Personalized Pricing and Brand Distribution[*]

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Abstract

This paper examines the effects of personalized pricing on brand distribution. We explore whether a brand manufacturer prefers to sell through its own retail outlet only (mono distribution) or through an independent retailer as well (dual distribution). Personalized pricing allows for higher rent extraction but also leads to more fierce intra-brand competition than does uniform pricing. Due to the latter effect, a brand manufacturer may prefer mono distribution even if the retailer broadens the demand of the manufacturer's product. By contrast, with uniform pricing, selling through both channels is always optimal. This result holds for wholesale contracts consisting of two-part tariffs as well as for linear wholesale tariffs. We also show that the manufacturer may obtain its largest profit in a hybrid pricing regime, in which only the retailer charges personalized prices.

Keywords: personalized pricing, distribution channels, dual distribution, vertical contracting, downstream competition.

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1 Introduction

The growing use of the Internet and advances in information technologies enable firms to gather unprecedented volumes of consumer data. This has led to important changes in their pricing policy, by enabling them to practice price discrimination at finely-tuned levels. For example, firms tailor their prices according to consumers' purchase history, their physical location, the device they are using, their online search behavior, their social network activity, and so on.[1] For example, Tanner (2014) reports that buyers using a discount site, such as Nextag.com, receive prices as much as 23% lower than direct visitors. Also, large Internet stores, such as Amazon and Staples, vary their prices according to customers' geographic locations by up to 166%. Firms often implement these different prices through coupons and promotions, thereby moving closer to personalized pricing.[2]

At the same time, technological advances have also made possible the entry of many new online retail companies. For example, Amazon (founded in 1994) now sells over 560 million products in the US, ranging from clothing to grocery items,[3] and iHerb (founded in 1996) distributes over 35,000 nutritional and organic food products. By contrast, most apparel and fashion brand manufacturers, such as American Apparel or Guess, offer their full range of products on their own websites but sell only part of them on Amazon.com or Alibaba.com. In the same vein, the organic coffee and food producer Equal Exchange sells exclusively through its own stores (i.e., it relies solely on direct distribution). Therefore, a core question for brand manufacturers is whether or not to sell their products through independent retail outlets. This question is relevant beyond the online market. For example, in the market for mobile telephony, the advancement of technology has allowed mobile virtual network operators, such as Ting or ringplus$^+$, to enter the market for mobile phone services without rolling out their own networks. Established mobile operators were confronted with the issue of whether or not to grant these new operators access to their networks.[4]

These independent firms bring value to the industry. In particular, they broaden the customer base of the manufacturer's products. This occurs mainly for consumers in the low valuation segment, who do not buy from the manufacturer's direct channel

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[1]For example, according to TRUSTe—a consulting firm on privacy and technological tools—the 100 most widely used sites on the Internet are monitored by more than 1,300 firms. Also, established companies, such as Bloomberg and Axciom or more recent ones such as PubMatic or Freshplum, which specialize in developing machine learning algorithms, act as data brokers and help firms to predict a consumer's willingness-to-pay (*The Economist*, 2014).

[2]In practice, firms may not know a consumer's valuation precisely. Although we will consider in this paper the benchmark of perfect information, the insights apply as well to fine-tuned price discrimination.

[3]See Scrapehero (2018).

[4]The telephone industry is one of first industries where firms used customized pricing to a large extent, see Chen and Iyer (2002).

regularly but, for example, become aware of the manufacturer's product through the retailer's site or prefer to buy from a younger company.[5] At the same time, these independent retailers compete with the manufacturer's own retail outlets, as consumers are able to compare the prices of the products at both stores. Both the benefit of dual distribution through increased demand and the costs through increased competition are affected by the possibility of price discrimination. Targeted prices allow for a larger rent extraction, therefore increasing the benefit of demand expansion, but induce firms to compete on multiple margins.

These observations indicate that personalized pricing and consumer tracking does not only affect firms' pricing strategy but also their distribution strategy. Although personalized pricing has itself received substantial attention in the literature (see e.g., Shaffer and Zhang, 1995, 2002; Choudhary *et al.*, 2005; Ghose and Huang, 2009), its interaction with other marketing decisions is not well understood.[6]

Building on these considerations, the objective of this paper is to identify the implications of price discrimination for the optimal distribution strategy of a manufacturer. Does personalized pricing change the incentives of a manufacturer to sell through an independent retailer? How does the trade-off between more fierce competition and higher rent extraction affect the wholesale contract? Does a manufacturer always prefer personalized pricing? Can the manufacturer benefit when only the retailer has access to personalized pricing?

To answer these questions, we set up a simple model with one brand manufacturer selling directly to final consumers and one independent retailer. The retailer competes with the manufacturer in the downstream market but also adds value to the industry: consumers with lower valuations prefer to buy from the independent retailer, and so the retailer is able to serve consumers that the manufacturer cannot. This captures in a simple and tractable way the structure of consumer preferences in the examples above.

We consider three different scenarios. In the first scenario, the manufacturer and the retailer offer uniform prices to final consumers. This represents a market in which consumer tracking is not possible. In the second scenario, both firms engage in personalized pricing. This reflects the situation in which they both have highly-frequented (e.g., online) stores, allowing them to gather consumer data at very precise levels. Although personalized pricing is an extreme form, it allows us to highlight the effect of price discrimination on distribution choices in the clearest way possible. In the third scenario, only the retailer can set personalized prices.[7] This represents a situa-

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[5]By contrast, consumers with a high valuation will usually buy directly from the manufacturer's own retail outlet, as they enjoy a larger variety there.

[6]We provide a detailed literature overview in the next section.

[7]In principle, we could also analyze a fourth scenario, in which only the manufacturer has access to personalized pricing. However, this is arguably the least relevant situation in practice, and therefore we

tion where a large retailer, such as Amazon, is patronized by many consumers and is thereby able to collect more consumer data than a brand manufacturer.[8]

We first show that dual distribution is optimal when the manufacturer only offers a uniform price downstream. This holds regardless of whether the retailer also sets a uniform price or has access to personalized pricing. The intuition is that by setting the wholesale price at a sufficiently high level, the manufacturer can partly control the intensity of downstream competition. The retailer will then also set its downstream price(s) at a relatively high level. The manufacturer nevertheless benefits from the value that the retailer brings in the low-valuation segment and, in addition, increases its own uniform price to obtain a larger rent from the high-value consumers.

By contrast, the manufacturer may prefer mono distribution (i.e., selling only through the direct channel) when it has access to personalized pricing. Specifically, if the retailer does not substantially expand demand, relying exclusively on direct distribution is optimal, as the effect of increased competition then dominates the rent extraction benefit. Indeed, when both firms can price discriminate, each one can price aggressively in the strong segment of the other without sacrificing margins in its own core business. This results in low prices for the most valuable consumers. As a consequence, the manufacturer can be better off with direct distribution only, thereby forgoing the value the retailer brings to the market.

We then compare the profitability of the three scenarios. We find that profits can be largest in the hybrid scenario, in which only the retailer can practice personalized pricing. The reason is that this scenario can achieve the right balance between rent extraction (within each channel) and the avoidance of fierce competition (between channels). Interestingly, this implies that the manufacturer may have the incentive to commit not to charge personalized prices (e.g., by not investing in technologies allowing it to do so).

In addition, we show that the scenarios in which both firms charge personalized prices unambiguously leads to higher profits than the one with uniform pricing for both firms, even though the former regime may reduce the number of distribution channels.

Finally, we show that our qualitative findings regarding the optimal distribution strategy do not depend on the form of the wholesale contract. They hold for two-part tariffs and for linear tariffs, and similar intuitions apply in both cases. However, with a linear tariff, mono distribution is optimal for a larger parameter range, as the manufacturer can extract less profit from the retailer.

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do not consider it here.

[8]This also applies when the manufacturer is a dominant firm (e.g., an incumbent telecom or energy operator) subject to antitrust or regulatory supervision.

We discuss in the Conclusion the lessons from our analysis, which may provide guidance as to how marketing managers should structure the wholesale contracts to pure retailers. A key insight is that price discrimination and consumer addressability—which is feasible in many modern industries—not only affects the pricing strategy but also the optimal distribution network. In fact, mono distribution may be optimal even when a retailer adds value to the market. The reason is that competition for final consumers can destroy profits in the manufacturer's core segment.

The rest of the paper is organized as follows: Section 2 discusses the related literature. Section 3 presents the baseline model. Section 4 analyzes uniform pricing. Section 5 considers personalized pricing of both firms and determines the conditions for dual distribution to be optimal. Section 6 analyzes the hybrid pricing regime and compares the three pricing scenarios. Section 7 extends the analysis to linear wholesale tariffs, and Section 8 concludes.

2   Related Literature

The literature on personalized pricing—and more generally on price discrimination—and competition has almost exclusively focused on retail markets.[9] This literature usually distinguishes between models of horizontal and vertical differentiation. In their seminal paper, Thisse and Vives (1988) analyze the effects of price discrimination for horizontally differentiated firms competing on a Hotelling line. They demonstrate that this leads to a prisoner's dilemma: firms adopt price discrimination but profits fall due to increased competition.[10] Shaffer and Zhang (1995) allow firms to discriminate through coupon targeting when consumers differ in relation to the cost of redeeming their coupon. They find that this still leads to a prisoner's dilemma situation. Chen and Iyer (2002) allow firms to choose the proportion of consumers for whom they acquire information. In this case, firms may benefit from consumer addressability and may refrain from acquiring full information. Chen *et al*. (2018) allow consumers for whom firms can charge personalized prices to bypass price discrimination and buy at a uniform price. They show that this possibility can collectively harm consumers and allow firms to benefit from price discrimination.[11]

Choudhary *et al*. (2005), in one of the first papers explicitly using the expression "personalized pricing", consider instead competition between vertically differentiated

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[9]See Stole (2007) and Zhang (2009) for an overview of different forms of price discrimination and targeted pricing and how they affect competitive outcomes.

[10]Liu and Serfes (2013) extend the framework of Thisse and Vives (1988) by studying the effects of price discrimination in two-sided markets. Matsumura and Matsushima (2015) show instead that firms may choose not to price discriminate in order to limit rivals' incentives to engage in cost reduction.

[11]Shaffer and Zhang (2000) consider asymmetric customer bases and provide conditions under which a firm may offer a lower price to its own consumer base.

firms, and find that pricing strategies can be non-monotonic in consumer valuations. In addition, they show that personalized pricing can lead to an increase or decrease in quality levels. Two papers combine vertical and horizontal differentiation. Shaffer and Zhang (2002) show that firms with a higher quality product may benefit from personalized pricing, even though competition is more fierce. This is due to a gain in market share, which dominates the effect of lower prices. Ghose and Huang (2009) consider the case in which firms can customize the product quality to consumer preferences, and find that this can also lead to firms benefitting from personalized pricing.[12]

Our paper contributes to this literature by studying the implications of personalized pricing on the choice of distribution channels. To the best of our knowledge, the only paper that also analyzes the interplay between personalized pricing and distribution strategy is Liu and Zhang (2006). They consider a setting in which the retailer has access to personalized pricing, and the manufacturer—who charges a linear wholesale tariff—can enter through direct marketing with a uniform price. They show that the adoption of personalized pricing harms the retailer by inducing the manufacturer to charge a higher wholesale price, but can nevertheless be profitable by deterring the manufacturer from entering the downstream market. Their focus is on the retailer's pricing strategy and its implications for downstream entry by the manufacturer. We focus instead on an integrated manufacturer's decision to allow a retailer to enter the market, and study the implications of pricing strategies on this decision. We also allow for personalized pricing by both firms and consider non-linear as well as linear wholesale tariffs.

Our paper also contributes to the literature on Internet channel entry (Chiang *et al.*, 2003; Yoo and Lee, 2011) by determining the conditions under which an incumbent who directly markets its products deters entry by a pure retailer.[13] On a broader level, our paper establishes that the possibility of price discrimination not only has short-run effects on competition but also influences other important marketing decisions, such as the optimal distribution. It is therefore in line with Jing (2016), who shows that behavior-based price discrimination affects quality differentiation between firms.[14]

Finally, our paper also contributes to the literature on market foreclosure. Several

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[12]For empirical papers on how firms can implement personalized pricing, see, for example, Wertenbroch and Skiera (2002) or Elsner *et al.* (2004). Rossi *et al.* (1996), Shiller (2014), and Dubé and Misra (2017), among others, provide estimates for the profitability of personalized pricing relative to uniform pricing in different set-ups.

[13]The literature on channel coordination (McGuire and Staelin, 1983; Moorthy, 1987, 1988; Rey and Stiglitz 1988, 1995) usually focuses on channel coordination without suppliers being able to sell directly to final consumers.

