

POLICY CHOICES BY AN INCUMBENT - A CASE WITH DOWN-UP PROBLEM,
BIASED BELIEFS AND RETROSPECTIVE VOTING^{*}

Carlos Seixas (cseixas.phd@fep.up.pt), António Brandão (abrandao@fep.up.pt),
Manuel Luís Costa (mlcosta@fep.up.pt)

University of Porto, Faculty of Economics, CEF.UP
Address: R. Roberto Frias, 4200-464 Porto, Portugal

ABSTRACT

Politicians often happen to choose bad policies in democracies. The main question addressed in our model regards which type of incentives an elected politician has to choose bad over good policies. In order to answer it, we focus on two inefficiencies, recently considered in the literature: the down-up problem and voters having biased beliefs about how public policies affect economic outcomes. Additionally, we consider that voters vote retrospectively. The down up problem is caused by the fact that democratic elections alone cannot motivate a politician to undertake socially beneficial projects that do not perform well in the short-run. Biased beliefs and retrospective voting are two well-established empirical regularities about voters. To the extent of our knowledge, we are the first to combine all these effects. Moreover, we assume that the politician receives utility from holding office and from the success of his projects and as to his policy platform, we consider that he can choose any combination of bad (yet popular) policies and good (yet less popular) policies. We are able to show that politicians can choose good long term policy platforms even when those policies have bad short term results. Motivation regarding the success of the projects or an incumbent bias tend to induce the politician to implement a good policy. Unclear responsibilities or campaign promises will have mixed effects on the type of policy implemented.

Keywords: Policy choice; Elections; Voting behavior; Down-up problem; Biased beliefs; Retrospective voting

JEL Classification: D72, D78, D83, D90, P16

^{*} This paper is financed by a FCT (Fundação para a Ciência e Tecnologia) PhD scholarship

1 INTRODUCTION

Politicians often happen to choose bad policies in democracies. Some questions arise. Why do they do it? How can this be explained? What type of incentives can lead to good or bad policies? In order to answer these questions, the literature has considered several inefficiencies, but we focus on two recently under theoretical research: the down-up problem and voters having biased beliefs about how public policies affect economic outcomes. Additionally, we consider that voters vote retrospectively.

The down up problem is caused by the fact that democratic elections alone cannot motivate a politician to undertake socially beneficial projects that do not perform well in the short-run. Gersbach (2004a) and Mueller (2008) deal with this problem. We follow their idea that there is a socially beneficial policy that does not perform well in the short run; the alternative policy is one that leads to good results in the short run, but it is not efficient in the long run.

The biased beliefs has been studied by Beilharz and Gersbach (2004); Caplan (2007), Bischoff and Siemers (2011). As in Bischoff and Siemers (2011), our model micro-founds biased beliefs using mental models. As well, it assumes that voters vote retrospectively, hence, the incumbent evaluation by voters will depend on the current performance of the economy: an incumbent will have a higher (lower) share of votes if the performance is good (bad) (e.g., Paldam 2004). In our case, contrary to Bischoff and Siemers (2011), the bad policy (and not efficient in the long run) allows the politician to build valence. Thus, retrospective voting also becomes part of the explanation of the inefficiency.

To the extent of our knowledge, we are the first to combine these effects: the down-up problem with biased and retrospective voters. Bearing in mind this, our model considers a politician that can either be more motivated by the benefits from holding office, so he pursues the policy that gives him the best chance of being reelected, or he can be mainly motivated by the benefits from the positive results of the policy implemented; the first type is an “office seeker” politician, the second a “policy success-seeker”. Moreover, the political process in the model consists basically in two steps. To start with, the elected politician will choose policy platforms including any mixture of bad yet popular policies and good yet less popular policies. Later, he decides whether he wants to run for reelection or not and the voters decides on the reelection of the politician.

In addition, the basic model is extended in three different ways: adding an incumbent bias, since when a certain politician has won the election his chances of reelection automatically tend to increase; introducing limited accountability, because the government does not fully control the economic outcome; finally, considering penalization of deviations from campaign promises, given that voters do not like to be deceived.

The main result of our paper is that it is possible to implement a purely good policy if its weighted long term gains are higher enough. In addition, motivation plays a role, since the more a politician is “policy success-seeker”, the more he will choose good policy platform. Therefore, we show that the implementation of a good policy does not depend on external incentives as in Gersbach (2003), Gersbach (2004a) and Mueller (2011) or some self-correction mechanism as in Bischoff and Siemers (2011), but depends intrinsically on the benefits and motivation to apply that policy.

The paper is organized as follows. In section 2, we cover the related literature. The model is presented in section 3 and the results in section 4. Section 5 concludes.

2 LITERATURE

In our model, voters behave as in Bischoff and Siemers (2011). Bischoff and Siemers (2011) use the concept of mental models from cognitive psychology in order to micro-found biased beliefs. A mental model is a simplified representation of a real system (e.g., Johnson-Laird 1983; Legrenzi and Girotto 1996).¹ Mental models are different for each individual, so each one will reach particular conclusions when he simulate how a certain policy affects the economy (e.g., Tetlock 1989, 1999; Green et al. 1998; Leiser and Drori 2005). In our model, biased beliefs stem from the way voters assess politicians’ actions, which is done by means of mental simulations.

Again, we follow Bischoff and Siemers’ paper in their use of the notion of valence: incumbents are assessed as competent and assigned a high competence-related valence if they are seen as having reached good macroeconomic results. Their model uses retrospective voting as a way to make valence endogenous, which does not compromise the notion of mental models and biased beliefs.

