P_{F,t} Y_t^F \tag{15F}$$

where B_t^H and B_t^F represent the *per capita* nominal government debt of country H and F, respectively.[6]

Equivalently,

$$b_t^i = (1+i_t)\left(b_{t-1}^i \frac{P_{t-1}}{P_t} + \frac{P_{i,t}}{P_t} G_t^i - \tau_t^i \frac{P_{i,t}}{P_t} Y_t^i\right), \quad i = H, F \tag{16}$$

where the variable $b_t^i \equiv \frac{(1+i_t)B_t^i}{P_t}$ denotes the real value of debt at maturity in *per capita* terms.

2.3.2 Equilibrium Conditions

To solve for the optimal policy, authorities have to take into account both the private sector behaviour as well as the budget constraints, described above. These conditions can be log-linearized and written in gap form as

$$E_t c_{t+1}^w = c_t^w + \sigma\left(i_t - E_t \pi_{t+1}^w\right) \tag{17}$$

$$y_t^H = s_c \rho n_F q_t + (1 - s_c) g_t^H + s_c c_t^w \tag{18H}$$
$$y_t^F = -s_c \rho n_H q_t + (1 - s_c) g_t^F + s_c c_t^w \tag{18F}$$

[5] For simplicity, we admit that government debt is zero in this scenario.

[6] With asset markets clearing only at the monetary union level, the sole public sector inter-temporal budget constraint is the union-wide consolidated debt. However, in the context of a monetary union with an institutional arrangement like the EMU, there are arguments to impose the verification of this inter-temporal budget constraint at the national levels.

$$\pi_t^H = k^H \left[(1 + s_c \rho\eta)\, n_{\scriptscriptstyle F}\, q_t + \frac{1 + s_c \sigma\eta}{\sigma} c_t^w + (1 - s_c)\, \eta g_t^H + \frac{\overline{\tau}^H}{\left(1 - \overline{\tau}^H\right)} \tau_t^H \right] + \beta E_t \pi_{t+1}^H \tag{19H}$$

$$\pi_t^F = k^F \left[-(1 + s_c \rho\eta)\, n_{\scriptscriptstyle H}\, q_t + \frac{1 + s_c \sigma\eta}{\sigma} c_t^w + (1 - s_c)\, \eta g_t^F + \frac{\overline{\tau}^F}{\left(1 - \overline{\tau}^F\right)} \tau_t^F \right] + \beta E_t \pi_{t+1}^F \tag{19F}$$

$$q_t = q_{t-1} + \pi_t^F - \pi_t^H - \left(\widetilde{T}_t - \widetilde{T}_{t-1} \right) \tag{20}$$

$$\widehat{b}_t^H = \frac{1}{\beta} \left\{ \widehat{b}_{t-1}^H - \pi_t + n_{\scriptscriptstyle F} (1 - \beta)\, q_t + \frac{\overline{Y}}{\overline{b}^H} \left[(1 - s_c)\, g_t^H - \overline{\tau}^H y_t^H - \overline{\tau}^H \tau_t^H \right] \right\} + i_t + \widehat{\varepsilon}_{b^H,t} \tag{21H}$$

$$\widehat{b}_t^F = \frac{1}{\beta} \left\{ \widehat{b}_{t-1}^F - \pi_t - n_{\scriptscriptstyle H} (1 - \beta)\, q_t + \frac{\overline{Y}}{\overline{b}^F} \left[(1 - s_c)\, g_t^F - \overline{\tau}^F y_t^F - \overline{\tau}^F \tau_t^F \right] \right\} + i_t + \widehat{\varepsilon}_{b^F,t} \tag{21F}$$

where

$$k^H \equiv \frac{\left(1 - \alpha^H\right)\left(1 - \alpha^H \beta\right)}{\alpha^H (1 + \theta\eta)}; \; k^F \equiv \frac{\left(1 - \alpha^F\right)\left(1 - \alpha^F \beta\right)}{\alpha^F (1 + \theta\eta)},$$

$\widehat{\varepsilon}_{b^H,t}$ and $\widehat{\varepsilon}_{b^F,t}$ are composite shocks defined as

$$\begin{aligned}
\widehat{\varepsilon}_{b^H,t} &= \widetilde{i}_t + \frac{1}{\beta} \left\{ n_F (1 - \beta)\, \widetilde{T}_t + \frac{\overline{Y}}{\overline{b}} \left[(1 - s_c)\, \widetilde{G}_t^H - \overline{\tau}^H \widetilde{Y}_t^H + \left(1 - \overline{\tau}^H\right) \widehat{\mu}_{w,t}^H \right] \right\} \\
\widehat{\varepsilon}_{b^F,t} &= \widetilde{i}_t + \frac{1}{\beta} \left\{ -n_H (1 - \beta)\, \widetilde{T}_t + \frac{\overline{Y}}{\overline{b}} \left[(1 - s_c)\, \widetilde{G}_t^F - \overline{\tau}^F \widetilde{Y}_t^F + \left(1 - \overline{\tau}^F\right) \widehat{\mu}_{w,t}^F \right] \right\}
\end{aligned}$$

and where lower case variables refer to variables in gaps. For a generic variable, X_t, its gap is defined as $x_t = \widehat{X}_t - \widetilde{X}_t$, where $\widehat{X}_t$ and $\widetilde{X}_t$ denote, respectively, their effective and efficient values, in log-deviations from the zero-inflation efficient steady state (see, section 2.3.3, below).[7] A "union-wide" variable, X^w, is defined as $X^w \equiv nX^H + (1 - n)\, X^F$.

Equation (17) refers to the IS equation, written in terms of the union consumption[8] and nominal interest-rate gaps. Equations (18H) and (18F) are country-specific aggregate demand equations, with s_c being the steady-state consumption share of output and q_t being the terms-of-trade gap ($\equiv \widehat{T}_t - \widetilde{T}_t$). These three equations constitute the aggregate demand-side block of the model and were derived from log-linearization of equations (7H), (7F), (8), (9) and (10).

The aggregate supply-side block of the model was obtained from the log-

[7] This definition does not apply for the inflation rates, as stable prices are optimal under sticky prices.

[8] The risk-sharing condition implies that $c_t^w = c_t^H = c_t^F$.

linear approximation of equations (13) and (14), as well as from their Foreign counterparts, around the efficient steady state equilibrium. Equations (19H) and (19F) are open-economy pure New-Keynesian aggregate supply (AS) curves. Positive gaps on the terms-of-trade, consumption and public spending have inflationary consequences at H: an increase in the demand for H-produced goods leads to more work effort, and, thus, raises marginal costs. Moreover, the positive gaps on the terms-of-trade and on the consumption exert an additional inflationary pressure as they reduce the marginal utility of nominal income for households. The efficient tax rate $\widetilde{\tau}_t^i$, used to compute the tax rate gap ($\tau_t^i = \widehat{\tau}_t^i - \widetilde{\tau}_t^i$) in country i, is defined as the tax rate required to fully offset the impact of an idiosyncratic "cost-push" (wage markup) shock.[9] Equation (20) is the terms-of-trade gap's identity, reflecting the inflation differential and the one-period change in the efficient level of the terms-of-trade ($\widetilde{T}_t - \widetilde{T}_{t-1}$).

The final equations, (21H) and (21F), are the government budget constraints relevant for the equilibrium allocation only in the second fiscal policy scenario. Shocks impinge on debt accumulation and create "fiscal stress" through their effects on the efficient equilibrium.[10]

In sum, in the first balanced-budget policy scenario, given the path for policy instruments and the initial value of $\widehat{T}_{t-1}$, the system including equations (17)-(20) provides solutions for the endogenous variables c_t^w, y_t^H, y_t^F, π_t^H, π_t^F and q_t. In the second policy scenario, where policymakers are constrained to ensure debt sustainability, equations (21H) and (21F) add to the previous system to describe the economic structure of the economy.

2.3.3 Policy Objectives - The Social Planner's Problem

The optimal allocation for the monetary union as a whole, in any given period t, can be described as the solution to the following social planner's problem, where the single policy authority is willing to maximize the discounted sum of the utility flows of the households belonging to the whole union (W):

[9] The steady-state tax rates are given by $\overline{\tau}^i = (1-\beta)\,\frac{\overline{b}^i}{\overline{Y}} + (1-s_c)$ and the efficient tax rates by $\widetilde{\tau}_t^i = -\frac{1-\overline{\tau}^i}{\overline{\tau}^i}\widehat{\mu}_{w,t}^i$, for $i = H, F$.

[10] The derivations of all these equations are available upon request.

$$\begin{aligned}
\max_{C_{H,t}^{H},\, C_{H,t}^{F},\, C_{F,t}^{H}, C_{F,t}^{F}, G_t^H, G_t^F} \quad W &= E_0\left\{\sum_{t=0}^{\infty}\beta^t[n_H w_t^H + n_F w_t^F]\right\}, && (22)\\
\text{with } w_t^H &= u\left(C_t^H, \overline{C}_t^H\right) + V\left(G_t^H\right) - \frac{1}{n_H}\int_0^{n_H} v\left(L_t^j\right) dj\\
\text{and } w_t^F &= u\left(C_t^F, \overline{C}_t^F\right) + V\left(G_t^F\right) - \frac{1}{n_F}\int_{n_H}^{1} v\left(L_t^j\right) dj\\
s.t. &\\
\text{(production functions)} \quad & Y_t^H = a_t^H L_t^H\\
& Y_t^F = a_t^F L_t^F\\
\text{(resource constraints)} \quad & n_H Y_t^H = n_H C_{H,t}^H + n_F C_{H,t}^F + n_H G_t^H\\
& n_F Y_t^F = n_H C_{F,t}^H + n_F C_{F,t}^F + n_H G_t^F\\
\text{(consumption indexes)} \quad & C_t^H \equiv \left[n_H^{\frac{1}{\rho}}\left(C_{H,t}^H\right)^{\frac{\rho-1}{\rho}} + n_F^{\frac{1}{\rho}}\left(C_{F,t}^H\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}\\
& C_t^F \equiv \left[n_H^{\frac{1}{\rho}}\left(C_{H,t}^F\right)^{\frac{\rho-1}{\rho}} + n_F^{\frac{1}{\rho}}\left(C_{F,t}^F\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}
\end{aligned}$$

The social planner will choose to produce equal quantities of the different goods in each country. Moreover, the aggregation over all agents (households, governments and central bank) cancels out the budget constraints and, thus, the social planner's solution is not constrained by them.

Maximization program in (22) yields the following optimallity conditions

$$u_c\left(C_t^H, \overline{C}_t^H\right) n_H^{\frac{1}{\rho}}\left(\frac{C_{H,t}^H}{C_t^H}\right)^{-\frac{1}{\rho}} = v_y\left(Y_t^H; a_t^H\right) \tag{23}$$

$$u_c\left(C_t^H, \overline{C}_t^H\right) n_F^{\frac{1}{\rho}}\left(\frac{C_{F,t}^H}{C_t^H}\right)^{-\frac{1}{\rho}} = v_y\left(Y_t^F; a_t^F\right) \tag{24}$$

$$u_c\left(C_t^F, \overline{C}_t^F\right) n_H^{\frac{1}{\rho}}\left(\frac{C_{H,t}^F}{C_t^F}\right)^{-\frac{1}{\rho}} = v_y\left(Y_t^H; a_t^H\right) \tag{25}$$

$$u_c\left(C_t^F, \overline{C}_t^F\right) n_F^{\frac{1}{\rho}}\left(\frac{C_{F,t}^F}{C_t^F}\right)^{-\frac{1}{\rho}} = v_y\left(Y_t^F; a_t^F\right) \tag{26}$$

$$V_G\left(G_t^H\right) = v_y\left(Y_t^H, a_t^H\right) \tag{27}$$

$$V_G\left(G_t^F\right) = v_y\left(Y_t^F, a_t^F\right) \tag{28}$$

Efficient equilibrium In a symmetric efficient steady state equilibrium, it follows that $\overline{Y}^H = \overline{Y}^F = \overline{Y}$; $\overline{C^H} = \overline{C^F} = \overline{C}$; $\overline{C_H^H} = \overline{C_H^F} = n_H\overline{C}$; $\overline{C_F^H} = \overline{C_F^F} =$

$n_F \overline{C}$ and $\overline{G^H} = \overline{G^F} = \overline{G}$.