[14]Behavior-based price discrimination refers to the practice of charging consumers different prices dependent on their purchase history. For papers analyzing how this affects dynamic pricing, see Acquisti and Varian (2005) for the monopoly case and, for example, Villas-Boas (1999), Fudenberg and Tirole (2000), Zhang (2011), Rhee and Thomadsen (2017), and Choe *et al.* (2018) for the case of (imperfect) competition.

papers analyze why a vertically integrated firm has the incentive to raise wholesale prices to a non-integrated downstream rival to dampen price competition (see e.g., Salinger, 1988, Ordover *et al.*, 1990, Hart and Tirole, 1990, Chen, 2001, and Bourreau *et al.*, 2011).[15] However, if the rival adds value to the industry, for example, by offering a differentiated product, foreclosure takes place only partly, as the integrated firm benefits from entry through the wholesale revenue.[16] Instead, our paper shows that an integrated firm may fully deny access to its products if price discrimination downstream is feasible.

3 The Model

Supply: A monopoly brand manufacturer, firm A, sells its good through a direct distribution channel and can also rely on an independent retailer, firm B. In order to highlight the strategic motive for mono or dual distribution, we assume away any fixed costs of opening a new distribution channel. For simplicity, all variable costs are also assumed to be zero.[17]

Wholesale contracts: In the baseline model, we focus on two-part tariffs of the form $T(q) = F + wq$, where F denotes the fixed fee and w the uniform wholesale price charged by A to B, and q is the quantity bought by B. In Section 7, we consider the case of linear wholesale contracts.

Demand: The two firms offer differentiated distribution services: A's offering better suits high-value consumers, whereas B can reach out to additional, lower-value consumers. Specifically, final consumers are indexed by their location x, which is uniformly distributed along the real line $[0, +\infty[$, and the willingness-to-pay of a consumer located at x for the offering (the "product", thereafter) of firm $i = A, B$ is given by:

$$u_i(x) = \max\{r_i - s_i x, 0\},$$

where $r_A > r_B > 0$ and $s_A > s_B > 0$, as illustrated by Figure 1. The willingness-to-pay for the two products is thus positively correlated across consumers, which is consistent with vertical differentiation.[18] By contrast, this setting departs from standard horizontal differentiation models, such as Hotelling, which rely instead on negative

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[15]Rey and Tirole (2007) provide an overview of this literature.

[16]An exception is Weeds (2016) who shows that vertically integrated content providers may fully foreclose rival distributors due to dynamic considerations. If consumers have switching costs, exclusivity confers a market share advantage, which is beneficial in the future.

[17]Introducing positive unit costs would not affect the results.

[18]Consider for example a classic model à la Shaked and Sutton (1982), in which firm i can supply quality q_i at unit cost c_i, where $q_A - c_A > q_B - c_B$, and consumers' valuations are θq_i, where θ is uniformly distributed over $[0, 1]$. Using firm i's margin m_i as its strategic pricing decision, $x = 1 - \theta$, $r_i = q_i - c_i$, and $s_i = q_i$, we then have: $\theta q_i - p_i = u_i(x) - m_i$, where $u_i(x) = r_i - s_i x$.

correlation. We discuss in Section 5 how different shapes of the willingness-to-pay functions affect the results.
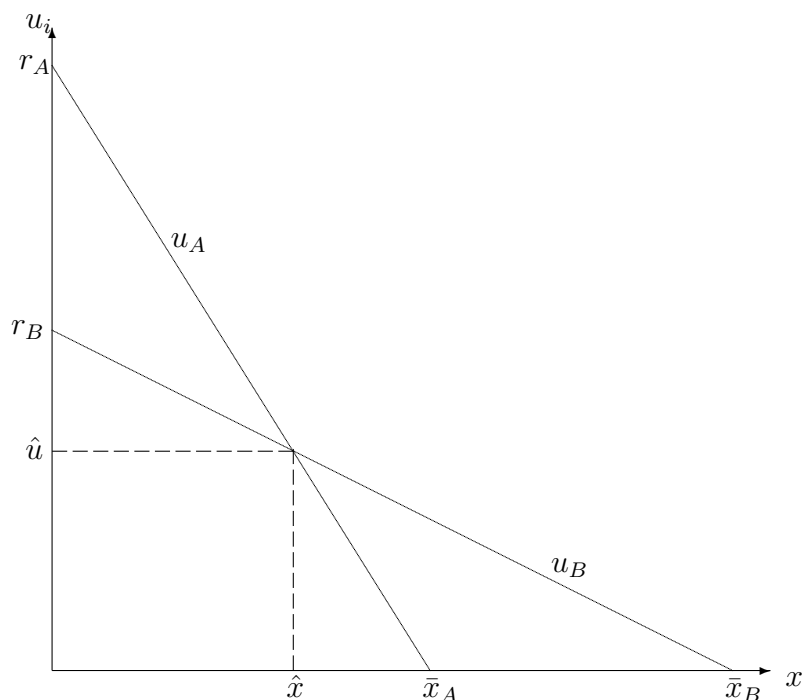

Figure 1: Consumers' valuations

The consumer located at:

$$x = \hat{x} \equiv \frac{r_A - r_B}{s_A - s_B},$$

receives the same gross utility from buying either firm's product, given by:

$$\hat{u} \equiv \frac{s_A r_B - s_B r_A}{s_A - s_B}.$$

If $\hat{u}$ was negative, mono distribution would be the optimal strategy for A, as any consumer willing to pay a positive price prefers A's product (i.e., B does not add value to the industry). Therefore, we focus on $\hat{u}$ being positive,[19] which amounts to:

$$\frac{r_B}{r_A} \equiv \rho > \sigma \equiv \frac{s_B}{s_A}. \tag{1}$$

Consumers with $x < \hat{x}$ then favor A whereas consumers with $x > \hat{x}$ favor B. In addition, consumers are willing to buy firm i's product at a positive price as long as:

$$x < \bar{x}_i \equiv \frac{r_i}{s_i}.$$

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[19]When showing the equilibrium distribution regimes graphically (i.e., Figures 4 and 6), however, we cover both cases, $\hat{u} > 0$ and $\hat{u} \leq 0$.

Retail competition: Firms A and B compete in prices for consumers. We consider the cases where they charge uniform prices (non-discrimination) and where they can charge personalized prices according to consumers' locations (perfect discrimination). In Section 5, we also consider the case where only B can charge personalized prices. Firm i's price is denoted by p_i under uniform pricing, and by $p_i(x)$ under personalized pricing.

Timing: The timing of the game is as follows. In the first stage, A offers a wholesale contract, which B then accepts or rejects. B becomes active if it accepts the offer, otherwise A remains the only active firm. In the second stage, active firms simultaneously set their prices. Each consumer x then observes all prices and decides whether or not to buy, and from which firm to buy. Finally, if active, B orders the quantity from A to satisfy its demand and pays the wholesale tariff, and profits realize.

Our solution concept is subgame perfection. In the case of price discrimination, asymmetric Bertrand competition for each consumer x is known to generate multiple equilibria. Following the literature, we focus on the equilibrium in which the firm offering the lower value prices at cost.[20]

In what follows, we say that dual distribution is optimal for A when B is active in equilibrium, that is, if A offers and B accepts a wholesale contract that enables B to serve some consumers.

Remark: Bargaining power. We assume that A has all the bargaining power at the wholesale stage (namely, it makes a take-it-or-leave-it offer to B). However, in some industries retailers may have some bargaining power as well.[21] As we will show below, allowing for such bargaining power does not affect the analysis: distribution strategies are the same as in our baseline setting.

Remark: Wholesale personalized pricing. We focus on the case in which personalized pricing is possible at the retail but not at the wholesale level. That is, the wholesale contract cannot be conditioned on consumers' types. While this would allow the firms to maximize the industry profit, it is usually infeasible. Manufacturers are often unable to monitor which consumers their distributors are selling to; and even if they could obtain that information, it would be difficult to verify in a court of law.

4 Uniform Pricing

We first analyze the situation in which personalized pricing is not possible. If only A is active, it faces the monopoly demand $(r_A - p_A)/s_A$; it thus charges the monopoly price

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[20]This is the unique Coalition-Proof Nash equilibrium (in particular, it is the Pareto-dominant equilibrium from the firms' standpoint) and is also the unique trembling-hand perfect equilibrium.

[21]This is especially true when the retailer is a big platform, such as Amazon.

$r_A/2$ and obtains a profit of (where the subscript U stands for *Uniform pricing*):

$$\Pi_U^m = \frac{r_A^2}{4s_A}.$$

If instead A and B share the demand, retail prices p_A and p_B must be such that some consumers favor A whereas others favor B. Let x_{AB} denote the consumer type indifferent between buying from A and B, and x_B denote the consumer type indifferent between buying from B and not buying. They are given by:

$$x_{AB}(p_A, p_B) = \frac{r_A - p_A - r_B + p_B}{s_A - s_B} \quad \text{and} \quad x_B(p_B) = \frac{r_B - p_B}{s_B}.$$

If the market is shared between the two firms, $x_B > x_{AB}$. The profit functions of the two firms are then $\Pi_A = D_A(p_A, p_B)p_A + D_B(p_A, p_B)w + F$ and $\Pi_B = D_B(p_A, p_B)(p_B - w) - F$, with:[22]

$$D_A(p_A, p_B) = \frac{r_A - p_A - r_B + p_B}{s_A - s_B} \quad \text{and} \quad D_B(p_A, p_B) = \frac{r_B - p_B}{s_B} - \left(\frac{r_A - p_A - r_B + p_B}{s_A - s_B} \right).$$

In the first stage, A sets w taking into account that, in the second stage, each firm sets its retail price so as to maximize its own profit. We obtain that A optimally chooses a wholesale price such that $p_A^* > p_B^*$ and both firms are active:

Proposition 1: *Under uniform pricing, dual distribution is the unique optimal strategy for A.*

Proof: See Appendix A.

The intuition is illustrated by Figure 2. Under mono distribution, A faces an inverse monopolistic demand $p_A = u_A(x) = r_A - s_A x$; maximizing its profit $u_A(x)x$ leads it to serve consumers $x \leq x_A^m = r_A/(2s_A)$; the resulting profit is then $\Pi_U^m = p_A^m x_A^m$, where $p_A^m = u_A(x_A^m)$. As A can appropriate B's profit through the fixed fee, dual distribution is profitable if it increases the industry profit. To see that this is indeed the case, note first that A can replicate the outcome of mono distribution by charging $w^m = u_B(x_A^m)$. This induces A to charge the monopoly price p_A^m and prevents B, which must charge at least w^m, from profitably attracting any consumer. Indeed, consumers with $x > x_A^m$ are not willing to pay w^m, and those with $x < x_A^m$ prefer A's product at price $p_A^m = u_A(x_A^m)$ to B's product at price $w^m = u_B(x_A^m)$. Consider now a small reduction in the wholesale price that induces B to serve some additional consumers by charging $p_B = w^m - dp$. In the resulting equilibrium, B cannot obtain a negative profit (it could avoid any loss by charging $p_B = w$) and A cannot obtain less than what it would earn by charging $\hat{p}_A = p_A^m - dp$, so as to maintain A's market share: $x_A = x_A^m$. In this case, B would sell

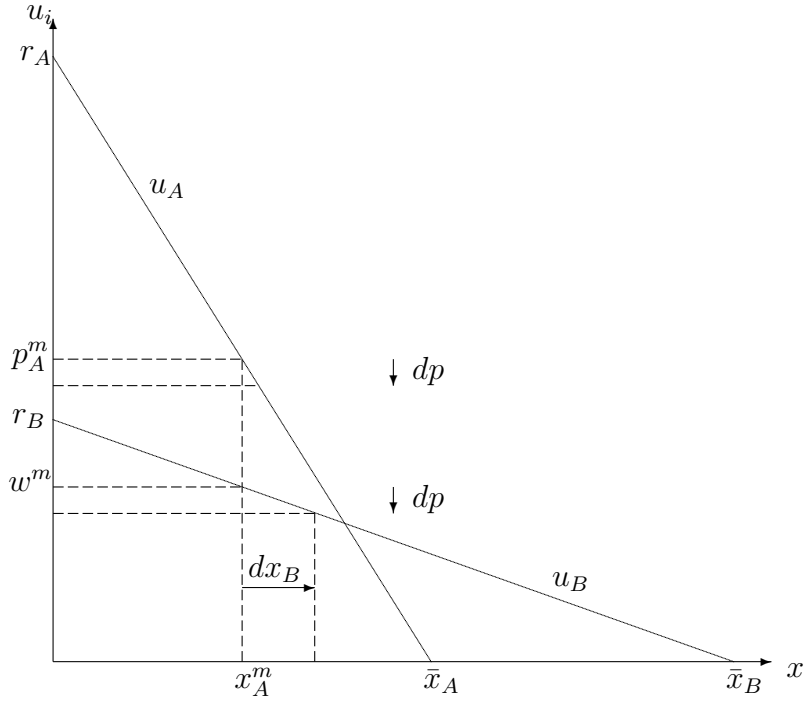[22]Without loss of generality, we restrict attention to $p_i \leq r_i$.