The complete solution for the efficient equilibrium is given by the following expressions (29-32)

$$\widetilde{C}_t^w = \frac{1}{1+\eta\left[s_c\sigma + (1-s_c)\,\psi\right]}\left\{\left[1+(1-s_c)\,\psi\eta\right]\widehat{\overline{C}}_t^w + (1+\eta)\,\sigma\widehat{a}_t^w\right\} \tag{29}$$

$$\widetilde{C}_{H,t}^H - \widetilde{C}_{F,t}^H = \widetilde{C}_{H,t}^F - \widetilde{C}_{F,t}^F = -\frac{\rho\,(1+\eta)}{1+\eta\left[s_c\rho + (1-s_c)\,\psi\right]}\left(\widehat{a}_t^F - \widehat{a}_t^H\right) \tag{30}$$

$$\widetilde{G}_t^w = \frac{\psi}{1+\eta\left[s_c\sigma + (1-s_c)\,\psi\right]}\left[-\eta s_c\widehat{\overline{C}}_t^w + (1+\eta)\,\widehat{a}_t^w\right] \tag{31}$$

$$\widetilde{G}_t^F - \widetilde{G}_t^H = \frac{(1+\eta)\,\psi}{1+\eta\left[s_c\rho + (1-s_c)\,\psi\right]}\left(\widehat{a}_t^F - \widehat{a}_t^H\right) \tag{32}$$

To fully define the gap variables described in section above, we need to determine the efficient interest rate and terms-of-trade levels. The former follows directly from the Euler equation, while the latter results from the combination of equation (30) with the optimal intratemporal household's allocations

$$\widetilde{i}_t = \frac{1}{\sigma}E_t\left[\left(\widetilde{C}_{t+1}^w - \widetilde{C}_t^w\right) - \left(\widehat{\overline{C}}_{t+1}^w - \widehat{\overline{C}}_t^w\right)\right] \tag{33}$$

$$\widetilde{T}_t = -\frac{1+\eta}{1+\eta\left[s_c\rho + (1-s_c)\,\psi\right]}\left(\widehat{a}_t^F - \widehat{a}_t^H\right). \tag{34}$$

In the first fiscal policy scenario (lump-sum taxes warrant balanced budgets) this efficient allocation corresponds to the decentralized flexible-price equilibrium when monopolistic and tax distortions are removed through an employment subsidy and the implemented government spending rules agree with those derived under the social planner's optimization. However, in the second fiscal policy scenario, that union-wide optimal allocation may not be supported as a flexible-price equilibrium, since fiscal policy instruments may have to deviate from those rules to ensure fiscal solvency. Anyway, the policy problem will be formulated with variables in gaps defined in terms of the efficient outcomes and the two steady state equilibriums coincide.

Steady state equilibrium In order to avoid the traditional inflationary bias problem arising from an inefficiently low steady-state output level, we will assume the existence of an employment subsidy that removes average monopolistic and tax rate distortions. To compute this employment subsidy, observe that the profit-maximizing H-firms, in a flexible-price setup, choose the same price

$p_t(h) = P_{H,t}$ such that

$$u_c\left(C_t^H, \overline{C}_t^H\right) = \frac{\theta}{(\theta-1)\left(1-\tau_t^H\right)} \mu_{w,t}^H \left(1-\zeta^H\right) \left[n_{_H} + n_{_F} T_t^{1-\rho}\right]^{\frac{1}{1-\rho}} v_y\left(Y_t^H, a_t^H\right)$$

and, the F counterpart of this price-setting behaviour is given by

$$u_c\left(C_t^F, \overline{C}_t^F\right) = \frac{\theta}{(\theta-1)\left(1-\tau_t^F\right)} \mu_{w,t}^F \left(1-\zeta^F\right) \left[n_{_H} T_t^{\rho-1} + n_{_F}\right]^{\frac{1}{1-\rho}} v_y\left(Y_t^F; a_t^F\right)$$

To get symmetry in the steady-state levels of the output, consumption, government spending and prices in both countries, we need to impose that $\frac{\theta}{(\theta-1)\left(1-\overline{\tau}^H\right)}\overline{\mu}_w\left(1-\zeta^H\right) = \frac{\theta}{(\theta-1)\left(1-\overline{\tau}^F\right)}\overline{\mu}_w\left(1-\zeta^F\right) = \overline{\mu}$ where, as we have already remarked, the employment subsidy ζ^i is fully financed by lump sum taxes.

In steady state, we verify that

$$u_c\left(\overline{C}, \overline{C}\right) = \overline{\mu} v_y\left(\overline{Y}, \overline{a}\right)$$

and, if the employment subsidy ζ^i is set to match $\overline{\mu} = 1$, the efficient steady-state output level holds. Hence, the employment subsidy in country $i = H, F$ is assumed to take the value

$$\zeta^i = 1 - \frac{(\theta-1)\left(1-\overline{\tau}^i\right)}{\theta\overline{\mu}_w} \tag{35}$$

The steady-state nominal (and real) interest rate is $\overline{i} = \frac{1-\beta}{\beta}$.

2.3.4 Policy Objectives - The Social Loss Function

Benevolent authorities, under full cooperation, seek to maximize welfare for the monetary union as a whole, W, given, now, the set of equations describing the effective economic structure dynamics: (17)-(20), in the first policy scenario; and (17)-(21F), in the second policy scenario. This environment enables the derivation of union-wide optimal stabilization policies, but serves also as a benchmark to assess alternative policy regimes.

Following Woodford (2003), we compute the second-order approximation of W around a deterministic steady state. Ignoring the terms independent of policy and terms of three or higher order, the welfare objective takes the form:

$$W \simeq -\Omega E_0 \left\{\sum_{t=0}^{\infty} \beta^t L_t\right\}, \tag{36}$$

where the per-period social loss function (L_t), similar to the one derived by Beetsma and Jensen (2004, 2005), is defined as[11]

[11] The derivation of the social loss function is available upon request.

$$
\begin{aligned}
L_t &= \Lambda_c \left(c_t^w\right)^2 + \Lambda_g \left[n_{\scriptscriptstyle H} \left(g_t^H\right)^2 + n_{\scriptscriptstyle F} \left(g_t^F\right)^2\right] + \Lambda_{gc} c_t^w \left(n_{\scriptscriptstyle H} g_t^H + n_{\scriptscriptstyle F} g_t^F\right) \\
&\quad + \Lambda_T q_t^2 - \Lambda_{gT} \left(g_t^F - g_t^H\right) q_t + n_{\scriptscriptstyle H} \Lambda_\pi^H \left(\pi_t^H\right)^2 + n_{\scriptscriptstyle F} \Lambda_\pi^F \left(\pi_t^F\right)^2 \qquad (37)
\end{aligned}
$$

and

$$
\begin{aligned}
\Lambda_c &\equiv s_c \left(\frac{1}{\sigma} + s_c \eta\right), \;\; \Lambda_g \equiv (1 - s_c) \left(\frac{1}{\psi} + (1 - s_c)\eta\right), \;\; \Lambda_{gc} \equiv 2 s_c (1 - s_c) \eta, \\
\Lambda_T &\equiv n_{\scriptscriptstyle H} n_{\scriptscriptstyle F} s_c \rho (1 + s_c \rho \eta), \;\; \Lambda_{gT} \equiv 2 n_{\scriptscriptstyle H} n_{\scriptscriptstyle F} s_c (1 - s_c) \rho \eta, \\
\Lambda_\pi^H &\equiv \frac{\theta (1 + \theta\eta) \alpha^H}{(1 - \alpha^H \beta)(1 - \alpha^H)}, \;\; \Lambda_\pi^F \equiv \frac{\theta (1 + \theta\eta) \alpha^F}{(1 - \alpha^F \beta)(1 - \alpha^F)}
\end{aligned}
$$

Fluctuations in the consumption and the public spending gaps imply welfare losses in line with the respective households' risk aversions ($1/\sigma$ and $1/\psi$) and with the elasticity of disutility with respect to work effort (η). Inflation at H is more costly the higher the degree of nominal rigidity $\left(\alpha^H\right)$, the higher the elasticity of substitution between H-produced goods (θ) and the higher η. The welfare cost of inflation $\left(\Lambda_\pi^H\right)$ vanishes when prices are fully flexible $\left(\alpha^H = 0\right)$.

At the monetary union level, misallocation of goods also applies for deviations of the terms-of-trade from the respective efficient level. The costs of this distortion (Λ_T) increase with the elasticity of substitution between Home and Foreign produced goods (ρ), with the steady-state consumption share on output (s_c), with η and decrease with country-size asymmetry. Following an asymmetric technology shock, efficiency requires prices to change as to shift the adjustment burden "equally" across the two countries (Benigno and López-Salido, 2006). This creates a trade-off between the stabilization of relative prices to the correspondent efficient levels and the stabilization of inflation in both countries and it provides a rationale for the stabilization role of fiscal policy.

The cross-term between the consumption gap and the weighted average government spending gap occurs because positive co-movements between these two variables cause undesirable fluctuations in the work effort for the monetary union as a whole, in addition to the effort fluctuations caused by each of these variables *per se*. There is also a negative cross-term between the terms of trade gap and the relative spending gap that is increasing (in absolute value) with η and ρ, while decreasing with country-size asymmetry. This negative co-movement arises because a positive terms-of-trade gap rises H-competitiveness which, combined with a negative relative public spending gap (higher public spending at H than at F), shifts demand towards H-produced goods. As a consequence, work effort shifts from F- towards H-households (cf. Beetsma and Jensen 2004 and 2005, for these arguments).

2.3.5 Other policy objectives

We also consider that policymakers may have divergent policy objectives. This is a valid assumption since it is reasonable to conjecture that national (fiscal) authorities are mainly concerned with their own citizens and so, their objective functions should only comprise the utility of the respective constituencies. Pragmatically, we approximate the national welfare criteria through welfare losses obtained from splitting the union-wide loss function.[12]

We will also consider the case of the delegation of monetary policy to a weight-conservative central bank by distorting the weights on the inflation and the output terms of the social loss function. This is usually seen as a potential solution to reduce the time-inconsistency problems of policy stabilization, which can be aggravated by specific incentives of the fiscal authorities.

The table below summarizes the policy environments we will analyze.

Benevolent Cooperative Policymakers
$L_t^{H,F} = L_t^M = L_t$
Benevolent non-Cooperative Policymakers
$L_t^H = \Lambda_c \left(c_t^H\right)^2 + \Lambda_g \left(g_t^H\right)^2 + \Lambda_{gc} c_t^w g_t^H + \Lambda_T q_t^2 + \frac{1}{n_H} \Lambda_{gT} g_t^H q_t + \Lambda_\pi^H \left(\pi_t^H\right)^2$ $L_t^F = \Lambda_c \left(c_t^F\right)^2 + \Lambda_g \left(g_t^F\right)^2 + \Lambda_{gc} c_t^w g_t^F + \Lambda_T q_t^2 - \frac{1}{n_F} \Lambda_{gT} g_t^F q_t + \Lambda_\pi^F \left(\pi_t^F\right)^2$ $L_t^M = L_t$
Conservative Central Bank
L_t^H; L_t^F $L_t^M = (1-\rho^c) \left\{ \Lambda_c \left(c_t^w\right)^2 + \Lambda_g \left[n_H \left(g_t^H\right)^2 + n_F \left(g_t^F\right)^2 \right] + \Lambda_{gc} c_t^w \left(n_H g_t^H + n_F g_t^F \right) \right.$ $\left. + \Lambda_T q_t^2 - \Lambda_{gT} \left(g_t^F \text{-} g_t^H \right) q_t \right\} + \rho^c \left\{ n_H \Lambda_\pi^H \left(\pi_t^H\right)^2 + n_F \Lambda_\pi^F \left(\pi_t^F\right)^2 \right\}$

2.4 Policy Games

We assume that fiscal and monetary authorities set their policy instruments in order to minimize the respective loss functions, given the dynamic structure of the economies, and that they can engage in various policy games. We will consider, as a benchmark case for policy analysis, that policymakers are benevolent and cooperate under discretion. To assess the importance of time-consistency, we also compute the optimal policy solution under commitment. These two optimizing problems will be solved by using the algorithms in Söderlind (1999).