Figure 2: Uniform prices

a quantity dx_B implicitly given by $dp = u'_B(x^m_A) dx_B$. Hence, the industry profit cannot be lower than $(p^m_A - dp) x^m_A + w dx_B \simeq \Pi^m_U + w^m dx_B - x^m_A dp$, which exceeds Π^m_U: as B faces a more elastic monopolistic demand (that is, $|u'_B(x)| / u_B(x) < |u'_A(x)| / u_A(x)$), it has an incentive to increase the market demand beyond $x_B = x^m_A$ by slightly lowering its price below $w^m = u_B(x^m_A)$; hence, $w^m dx_B > x^m_A dp$.

This argument shows that A always benefits from dual distribution. We note that the optimal wholesale price may be substantially lower than w^m, in which case B's market share can be significant.

The intuition does not hinge on demand being linear, and the result holds more generally as long as B's monopolistic output exceeds that of A.[23] We also note that the result does not hinge on the assumption that A has the bargaining power in the first stage. As the firms can share the industry profit through the fixed fee F, they both have an incentive to choose the wholesale price w so as to maximize this industry profit, given the retail price response in the second stage. Hence, even if bargaining power is more balanced, firms will still choose the same wholesale price w and dual distribution will still arise.

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[23]We provide a proof of this statement in Appendix H.

5 Personalized pricing

We now consider the situation in which both firms can charge personalized prices. If B rejects A's offer, then A charges each consumer x a price equal to her utility $u_A(x)$, and thus obtains a profit of (where the subscript P stands for *Personalized pricing*):

$$\Pi_P^m = \int_0^{\bar{x}_A} (r_A - s_A x)\, dx = \frac{r_A^2}{2 s_A}. \tag{2}$$

We now turn to equilibria in which B accepts A's offer, starting with the retail stage.

5.1 Retail competition

As firms now compete for each consumer x, three cases can arise.

If $u_B(x) < w$, then B cannot offer a positive value to consumers without incurring a loss; A then charges the monopoly price $p_A = u_A(x)$.

If instead $u_A(x) < w \le u_B(x)$, A would have to price below w to win the consumer, and is therefore better off letting B serve this consumer; hence, B wins the competition by charging the monopoly price $u_B(x)$ (and A charges a price equal to its opportunity cost w, or any other price exceeding $u_A(x)$).

The most interesting case occurs when $w \le u_A(x), u_B(x)$. A's profit from such a consumer type is either $p_A(x)$, if A serves the consumer itself, or w, if instead B serves the consumer. As a consequence, w constitutes A's opportunity cost from serving the consumer. As w is B's real cost, a standard Bertrand argument applies: for consumers x with $u_i(x) > u_j(x)$, for $i \neq j \in \{A, B\}$, firm i wins the competition and sells to the consumer at price $p_i(x) = w + u_i(x) - u_j(x)$, whereas the other firm sets $p_j(x) = w$.[24]

5.2 Wholesale negotiation

We now turn to the determination of the wholesale contract. We first note that, as long as A charges a wholesale price w above $\hat{u}$, B is inactive in equilibrium: it is dominated by A in the consumer segment $x < \hat{x}$, and cannot offer a positive value at a profitable price in the segment $x > \hat{x}$. The profit thus cannot exceed Π_P^m.

If A sets $w \le \hat{u}$, both firms are active in the continuation equilibrium. Let:

$$\tilde{x}_i(w) \equiv \frac{r_i - w}{s_i}, \tag{3}$$

denote the marginal consumer willing to buy product i at price w. The profits of the

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[24]For the consumer type x with $u_A(x) = u_B(x)$, both firms set a price of w.

two firms can be expressed as $\Pi_A + F$ and $\Pi_B - F$, where:[25]

$$\Pi_A = \int_0^{\hat{x}} [w + u_A(x) - u_B(x)]\, dx + w\,[\tilde{x}_B(w) - \hat{x}],$$

and:

$$\Pi_B = \int_{\hat{x}}^{\tilde{x}_A(w)} [u_B(x) - u_A(x)]\, dx + \int_{\tilde{x}_A(w)}^{\tilde{x}_B(w)} [u_B(x) - w]\, dx.$$

These profit functions are illustrated by Figure 3, where the hatched area represents the industry profit. The first term in A's profit comes from consumers $x < \hat{x}$
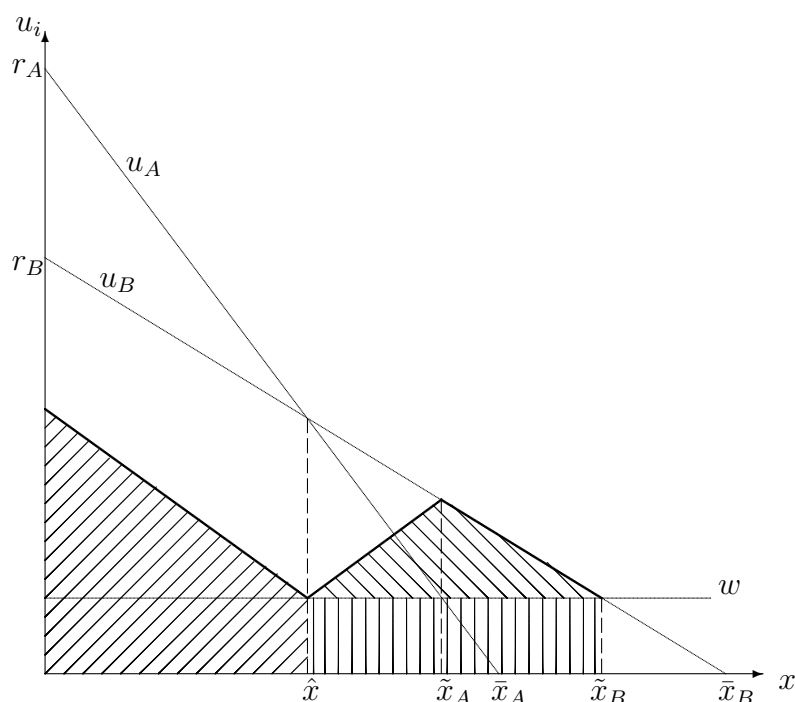

Figure 3: Profits

(first region in Figure 3): A offers them a higher value, and serves them at price $w + u_A(x) - u_B(x)$. The second term in A's profit reflects the wholesale revenue generated by consumers served by B (the other two rectangles in the Figure). B's profit comes from consumers for whom it offers a higher value, and can also be split in two parts. The first term corresponds to consumers $\hat{x} < x < \tilde{x}_A(w)$ (second triangle), for whom both firms compete, and so B only earns a margin $u_B(x) - u_A(x)$. The second term corresponds to consumers $\tilde{x}_A(w) < x < \tilde{x}_B(w)$ (third triangle), to whom A offers a lower value than w, and so B can extract the full value and earn a margin $u_B(x) - w$.

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[25] As we show in Appendix B, conditional on opting for dual distribution (i.e., $w < \hat{u}$), A finds it optimal to allow B to expand potential demand; that is, B sells to consumers who would not be interested in buying from A at any positive price (i.e., w is sufficiently low that $\tilde{x}_B(w) > \bar{x}_A$).

As A can use the fee to appropriate B's profit in full (i.e., $F = \Pi_B$), it maximizes the industry profit:

$$\Pi(w) = \int_0^{\tilde{x}_A(w)} \left[w + |u_B(x) - u_A(x)| \right] dx + \int_{\tilde{x}_A(w)}^{\tilde{x}_B(w)} u_B(x) dx.$$

Taking the derivative with respect to w (and using $u_i(\tilde{x}_i(w)) = w$ for $i = A, B$) yields:

$$\Pi'(w) = \tilde{x}_A(w) + w\tilde{x}'_B(w). \tag{4}$$

When setting the wholesale price w, A faces the following trade-off. By increasing w, the firm obtains a higher benefit from its inframarginal consumers in the range $x < \tilde{x}_A(w)$: as the two firms compete for these consumers, an increase in w increases the final consumer price by the same amount. However, increasing w has also a negative effect on the marginal consumer, $\tilde{x}_B(w)$, for whom B can charge the full value, $u_B(\tilde{x}_B(w)) = w$. By contrast, the revenue from consumers between $\tilde{x}_A(w)$ and $\tilde{x}_B(w)$ is unchanged, as these consumers continue buying from B and pay their reservation price.

Using (3), the first-order condition $\tilde{x}_A(w) + w\tilde{x}'_B(w) = 0$ yields:[26]

$$w = \frac{s_B r_A}{s_A + s_B}.$$

Comparing the associated profit with the monopoly profit Π_P^m given by (2) yields (recalling the notation $\rho \equiv r_B/r_A \in (0, 1)$ and $\sigma \equiv s_B/s_A \in (0, \rho)$):

Proposition 2: *Under personalized pricing, mono distribution is the unique optimal strategy for A if and only if:*

$$\rho < \sigma \frac{2 + \sqrt{1 - \sigma}}{1 + \sigma}. \tag{5}$$

Proof: See Appendix B.

In contrast to the case with uniform pricing, mono distribution may indeed occur when downstream firms can price discriminate. This result holds despite the fact that, with personalized pricing, the two firms share the market efficiently: consumers $x < \hat{x}$ (resp., $x > \hat{x}$) buy from A (resp., B). This was not true under uniform pricing, as B then sets a lower price and therefore also sells to consumers who have a relative preference for A's product. However, personalized pricing also allows a firm to lower the price charged to marginal consumers down to marginal cost without sacrificing profit on inframarginal ones. This has two implications: first, B can serve additional consumers,

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[26]The profit function is concave as $\Pi''(w) = -(1/s_A + 1/s_B) < 0$.

and thereby expand the market, and second, B prices more aggressively in A's "core market" (i.e., the market in which A has a comparative advantage). This in turn makes A more aggressive. Competition is thus more intense, which dissipates profits, and when this effect prevails, A favors mono distribution. Instead, under uniform pricing, firms tend to focus on exploiting their market power over consumers in their respective core markets, which leads to relatively high prices, and dual distribution.
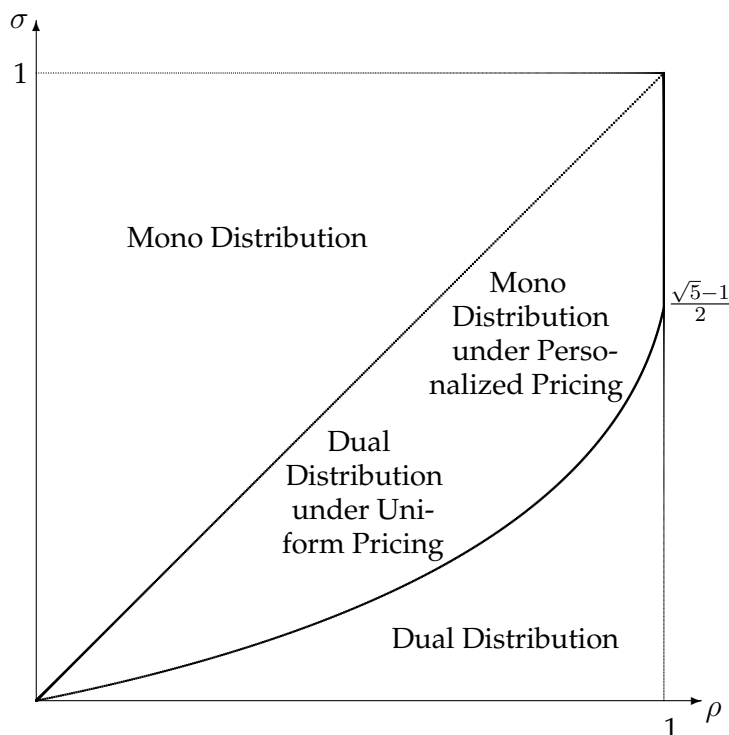

Figure 4: Equilibrium configurations

The equilibrium configurations under uniform and personalized pricing are depicted in Figure 4. Whereas dual distribution is optimal under uniform pricing in the whole range $\rho > \sigma$ (i.e., the range in which B adds value to the industry), the optimal distribution choice under personalized pricing depends on the specific values of ρ and σ. The condition stated in Proposition 2 shows that mono distribution is more likely to be optimal, the lower the net additional value being brought by B, namely, the lower the relative intercept of B's demand function (as measured by ρ) and the steeper the relative slope (as measured by σ). In fact, condition (5) always holds if $\sigma \geq (\sqrt{5}-1)/2 \approx 0.618$, as the right-hand side is then larger than 1. For $\sigma < (\sqrt{5}-1)/2$, the right-hand side is strictly increasing in σ.

Interestingly, when A opts for dual distribution, B always has a significant market share under personalized pricing, but not necessarily so under uniform pricing. In the latter case, B's sales are small if, for example, ρ is close to 0 or σ is close to 1. Instead, with personalized pricing, A chooses mono distribution in such cases. The reason is

that if A sets a wholesale price such that B is barely active (i.e., w slightly below $\hat{u}$), B would add little value to the industry but would still compete for high-valuation consumers. As a consequence, A opts for dual distribution only when the independent retailer serves a sufficiently large share of the market. This result is illustrated with a numerical example in Table 1. We set $\sigma = 0.4$ and then report B's market shares for different values of ρ under uniform pricing and personalized pricing. Notice that B's market share is higher with personalized pricing than with uniform pricing whenever ρ is sufficiently high, so that dual distribution occurs under personalized pricing.

ρ	Uniform Pricing	Personalized Pricing
0.5	0.27	0
0.6	0.45	0
0.7	0.59	0
0.8	0.69	0.74
0.9	0.77	0.89

Table 1: Market shares of firm B in the different regimes

Finally, we note that the correlation between the utility functions is a crucial driver in the choice of distribution channels. If, instead, u_A and u_B were evolving in opposing directions as a function of consumers' types, as in a standard Hotelling model, then dual distribution would always be optimal. Setting w equal to $\hat{u}$ (i.e., the utility of the consumer indifferent between the two firms) would prevent B from competing in A's core market while allowing it to sell in its own core market. Therefore, A would always benefit from dual distribution, even under personalized pricing.

5.3 Comparison between personalized and uniform pricing

As personalized pricing may reduce the number of distribution channels, whether or not A prefers it to uniform pricing may seem unclear. The next proposition shows, however, that personalized pricing always dominates, regardless of whether it leads to mono or dual distribution:

Proposition 3: *A's profit is unambiguously higher under personalized pricing than under uniform pricing.*

Proof: See Appendix C.

The intuition is illustrated by Figure 5, which depicts the equilibrium prices under uniform pricing, p_A^* and p_B^*, and the retail prices that would emerge under personalized pricing if $w = p_B^*$.[27] These latter prices are equal to $p_B^* + u_A(x) - u_B(x)$ for $x \leq \hat{x}$ (i.e., when A serves the consumers) and to $\min \{p_A^* + u_B(x) - u_A(x), u_B(x)\}$ for $\hat{x} < x \leq x_B^*$

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[27] The figure considers the case where $p_B^* < \hat{u}$; a similar reasoning applies when $p_B^* \geq \hat{u}$.
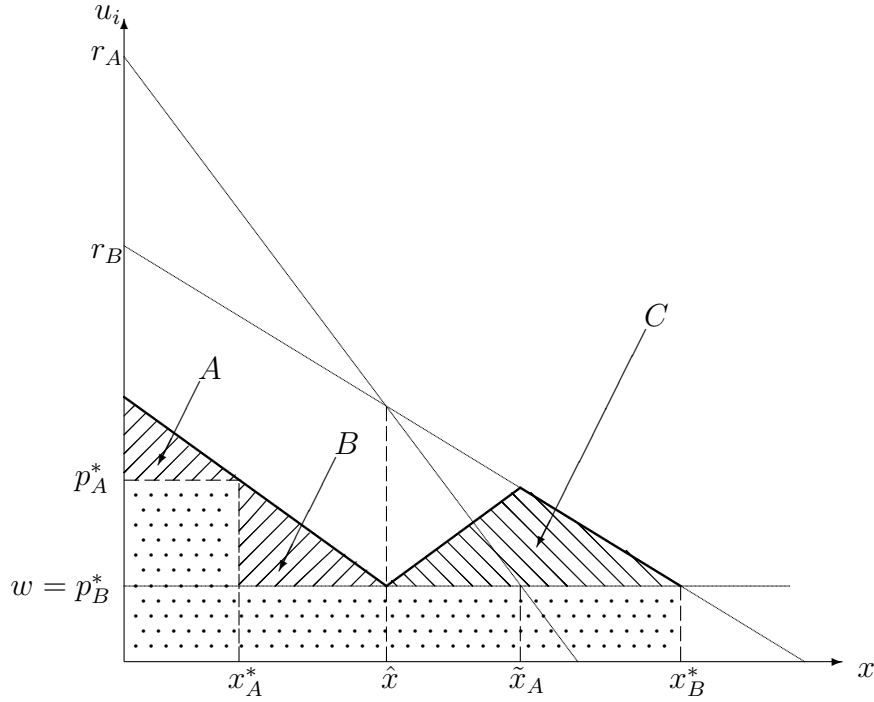
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Figure 5: Profit under personalized pricing for $w = p_B^*$

(i.e., when B serves the consumers). The figure shows that the industry profit (and therefore also A's profit) is unambiguously larger with personalized pricing. The industry profit under uniform pricing corresponds to the dotted area, and the additional profit from personalized pricing corresponds to the hatched area. Consider, for example, the first region (i.e., $x \in [0, x_A^*]$), in which consumers buy from A in both regimes. With uniform pricing, consumers pay $p_A^* = p_B^* + u_A(x_A^*) - u_B(x_A^*)$, whereas with personalized pricing, they pay $p_A(x) = p_B^* + u_A(x) - u_B(x)$. The latter is unambiguously larger because the difference between $u_A(\cdot)$ and $u_B(\cdot)$ is strictly larger for all $x < x_A^*$ than for $x = x_A^*$; the resulting profit increase is depicted by area A. Similar arguments hold in the second region ($x \in [x_A^*, \hat{x}]$), in which the profit increase corresponds to area B, and in the third region ($x \in [\hat{x}, x_B^*]$), in which the profit increase corresponds to area C. By a revealed preference argument, if A chooses to charge a wholesale price other than p_B^* (including one that would lead to mono distribution), its profit must be even larger. It follows that A's profit is higher under personalized pricing than under uniform pricing.

6 Personalized pricing by the independent retailer

In this section, we consider the situation where only the independent retailer can offer personalized prices. For instance, the retailer may be selling products from several different categories, which allows it to gather more data than the brand manufacturer. Alternatively, regulations or antitrust laws may prevent the brand manufacturer from engaging in price discrimination.

Following Thisse and Vives (1988), Liu and Zhang (2006), and Choe *et al*. (2018), we assume that in this hybrid regime A acts as a price leader: it sets p_A before B sets its personalized prices $p_B(x)$. This assumption ensures the existence of a pure-strategy Nash equilibrium. As pointed out by Thisse and Vives (1988), it is natural in the hybrid regime, as A can announce and advertise its uniform price in advance, whereas this may be too complex or costly for B. In addition, as noted by Choe *et al*. (2018) and Chen *et al*. (2018), as the uniform price is public whereas personalized prices are private, the adjustment of the former price is a higher-level managerial decision, and is relatively slower in practice than the adjustment of personalized prices.[28]

We characterize the equilibrium for the hybrid pricing regime following similar steps as before. The detailed analysis is presented in Appendix D, and yields the following result:

Proposition 4: *In the hybrid regime, dual distribution is the unique optimal strategy for A, and the resulting profit is larger than the profit under uniform pricing.*

Proof: See Appendix D.

The intuition behind the result follows the same reasoning as for Proposition 3. Suppose that, in the hybrid regime, A opts for dual distribution and sets $w = p_B^*$ and $p_A = p_A^*$ (i.e., the equilibrium prices under uniform pricing).[29] The industry profit is then larger in the hybrid regime than under uniform pricing: in the region in which A serves consumers, the profit is the same because $p_A = p_A^*$; instead, in the regions in which B sells to consumers, B charges a strictly higher price than p_B^*. Because opting for dual distribution was already optimal with uniform pricing, and yields even more profits in the hybrid regime, it also dominates mono distribution in the latter regime.

So far, we have shown that A benefits if B can set personalized prices, given that A can only charge a uniform price. We now analyze whether or not it is profitable for A

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[28]Our qualitative insights would not be affected if we considered a simultaneous-move game. However, the derivation is more involved as it requires solving for a mixed-strategy equilibrium in prices. It is possible to show that, in this mixed-strategy equilibrium, the upper bound of A's price distribution is the same as the optimal retail price in the sequential game. A then obtains the same demand in the simultaneous game as in the sequential game. As all prices chosen with positive probability in equilibrium must give the same profit, A's equilibrium profit is the same in the two games.

[29]As A acts as a price leader in the hybrid regime, it can now pick w and p_A instead of only w.

to set personalized prices as well. The next proposition shows that this may not be the case:

Proposition 5: *A's profit in the hybrid regime is larger than under personalized pricing if and only if:*

$$\sigma \leq \tilde{\sigma} \quad and \quad \rho > \frac{1 + 4\sigma}{3 + 2\sigma},$$

or:

$$\tilde{\sigma} < \sigma < \frac{\sqrt{5} - 1}{2} \quad and \quad \rho > \sigma + \sqrt{\frac{\sigma(1 - \sigma^2)}{2 + \sigma}},$$

where $\tilde{\sigma}$ is the unique solution to the equation $\sqrt{1 - \sigma} = \sigma(3 + 2\sigma)$ in the range $(0, (\sqrt{5} - 1)/2)$, and is approximately equal to 0.248.

Proof: See Appendix E.

The proposition shows that A does not necessarily benefit if both firms have access to personalized pricing. Instead, it does better in the hybrid regime if r_B (i.e., the intercept of B's demand) is relatively high (ρ large) and s_B (i.e., the slope) relatively low (σ small). The intuition is as follows: when both firms engage in personalized pricing, competition becomes more fierce on the retail market. When, instead, A must charge a uniform price, it focuses on its own core market and charges a relatively high p_A. This dampens the competitive pressure on B, allowing it to extract a larger part of the surplus from low-valuation consumers and from consumers unwilling to buy from A at price p_A (medium range consumers). This however comes at a cost, as A's ability to extract rents from the highest valuation consumers is impeded. The hybrid regime yields higher industry profit whenever there is limited scope for rent extraction by A under personalized prices (i.e., when ρ is large).

From the preceding analysis, we know that three combinations of distribution networks and pricing regimes can be optimal for A. These are: (i) mono distribution and personalized pricing; (ii) dual distribution and personalized pricing; and (iii) dual distribution and hybrid regime. Figure 6 graphically shows which combination is the most profitable. If distributing via B does not expand demand significantly (σ large and/or ρ small), then A opts for mono distribution in order to avoid downstream competition and better exploit price discrimination. When instead the independent retailer brings enough value, the firm opts for dual distribution. It chooses personalized pricing if the independent retailer is not too competitive at the high end of the market (i.e., when ρ and σ are small). This is represented by the lower-left area in the figure. If instead the independent retailer is a relatively strong competitor for high-valuation consumers (that is, if σ is small—and so B expands demand substantially—but ρ is high), A optimally commits to uniform pricing (lower-right area).

Finally, we compare wholesale prices across pricing regimes. Let w_U^* denote the
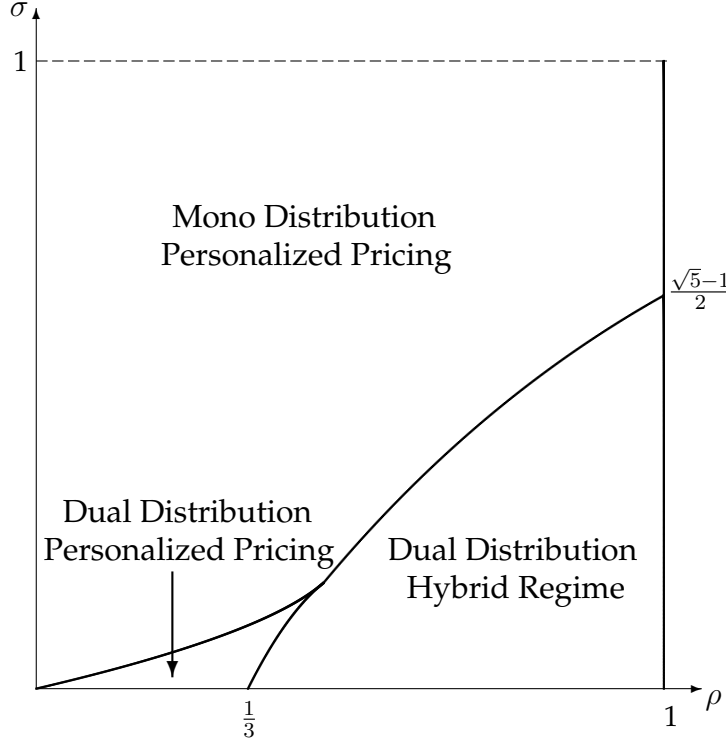
Figure 6: Most profitable regime

equilibrium wholesale price when both firms charge uniform prices, w_P^* denote that when both firms offer personalized prices, and w_H^* denote that in the hybrid regime where only B can offer personalized prices. We have:

Proposition 6: *Comparing the equilibrium wholesale prices across pricing regimes yields:*

$$w_U^* < w_H^* < w_P^*.$$

Proof: See Appendix F.

As the proposition shows, the equilibrium wholesale price is lowest under uniform pricing, and largest under personalized pricing. In the light of the preceding discussion, this result is intuitive. When setting the wholesale price, A faces a trade-off: a lower wholesale price enables B to attract additional low-value consumers, but it also fosters competition for A's high-value consumers. As competition is fiercest when both firms offer personalized prices, the wholesale price is highest in this scenario. Conversely, competition is weakest under uniform pricing, and the wholesale price is therefore lowest in that scenario.