We also consider discretionary non-cooperative policy games and, depending on the time of events, we obtain Nash or leadership equilibria. In these different setups, the timing of the events is as following: 1) the private sector forms expectations; 2) the shocks are realized; 3a) the central bank sets the interest rate; 3b) the fiscal authorities choose simultaneously the right amount of fiscal policy instruments. There is a Nash equilibrium, if 3a) and 3b) occur simultane-

[12] Forlati (2009) provides fully micro-founded welfare criteria for the case of non-coordinated fiscal and monetary policies in a monetary union.

ously; there is monetary leadership if 3a) occurs before 3b); and, if the order of these occurrences is reversed, there is a fiscal leadership. To solve for these dynamic policy games we use the methodology developed by Blake and Kirsanova (2009) and Kirsanova *et al.* (2005). The derivation of a numerical algorithm for the solution of the non-cooperative monetary leadership discretionary game is deferred to a separate appendix, available upon request.

2.5 Calibration

Our baseline calibration was chosen taking as reference Beetsma and Jensen (2004, 2005), Benigno (2004), Benigno and López-Salido (2006) and Ferrero (2009).

The discount factor β is 0.99, which implies a 4% annual basis steady-state interest rate. The parameter θ, the elasticity of substitution between goods produced in the same country, is equal to 11, implying a price mark-up of 10%. In turn, the elasticity of substitution between H and the F produced goods, ρ, is set at 4.5. We assume $\sigma = \psi = 0.4$, which implies a coefficient of risk aversion for private and public consumption equal to 2.5. The steady-state share of public consumption in output $(1 - s_c)$ is set at 0.25.We parameterize $\eta = 0.47$, implying a labour supply elasticity of $1/0.47$.

Our benchmark calibration aims to reflect a perfectly symmetric setup from which we can diverge and assess how country-size asymmetry affects the results. Hence, we begin by assuming that the two economies in the monetary union are of equal size ($n_i = 0.5$) and have identical degrees of nominal rigidities $\left(\alpha^H = \alpha^F\right)$. We select a value for α equal to 0.75, in order to get an average length of price contracts equal to one year. While allowing the relative dimension of country H to vary from $n_{_H} = 0.5$ to $n_{_H} = 0.9$, country-size asymmetry is illustrated for $n_{_H} = 0.8$.

To reach a high-debt policy scenario and match the numerical constraint of the Maastricht Treaty, the yearly steady-state debt-to-output ratio $(\overline{b}/4\overline{Y})$ is calibrated to 60%. Finally, we assume that the consumption and the technology shocks follow an uncorrelated AR(1) process with common persistence of 0.85, while the wage mark-up shocks are i.i.d., and the standard deviation of the innovations are equal to 0.01.

3 Policy Analysis

In what follows, we will broadly assume that policymakers engage in optimizing discretionary policy games. We attempt to draw welfare implications arising from different policy regimes, under the two fiscal policy scenarios - with and without debt constraints.

3.1 Discretionary policy outcomes under cooperation

Strategic interactions between policymakers are absent when they minimize the social loss function. However, if policymakers are unable to commit relative to the private sector, strategic interactions between the former and the latter can lead to meaningful discrepancies between discretionary and commitment cooperative policy outcomes.

Balanced-budget scenario In the balanced-budget scenario, the traditional stabilization bias does not occur under discretion because, in every period, distortionary tax rates can freely adjust to optimally control for national inflation rates. Monetary policy does not face stabilization trade-offs; and an active fiscal policy is only required to stabilize asymmetric technology shocks.[13] Furthermore, given that changes in the relative prices have more effect on the marginal costs and inflation rates of the smaller (and more open) economies, small countries have to engage in more active fiscal policies than the larger ones and, even so, they achieve a worse stabilization performance. Figure 1 details the responses of key endogenous variables to a 1% negative technology shock hitting the large country.[14] It is apparent that this shock, with a direct positive effect on the terms-of-trade gap and inefficiently shifting demand from the small (F) to the large (H) country, requires a larger adjustment (increase) of the fiscal policy instruments in the small country. Notwithstanding, this is not enough to prevent higher inflation variability relative to the large country.

[13] Since variables are defined in gaps, an active policy means that policy instruments deviate from their efficient values. Tables 1A and 1B show that, with the exception of asymmetric technology shocks, the feedback coefficients of the fiscal and monetary policy rules on shocks are zero. They also show that only fiscal policy instruments react to asymmetric technology shocks.

[14] This is equivalent to a positive technology shock in the small country, in this policy scenario.

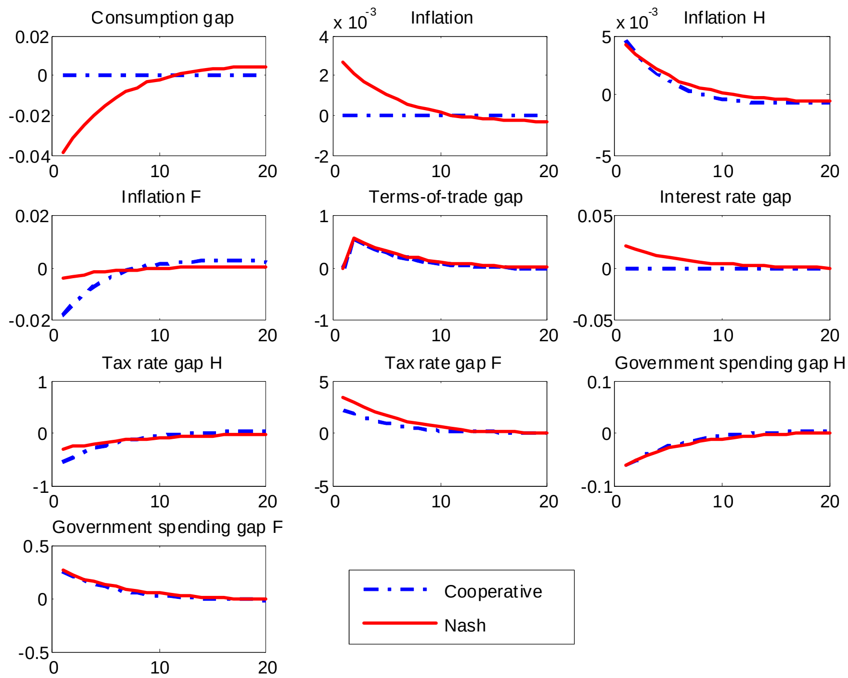

Figure 1: Responses to a 1% negative technology shock at a Large Country (H)

Debt scenario In turn, in the debt scenario, because policy instruments need to ensure government solvency conditions, policy reactions to shocks face a trade-off between short-run stabilization and permanent effects on the welfare-relevant variables. Consequently, the discretionary outcome exhibits a stabilization bias and the solutions under discretion and commitment diverge. In this scenario, symmetric shocks and idiosyncratic cost-push shocks produce welfare costs, because budgetary consequences prevent policy instruments from being set at their efficient levels. Likewise, policy response to country-specific technology shocks now has to balance terms-of-trade distortions against inefficiencies arising from the need to ensure the government inter-temporal solvency conditions.

Indeed, policy stabilization of current effects inflicts budgetary consequences and requires fiscal policy instruments to be permanently adjusted to sustain the new debt stocks. This leads to permanent effects on real welfare-relevant variables (consumption and government spending gaps), which can be lessened only at the expenses of higher short-run volatility (stabilization trade-off). Given the discounting structure embedded in welfare, the optimal policy solution (commitment) requires that permanent effects remain, in order to accomplish better short-run stabilization.[15] Nevertheless, in the first period, given that private sector expectations have already been formed, it is optimal to implement a policy-mix that generates higher inflation volatility, but reduces debt consequences and, thus, also allows smaller consumption and government spending gaps, thereafter. This policy is time-inconsistent because, at any later stage, policymakers would face the same incentive as that of the first period. Time-inconsistency vanishes only when permanent effects are fully eliminated and all variables return to their pre-shock levels (discretion).

Furthermore, since the level of government indebtedness affects the relative effectiveness of the fiscal and monetary policy instruments on debt stabilization, the elimination of permanent effects is achieved diversely when public debts are small or large, as Leith and Wren-Lewis (2007a) and Stehn and Vines (2008a) remarked. The larger the steady-state debt-to-output ratios are, the larger is the impact of monetary policy in the debt-service costs and, thus, the higher is the incentive to shift monetary policy conduct towards debt stabilization; conversely, fiscal policy instruments – particularly, the tax rate gaps – become less effective in controlling debt while they become relatively more apt to offset the inflationary consequences. For the considered (large) debts, Figure 2 shows that, in face of a symmetric shock simultaneously raising debt and inflation optimal discretionary policy requires a first-period cut in the interest rate gap.[16] This policy response is complemented, initially, with a decrease of the government spending gaps while, depending on the debt-to-output values, tax rate gaps may increase, to help debt stabilization, or may decrease, to offset inflationary consequences. The resulting debt decline induces a subsequent and anticipated deflationary policy-mix that also assists the control of inflation in the first period.

[15] This result is reminiscent of the tax smoothing result of the optimal taxation literature (Barro, 1979 and Lucas and Stokey, 1983).

Schmitt-Grohé and Uribe (2004) or Leith and Wren-Lewis (2007a), on closed economy models, and Ferrero (2009) or Leith and Wren-Lewis (2007b), on open economy models, show that the optimal policy response to shocks requires permanent variations in the public debt.

[16] Under our calibration, the interest rate gap rises only for steady-state debt-to-output ratios lower than 20%.

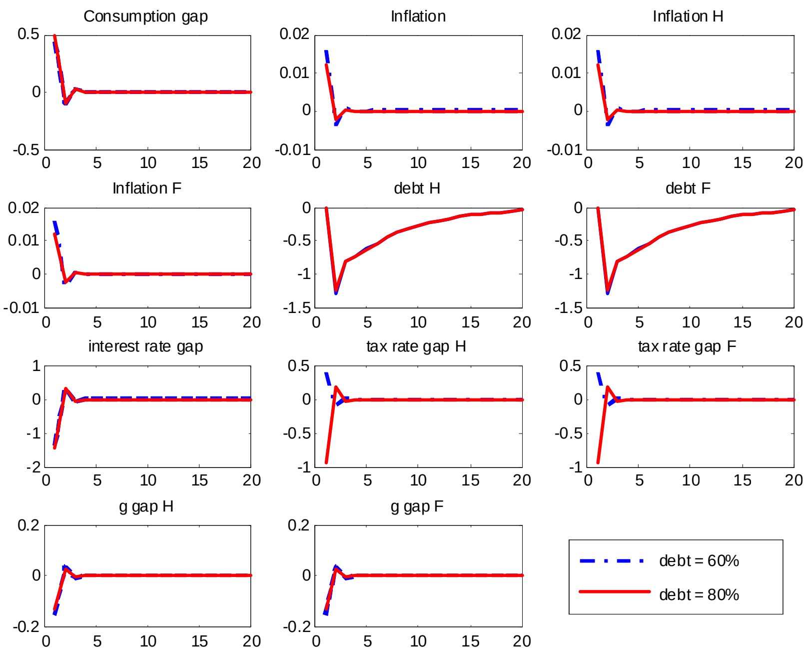

Figure 2: Responses to a 1% negative symmetric technology shock, Cooperative

Thus, in the debt scenario, a time-consistency problem emerges, materializing in a debt stabilization bias, under discretion. This debt stabilization bias is responsible for the meaningful divergence between debt and balanced-budget policies.[17] The example of a negative technology shock at H is instructive to better assess this discrepancy. The first-period policy response now requires an increase in the tax rate gap at H and a fall in the interest rate gap and in the tax rate gap at F that magnifies the effects on the inflation rates and on the consumption gap. In a country-size symmetric monetary union, the monetary policy response increases inflation variability at H while reducing it at F. In general, the country that suffers a domestic idiosyncratic technology shock experiences a worse stabilization performance than the other country, in contrast with the balanced-budget policy scenario, where domestic and foreign shocks deliver equal stabilization costs (see Figure 3).

[17] Under commitment, the solutions for the stabilization problem diverge only slightly between the two policy scenarios.