7 Linear Wholesale Tariff

In this section, we study the case of linear wholesale tariffs. Non-linear tariffs such as the two-part tariffs considered so far are efficient, as they enable the upstream and downstream partners to align their interests; by contrast, linear tariffs create double marginalization problems and tend to generate inefficiently high prices. Yet, because of their simplicity or for fairness reasons,[30] linear tariffs are sometimes used in practice.[31] The previous literature on personalized pricing also sometimes focuses on this type of contract.[32] It is therefore useful to check the robustness of our insights when wholesale contracts consist of linear tariffs. The next proposition shows that our qualitative insights indeed carry over in such a setting:

Proposition 7: *Suppose that wholesale contracts are restricted to a simple wholesale price. A's unique optimal strategy is dual distribution whenever it charges uniform prices (regardless of whether B offers uniform or personalized prices). By contrast, when both firms offer personalized prices, mono distribution is optimal if and only if:*

$$\rho \leq \sigma \frac{2 + \sqrt{2(1-\sigma)}}{1+\sigma}.$$

A's equilibrium profit when both firms offer personalized prices is however always higher than in the regime where both firms offer uniform prices and in the hybrid regime where only B offers personalized prices.

Proof: See Appendix G.

The first part of the proposition, which characterizes the optimal distribution strategy in the three pricing regimes, shows that our main insights carry over when considering linear wholesale prices instead of two-part tariffs. The intuition is the same as before. Dual distribution expands demand in the low-value consumer segment but triggers competition with the manufacturer's own distribution channel. As long as that channel charges a uniform price, this competition is not too fierce and can be sufficiently mitigated through an appropriate wholesale price. Dual distribution is therefore optimal. When instead both firms offer personalized prices, competition is tougher; mono distribution is then optimal if B does not add enough value to the industry. Compared with the previous setting, the range where mono distribution is optimal is now even larger, as a linear wholesale price contract does not allow A to extract as much profit from B.

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[30]Cui *et al.* (2007) show that a linear wholesale price contract can be efficient if the retailer is inequity averse when comparing its profit with that of the manufacturer.

[31]This is, for example, the case of the U.S. pay-TV industry; see Crawford and Yurukoglu (2012) and Crawford *et al.* (2018).

[32]See, for example, Liu and Zhang (2006).

Turning to the second part of the proposition on the profitability of the pricing regimes, the result is slightly different than with two-part tariffs: the profit is now unambiguously highest when both firms offer personalized prices. The reason is that, with linear wholesale pricing, A cannot extract the profit of B to the same extent as with a two-part tariff. This is more detrimental in the hybrid regime, because A's benefit from committing itself to non-personalized pricing is precisely to increase the profit obtained from B's sales.

8   Conclusion

This paper analyzes the effects of personalized pricing on the incentives of a brand manufacturer to opt for dual distribution. Adding an independent distribution channel enables the manufacturer to reach out to different consumer groups but also triggers retail competition with its own distribution channel. We show that when charging a uniform retail price, the manufacturer opts for dual distribution regardless of whether or not the independent retailer uses personalized pricing. This is no longer true when the manufacturer can also offer personalized prices through its own distribution channel. In this case, the two channels compete more intensely for each type of consumer, which dissipates profit to such an extent that the manufacturer opts for dual distribution only when the independent retailer expands demand significantly. These insights hold regardless of whether the wholesale contract consists of a two-part or a linear tariff.

We also find that the hybrid regime—in which only the retailer can offer personalized prices—may lead to a higher profit for the manufacturer than the scenario in which both firms can do so. Although the manufacturer can extract less surplus from its high-value consumers, it benefits from reduced competition in this scenario.

Our most important implication for marketing and distribution managers is that the extent to which price discrimination is feasible not only affects the pricing strategy but also the optimal distribution network. With prices becoming more and more finely tuned to consumer tastes, brand manufacturers risk more fierce competition with pure retailers, even if these retailers may appeal to different consumer groups than do the manufacturers. This calls for a cautious use of new distribution channels when price discrimination is possible at a finely-tuned level. This holds particularly for products which are relatively easy to sell online and for which consumer data and purchase history is available. It can then be more profitable to rely solely on direct distribution and avoid intra-brand competition. By contrast, dual distribution is beneficial for products where price discrimination is hard to achieve.

Another implication is that adopting a non-discriminatory pricing policy can be a

profitable strategy. This is particularly true for companies facing the opportunity of distributing their products through a big data-intensive retailer, which can perform price discrimination. In that case, not using consumer data for its own distribution channel can achieve the right balance between rent extraction (by the retailer) and the avoidance of fierce intra-brand competition. This strategy is more profitable the more a manufacturer can appropriate the profits of the retailer (for example, with a fixed payment in the wholesale contract).

We conclude by briefly discussing an interesting avenue for future research emerging from our model. Our analysis takes consumers' preferences as given. However, firms can influence them by adjusting the selection of their products or by investing in quality. In addition, consumers' preferences for the retailer's offering are likely to be a combination of the decisions made by the manufacturer and by the retailer. Analyzing whether the incentives to shape demands are aligned, and the resulting implications for wholesale contracts in such an extended scenario, constitutes a fruitful direction for future research.

Proofs

Appendix A: Proof of Proposition 1

To solve for the subgame-perfect equilibrium, we proceed by backward induction and first determine the reaction functions in the downstream stage. To simplify the exposition, we proceed under the assumption that both demands are positive in equilibrium and verify later that this is in fact true. The linearity of the demand functions ensures that firms' profit functions are strictly concave in their prices; hence, firms' reaction functions are characterized by the first-order conditions, which yield:

$$
\begin{aligned}
p_A(p_B; w) &= \frac{r_A - r_B + p_B + w}{2}, \\
p_B(p_A; w) &= \frac{r_B + w}{2} + \frac{s_B(p_A - r_A)}{2s_A}.
\end{aligned}
$$

Combining these reaction functions yields the equilibrium retail prices, as a function of the wholesale price w:

$$
\begin{aligned}
p_A(w) &= \frac{r_A(2s_A - s_B) + s_A(3w - r_B)}{4s_A - s_B}, \\
p_B(w) &= \frac{r_B(2s_A - s_B) + w(2s_A + s_B) - r_A s_B}{4s_A - s_B}.
\end{aligned}
\tag{6}
$$

The associated demands are $D_i(w) = D_i(p_A(w), p_B(w))$.

We now turn to the first stage. It is optimal for A to appropriate the profit generated by B through the fixed fee: $F(w) = [p_B(w) - w] D_B(w)$. It follows that A chooses w in the first stage so as to maximize the industry profit, $\Pi(w) = p_A(w) D_A(w) + p_B(w) D_B(w)$. This profit is again a strictly concave function of w, as its second-order derivative is given by:

$$
\Pi''(w) = -\frac{2s_A(4s_A + 5s_B)}{s_B(4s_A - s_B)^2} < 0.
$$

Hence, the equilibrium wholesale price is characterized by the first-order condition, leading to:

$$
w_U^* = \frac{s_B(4s_A(r_A + r_B) + r_A s_B)}{2s_A(4s_A + 5s_B)}.
$$

Inserting the equilibrium prices into the demand functions D_A and D_B yields:

$$
\begin{aligned}
D_A^* &= \frac{2s_A^2(2r_A - r_B) + s_B(3s_A r_A - 4s_A r_B - s_B r_A)}{2s_A(4s_A + 5s_B)(s_A - s_B)}, \\
D_B^* &= \frac{(2s_A + s_B)(r_B s_A - r_A s_B)}{s_B(4s_A + 5s_B)(s_A - s_B)}.
\end{aligned}
$$

The assumption that the two demand functions intersect at a positive valuation (i.e.,

$r_A/s_A < r_B/s_B$) ensures that both equilibrium demands are positive. Indeed, D_A^* is strictly falling in r_B and is equal to $r_A(2s_A + s_B)/(2s_A(4s_A + 5s_B)) > 0$ at the highest possible value of r_B, which is $r_B = r_A$. Direct inspection of D_B shows that it is positive for $r_A/s_A < r_B/s_B$. As w^* constitutes a global maximum in the relevant range, and achieving $D_B = 0$ is feasible with a high enough w, it follows that in equilibrium it is optimal for A to generate positive sales for B. Indeed, the resulting profit, equal to:

$$\Pi_U^* = \frac{r_A^2 s_B \left(5 s_A s_B + 4 s_A^2 - s_B^2\right) + 4 s_A r_B \left(s_A + s_B\right) \left(s_A r_B - 2 s_B r_A\right)}{4 s_A s_B \left(s_A - s_B\right) \left(4 s_A + 5 s_B\right)}, \tag{7}$$

exceeds the monopoly profit that A can obtain with mono distribution, Π_A^m:

$$\Pi_U^* - \Pi_U^m = \frac{(s_A + s_B)(s_A r_B - r_A s_B)^2}{s_A s_B (4 s_A + 5 s_B)(s_A - s_B)} > 0.$$

Appendix B: Proof of Proposition 2

As noted in the main text, if A's wholesale price w is such that $w \geq \hat{u}$, B will be inactive;[33] hence, A cannot obtain more than Π_P^m. Using the notation $\rho \equiv r_B/r_A \in (0,1)$ and $\sigma \equiv s_B/s_A \in (0, \rho)$, the threshold $\hat{u}$ is:

$$\hat{u} \equiv \frac{s_A r_B - s_B r_A}{s_A - s_B} = r_A \frac{\rho - \sigma}{1 - \sigma}.$$

We now focus on $w \leq \hat{u}$. We need to distinguish whether B finds it profitable to supply (some) consumers uninterested in A's product. From Figure 3, such consumers exist if and only if $\tilde{x}_B(w) > \bar{x}_A$. The latter inequality can only hold if w is sufficiently low, that is:

$$w < \underline{w} \equiv r_B - \frac{s_B}{s_A} r_A = r_A (\rho - \sigma).$$

Note that $\underline{w} = (1 - \sigma) \hat{u} < \hat{u}$.

Region $w \leq \underline{w}$

In this region, where $\tilde{x}_B(w) \geq \bar{x}_A$, as shown in the text, the industry profit is given by:

$$\Pi(w) = \int_0^{\tilde{x}_A(w)} \left[w + |u_B(x) - u_A(x)|\right] dx + \int_{\tilde{x}_A(w)}^{\tilde{x}_B(w)} u_B(x) dx.$$

It is strictly concave in w: using $u_A(\tilde{x}_A(w)) = u_B(\tilde{x}_B(w)) = w$, we have:

$$\Pi'(w) = \tilde{x}_A(w) + w \frac{d\tilde{x}_B}{dw}(w) = \frac{r_A - w}{s_A} - \frac{w}{s_B} = \frac{r_A}{s_A}\left(1 - \frac{1 + \sigma}{\sigma}\frac{w}{r_A}\right),$$

---

[33]Recall that $\hat{u} = u_A(\hat{x}) = u_B(\hat{x})$.

and thus (as $\tilde{x}_A(w)$ and $\tilde{x}_B(w)$ are both linear and strictly decreasing in w):

$$\Pi''(w) = \frac{d\tilde{x}_A}{dw}(w) + \frac{d\tilde{x}_B}{dw}(w) < 0.$$

Region $\underline{w} < w \leq \hat{u}$

When instead $w > \underline{w}$, A's profit includes an additional term, as illustrated by Figure 7. This term corresponds to consumers in the region $\tilde{x}_B(w) < x \leq \bar{x}_A$: B does not find it
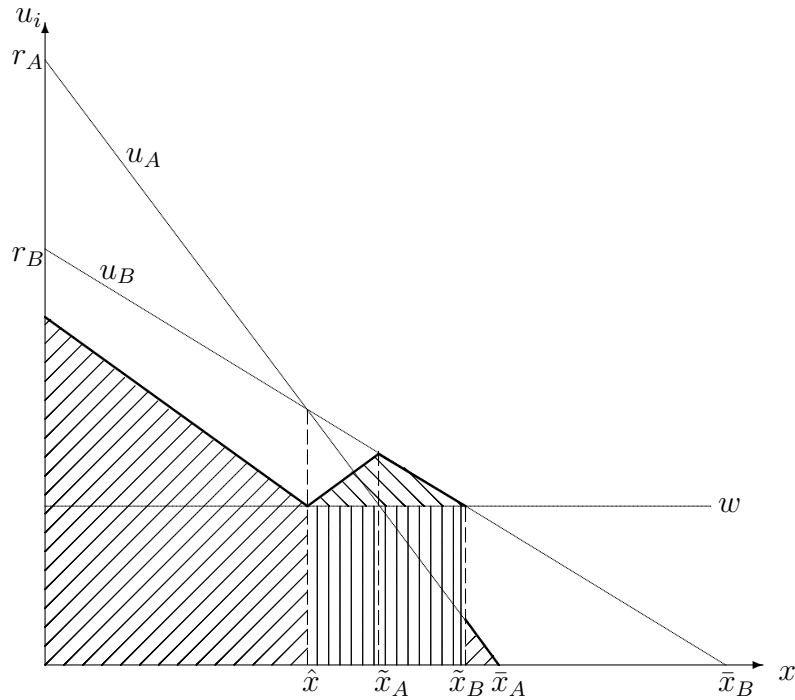