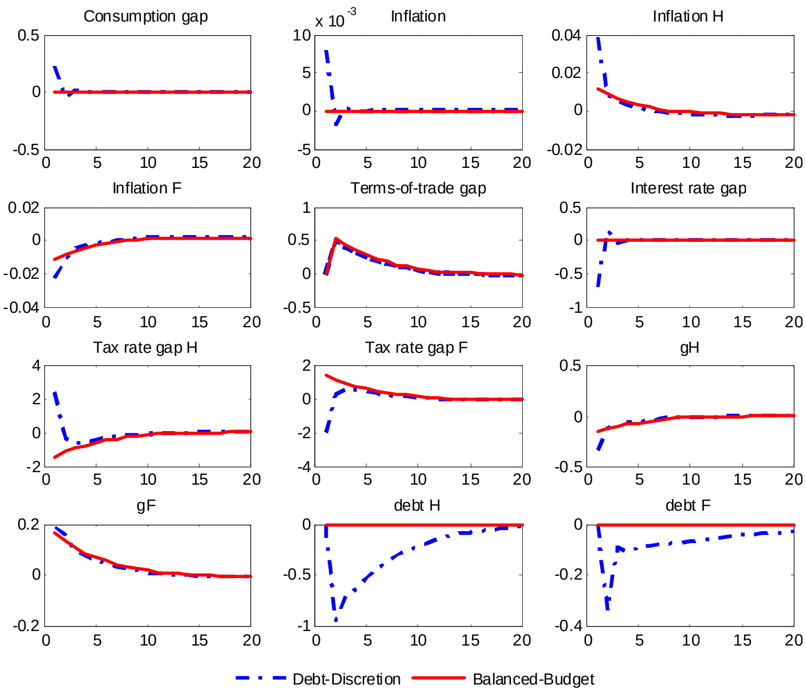

Figure 3: Responses to a 1% negative technology shock at H ($n_H = 0.5$)

Moreover, as a smaller country can benefit less from monetary policy debt accommodation than a larger one, it has to perform a relatively more active fiscal policy towards debt stabilization, with negative welfare consequences. Hence, the presence of the debt stabilization bias further aggravates the stabilization performance of a small country relative to that of a large country.

The computations of the social loss under the two policy scenarios (Tables 4A-4B) confirm that: i) welfare costs are larger for both countries under the debt-constrained scenario; ii) these costs are smaller the higher the degree of country-size asymmetry is;[18] iii) and, the distribution of welfare costs across countries is even more unfavourable to the small country under the debt scenario.

3.2 Discretionary policy outcomes under non-cooperative regimes

Non-cooperation allows for strategic interactions between policymakers. Different policy objectives, the order of playing (Nash, monetary leadership or fiscal

[18] In fact, larger country-size asymmetry implies a more symmetric structure of shocks at the union level.

leadership) and the relative size of each country crucially shape such interactions.

Relative to cooperation, fiscal authorities now face the following incentives (I): (I_1) they use more (less) actively fiscal policy instruments that cause negative (positive) cross-border effects; (I_2) fiscal policy is more (less) active when it causes a negative (positive) externality on the aggregate variables to which monetary policy reacts; and, (I_3) a larger country, causing larger externalities, moderates its fiscal policy while the smaller one faces the reverse incentive.

Balanced-budget scenario As in the cooperative arrangement, the asymmetric technology shock is the only one causing policy trade-offs. In face of such a shock, the tax rate and the government spending responses alleviate the impact on the domestic inflation rates but accentuate the effect of the shock on the terms-of-trade gap. The latter produces a negative effect in the other country which, by not being fully internalized, implies a more active use of fiscal policy instruments (I_1).

With equal-size countries, the non-internalization of these cross-border effects does not generate a free-riding problem between national fiscal authorities and the central bank: the effects of their (symmetric) actions on union-wide variables cancel out. When it leads, the central bank anticipates this outcome and, thus, the monetary leadership and the Nash solutions coincide. On the other hand, under fiscal leadership, each fiscal authority perceives that the central bank, internalizing the negative fiscal policy externalities, will react to an excessive policy response. As a consequence, both governments moderate their fiscal policy responses, moving closer to the cooperative outcome (cf. the fiscal policy feedback coefficients on a^H in Table 1A). Therefore, among the non-cooperative regimes, the fiscal leadership delivers the lowest welfare stabilization costs (Table 4A, $n_H = 0.5$).

Country-size asymmetry is the only reason for national fiscal authorities to experience differentiated incentives: as a larger country has more impact on the union-wide variables, to which central bank reacts, fiscal authorities of large countries moderate their policies while those of small countries face the reverse incentive (I_3). This asymmetric conduct impinges on union-wide variables and forces monetary policy to complement the large (H) country's fiscal policy response to idiosyncratic technology shocks (Table 1B). Even so, the small country (by making use of a socially costless policy instrument - the tax rate) achieves a better stabilization of its inflation rate, under non-cooperation (see Figure 1).

Columns 1 and 2 of Table 1B show that, relative to Nash, fiscal leadership further exacerbates the activism of the small country, particularly with respect to the use of the tax rate, while it further restrains that of the large country. As a result, monetary policy has to become relatively more active. On the other hand, under monetary leadership, the central bank, perceiving the opposite incentives for each country and how they impact on aggregate variables, lessens its response to shocks to force a more (less) active fiscal policy by the large (small) country.

Table 4A shows that policy cooperation always dominates non-cooperation for the monetary union, but it is the worst outcome for the small country. The table also shows that, among the non-cooperative policy regimes and for all countries, fiscal leadership delivers the best stabilization performance, when country-size asymmetry is not excessive; in turn, the monetary leadership outcome is superior only for a sufficiently high degree of country-size asymmetry ($n_H \geqslant 0.85$).

Debt scenario In this scenario, the need to ensure fiscal solvency amplifies the sources of strategic interactions between policies. Now, even with equal-size countries, fiscal policies always impinge on aggregate variables to which monetary policy reacts. Consider a negative technology shock at H which, because it leads to stronger policy trade-offs and exhibits more persistence, is key to welfare results. Compared with cooperation, the first-period reaction to such a shock implies a smaller variation of the tax rate gap and a larger response of the government spending gap in both countries, because of their opposite cross-border effects (I_1). Furthermore, since the domestic (foreign) fiscal policy reaction causes a positive (negative) externality on the union-wide debt (I_2), non-cooperation leads to a relatively less (more) active policy (towards debt adjustment) at H (F). Overall, aggregate fiscal policy ends up looser than in cooperation and, thus, the central bank is forced into a more expansionary monetary policy in order to ensure aggregate debt adjustment.[19] Hence, the inflationary stance of monetary policy, in a high-debt monetary union, aggravates under non-cooperation. Relative to Nash, fiscal leadership magnifies this problem while monetary leadership mitigates it. Under fiscal leadership, being aware of the monetary policy reaction against debt misalignments, fiscal authorities become less disciplined. In turn, a central bank with a first-mover advantage, anticipating this free-riding behaviour, restricts monetary policy and compels national governments to act closer to the cooperative outcome.[20] Consequently, because fiscal policy cross-border effects are not internalized and because monetary policy's time-consistency problem is amplified, non-cooperative regimes inflict larger welfare stabilization costs. However, and unlike the balanced-budget scenario, fiscal leadership delivers the worst welfare outcome, as the benefit from fiscal policy moderation (less fiscal discipline) is overturned by the time-consistency requirement to adjust aggregate debt to its pre-shock level. In turn, monetary leadership pushes non-cooperative towards the cooperative outcome, yielding lower welfare costs (see Table 4B, $n_H = 0.5$).

Considering now country-size asymmetry, the incentives faced by each government depend not only on the type but also on the size of the externalities

[19] This can be checked by computing, across policy regimes, the aggregate government spending and tax rate responses to an idiosyncratic negative technology shock at H, using the feedback coefficients on Table 2. Similar conclusions apply to the case of a country-specific positive cost-push shock.

[20] This manifests, relative to Nash, in a more active fiscal policy at H and less active fiscal policy at F (cf. feedback coefficients on a^H in Table 2, across policy regimes).

caused by fiscal policy. As in the balanced-budget scenario, small countries, causing small externalities, have incentives to engage in more active fiscal policies than under cooperation (I_3). However, this additional activism moves towards debt-stabilization, with negative consequences for macroeconomic stabilization. Large countries, expecting domestic debt-accommodation from the common monetary policy, face the reverse incentives. As a consequence, they undertake less active fiscal policies (towards debt management) under non-cooperation, achieving a better stabilization performance.

Since it aggravates the debt stabilization bias of the fiscal policy of the small country while mitigating that of the large country, non-cooperation makes the stabilization burden across the union countries more asymmetric. To reduce such asymmetry, the central bank accommodates the budgetary consequences of the small country relatively more than it would do under cooperation, while taking the converse attitude relative to the large country (cf. the monetary feedback coefficients on debts and shocks at Table 3, cooperation vs. non-cooperation). In effect, non-cooperation alleviates the time-consistency problems associated with the stabilization of a shock hitting the large country while it aggravates those of a shock hitting the small country, as is apparent from examination of Figures 4 and 5. In fact, Figure 4 shows that, relative to cooperation, the Nash monetary policy response to a negative technology shock hitting a large country is less debt-accommodative.[21] As a consequence, the union-wide and the H (large country) inflation rates exhibit lower volatility. Conversely, the monetary policy response to such a shock in the small country (Figure 5) is looser under Nash, causing higher volatility on the union-wide and F (small country) inflation rates. The net effect for the welfare of the large (H) country is positive because, in both cases, this country achieves better stabilization of its inflation rate, under Nash.

[21] This occurs because although this shock has a positive effect on aggregate debt it has a negative impact on the small country's debt, which determines a small first-period reduction on the interest rate gap, under non-cooperation.

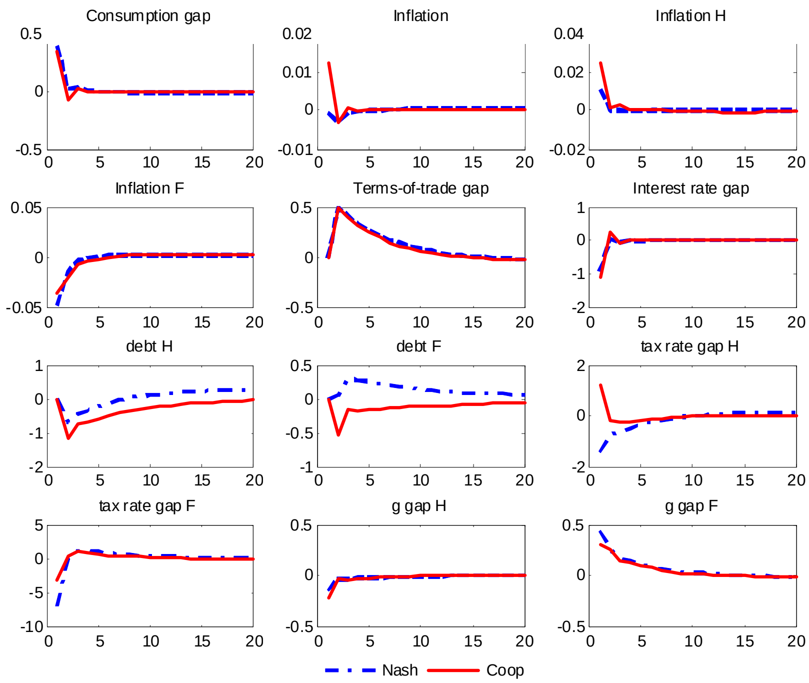

Figure 4: Responses to a 1% negative technology shock at H

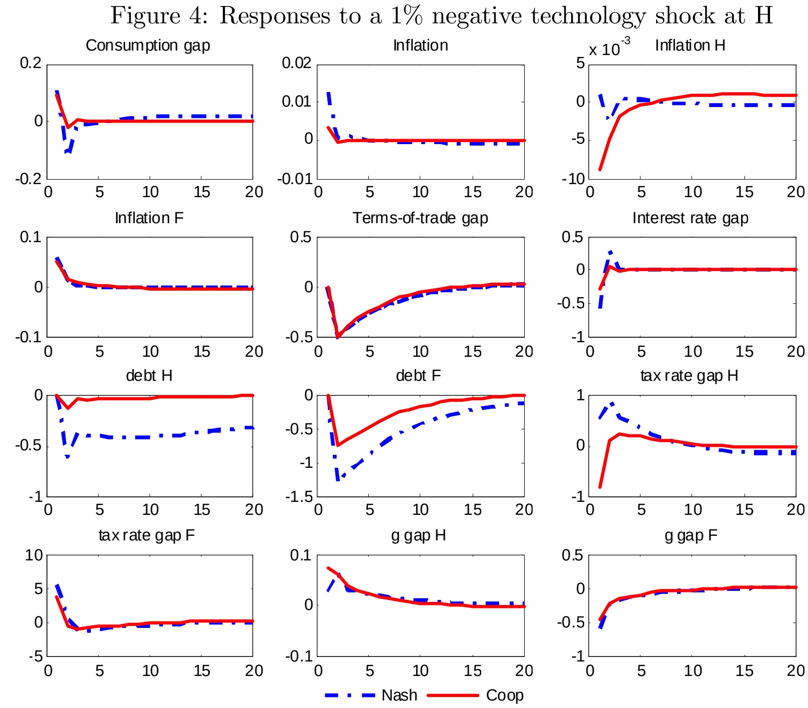

Figure 5: Responses to a 1% negative technology shock at F

Fiscal leadership further moderates the fiscal policy response of the large country to debt consequences, since monetary policy is expected to adjust domestic debt. In turn, monetary leadership enhances fiscal discipline for the large country, as the central bank, anticipating the incentives, accommodates its budgetary consequences less.[22]

The welfare losses reported in Table 4B show that, in general, both the union and the small country lose with non-cooperation when country-size asymmetry is not too high ($n_H < 0.85$). Fiscal leadership delivers the worst stabilization performance for the union and the small country, while it is the most favoured regime for the large country, which benefits from a larger strategic position vis-à-vis the central bank. Under non-cooperation, there are obvious social welfare stabilization gains from having a benevolent central bank as a first mover.