Figure 7: Profits when $w > \underline{w}$

profitable to supply these consumers (as $u_B(x) < w$), but they are still willing to buy from A, which can extract their full surplus. The industry profit can then be written as:

$$\Pi(w) = \int_0^{\tilde{x}_A(w)} \left[w + |u_B(x) - u_A(x)| \right] dx + \int_{\tilde{x}_A(w)}^{\tilde{x}_B(w)} u_B(x) dx + \int_{\tilde{x}_B(w)}^{\bar{x}_A} u_A(x) dx.$$

The first-order derivative is equal to:

$$\begin{aligned}
\Pi'(w) &= \tilde{x}_A(w) + [w - u_A(\tilde{x}_B(w))]\frac{d\tilde{x}_B}{dw}(w) \\
&= (r_A - w)\left(\frac{1}{s_A} + \frac{1}{s_B}\right) - (r_B - w)\frac{s_A}{s_B^2} \\
&= \frac{r_A}{s_A} \frac{\sigma^2 + \sigma - \rho + (1 - \sigma - \sigma^2)\frac{w}{r_A}}{\sigma^2}.
\end{aligned}$$

25

Hence:

$$\Pi'_- \left(\hat{u} \right) = \frac{r_A}{s_A} \frac{1 - \rho}{1 - \sigma},$$

$$\Pi'_+ \left(\underline{w} \right) = \frac{r_A}{s_A} \frac{1 + \sigma}{\sigma} \left(\sigma \frac{2 + \sigma}{1 + \sigma} - \rho \right),$$

$$\Pi'' \left(w \right) = \frac{1 - \sigma - \sigma^2}{s_A \sigma^2}.$$

It follows that $\Pi \left(w \right)$ is strictly concave in w if:

$$\sigma > \hat{\sigma} = \frac{\sqrt{5} - 1}{2} \simeq 0.62,$$

and is instead weakly convex if $\sigma \leq \hat{\sigma}$; in addition, $\Pi' \left(\hat{u} \right) > 0$ whereas $\Pi' \left(\underline{w} \right) \geq 0$ if and only if:

$$\rho \leq \hat{\rho} \left(\sigma \right) \equiv \sigma \frac{2 + \sigma}{1 + \sigma},$$

where $\hat{\rho} \left(\sigma \right)$ increases with σ and exceeds 1 for $\sigma \geq \hat{\sigma}$. Furthermore, not only is the profit function $\Pi \left(w \right)$ continuous at $w = \underline{w}$, its derivative $\Pi' \left(w \right)$ is also continuous:

$$\Pi'_- \left(\underline{w} \right) = \left. \frac{r_A}{s_A} \left(1 - \frac{1 + \sigma}{\sigma} \frac{w}{s_A} \right) \right|_{w = r_A(\rho - \sigma)} = \frac{r_A}{s_A} \frac{1 + \sigma}{\sigma} \left(\sigma \frac{2 + \sigma}{1 + \sigma} - \rho \right) = \Pi'_+ \left(\underline{w} \right).$$

Optimal distribution policy

As long as A charges $w \geq \hat{u}$, B cannot attract any consumer at any profitable price: hence, doing so cannot be more profitable than mono distribution. Furthermore, if $\rho \leq \hat{\rho} \left(\sigma \right)$, then $\Pi' \left(\underline{w} \right) \geq 0$, implying that dual distribution cannot be more profitable than mono distribution:

- in the range $\underline{w} \leq w \leq \hat{u}$, the profit function $\Pi \left(w \right)$ is increasing, as it is quadratic and its derivative is non-negative at both ends of the range (namely, $\Pi' \left(\underline{w} \right) \geq 0$ and $\Pi' \left(\hat{u} \right) > 0$);

- in the range $w \leq \underline{w}$, the profit function $\Pi \left(w \right)$ is again increasing, as it is concave and its derivative is non-negative at the upper end of the range (namely, $\Pi' \left(\underline{w} \right) \geq 0$);

- it follows that the profit achieved under dual distribution cannot exceed $\Pi \left(\hat{u} \right)$, which is less profitable than mono distribution.

As already noted, $\hat{\rho} \left(\sigma \right)$ is increasing in σ in the range $\sigma \in [0, 1]$, and satisfies $\hat{\rho} \left(\sigma \right) \geq 1$ for $\sigma \geq \hat{\sigma}$. It follows that, if $\sigma \geq \hat{\sigma}$, then dual distribution cannot be more profitable than mono distribution, as we then have $\hat{\rho} \left(\sigma \right) \geq 1 \left(> \rho \right)$.

If instead $\sigma < \hat{\sigma}$ and $\rho > \hat{\rho}(\sigma)$, then $\Pi'(\underline{w}) < 0$. From the analysis for the region $w \leq \underline{w}$ above, the first-order condition $\Pi'(w) = 0$ then determines the candidate optimal wholesale price, which is given by:

$$w = w_P^* = \frac{s_B r_A}{s_A + s_B} \equiv \frac{\sigma r_A}{1 + \sigma} \in (0, \underline{w}).$$

The corresponding profit is:

$$\Pi_P^* = \frac{r_A^2}{2 s_A} \frac{\sigma(1 + 3\sigma) - 4\sigma(1 + \sigma)\rho + (1 + \sigma)^2 \rho^2}{\sigma(1 - \sigma^2)}.$$

Compared with the profit from mono distribution, Π_P^m, dual distribution introduces a change in profit equal to:

$$\frac{r_A^2}{2 s_A} \frac{\sigma^2(\sigma + 3) - 4\sigma(1 + \sigma)\rho + (1 + \sigma)^2 \rho^2}{\sigma(1 - \sigma^2)}.$$

The numerator of this expression is a convex quadratic polynomial of ρ and its roots are:

$$\sigma \frac{2 - \sqrt{1 - \sigma}}{1 + \sigma} \quad \text{and} \quad \sigma \frac{2 + \sqrt{1 - \sigma}}{1 + \sigma}.$$

Furthermore, $\hat{\rho}(\sigma)$ lies between these two roots in the relevant range $\sigma < \hat{\sigma}$:

$$\frac{\sigma \frac{2 - \sqrt{1 - \sigma}}{1 + \sigma}}{\hat{\rho}(\sigma)} = \frac{\sigma \frac{2 - \sqrt{1 - \sigma}}{1 + \sigma}}{\sigma \frac{2 + \sigma}{1 + \sigma}} = \frac{2 - \sqrt{1 - \sigma}}{2 + \sigma} < 1,$$

$$\frac{\sigma \frac{2 + \sqrt{1 - \sigma}}{1 + \sigma}}{\hat{\rho}(\sigma)} = \frac{\sigma \frac{2 + \sqrt{1 - \sigma}}{1 + \sigma}}{\sigma \frac{2 + \sigma}{1 + \sigma}} = \frac{2 + \sqrt{1 - \sigma}}{2 + \sigma} > 1,$$

where the last inequality stems from $\sqrt{1 - \sigma} > \sigma$ in the relevant range $\sigma < \hat{\sigma}$. It follows that dual distribution is more profitable than mono distribution if and only if $\sigma < \hat{\sigma}$ and ρ exceeds the larger root, that is, if:

$$\rho > \tilde{\rho}(\sigma) \equiv \sigma \frac{2 + \sqrt{1 - \sigma}}{1 + \sigma}.$$

Note that $\tilde{\rho}(\sigma)$ is increasing in σ in the range $\sigma \leq \hat{\sigma}$, and exceeds 1 in the range $\sigma \geq \hat{\sigma}$. Hence, as $\rho < 1$, the condition $\rho > \tilde{\rho}(\sigma)$ implies $\sigma < \hat{\sigma}$.

Appendix C: Proof of Proposition 3

We start with the case in which B's equilibrium price with uniform pricing is below $\hat{u}$, that is $p_B^* < \hat{u}$. Suppose that A sets the wholesale price w under personalized pricing equal to the p_B^*. B can then profitably serve as many consumers as in the equilibrium

with uniform pricing – i.e., all consumers $x \leq x_B^*$. As a consequence, total demand with personalized pricing is at least as high as with uniform pricing: if $\bar{x}_A \leq x_B^*$, the total demand is the same in both cases, whereas for $\bar{x}_A > x_B^*$, total demand is larger with personalized pricing. In addition, the two consumer types $x = x_A^*$ and $x = x_B^*$ face the same prices in both regimes. The price of the former consumer is $w + u_A(x_A^*) - u_B(x_A^*) = p_A^*$ and the price of the latter consumer is $w = p_B^*$. A therefore obtains the same profit from these consumers.

We now consider consumers $x < x_A^*$. Let us denote the price to consumer type x in the personalized pricing regime with $w = p_B^*$ by $p^{**}(x)$. The price paid by consumers $x < x_A^*$ is $p^{**}(x) = w + u_A(x) - u_B(x) = p_B^* + u_A(x) - u_B(x) > p_B^* + u_A(x_A^*) - u_B(x_A^*) = p_A^*$, where the inequality is due to the fact that the difference between $u_A(x)$ and $u_B(x)$ is falling in x; hence, A obtains a higher profit from consumers $x < x_A^*$ with personalized pricing than with uniform pricing.

Consider next consumers $x_A^* < x < \hat{x}$. With personalized pricing, these consumers pay $p^{**}(x) = w + u_A(x) - u_B(x)$. By contrast, with uniform pricing, they pay $p_B^* = w$, which is lower than $w + u_A(x) - u_B(x)$, as $u_A(x) > u_B(x)$ for these consumers; hence A's profit with personalized pricing is higher than with uniform pricing for consumers $x_A^* < x < \hat{x}$.

Finally, consider consumers $\hat{x} < x < x_B^*$. With personalized pricing, these consumers pay $p^{**}(x) = \min\{w + u_B(x) - u_A(x), u_B(x)\} = \min\{p_B^* + u_B(x) - u_A(x), u_B(x)\} > p_B^*$, where the inequality follows because $u_B(x) > u_A(x)$ for these consumers. This implies that A also obtains a higher profit from consumers $\hat{x} < x < x_B^*$. As a consequence, the profit with personalized pricing is strictly larger than the one with uniform pricing.

We now turn to the case $p_B^* \geq \hat{u}$. Then, the overall amount of sales under uniform pricing will not be larger than $\hat{x}$, which implies that A's profit is strictly lower than $\int_0^{\hat{x}} (r_A - s_A x) dx$. Instead, with price discrimination, A obtains a profit of $\int_0^{\bar{x}_A} (r_A - s_A x) dx$ when choosing mono distribution. Because $\bar{x}_A > \hat{x}$, the latter is larger than the former.

Appendix D: Proof of Proposition 4

We solve the game by backward induction. Given w and p_A, B's price response is such that consumers x with $u_A(x) - p_A > u_B(x) - w$, or:

$$x < \check{x}(w, p_A) = \frac{r_A - p_A - r_B + w}{s_A - s_B},$$

end-up buying from A. Instead, consumers $\check{x}(w, p_A) < x < \tilde{x}_B(w)$ end-up buying from B at price $p_B(x) = u_B(x) - \max\{u_A(x) - p_A, 0\}$. A's variable profit (gross of the fee F)

is therefore given by:

$$p_A \check{x}\left(w, p_A\right) + w\left[\tilde{x}_B\left(w\right) - \check{x}\left(w, p_A\right)\right].$$

Optimizing this with respect to p_A yields:

$$p_A\left(w\right) = w + \frac{r_A - r_B}{2}.$$

We now turn to the wholesale stage. $A's$ seeks to maximize:

$$
\begin{aligned}
\Pi_A\left(w\right) &= p_A\left(w\right)\check{x}\left(w, p_A\left(w\right)\right) + w\left[\tilde{x}_B\left(w\right) - \check{x}\left(w, p_A\left(w\right)\right)\right] + F \\
&= \left(w + \frac{r_A - r_B}{2}\right)\left(\frac{r_A - r_B}{2(s_A - s_B)}\right) + w\left(\frac{r_B - w}{s_B} - \frac{r_A - r_B}{2(s_A - s_B)}\right) + F,
\end{aligned}
$$

subject to (noting that $u_A\left(x\right) = p_A\left(w\right)$ for $x = \left(r_A + r_B - 2w\right)/2s_A$):

$$
\begin{aligned}
F &\leq \int_{\check{x}(w, p_A(w))}^{\frac{r_A + r_B - 2w}{2s_A}}\left[u_B\left(x\right) - u_A\left(x\right) + p_A\left(w\right) - w\right]dx + \int_{\frac{r_A + r_B - 2w}{2s_A}}^{\tilde{x}_B(w)}\left[u_B\left(x\right) - w\right]dx \\
&= \int_{\frac{r_A - r_B}{2(s_A - s_B)}}^{\frac{r_A + r_B - 2w}{2s_A}}\left[(s_A - s_B)x - \frac{r_A - r_B}{2}\right]dx + \int_{\frac{r_A + r_B - 2w}{2s_A}}^{\frac{r_B - w}{s_B}}\left[r_B - s_B x - w\right]dx.
\end{aligned}
$$

It is again optimal to set F so as to appropriate $B's$ profit. Therefore, A maximizes the industry profit given by:

$$\Pi = p_A(w)\check{x}\left(w\right) + \int_{\check{x}(w)}^{\hat{x}(w)}\left[p_A(w) + u_B\left(x\right) - u_A\left(x\right)\right]dx + \int_{\hat{x}(w)}^{\tilde{x}_B(w)}u_B\left(x\right)dx, \qquad (8)$$

where $p_A(w) = w + (r_A - r_B)/2$, $\check{x}(w) = \left(r_A - p_A - r_B + w\right)/\left(s_A - s_B\right)$, $\hat{x}(w) = \left(r_A + r_B\right)/(2s_A) - w/s_A$, and $\tilde{x}_B(w) = \left(r_B - w\right)/s_B$. Maximizing the industry profit with respect to w yields (where the subscript H stands for *Hybrid Regime*):[34]

$$w_H^* = \frac{s_B(r_A + r_B)}{2(s_A + s_B)}.$$

Inserting $w = w_H^*$ into (8), we obtain that the industry profit is given by:

$$
\begin{aligned}
\Pi_H^* &= \frac{r_A^2 s_A s_B + 2r_A^2 s_B^2 - 4r_A r_B s_A s_B - 2r_A r_B s_B^2 + 2r_B^2 s_A^2 + r_B^2 s_A s_B}{4s_A^2 s_B - 4s_B^3} \\
&= \frac{r_A^2}{s_A}\frac{\sigma + 2\sigma^2 - 4\rho\sigma - 2\rho\sigma^2 + 2\rho^2 + \rho^2\sigma}{4\sigma\left(1 - \sigma^2\right)}.
\end{aligned}
$$

We can now show that this profit exceeds the profit obtained under uniform pric-

---

[34]It is straightforward to check that the industry profit is a concave function of w.

ing, which, from (7), can be written as:

$$\Pi_U^* = \frac{r_A^2}{s_A} \frac{4\sigma + 5\sigma^2 - \sigma^3 - 8\rho\sigma - 8\rho\sigma^2 + 4\rho^2 + 4\rho^2\sigma}{4\sigma(1-\sigma)(4+5\sigma)}.$$

We have:

$$\Pi_H^* - \Pi_U^* = \frac{(\sigma - \rho)^2 r_A^2}{4\sigma s_A} \frac{4 + 6\sigma + \sigma^2}{(4+5\sigma)(1-\sigma^2)} > 0.$$

This establishes that the profit in the hybrid regime is larger than under uniform pricing.

From Proposition 1, we know that A optimally chooses dual distribution in case both firms set uniform prices, which leads to a profit of Π_U^*. Because A's profit in the hybrid regime with dual distribution (i.e., Π_H^*) is larger than Π_U^*, it must also be larger than the profit with mono distribution.

Appendix E: Proof of Proposition 5

As shown in the the proof of Proposition 4, A's profit in the hybrid regime is:

$$\Pi_H^* = \frac{r_A^2}{s_A} \frac{\sigma + 2\sigma^2 - 4\rho\sigma - 2\rho\sigma^2 + 2\rho^2 + \rho^2\sigma}{4\sigma(1-\sigma^2)}.$$

We now turn to the profit obtained when both firms offer personalized prices, and first consider the region in which A then opts for dual distribution. From Proposition 2, this is the case when:

$$\rho > \tilde{\rho}(\sigma) = \sigma \frac{2 + \sqrt{1-\sigma}}{1+\sigma},$$

which, together with $\rho < 1$, implies $\sigma < \hat{\sigma}$; the industry profit from dual distribution is then equal to:

$$\Pi_P^* = \frac{r_A^2}{s_A} \frac{\sigma(1+3\sigma) - 4\rho\sigma(1+\sigma) + \rho^2(1+\sigma)^2}{2\sigma(1-\sigma^2)}.$$

Therefore:

$$\Pi_H^* > \Pi_P^* \iff \frac{r_A^2}{s_A} \frac{\sigma + 2\sigma^2 - 4\rho\sigma - 2\rho\sigma^2 + 2\rho^2 + \rho^2\sigma}{4\sigma(1-\sigma^2)} > \frac{r_A^2}{s_A} \frac{\sigma(1+3\sigma) - 4\rho\sigma(1+\sigma) + \rho^2(1+\sigma)^2}{2\sigma(1-\sigma^2)}$$

$$\iff \rho > g(\sigma) \equiv \frac{1+4\sigma}{3+2\sigma}.$$

If instead $\rho \le \tilde{\rho}(\sigma)$, then:

$$\Pi_P^* = \Pi_P^m = \frac{r_A^2}{2s_A},$$

30

and thus:

$$\Pi_H^* \quad > \quad \Pi_P^* \iff \frac{r_A^2}{s_A} \frac{\sigma + 2\sigma^2 - 4\rho\sigma - 2\rho\sigma^2 + 2\rho^2 + \rho^2\sigma}{4\sigma(1 - \sigma^2)} > \frac{r_A^2}{2s_A}$$

$$\iff \quad \rho > \sigma + \sqrt{\frac{\sigma(1 - \sigma^2)}{2 + \sigma}} \text{ or } \rho < \sigma - \sqrt{\frac{\sigma(1 - \sigma^2)}{2 + \sigma}}.$$

As $\rho > \sigma$ by assumption, the only relevant case is:

$$\rho > h(\sigma) \equiv \sigma + \sqrt{\frac{\sigma(1 - \sigma^2)}{2 + \sigma}}.$$

It is easy to check that $\tilde{\rho}(\sigma) < g(\sigma) < h(\sigma)$ (resp., $\tilde{\rho}(\sigma) > g(\sigma) > h(\sigma)$) for $\sigma < \tilde{\sigma}$ (resp., $\sigma > \tilde{\sigma}$), where $\tilde{\sigma} \simeq 0.248$ is the unique solution in $(0, \hat{\sigma})$ to:

$$\sqrt{1 - \tilde{\sigma}} = \tilde{\sigma}(3 + 2\tilde{\sigma}).$$

It follows that A prefers the hybrid regime over the regime in which both firms engage in personalized pricing if and only if:

$$\rho > \min\{g(\sigma), h(\sigma)\} = \begin{cases} g(\sigma) & \text{if } \sigma \le \tilde{\sigma}, \\ h(\sigma) & \text{if } \tilde{\sigma} < \sigma < \hat{\sigma}. \end{cases}$$

Appendix F: Proof of Proposition 6

Under uniform pricing, the equilibrium wholesale price is given by:

$$w_U = \frac{s_B}{2s_A} \frac{4s_A(r_A + r_B) + r_A s_B}{4s_A + 5s_B} = r_A \frac{\sigma}{2} \frac{4(1 + \rho) + \sigma}{4 + 5\sigma},$$

whereas in the hybrid regime it is equal to:

$$w_H = \frac{s_B}{2} \frac{r_A + r_B}{s_A + s_B} = r_A \frac{\sigma}{2} \frac{1 + \rho}{1 + \sigma}.$$

Hence:

$$w_H - w_U = \frac{r_A \sigma^2(\rho - \sigma)}{2(1 + \sigma)(4 + 5\sigma)} > 0,$$

where the inequality follows from $\rho > \sigma$.

By contrast, when both firms offer personalized prices, the wholesale price is either so large that B does not serve any consumer, or equal to:

$$w_P = \frac{s_B r_A}{s_A + s_B} = \frac{r_A \sigma}{1 + \sigma},$$

which satisfies:

$$w_P - w_H = \frac{r_A \sigma}{1 + \sigma} - r_A \frac{\sigma}{2} \frac{1 + \rho}{1 + \sigma} = \frac{r_A \sigma}{2} \frac{1 - \rho}{1 + \sigma} > 0.$$

Therefore, regardless of whether mono or dual distribution is optimal when both firms offer personalized prices, the wholesale price is there higher than in the hybrid regime.

Appendix G: Proof of Proposition 7

With dual distribution, the second stage of the game leads, as before, to downstream prices given by (6). We now consider the first stage for the three pricing regimes.

Under uniform pricing, the profit function of A is now $\Pi_A = D_A p_A + D_B w$. Inserting the corresponding demand functions, p_A and p_B from (6), and maximizing with respect to w, we obtain that the equilibrium wholesale price is (using "**" to distinguish from the equilibrium that arises with two-part tariffs):[35]

$$w_U^{**} = \frac{r_A s_B^2 + 8 r_B s_A^2}{2 s_A (8 s_A + s_B)} = \frac{r_A}{2} \frac{8\rho + \sigma^2}{8 + \sigma}.$$

Inserting w_U^{**} into the profit yields the equilibrium profit with dual distribution:

$$\begin{aligned}
\Pi_U^{**} &= \frac{4 s_A^3 r_B^2 + 8 s_A^2 s_B r_A (r_A - r_B) - r_A^2 s_B^2 (3 s_A + s_B)}{4 s_A s_B (8 s_A + s_B)(s_A - s_B)} \\
&= \frac{r_A^2}{4 s_A} \frac{4\rho^2 + 8\sigma(1 - \rho) - \sigma^2(3 + \sigma)}{\sigma(1 - \sigma)(8 + \sigma)}.
\end{aligned} \tag{9}$$

As in Section 4, it can be checked that demands D_A and D_B are both positive at $w = w_U^{**}$, implying that dual distribution is optimal. Indeed, comparing Π_U^{**} with the profit with mono distribution, $\Pi_U^m = r_A^2 / 4 s_A$, yields:

$$\Pi_U^{**} - \Pi_U^m = \Pi_U^m \frac{4(\sigma - \rho)^2}{\sigma(1 - \sigma)(8 + \sigma)} > 0.$$

Following the same steps for the hybrid regime, we obtain that the optimal wholesale price is $r_B/2$, leading to a profit of:

$$\Pi_H^{**} = \frac{s_A r_B^2 + s_B r_A^2 - 2 s_B r_A r_B}{4 s_B (s_A - s_B)} = \Pi_U^m \frac{\rho^2 - 2\sigma\rho + \sigma}{\sigma(1 - \sigma)}. \tag{10}$$

---

[35]The second-order condition is $-2 s_A (8 s_A + s_B)/(s_B (4 s_A - s_B)^2) < 0$, implying that the profit function is concave.

This profit exceeds that from mono distribution:

$$\Pi_H^{**} - \Pi_U^m = \Pi_U^m \frac{(\sigma - \rho)^2}{\sigma(1 - \sigma)} > 0.$$

We now turn to personalized pricing. As in the case of two-part tariffs, in the range $w \geq \hat{u}$, B is inactive and so A cannot obtain more than the mono distribution profit Π_P^m. We thus focus on $w \leq \hat{u}$, distinguishing again between $w \leq \underline{w} = r_A(\rho - \sigma)$ and $w > \underline{w}$. We start with the former case. With linear tariffs, the profit function of A is:

$$\Pi_A(w) = \int_0^{\hat{x}} [w + u_A(x) - u_B(x)] \, dx + \int_{\hat{x}}^{\tilde{x}_B(w)} w \, dx,$$

which is strictly concave in w:

$$\Pi_A'(w) = \tilde{x}_B(w) + w \frac{d\tilde{x}_B}{dw}(w) = \frac{r_B - w}{s_B} - \frac{w}{s_B} = \frac{r_B - 2w}{s_B}, \tag{11}$$

and thus $\Pi_A'' = -2/s_B < 0$. When instead $w > \underline{w}$, A's profit can be written as:

$$\Pi_A(w) = \int_0^{\hat{x}} w + u_A(x) - u_B(x) \, dx + \int_{\hat{x}}^{\tilde{x}_B(w)} w \, dx + \int_{\tilde{x}_B(w)}^{\bar{x}_A} u_A(x) \, dx.$$

The first derivative is equal to:

$$
\begin{aligned}
\Pi_A'(w) &= \tilde{x}_B(w) + [w - u_A(\tilde{x}_B(w))] \frac{d\tilde{x}_B}{dw}(w) \\
&= \frac{r_B - 2w + r_A}{s_B} - \frac{s_A}{s_B} \frac{r_B - w}{s_B} \\
&= \frac{r_A}{s_A \sigma^2} \left[\sigma - \rho(1 - \sigma) + (1 - 2\sigma) \frac{w}{r_A} \right].
\end{aligned}
$$

Hence:

$$
\begin{aligned}
\Pi_{A-}'(\hat{u}) &= \frac{r_A}{s_A} \frac{1 - \rho}{1 - \sigma}, \\
\Pi_{A+}'(\underline{w}) &= \Pi_-'(\underline{w}) = \frac{r_A}{s_A \sigma}(2\sigma - \rho), \\
\Pi_A''(w) &= \frac{1 - 2\sigma}{s_A \sigma^2}.
\end{aligned}
$$

It follows that $\Pi(w)$ is strictly concave in w if $\sigma > \hat{\sigma}^{**} = 1/2$ and is weakly convex otherwise; in addition, $\Pi_A'(\hat{u}) > 0$ whereas $\Pi_A'(\underline{w}) \geq 0$ if and only if:

$$\rho \leq \hat{\rho}^{**}(\sigma) \equiv 2\sigma,$$

where $\hat{\rho}^{**}(\sigma)$ increases with σ and exceeds 1 for $\sigma \geq \hat{\sigma}^{**}$. Furthermore, the profit function $\Pi_A(w)$ and its derivative $\Pi'_A(w)$ are both continuous at $w = \underline{w}$.