3.3 The case for a conservative central bank

Either because of the opposition of the larger country in the debt-constrained framework, or because it may be politically unappealing, the cooperative solution may be unfeasible; furthermore, time-consistency problems cause expressive welfare stabilization costs, under the debt scenario. In this context, an analysis of wether alternative institutional devices could improve on the non-cooperative discretionary outcomes for the whole union is, thus, in order.

A typical institutional solution is to delegate monetary policy to a conservative central bank. According to the literature, and in the context of pure monetary policy models, a conservative central bank unambiguously delivers welfare gains (see, among others, Rogoff, 1985, and Clarida *et al.*, 1999). However, in the context of models combining monetary and fiscal policies, the presence of a conservative central bank may not be strictly welfare-enhancing (see, for instance, Dixit and Lambertini, 2003, Adam and Billi, 2006, Blake and Kirsanova, 2009, and Stehn and Vines, 2008b).

In the balanced-budget scenario, where a cooperative solution under commitment coincides with that under discretion, a weight-conservative central bank may only correct distortions arising from the lack of policy cooperation. However, such welfare gains proved to be null under monetary leadership while, under fiscal leadership, monetary conservatism turns out to be welfare-improving only if the degree of country-size asymmetry is not too high[23] ($n_H < 0.7$, Table 4A).

In the debt scenario, delegating monetary policy to a conservative central bank gains an additional rationale: it can reduce distortions generated by the lack of commitment of fiscal and monetary policies. Intuitively, an inflation-averse central bank is more effective in controlling for inflation expectations

[22] It is clear from Table 3 that, relative to Nash and in response to a negative technology shock at H, the government spending gap falls less (more) and the tax rate gap decreases by more (less) at H in fiscal leadership (monetary leadership). Hence, in fiscal leadership (monetary leadership) the fiscal policy of the large country is globally more loose (tight).

[23] The conservative central bank moderates the large country's fiscal policy reaction to shocks, but exacerbates that of the small country.

and, thus, it may improve the short-run trade-off between inflation and output stabilization. However, central bank conservatism can have a perverse effect as it may strengthen the incentives to reduce the permanent effects on debt, because inflation costs diminish in the first period. A more aggressive monetary policy response to debt displacement, further cutting debt below its pre-shock level in the first period, allows a more effective deflationary policy in subsequent periods and, thus, enables better control of inflation in the first period. Therefore, delegating monetary policy to a weight-conservative central bank in a high-debt monetary union may aggravate the time-consistency problem of monetary policy, as it becomes more reactive to debt misalignments; moreover, it may exacerbate the strategic interactions between fiscal and monetary authorities, due to conflicting objectives.

In fact, our experiments confirm that, in an equal-size country monetary union, a conservative central bank has tighter control over debt while overall fiscal policy indiscipline increases (cf. the monetary policy feedback coefficients on shocks under a benevolent and a conservative central bank in Table 2). Each country experiences better stabilization of domestic idiosyncratic shocks, because the stabilization of its own inflation benefits from the central bank's conservative reputation and from the lessening of domestic fiscal discipline; on the other hand, external shocks cause higher welfare stabilization costs, under a conservative central bank. This is welfare-decreasing under monetary leadership, where monetary policy overreaction to debt displacement is further exacerbated. In turn, to counteract the monetary authority's excessive concern with inflation volatility, leading fiscal authorities restrict their free-riding behaviours; therefore, fiscal leadership reduces the union-wide fiscal indiscipline and moderates the budgetary accommodation stance of the monetary policy.[24] Comparing welfare losses under a benevolent and a conservative central bank, Table 4B shows that the latter is welfare-enhancing only under fiscal leadership: a conservative central bank contributes meaningfully to reduce distortions generated by home-biased fiscal policy objectives while not excessively aggravating the monetary policy debt-stabilization bias. [25]

In general, these results also apply to the case of country-size asymmetry: delegating monetary policy to a conservative central bank improves the welfare of the union only under fiscal leadership. However, in this case, the incentives each fiscal authority faces do not parallel and, therefore, the welfare implications do not spread proportionally across countries. For instance, with our calibra-

[24] Looking at the policy feedback coefficients on shocks (Table 2), it is easy to check that, with a conservative central bank, monetary policy is relatively more debt-accommodative under monetary leadership than under fiscal leadership while the reverse occurs with a benevolent monetary authority. The costly game between a leading conservative central bank and national fiscal authorities perversely generates higher inflation variability than in the corresponding benevolent policy scenario.

[25] We have also computed the welfare losses when fiscal authorities cooperate against a conservative central bank and we found that the losses are higher than with a benevolent central bank. We can infer that the gain from a conservative central bank under fiscal leadership with non-cooperative fiscal authorities follows exclusively from the attenuation of the fiscal policy free-riding problem.

tion, a conservative central bank may produce welfare gains for the union as a whole, as well as for the small country, at the expense of a worse stabilization performance for the larger one (Table 4B).

4 Concluding Remarks

This paper has explored the interactions between monetary and fiscal stabilization policies in a micro-founded macroeconomic dynamic model for a monetary union with country-size asymmetry, under two opposite policy scenarios: a balanced-budget and a high-debt scenario. The former, magnifies the fiscal policy stabilization role and minimizes the sources of strategic interactions; the latter, substantially lessens the fiscal policy stabilization effort and, because of the enhanced effectiveness of monetary policy to control public debt, amplifies the risk of harmful policy-mixes.[26]

We found that a small country performs a more active fiscal policy than a large one and that non-cooperation accentuates this activism while moderating that of a large country. This discloses higher stabilization costs for the union as a whole, while producing opposite welfare consequences for the small and the large country. In a debt-unconstrained policy scenario, small countries benefit from their extra fiscal policy activism, under non-cooperation, but cooperation dominates for the larger ones; thus, the best outcome for the union (cooperation) would be more likely to emerge. Conversely, in a high debt-constrained scenario welfare decreases for the small countries and improves for the larger ones, under non-cooperation, as a more active fiscal policy means a more active policy towards debt stabilization (debt stabilization bias); thus, the best outcome for the union (cooperation) would hardly emerge. Indebted large countries may strongly oppose to a cooperative arrangement in favour to fiscal leadership where they can explore a larger strategic power vis-à-vis a debt-accommodative central bank. In this case, delegation of monetary policy to a more conservative central bank could be a fruitful device to improve the welfare of the union as a whole.

In future research we intend to derive the benevolent non-cooperative country-specific loss functions and, additionally, include micro-founded political economy motivations to mimic the actual behaviour of fiscal policy authorities. Another possible extension stems from the need to represent more realistically a monetary union composed of many small countries and few large ones. Our two-country model is a good starting point in accounting for country-size asymmetry, but it can be improved by describing part of the union as a continuum of small open economies, as Galí and Monacelli (2008) do for a monetary union as a whole. In the EMU, the majority of the country-members are small compared with the union as a whole, and so, taken in isolation, their policy decisions have negligible aggregate impacts.

[26] In our model, this second effect is possibly overestimated, since all government debt has a one-period maturity; this lends monetary policy high leverage over debt service.

Appendix: Monetary leadership and Nash between the fiscal authorities

This appendix summarizes the iterative dynamic programming algorithm for the discretionary monetary leadership case when fiscal authorities play a Nash between them. This is an extension of the algorithms developed by Oudiz and Sachs (1985) and Backus and Driffill (1986) and popularized by Söderlind (1999). It closely follows the one developed by Kirsanova *et al.* (2005).

There are five strategic agents in the game: three explicit players - the monetary and the two fiscal authorities - and two implicit players - the private sector of both countries - that always act in last. In this type of game, the monetary authority moves first and sets the interest rate. Then the two fiscal authorities decide the levels of their fiscal policy instruments. Finally, the private sector in both countries reacts being the ultimate follower.

To solve this type of game, one inverts the order of playing and begins by solving the optimization of the last player, ending up with the optimization of the leader (the first player). The private sector's optimization problem is already solved out - the system of equations in section 2 - and can be represented by the following system, written in a state space form:

$$\widetilde{A}_0 \begin{bmatrix} I_{n_1} & \mathbf{O}_{n_1 x n_2} \\ \mathbf{O}_{n_2 x n_1} & H_{n_2 x n_2} \end{bmatrix} \begin{bmatrix} Y_{t+1} \\ E_t X_{t+1} \end{bmatrix} = \widetilde{A} \begin{bmatrix} Y_t \\ X_t \end{bmatrix} + \widetilde{B} \begin{bmatrix} U_t^H \\ U_t^F \end{bmatrix} + \widetilde{D} U_t^M + \widetilde{C} \widetilde{\varepsilon}_{t+1} \tag{38}$$

where Y_t is an n_1-vector of predetermined state variables, Y_0 is given, and X_t are the effective instruments of the private sector, an n_2-vector of non-predetermined or forward-looking variables ($n = n_1 + n_2$). The policy instruments are represented by U_t^H, U_t^F and U_t^M. U_t^H and U_t^F stand for the instruments of the followers which are, respectively, the Home and the Foreign fiscal authorities, while U_t^M represents the instrument of the leader, which is the monetary authority. ε_{t+1} is an n_ε-vector of exogenous zero-mean *iid* shocks with an identity covariance matrix. Premultiplying (38) by $\widetilde{A}_0^{-1}$ we get

$$\begin{bmatrix} Y_{t+1} \\ H E_t X_{t+1} \end{bmatrix} = A \begin{bmatrix} Y_t \\ X_t \end{bmatrix} + B \begin{bmatrix} U_t^H \\ U_t^F \end{bmatrix} + D U_t^M + C \varepsilon_{t+1} \tag{39}$$

where $A = \widetilde{A}_0^{-1}\widetilde{A}$, $B = \widetilde{A}_0^{-1}\widetilde{B}$, $D = \widetilde{A}_0^{-1}\widetilde{D}$ and $C = \widetilde{A}_0^{-1}\widetilde{C}$. The covariance matrix of the shocks to Y_{t+1} is CC' and matrices A, B, C, and D are partioned conformably with Y_t and X_t as

$$\begin{aligned} A &\equiv \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix};\ B \equiv \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \\ D &\equiv \begin{bmatrix} D_1 \\ D_2 \end{bmatrix};\ C \equiv \begin{bmatrix} C_1 \\ \mathbf{O} \end{bmatrix} \end{aligned}$$

A common special case is when $H \equiv I$, but in general this matrix need not to be invertible. This system describes the evolution of the economy as observed by policymakers.

The followers' optimization problem

In the discretionary case, the three policymakers reoptimize every period by taking the process by which private agents form their expectations as given - and where the expectations are consistent with actual policies (Söderlind 1999). The two Nash fiscal authorities minimize their loss functions treating the monetary policy instrument as parametric but incorporating the reaction functions of the private sectors. Assuming that the fiscal authority of the Home country has the following objective function:

$$\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(G_t^{H'}Q^HG_t^H\right)=\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(Z_t'\mathcal{Q}^HZ_t+Z_t'\mathcal{P}^HU_t+U_t'\mathcal{P}^{H\prime}Z_t+U_t'\mathcal{R}^HU_t\right) \tag{40}$$

where $G_t^{H'}$ is the target variables for the Home fiscal authority while Q^H is the corresponding matrix of weights. The target variables can be rewritten in terms of the predetermined and non-predetermined state variables collected on vector Z_t, in terms of the policy instruments (U_t) and in terms of combinations of these two variables.

The fiscal authority in H optimizes every period, taking into account that she will be able to reoptimize next period. The model is linear-quadratic, thus the solution in $t+1$ gives a period return which is quadratic in the state variables, $W_{t+1}^H \equiv Y_{t+1}^{'}S_H^{t+1}Y_{t+1}+w_{t+1}^H$, where S_H^{t+1} is a positive semidefinite matrix and w_{t+1}^H is a scalar independent of Y_{t+1}. Moreover, the forward looking variables must be linear functions of the state variables, $X_{t+1}=-N_{t+1}Y_{t+1}$. Hence, the value function of the fiscal authority of H in t will then satisfy the Bellman equation:

$$W_t^H=\min_{U_t^H}\frac{1}{2}\left[\left(Z_t'\mathcal{Q}^HZ_t+Z_t'\mathcal{P}^HU_t+U_t'\mathcal{P}^{H\prime}Z_t+U_t'\mathcal{R}^HU_t\right)+\beta E_t\left(W_{t+1}^H\right)\right] \tag{41}$$

s.t. $E_tX_{t+1}=-N_{t+1}E_tY_{t+1}$, $W_{t+1}^H\equiv Y_{t+1}^{'}S_H^{t+1}Y_{t+1}+w_{t+1}^H$, eq. (39) and Y_t given.

Rewriting the system by using $E_tX_{t+1}=-N_{t+1}E_tY_{t+1}$ Using the expression above to substitute into the upper block of (39), we get

$$E_tX_{t+1}=-N_{t+1}\left[A_{11}Y_t+A_{12}X_t+B_{11}U_t^H+B_{12}U_t^F+D_1U_t^M\right]$$

while the lower block of (39) is

$$HE_tX_{t+1} = A_{21}Y_t + A_{22}X_t + B_{21}U_t^H + B_{22}U_t^F + D_2U_t^M$$

Multiplying the former equation by H, setting the result equal to the latter equation and solving for X_t we obtain

$$\begin{aligned} X_t \quad = \quad & \text{-}\underbrace{(A_{22}{+}HN_{t+1}A_{12})^{-1}(A_{21}{+}HN_{t+1}A_{11})}_{J_t}Y_t\text{-}\underbrace{(A_{22}{+}HN_{t+1}A_{12})^{-1}(B_{21}{+}HN_{t+1}B_{11})}_{K_t^H}U_t^H \\ & \text{-}\underbrace{(A_{22}{+}HN_{t+1}A_{12})^{-1}(B_{22}{+}HN_{t+1}B_{12})}_{K_t^F}U_t^F\text{-}\underbrace{(A_{22}{+}HN_{t+1}A_{12})^{-1}(D_2{+}HN_{t+1}D_1)}_{K_t^M}U_t^M \end{aligned}$$

$$X_t = -J_tY_t - K_t^HU_t^H - K_t^FU_t^F - K_t^MU_t^M \tag{42}$$

where J_t is n_2xn_1, K_t^H is n_2xk_H, K_t^F is n_2xk_F and K_t^M is n_2xk_M (k_H and k_F stand respectively for the number of fiscal policy instruments of H and F, while k_M stands for the number of monetary policy instruments)[27].

The evolution of Y_t Use (42) in the first n_1 equations in the system(39) to get the reduced form evolution of the predetermined variables

$$\begin{aligned} Y_{t+1} \quad = \quad & \underbrace{[A_{11} - A_{12}J_t]}_{O_{Y_t}}Y_t + \underbrace{\left[B_{11} - A_{12}K_t^H\right]}_{O_{H_t}}U_t^H \\ & + \underbrace{\left[B_{12} - A_{12}K_t^F\right]}_{O_{F_t}}U_t^F + \underbrace{\left[D_1 - A_{12}K_t^M\right]}_{O_{M_t}}U_t^M + C_1\varepsilon_{t+1} \end{aligned}$$

$$Y_{t+1} = O_{Y_t}Y_t + O_{H_t}U_t^H + O_{F_t}U_t^F + O_{M_t}U_t^M + C_1\varepsilon_{t+1} \tag{43}$$

Being a follower, the Home fiscal authority observes monetary authority's actions and reacts to them. In a linear-quadratic setup, the optimal solution belongs to the class of linear feedback rules of the form:

$$U_t^H = -F_t^HY_t - L_t^HU_t^M \tag{44}$$

where F_t^H denotes feedback coefficients on the predetermined state variables and L_t^H is the leadership parameter. The other fiscal authority solves a similar problem and get:

$$U_t^F = -F_t^FY_t - L_t^FU_t^M \tag{45}$$

Being in a Nash game, the two fiscal authorities do not respond to each other's actions.

[27] It is assumed that $A_{22} + HN_{t+1}A_{12}$ is invertible.

The monetary leadership authority takes into account these fiscal policy reaction functions as well as the private sector's optimal conditions, when solves its optimization problem. Thus, the leader can manipulate the follower by changing its policy instrument. The monetary leadership reaction function takes the form of:

$$U_t^M = -F_t^M Y_t \tag{46}$$

Reformulated optimization problem Therefore we can substitute eqs. (42) and (43) into (41) to obtain an equivalent minimization problem[28]:

$$\begin{aligned}
2\widetilde{W}_t^H \equiv \min_{U_t^H} \Big\{ & Y_t' \left[Q_H^S + \beta O_{Y_t}' S_H^{t+1} O_{Y_t}\right] Y_t + U_t^{H'} \left[\mathcal{U}_H^{S,H'} + \beta O_{H_t}' S_H^{t+1} O_{Y_t}\right] Y_t \\
& + Y_t' \left[\mathcal{U}_H^{S,H} + \beta O_{Y_t}' S_H^{t+1} O_{H_t}\right] U_t^H + U_t^{F'} \left[\mathcal{U}_F^{S,H'} + \beta O_{F_t}' S_H^{t+1} O_{Y_t}\right] Y_t \\
& + Y_t' \left[\mathcal{U}_F^{S,H} + \beta O_{Y_t}' S_H^{t+1} O_{F_t}\right] U_t^F + U_t^{M'} \left[\mathcal{U}_M^{S,H'} + \beta O_{M_t}' S_H^{t+1} O_{Y_t}\right] Y_t \\
& + Y_t' \left[\mathcal{U}_M^{S,H} + \beta O_{Y_t}' S_H^{t+1} O_{M_t}\right] U_t^M + U_t^{H'} \left[\mathcal{R}_H^{S,H} + \beta O_{H_t}' S_H^{t+1} O_{H_t}\right] U_t^H \\
& + U_t^{F'} \left[\mathcal{R}_F^{S,H} + \beta O_{F_t}' S_H^{t+1} O_{F_t}\right] U_t^F + U_t^{M'} \left[\mathcal{R}_M^{S,H} + \beta O_{M_t}' S_H^{t+1} O_{M_t}\right] U_t^M \\
& + U_t^{H'} \left[\mathcal{P}_{HF}^{S,H} + \beta O_{H_t}' S_H^{t+1} O_{F_t}\right] U_t^F + U_t^{F'} \left[\mathcal{P}_{HF}^{S,H'} + \beta O_{F_t}' S_H^{t+1} O_{H_t}\right] U_t^H \\
& + U_t^{H'} \left[\mathcal{P}_{HM}^{S,H} + \beta O_{H_t}' S_H^{t+1} O_{M_t}\right] U_t^M + U_t^{M'} \left[\mathcal{P}_{HM}^{S,H'} + \beta O_{M_t}' S_H^{t+1} O_{H_t}\right] U_t^H \\
& + U_t^{F'} \left[\mathcal{P}_{FM}^{S,H} + \beta O_{F_t}' S_H^{t+1} O_{M_t}\right] U_t^M + U_t^{M'} \left[\mathcal{P}_{FM}^{S,H'} + \beta O_{M_t}' S_H^{t+1} O_{F_t}\right] U_t^F \Big\}
\end{aligned} \tag{47}$$

where

$$\begin{aligned}
Q_H^S &= \mathcal{Q}_{11}^H - J_t^{'} \mathcal{Q}_{21}^H - \mathcal{Q}_{12}^H J_t + J_t^{'} \mathcal{Q}_{22}^H J_t \\
\mathcal{U}_H^{S,H} &= J_t^{'} \mathcal{Q}_{22}^H K_t^H - \mathcal{Q}_{12}^H K_t^H + \mathcal{P}_{12}^H - J_t^{'} \mathcal{P}_{22}^H \\
\mathcal{U}_F^{S,H} &= J_t^{'} \mathcal{Q}_{22}^H K_t^F - \mathcal{Q}_{12}^H K_t^F + \mathcal{P}_{13}^H - J_t^{'} \mathcal{P}_{23}^H \\
\mathcal{U}_M^{S,H} &= J_t^{'} \mathcal{Q}_{22}^H K_t^M - \mathcal{Q}_{12}^H K_t^M + \mathcal{P}_{11}^H - J_t^{'} \mathcal{P}_{21}^H \\
\mathcal{R}_H^{S,H} &= K_t^{H\prime} \mathcal{Q}_{22}^H K_t^H - K_t^{H\prime} \mathcal{P}_{22}^H - \mathcal{P}_{22}^{H\prime} K_t^H + \mathcal{R}_{22}^H \\
\mathcal{R}_F^{S,H} &= K_t^{F\prime} \mathcal{Q}_{22}^H K_t^F - K_t^{F\prime} \mathcal{P}_{23}^H - \mathcal{P}_{23}^{H\prime} K_t^F + \mathcal{R}_{33}^H \\
\mathcal{R}_M^{S,H} &= K_t^{M\prime} \mathcal{Q}_{22}^H K_t^M - K_t^{M\prime} \mathcal{P}_{21}^H - \mathcal{P}_{21}^{H\prime} K_t^M + \mathcal{R}_{11}^H \\
\mathcal{P}_{HF}^{S,H} &= K_t^{H\prime} \mathcal{Q}_{22}^H K_t^F - K_t^{H\prime} \mathcal{P}_{23}^H - \mathcal{P}_{22}^{H\prime} K_t^F + \mathcal{R}_{23}^H \\
\mathcal{P}_{HM}^{S,H} &= K_t^{H\prime} \mathcal{Q}_{22}^H K_t^M - K_t^{H\prime} \mathcal{P}_{21}^H - \mathcal{P}_{22}^{H\prime} K_t^M + \mathcal{R}_{21}^H \\
\mathcal{P}_{FM}^{S,H} &= K_t^{F\prime} \mathcal{Q}_{22}^H K_t^M - K_t^{F\prime} \mathcal{P}_{21}^H - \mathcal{P}_{23}^{H\prime} K_t^M + \mathcal{R}_{31}^H
\end{aligned}$$

[28] We have make use of the fact that w_{t+1}^H is independent of Y_{t+1} and $E_t \varepsilon_{t+1} = 0$.