As mentioned above, as long as A charges $w \geq \hat{u}$, it cannot obtain a higher profit than with mono distribution. Furthermore, if $\rho \leq \hat{\rho}^{**}(\sigma)$, then $\Pi'_A(\underline{w}) \geq 0$, implying that dual distribution cannot be more profitable than mono-distribution:

- in the range $\underline{w} \leq w \leq \hat{u}$, the profit function $\Pi_A(w)$ is increasing, as it is quadratic and its derivative is non-negative at both ends of the range (namely, $\Pi'_A(\underline{w}) \geq 0$ and $\Pi'_A(\hat{u}) > 0$);

- in the range $w \leq \underline{w}$, the profit function $\Pi_A(w)$ is again increasing, as it is concave and its derivative is non-negative at the upper end of the range (namely, $\Pi'_A(\underline{w}) \geq 0$);

- it follows that the profit achieved under dual distribution cannot exceed $\Pi_A(\hat{u})$, which is less profitable than mono distribution.

As already noted, $\hat{\rho}^{**}(\sigma)$ is increasing in σ, and satisfies $\hat{\rho}^{**}(\sigma) \geq 1$ for $\sigma \geq \hat{\sigma}^{**}$. It follows that, if $\sigma \geq \hat{\sigma}^{**}$, then dual distribution cannot be more profitable than mono distribution, as we then have $\hat{\rho}^{**}(\sigma) \geq 1 \, (> \rho)$.

If instead $\sigma < \hat{\sigma}^{**}$ and $\rho > \hat{\rho}^{**}(\sigma)$, then $\Pi'_A(\underline{w}) < 0$ and, in the range $w \leq \underline{w}$, from (11), $\Pi_A(w)$ is maximal for $w_P^{**} = r_B/2$, which lies below $\underline{w}$ and yields a profit equal to:

$$\Pi_P^{**} = \frac{2r_A^2 s_B + r_B^2(s_A + s_B) - 4r_A r_B s_B}{4s_B(s_A - s_B)} = \frac{r_A^2}{4s_A} \frac{2\sigma + \rho^2(1+\sigma) - 4\rho\sigma}{\sigma(1-\sigma)}.$$

Compared with the profit from mono-distribution, $\Pi_P^m = r_A^2/2s_A$, dual distribution introduces a change in profit equal to:

$$\Pi_P^{**} - \Pi_P^m = \Pi_P^m\left(\frac{2\sigma - 4\rho\sigma + \rho^2(1+\sigma)}{2\sigma(1-\sigma)} - 1\right) = \Pi_P^m\frac{2\sigma^2 - 4\sigma\rho + (1+\sigma)\rho^2}{2\sigma(1-\sigma)}.$$

The numerator of this expression is a convex quadratic polynomial of ρ and its roots are:

$$\sigma\frac{2 - \sqrt{2(1-\sigma)}}{1+\sigma} \quad \text{and} \quad \sigma\frac{2 + \sqrt{2(1-\sigma)}}{1+\sigma}.$$

Furthermore, $\hat{\rho}^{**}(\sigma)$ lies between these two roots in the relevant range $\sigma < \hat{\sigma}^{**}$:

$$\frac{\sigma\frac{2-\sqrt{2(1-\sigma)}}{1+\sigma}}{\hat{\rho}^{**}(\sigma)} = \frac{2 - \sqrt{2(1-\sigma)}}{2(1+\sigma)} < 1,$$

$$\frac{\sigma\frac{2+\sqrt{2(1-\sigma)}}{1+\sigma}}{\hat{\rho}^{**}(\sigma)} = \frac{\frac{2+\sqrt{2(1-\sigma)}}{1+\sigma}}{2} = \frac{2 + \sqrt{2(1-\sigma)}}{2(1+\sigma)} > 1,$$

where the last inequality stems from $\sqrt{2(1-\sigma)} > 2\sigma$ in the relevant range $\sigma < \hat{\sigma}^{**}$. It follows that dual-distribution is more profitable than mono-distribution if and only if $\sigma < \hat{\sigma}^{**}$ and ρ exceeds the larger root, that is, if:

$$\rho > \tilde{\rho}^{**}(\sigma) \equiv \sigma \frac{2 + \sqrt{2(1-\sigma)}}{1+\sigma}.$$

Note that $\tilde{\rho}^{**}(\sigma)$ is increasing in σ in the range $\sigma \le \hat{\sigma}^{**}$, and exceeds 1 in the range $\sigma \ge \hat{\sigma}^{**}$. Hence, as $\rho < 1$, the condition $\rho > \tilde{\rho}^{**}(\sigma)$ implies $\sigma < \hat{\sigma}^{**}$.

Finally, we turn to the comparison between the profits of the three regimes. We first check that the equilibrium profit is higher in the hybrid regime than under uniform pricing: indeed, subtracting (9) from (10) yields:

$$\Pi_H^{**} - \Pi_U^{**} = \Pi_U^m \frac{(4+\sigma)(\sigma-\rho)^2}{\sigma(8+\sigma)(1-\sigma)} > 0.$$

We now show that the equilibrium profit is even higher under personalized pricing. When dual distribution is then optimal, the equilibrium profit is Π_P^{**}, which exceeds that of the hybrid regime:

$$\Pi_P^{**} - \Pi_H^{**} = \Pi_U^m \frac{(1-\rho)^2}{1-\sigma} > 0.$$

If instead mono distribution is optimal, then personalized pricing yields a profit $\Pi_P^m = r_A^2/2s_A$ and:

$$\Pi_P^m - \Pi_H^{**} = \Pi_U^m \left(2 - \frac{\rho^2 - 2\sigma\rho + \sigma}{\sigma(1-\sigma)}\right) = \Pi_U^m \frac{\sigma - 2\sigma^2 - \rho^2 + 2\sigma\rho}{\sigma(1-\sigma)} > 0,$$

where the inequality follows from the fact that the numerator is decreasing in ρ[36] and, for the lower bound $\rho = \sigma$, is equal to $\sigma(1-\sigma) > 0$.

Appendix H: Generalization of Proposition 1

In this appendix, we generalize Proposition 1 to an extended setting in which consumers with unit demand have valuations $u_A(x)$ and $u_B(x)$ for the products of the two firms, where $u_A(\cdot)$ and $u_B(\cdot)$ are both twice continuously differentiable, x is distributed according to a twice continuously differentiable c.d.f. $G(x)$ over $\mathbb{R}_+$ and:

- $\forall x \in \mathbb{R}_+$, $u_A'(x) < u_B'(x) < 0$;

- $u_i(\bar{x}_i) = 0$ for some $\bar{x}_i > 0$; and

---

[36]Its derivative with respect to ρ is equal to $-2(\rho - \sigma)$, which is negative from (1).

- $u_A(\hat{x}) = u_B(\hat{x}) > 0$ for some $\hat{x} > 0$.

This implies that, as in our baseline model, the curves $u_A(\hat{x})$ and $u_B(\hat{x})$ intersect exactly once, and this intersection occurs in the positive quadrant.

Let:

$$D_i^m(p_i) \equiv G\left(u_i^{-1}(p_i)\right),$$

denote the monopolistic demand for firm i's product:

$$p_i^m \equiv \arg\max_{p_i} p_i D_i^m(p_i),$$

denote firm i's monopoly price:

$$x_i^m \equiv u_i^{-1}(p_i^m),$$

denote the location of the associated marginal consumer, and:

$$
\begin{aligned}
q_i^m &\equiv D_i^m(p_i^m) = G(x_i^m), \\
\pi_i^m &\equiv p_i^m q_i^m,
\end{aligned}
$$

denote the monopoly output and profit. Our working assumption is that B would seek to serve more consumers than A in these monopoly situations:

Assumption A: B's monopoly profit function is strictly quasi-concave and $q_B^m > q_A^m$.

Let $w^m \equiv u_B(x_A^m)$. For $w \geq w^m$, there exists a continuation equilibrium in which A charges its monopoly price, p_A^m, and B does not serve any consumer (e.g., by charging $p_B = w^m$). If instead $w < w^m$, both firms can obtain a positive market share: A then faces a demand:

$$D_A(p_A, p_B) \equiv G\left(\Delta^{-1}(p_A - p_B)\right),$$

where:

$$\Delta(x) \equiv u_A(x) - u_B(x),$$

whereas B faces a demand given by:

$$D_B(p_A, p_B) \equiv D_B^m(p_B) - D_A(p_A, p_B).$$

For the sake of exposition, we will assume that there then exists an equilibrium where both firms obtain a positive market share, which is moreover "well-behaved":

Assumptions B: For any $w \leq w^m$, there exists a unique downstream equilibrium, $(p_A^e(w), p_B^e(w))$, where $p_A^e(w)$ and $p_B^e(w)$ are continuous and increasing in w, and such

that $p_A^e(w^m) = p_A^m$ and $p_B^e(w^m) = w^m$.

We have:

Proposition 1': *Under Assumptions A and B, dual distribution is the unique optimal strategy for A under uniform pricing.*

Proof: Starting from a situation in which A charges $w = w^m$, and thus obtains Π_A^m, consider a small reduction in the wholesale price from w^m to $w < w^m$, together with a fixed fee, $F(w)$, designed to appropriate B's profit (or almost all of it, to ensure acceptance). A then obtains (almost all of) the industry profit, which can be expressed as:

$$\Pi(w) = \Pi_A(w) + \Pi_B(w),$$

where:

$$
\begin{aligned}
\Pi_A(w) &= p_A^e(w) D_A(p_A^e(w), p_B^e(w)) + w D_B(p_A^e(w), p_B^e(w)) + F(w), \\
\Pi_B(w) &= [p_B^e(w) - w] D_B(p_A^e(w), p_B^e(w)) - F(w).
\end{aligned}
$$

By deviating from the downstream equilibrium and charging:

$$\hat{p}_A(w) = p_B^e(w) - u_B(x_A^m) + u_A(x_A^m) = p_A^m + p_B^e(w) - w^m,$$

A would maintain its output of q_A^m, and generate an output $\hat{q}_B = D_B^m(p_B^e(w)) - q_A^m$ for B. Therefore:

$$
\begin{aligned}
\Pi_A(w) &\geq \hat{p}_A(w) D_A(\hat{p}_A(w), p_B^e(w)) + w D_B(\hat{p}_A(w), p_B^e(w)) + F(w) \\
&= [p_A^m + p_B^e(w) - w^m] q_A^m + w [D_B^m(p_B^e(w)) - q_A^m] + F(w) \\
&= \pi_A^m + [p_B^e(w) - w - w^m] q_A^m + w D_B^m(p_B^e(w)) + F(w).
\end{aligned}
$$

Likewise, noting that B could always choose to deviate from the downstream equilibrium and charge $p_B = w$, we have:

$$\Pi_B(w) \geq -F(w).$$

Adding these two inequalities yields (recalling that $\Pi(w) = \Pi_A(w) + \Pi_B(w)$):

$$\Pi(w) - \pi_A^m \geq \phi(w) \equiv [p_B^e(w) - w - w^m] q_A^m + w D_B^m(p_B^e(w)).$$

Note that $\phi(w^m) = 0$ because $p_B^e(w^m) = w^m$ and $D_B^m(w^m) = G(x_A^m) = q_A^m$. Taking the derivative of $\phi(w)$ and evaluating it at $w = w^m$, we obtain (again using $p_B^e(w^m) = w^m$

and $q_A^m = D_B^m (w^m)$):

$$
\begin{aligned}
\phi' (w^m) &= \left[\frac{dp_B^e}{dw} (w) - 1 \right] q_A^m + D_B^m (w^m) + w \frac{dD_B^m}{dp_B^e} (p_B^e (w)) \frac{dp_B^e}{dw} (w) \\
&= \frac{dp_B^e}{dw} (w) \left[D_B^m (w^m) + w^m \frac{dD_B^m}{dp_B^e} (w^m) \right],
\end{aligned}
$$

where the expression within bracket is negative from Assumption A.[37] It follows that a reduction of w below w^m is strictly profitable, implying that dual distribution is the unique optimal mode of distribution.

---

[37]Indeed, Assumption A implies that firm B's optimal monopoly demand is strictly larger than $q_A^m = D_B^m (w^m)$; hence, firm B's monopoly price is below w^m, which implies that the first-order condition evaluated at w^m is negative.

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