Hence, the problem faced by the Home fiscal authority has been transformed to a standard linear-quadratic regulator problem without forward looking variables but with time varying parameters. The first-order condition is

$$\begin{aligned} 0 &= \left[\mathcal{U}_H^{S,H\prime} + \beta O_{H_t}' S_H^{t+1} O_{Y_t}\right] Y_t + \left[\mathcal{R}_H^{S,H} + \beta O_{H_t}' S_H^{t+1} O_{H_t}\right] U_t^H \\ &\quad + \left[\mathcal{P}_{HF}^{S,H} + \beta O_{H_t}' S_H^{t+1} O_{F_t}\right] U_t^F + \left[\mathcal{P}_{HM}^{S,H} + \beta O_{H_t}' S_H^{t+1} O_{M_t}\right] U_t^M \end{aligned}$$

Since $U_t^H = -F_t^H Y_t - L_t^H U_t^M$ and $U_t^F = -F_t^F Y_t - L_t^F U_t^M$, the first-order condition can be solved for the feedback coefficients of the reaction function of the Home fiscal authority:

$$F_t^H \equiv \left[\mathcal{R}_H^{S,H} + \beta O_{H_t}' S_H^{t+1} O_{H_t}\right]^{-1} \left\{\left[\mathcal{U}_H^{S,H\prime} + \beta O_{H_t}' S_H^{t+1} O_{Y_t}\right] - \left[\mathcal{P}_{HF}^{S,H} + \beta O_{H_t}' S_H^{t+1} O_{F_t}\right] F_t^F\right\} \tag{48}$$

$$L_t^H \equiv \left[\mathcal{R}_H^{S,H} + \beta O_{H_t}' S_H^{t+1} O_{H_t}\right]^{-1} \left\{\left[\mathcal{P}_{HM}^{S,H} + \beta O_{H_t}' S_H^{t+1} O_{M_t}\right] - \left[\mathcal{P}_{HF}^{S,H} + \beta O_{H_t}' S_H^{t+1} O_{F_t}\right] L_t^F\right\} \tag{49}$$

Finding the recursive equation for S_H^t Substituting the decision rules (44) , (45) and (46) into (47) we obtain the recursive equations for

$$S_H^t \equiv T_{0,t}^H + \beta T_t^{H\prime} S_H^{t+1} T_t^H \tag{50}$$

$$\begin{aligned} T_{0,t}^H &= Q_H^S - \mathcal{U}_H^{S,H}\left(F_t^H - L_t^H F_t^M\right) - \left(F_t^H - L_t^H F_t^M\right)' \mathcal{U}_H^{S,H\prime} - \mathcal{U}_F^{S,H}\left(F_t^F - L_t^F F_t^M\right) \\ &\quad - \left(F_t^F - L_t^F F_t^M\right)' \mathcal{U}_F^{S,H\prime} - \mathcal{U}_M^{S,H} F_t^M - F_t^{M\prime} \mathcal{U}_M^{S,H\prime} + \left(F_t^H - L_t^H F_t^M\right)' \mathcal{R}_H^{S,H}\left(F_t^H - L_t^H F_t^M\right) \\ &\quad + \left(F_t^F - L_t^F F_t^M\right)' \mathcal{R}_F^{S,H}\left(F_t^F - L_t^F F_t^M\right) + F_t^{M\prime} \mathcal{R}_M^{S,H} F_t^M \\ &\quad + \left(F_t^H - L_t^H F_t^M\right)' \mathcal{P}_{HF}^{S,H}\left(F_t^F - L_t^F F_t^M\right) + \left(F_t^F - L_t^F F_t^M\right)' \mathcal{P}_{HF}^{S,H\prime}\left(F_t^H - L_t^H F_t^M\right) \\ &\quad + \left(F_t^H - L_t^H F_t^M\right)' \mathcal{P}_{HM}^{S,H} F_t^M + F_t^{M\prime\prime} \mathcal{P}_{HM}^{S,H\prime}\left(F_t^H - L_t^H F_t^M\right) \\ &\quad + \left(F_t^F - L_t^F F_t^M\right)' \mathcal{P}_{FM}^{S,H} F_t^M + F_t^{M\prime\prime} \mathcal{P}_{FM}^{S,H\prime}\left(F_t^F - L_t^F F_t^M\right) \end{aligned}$$

and

$$T_t^H = O_{Y_t} - O_{H_t}\left(F_t^H - L_t^H F_t^M\right) - O_{F_t}\left(F_t^F - L_t^F F_t^M\right) - O_{M_t} F_t^M$$

Similar formulae can be derived for country F.

The leader's optimization problem This part of the problem is the standard optimization problem when the system under control evolves as

$$\begin{bmatrix} Y_{t+1} \\ HE_tX_{t+1} \end{bmatrix} = \begin{bmatrix} A11\text{-}B11F_t^H\text{-}B12F_t^F & A12 \\ A21\text{-}B21F_t^H\text{-}B22F_t^F & A22 \end{bmatrix} \begin{bmatrix} Y_t \\ X_t \end{bmatrix} + \begin{bmatrix} D11\text{-}B11L_t^H\text{-}B12L_t^F \\ D21\text{-}B21L_t^H\text{-}B22L_t^F \end{bmatrix} U_t^M + C\varepsilon_{t+1} \tag{51}$$

The monetary authority loss function is

$$\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(G_t^{M\prime}\mathrm{Q}^M G_t^M\right)$$

But, since the leadership integrates the followers' reaction functions - $U_t^H = -F_t^H Y_t - L_t^H U_t^M$ and $U_t^F = -F_t^F Y_t - L_t^F U_t^M$ - into its optimization problem, the leadership's loss function as to be rewritten in terms of the relevant variables for the leadership authority. Since

$$\begin{bmatrix} Y_t \\ X_t \\ U_t^M \\ U_t^H \\ U_t^F \end{bmatrix} = \underbrace{\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ -F_t^H & 0 & -L_t^H \\ -F_t^F & 0 & -L_t^F \end{bmatrix}}_{\mathcal{C}} \begin{bmatrix} Y_t \\ X_t \\ U_t^M \end{bmatrix}$$

we can set $G_t^{M\prime}\mathrm{Q}^M G_t^M = \begin{bmatrix} Y_t' & X_t' & U_t^{M\prime} \end{bmatrix} \widetilde{\mathcal{K}}^M \begin{bmatrix} Y_t \\ X_t \\ U_t^M \end{bmatrix}$ where $\widetilde{\mathcal{K}}^M = \mathcal{C}'\underbrace{C^{M\prime}\mathrm{Q}^M C^M}_{\mathcal{K}^M}\mathcal{C}$ and $\widetilde{\mathcal{K}}^M$ have to partioned conformably with $\left(Y_t' \quad X_t' \quad U_t^{M\prime}\right)'$.

The iterative procedure We start with initial approximation for the monetary policy rule, $F_{(0)}^M$, symmetric positive definite matrices (usually, identity matrices), $S_H^{(0)}$ and $S_F^{(0)}$, some (e.g. a matrix of zeros) $N_{(0)}$ and solve the follower's problem, using Eq. $(2-50)$ for country H and equivalent equations for country F. We get $F_{(0)}^H$ and $L_{(0)}^H$, as well as $F_{(0)}^F$ and $L_{(0)}^F$ and updated matrices $S_H^{(1)}$ and $S_F^{(1)}$. We then take into account the policy reaction functions of fiscal authorities and compute new matrices in Eq. (51), updated target variable $\left(G_t^M = C^M\mathcal{C}\left(Y_t' \quad X_t' \quad U_t^{M\prime}\right)'\right)$ and solve the problem for the monetary authority. This will give us the monetary policy reaction function, $F_{(1)}^M$, and updated matrices $N_{(1)}$ and $S_M^{(1)}$. Then, we again solve the problem for the fiscal authorities to update $S_H^{(2)}$ and $S_F^{(2)}$ and $F_{(1)}^H$, $L_{(1)}^H$, $F_{(1)}^F$ and $L_{(1)}^F$ and so on. The fixed point is found when the policy rules and the matrices converge towards constants for a given level of tolerance.

Blake and Kirsanova (2010) have examined the existence of multiple discretionary equilibria in dynamic linear quadratic rational expectations models. They show that different initializations of the Oudiz and Sachs (1985) and of the Backus and Driffill (1986) algorithms can converge to different solutions.

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Tables

Table 1A: Policy reaction functions, Balanced-budget, $n_H = 0.5$

		a_t^H (1)	a_t^F (2)	μ_t^H (3)	μ_t^F (4)	a_{t-1}^H (5)	a_{t-1}^F (6)	q_{t-1} (7)
	i_t	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	g_t^H	0.1622	-0.1622	0.0000	0.0000	-0.1532	0.1532	-0.2745
Coop	τ_t^H	1.4001	-1.4001	0.0000	0.0000	-1.3224	1.3224	-2.3688
	g_t^F	-0.1622	0.1622	0.0000	0.0000	0.1532	-0.1532	0.2745
	τ_t^F	-1.4001	1.4001	0.0000	0.0000	1.3224	-1.3224	2.3688
	i_t	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	g_t^H	0.1655	-0.1655	0.0000	0.0000	-0.1600	0.1600	-0.2866
Nash	τ_t^H	1.7869	-1.7869	0.0000	0.0000	-1.7855	1.7855	-3.1985
	g_t^F	-0.1655	0.1655	0.0000	0.0000	0.1600	-0.1600	0.2866
	τ_t^F	-1.7869	1.7869	0.0000	0.0000	1.7855	-1.7855	3.1985
	i_t	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	g_t^H	0.1607	-0.1607	0.0000	0.0000	-0.1481	0.1481	-0.2653
FL	τ_t^H	1.7178	-1.7178	0.0000	0.0000	-1.7120	1.7120	-3.0668
	g_t^F	-0.1607	0.1607	0.0000	0.0000	0.1481	-0.1481	0.2653
	τ_t^F	-1.7178	1.7178	0.0000	0.0000	1.7120	-1.7120	3.0668
	i_t	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	g_t^H	0.1655	-0.1655	0.0000	0.0000	-0.1600	0.1600	-0.2866
ML	τ_t^H	1.7869	-1.7869	0.0000	0.0000	-1.7855	1.7855	-3.1985
	g_t^F	-0.1655	0.1655	0.0000	0.0000	0.1600	-0.1600	0.2866
	τ_t^F	-1.7869	1.7869	0.0000	0.0000	1.7855	-1.7855	3.1985

Table 1B: Policy reaction functions, Balanced-budget, $n_H = 0.8$

	i_t	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	g_t^H	0.0649	-0.0649	0.0000	0.0000	-0.0613	0.0613	-0.1098
Coop	τ_t^H	0.5601	-0.5601	0.0000	0.0000	-0.5289	0.5289	-0.9475
	g_t^F	-0.2596	0.2596	0.0000	0.0000	0.2452	-0.2452	0.4392
	τ_t^F	-2.2402	2.2402	0.0000	0.0000	2.1158	-2.1158	3.7900
	i_t	-0.0209	0.0209	0.0000	0.0000	0.0205	-0.0205	0.0367
	g_t^H	0.0615	-0.0615	0.0000	0.0000	-0.0489	0.0489	-0.0876
Nash	τ_t^H	0.3149	-0.3149	0.0000	0.0000	0.3419	-0.3419	0.6124
	g_t^F	-0.2719	0.2719	0.0000	0.0000	0.2746	-0.2746	0.4919
	τ_t^F	-3.4803	3.4803	0.0000	0.0000	4.0882	-4.0882	7.3232
	i_t	-0.0234	0.0234	0.0000	0.0000	0.0234	-0.0234	0.0419
	g_t^H	0.0592	-0.0592	0.0000	0.0000	-0.0422	0.0422	-0.0755
FL	τ_t^H	0.2580	-0.2580	0.0000	0.0000	0.4488	-0.4488	0.8040
	g_t^F	-0.2704	0.2704	0.0000	0.0000	0.2696	-0.2696	0.4830
	τ_t^F	-3.4857	3.4857	0.0000	0.0000	4.1320	-4.1320	7.4017
	i_t	-0.0021	0.0021	0.0000	0.0000	0.0074	-0.0074	0.0133
	g_t^H	0.0667	-0.0667	0.0000	0.0000	-0.0648	0.0648	-0.1161
ML	τ_t^H	0.6477	-0.6477	0.0000	0.0000	-0.6695	0.6695	-1.1993
	g_t^F	-0.2667	0.2667	0.0000	0.0000	0.2592	-0.2592	0.4643
	τ_t^F	-3.1542	3.1542	0.0000	0.0000	3.1011	-3.1011	5.5550

Table 2: Policy reaction functions, Debt, $n_H = 0.5$

		a_t^H (1)	a_t^F (2)	μ_t^H (3)	μ_t^F (4)	$\overline{c}_t^w$ (5)	a_{t-1}^H (6)	a_{t-1}^F (7)	q_{t-1} (8)	b_{t-1}^H (9)	b_{t-1}^F (10)
	i_t	0.6826	0.6826	-0.1734	-0.1734	0.2921	-0.0000	0.0000	-0.0000	-0.5733	-0.5733
	g_t^H	0.3391	-0.1862	-0.0497	0.0109	0.0327	-0.2053	0.2053	-0.3678	-0.1644	0.0359
C	τ_t^H	-2.4114	2.0082	1.0910	-0.9886	-0.0863	0.6206	-0.6206	1.1117	3.6068	-3.2681
	g_t^F	-0.1862	0.3391	0.0109	-0.0497	0.0327	0.2053	-0.2053	0.3678	0.0359	-0.1644
	τ_t^F	2.0082	-2.4114	-0.9886	1.0910	-0.0863	-0.6206	0.6206	-1.1117	-3.2681	3.6068
	i_t	0.7520	0.7520	-0.1911	-0.1911	0.3218	-0.0000	0.0000	-0.0000	-0.6316	-0.6316
	g_t^H	0.3594	-0.2256	-0.0471	0.0131	0.0286	-0.2269	0.2269	-0.4065	-0.1557	0.0433
N	τ_t^H	-0.0943	1.1465	0.5464	-0.8137	0.2251	-1.8627	1.8627	-3.3366	1.8061	-2.6899
	g_t^F	-0.2256	0.3594	0.0131	-0.0471	0.0286	0.2269	-0.2269	0.4065	0.0433	-0.1557
	τ_t^F	1.1465	-0.0943	-0.8137	0.5464	0.2251	1.8627	-1.8627	3.3366	-2.6899	1.8061
	i_t	0.7524	0.7524	-0.1912	-0.1912	0.3220	0.0000	-0.0000	0.0000	-0.6320	-0.6320
	g_t^H	0.3396	-0.2056	-0.0437	0.0097	0.0287	-0.1830	0.1830	-0.3278	-0.1445	0.0320
FL	τ_t^H	0.3780	0.6837	0.4446	-0.7143	0.2272	-2.6778	2.6778	-4.7968	1.4696	-2.3613
	g_t^F	-0.2056	0.3396	0.0097	-0.0437	0.0287	0.1830	-0.1830	0.3278	0.0320	-0.1445
	τ_t^F	0.6837	0.3780	-0.7143	0.4446	0.2272	2.6778	-2.6778	4.7968	-2.3613	1.4696
	i_t	0.6932	0.6932	-0.1761	-0.1761	0.2966	-0.0000	0.0000	-0.0000	-0.5823	-0.5823
	g_t^H	0.3627	-0.2208	-0.0479	0.0119	0.0304	-0.2350	0.2350	-0.4210	-0.1584	0.0392
ML	τ_t^H	-0.3669	1.0894	0.6066	-0.7901	0.1546	-1.5500	1.5500	-2.7766	2.0052	-2.6120
	g_t^F	-0.2208	0.3627	0.0119	-0.0479	0.0304	0.2350	-0.2350	0.4210	0.0392	-0.1584
	τ_t^F	1.0894	-0.3669	-0.7901	0.6066	0.1546	1.5500	-1.5500	2.7766	-2.6120	2.0052

Conservative central bank: $\rho = 0.75; 1 - \rho = 0.25$

	i_t	0.8456	0.8456	-0.2148	-0.2148	0.3618	-0.0000	0.0000	-0.0000	-0.7102	-0.7102
	g_t^H	0.3288	-0.1970	-0.0417	0.0082	0.0282	-0.1840	0.1840	-0.3297	-0.1377	0.0270
FL	τ_t^H	0.4267	1.0540	0.4307	-0.8069	0.3168	-2.4089	2.4089	-4.3150	1.4239	-2.6676
	g_t^F	-0.1970	0.3288	0.0082	-0.0417	0.0282	0.1840	-0.1840	0.3297	0.0270	-0.1377
	τ_t^F	1.0540	0.4267	-0.8069	0.4307	0.3168	2.4089	-2.4089	4.3150	-2.6676	1.4239
	i_t	0.8895	0.8895	-0.2260	-0.2260	0.3807	0.0000	-0.0000	0.0000	-0.7472	-0.7472
	g_t^H	0.3523	-0.2355	-0.0453	0.0156	0.0250	-0.2119	0.2119	-0.3796	-0.1497	0.0516
ML	τ_t^H	0.5041	1.3272	0.4082	-0.8735	0.3918	-2.3922	2.3922	-4.2853	1.3495	-2.8878
	g_t^F	-0.2355	0.3523	0.0156	-0.0453	0.0250	0.2119	-0.2119	0.3796	0.0516	-0.1497
	τ_t^F	1.3272	0.5041	-0.8735	0.4082	0.3918	2.3922	-2.3922	4.2853	-2.8878	1.3495

Table 3: Policy reaction functions, Debt, $n_H = 0.8$

		a_t^H (1)	a_t^F (2)	μ_t^H (3)	μ_t^F (4)	$\overline{c}_t^w$ (5)	a_{t-1}^H (6)	a_{t-1}^F (7)	q_{t-1} (8)	b_{t-1}^H (9)	b_{t-1}^F (10)
	i_t	1.0921	0.2730	-0.2775	-0.0694	0.2921	-0.0000	0.0000	-0.0000	-0.9173	-0.2293
	g_t^H	0.2274	-0.0745	-0.0432	0.0043	0.0327	-0.0821	0.0821	-0.1471	-0.1428	0.0144
Coop	τ_t^H	-1.2065	0.8033	0.4979	-0.3954	-0.0863	0.2482	-0.2482	0.4447	1.6459	-1.3072
	g_t^F	-0.2978	0.4508	0.0174	-0.0562	0.0327	0.3285	-0.3285	0.5885	0.0574	-0.1859
	τ_t^F	3.2131	-3.6163	-1.5817	1.6842	-0.0863	-0.9929	0.9929	-1.7786	-5.2289	5.5676
	i_t	0.9293	0.5793	-0.2338	-0.1495	0.3228	-3.1787	3.1787	-5.6940	-0.7730	-0.4942
	g_t^H	0.1456	-0.0272	-0.0242	-0.0059	0.0253	-0.3065	0.3065	-0.5490	-0.0800	-0.0194
Nash	τ_t^H	1.4286	-0.5334	-0.0721	-0.1553	0.1915	-3.5439	3.5439	-6.3482	-0.2384	-0.5135
	g_t^F	-0.4296	0.5744	0.0210	-0.0578	0.0310	0.4076	-0.4076	0.7302	0.0694	-0.1910
	τ_t^F	6.8594	-5.5344	-1.6558	1.3191	0.2835	-16.9330	16.9330	-30.3325	-5.4738	4.3608
	i_t	0.9368	0.5749	-0.2294	-0.1547	0.3234	-2.9852	2.9852	-5.3476	-0.7584	-0.5113
	g_t^H	0.1364	-0.0178	-0.0226	-0.0075	0.0254	-0.2692	0.2692	-0.4823	-0.0747	-0.0249
FL	τ_t^H	1.7470	-0.8120	-0.1151	-0.1225	0.2001	-4.0777	4.0777	-7.3045	-0.3805	-0.4048
	g_t^F	-0.4037	0.5484	0.0172	-0.0539	0.0310	0.3051	-0.3051	0.5466	0.0568	-0.1783
	τ_t^F	6.1141	-4.7821	-1.5078	1.1694	0.2850	-13.0557	13.0557	-23.3871	-4.9846	3.8657
	i_t	0.6527	0.7404	-0.1957	-0.1582	0.2981	-3.0052	3.0052	-5.3833	-0.6471	-0.5231
	g_t^H	0.1457	-0.0265	-0.0239	-0.0064	0.0255	-0.3578	0.3578	-0.6408	-0.0789	-0.0212
ML	τ_t^H	1.2850	-0.4787	-0.0305	-0.1744	0.1725	-3.7175	3.7175	-6.6592	-0.1008	-0.5765
	g_t^F	-0.4309	0.5880	0.0230	-0.0629	0.0336	0.3051	-0.3051	0.5465	0.0761	-0.2080
	τ_t^F	6.3608	-5.4950	-1.6520	1.4320	0.1853	-14.6393	14.6393	-26.2237	-5.4611	4.7339

Table 4A: Losses − H and F households (L^H, L^F) and union-wide (L) − Balanced-Budget

	n_H=0.5	n_H=0.55	n_H=0.6	n_H=0.65	n_H=0.7	n_H=0.75	n_H=0.8	n_H=0.85	n_H=0.9
LCoop	3.8078	3.7697	3.6555	3.4651	3.1985	2.8558	2.4370	1.9420	1.3708
LML	3.9479	3.9129	3.8075	3.6306	3.3800	3.0526	2.6438	2.1473	1.5534
LFL	3.8942	3.8617	3.7635	3.5973	3.3595	3.0450	2.6470	2.1569	1.5639
LN	3.9479	3.9137	3.8106	3.6371	3.3904	3.0665	2.6599	2.1631	1.5658
L^HCoop	3.8078	3.6989	3.5296	3.2999	3.0097	2.6592	2.2482	1.7768	1.2449
L^HML	3.9479	3.9334	3.8441	3.6790	3.4358	3.1114	2.7011	2.1982	1.5931
L^HFL	3.8942	3.8829	3.8013	3.6473	3.4173	3.1060	2.7063	2.2096	1.6046
L^HN	3.9479	3.9352	3.8490	3.6878	3.4487	3.1278	2.7194	2.2158	1.6065
L^FCoop	3.8078	3.8562	3.8442	3.7719	3.6390	3.4458	3.1921	2.8781	2.5036
L^FML	3.9479	3.8878	3.7526	3.5407	3.2498	2.8762	2.4148	1.8584	1.1966
L^FFL	3.8942	3.8358	3.7067	3.5043	3.2247	2.8622	2.4098	1.8586	1.1973
L^FN	3.9479	3.8874	3.7530	3.5429	3.2542	2.8824	2.4216	1.8640	1.1989
Conservative central bank: $\rho = 0.75; 1 - \rho = 0.25$									
LMLcons	3.9479	3.9129	3.8075	3.6306	3.3800	3.0526	2.6438	2.1473	1.5534
LFLcons	3.8523	3.8233	3.7346	3.5822	3.3597	3.0587	2.6694	2.1812	1.5827
L^HMLcons	3.9479	3.9334	3.8441	3.6790	3.4358	3.1114	2.7011	2.1982	1.5931
L^HFLcons	3.8523	3.8238	3.7391	3.5950	3.3839	3.0943	2.7130	2.2265	1.6214
L^FMLcons	3.9479	3.8878	3.7526	3.5407	3.2498	2.8762	2.4148	1.8584	1.1966
L^FFLcons	3.8523	3.8226	3.7279	3.5584	3.3033	2.9519	2.4948	1.9244	1.2346

Table 4B: Losses − H and F households (L^H, L^F) and union-wide (L) − Debt

	n_H=0.5	n_H=0.55	n_H=0.6	n_H=0.65	n_H=0.7	n_H=0.75	n_H=0.8	n_H=0.85	n_H=0.9
LCoop	4.8950	4.8533	4.7280	4.5192	4.2269	3.8511	3.3917	2.8489	2.2225
LML	5.0697	5.3056	5.2641	4.9940	4.5837	4.0676	3.4625	2.7794	2.0293
LFL	5.3826	6.2510	5.8882	5.4233	4.8733	4.2497	3.5623	2.8213	2.0405
LN	5.1264	5.9954	5.6737	5.2541	4.7494	4.1681	3.5174	2.8049	2.0416
L^HCoop	4.8950	4.6041	4.2850	3.9378	3.5624	3.1589	2.7272	2.2674	1.7795
L^HML	5.0697	4.2078	3.8818	3.5961	3.2889	2.9435	2.5531	2.1150	1.6287
L^HFL	5.3826	3.8105	3.6292	3.4088	3.1435	2.8309	2.4701	2.0614	1.6067
L^HN	5.1264	3.7865	3.6356	3.4368	3.1862	2.8819	2.5232	2.1101	1.6433
L^FCoop	4.8950	5.1578	5.3925	5.5990	5.7774	5.9276	6.0497	6.1436	6.2094
L^FML	5.0697	6.6474	7.3375	7.5900	7.6049	7.4398	7.1002	6.5449	5.6352
L^FFL	5.3826	9.2339	9.2766	9.1647	8.9096	8.5062	7.9314	7.1275	5.9452
L^FN	5.1264	8.6952	8.7308	8.6292	8.3971	8.0270	7.4944	6.7422	5.6260
Conservative central bank: $\rho = 0.75; 1 - \rho = 0.25$									
LMLcons	5.2812	5.4786	5.4105	5.1252	4.6957	4.1573	3.5302	2.8288	2.0665
LFLcons	5.2958	5.4233	5.3344	5.0430	4.6150	4.0824	3.4638	2.7725	2.0228
L^HMLcons	5.2812	4.4176	4.0945	3.7840	3.4398	3.0539	2.6257	2.1574	1.6530
L^HFLcons	5.2958	4.3213	3.9224	3.5913	3.2549	2.8940	2.5010	2.0719	1.6053
L^FMLcons	5.2812	6.7754	7.3844	7.6158	7.6261	7.4675	7.1485	6.6332	5.7886
L^FFLcons	5.2958	6.7701	7.4525	7.7389	7.7884	7.6477	7.3151	6.7424	5.7806