¹ To know more about mental and how they can be applied to economics see Bischoff and Siemers (2011).

In Bischoff and Siemers's paper, two competing parties are allowed to choose policy platforms including any combination of bad (yet popular) policies and good (yet less popular) policies. Assuming that the good yet unpopular policies produce immediately good results, and vice-versa, Bischoff and Siemers are able to find that it is possible to implement a purely good policy, despite persistently biased beliefs, since in their model retrospective voting serves as a self-correction mechanism in democracy.

Regarding this feature that a good policy does not always produce immediately good results, our approach is similar to Gersbach (2004a) and Mueller (2007). We assume that there is a socially beneficial policy that does not perform well in the short run; and that there is a policy that leads to good results in the short run, but it is not efficient in the long run; as well, they also allow the politician to choose the status quo. These two papers assume that voters are rational, that there are two competing candidates for office, and that they face an initial election period and a subsequent reelection. Candidates are motivated by the office they hold and by the policies they choose. Once a candidate is elected, he will choose which type of policy he will apply.

In Gersbach (2004a), the two politicians differ in their discount factor. The problem to the public is that the discount factor of the politician may be smaller than theirs. Then, the politician does not have incentives to undertake long-term beneficial policies, even if the voters commit to a reelection scheme.

In Mueller (2008), one politician is more "policy success-seeker" and the other more "office seeker". The problem is that some politicians, who are mainly motivated by the positive results of the policy they have implemented, do not have incentives to undertake long-term projects, even if the voters commit to a reelection scheme. This results from the shape of the politicians' utility function and the assumption that voters are fully rational. Voters will only reelect a politician if he chooses a long term policy, so the politician that cares more about office has higher incentives to undertake it in order to guarantee reelection. Thus, in Mueller (2008), a populist politician causes smaller inefficiencies in the political process.

In order to solve these inefficiencies, both Gersbach (2004a) and Mueller (2008) suggest the use of a hierarchy of incentive contracts and elections. The politicians could sign a contract before the first election (ex ante) that if he stands for reelection and it is actually reelected, his payoff will depend on some macroeconomic variable. This contract will motivate him to take the socially desirable projects.

Besides Mueller (2008), there is other literature related to populist politicians. One is Canovan (1981) who defends that popular decisions are vital for democracy and populists might be considered as democratic. Another is Gersbach (2004b) who considers that the (in)competence of the politician is not known to the voters. The populist politician tries to increase his chances of reelection by imitating the behavior of the statesmen. Thus, populism leads to undesirable outcomes since policy decisions are twisted. Our model also shows that populist politicians lead to higher inefficiencies in the outcomes of the political process.

3 THE MODEL

In the model, the government is elected by majority voting. The sequence of events is similar to Gersbach (2004a) and Mueller (2008). There are two periods, $t = 1, 2$.^{2,3} In the first period, an elected politician undertakes a policy project known to the voters.⁴ The policy project generates returns in both periods. The politician is perfectly informed and the government is assumed to be efficient in the sense that it is able to apply the policy chosen.

The game is given as follows:

Stage 1: An elected politician undertakes a policy project known to the voters.

Stage 2: The result from the policy in the first period is revealed to voters. The incumbent decides whether he runs for office again and the public decides on the reelection of the politician based on the policy platform and results. The incumbent is elected with probability Q ($0 \leq Q \leq 1$).

In order to determine the politician's strategy, his possible policy decision in the second stage is irrelevant. Therefore, the strategy will consist only in the definition of policy in the first stage. However, we consider stages in order to identify the timing of actions (stage 1) and outcomes (stage 1 and 2). A further explanation will be made later on.

² This assumption can be justified in three line of arguments: (i) party leads might not have the leadership position at a election in a possible period 3, since this can represent eight to ten years in the future; (ii) there are some democracies where the term limit for the incumbent is two, as the U.S. President; (iii) the next term's policy is likely to be lesser importance to voters.

³ An overview of the literature on term limits for politicians can be seen in Smart and Sturm (2004).

⁴ We can introduce an election between two politicians before the first period. Given the sequence of events, the results of the policy undertaken by the elected politician will be exactly the same.

3.1 Voting Decision

In our model, voters will behave as in Bischoff and Siemers (2011). There is a continuum of voters. Voter turnout is 100%. A voter's utility is affected by individual income and macroeconomic performance. The utility of an individual voter i is given by a utility function:

$$U_i = \sum_{t=1}^2 \delta_i^{t-1} U_{it}(y_{it}, a_t) \quad (1)$$

y_{it} is voter i 's income in period t , a_t is the indicator for the macroeconomic performance in t and $\delta_i \in [0,1)$ is the discount factor.⁵ As mentioned above, the economic policy taken in the first period influences the individual income and the macroeconomic performance in the two periods. η represents the policy vector with K policy dimensions.⁶ η directly influences both the voters' income and the macroeconomic performance. However, η can have different impacts in each voter since they have singular individual characteristics (employment status, sector of employment, education etc.). Besides, due to its influence on the macroeconomic performance a_t , η can have an indirect effect on y_{it} . Therefore:

$$y_{it} \equiv y_{it}(\eta) \text{ and } a_t \equiv a_t(\eta) \quad (2)$$

Thus, voter i 's utility is overall defined by the economic policy η :

$$U_i = \sum_{t=1}^2 \delta_i^{t-1} U_{it}(y_{it}(\eta), a_t(\eta)) \equiv U_i(\eta) \quad (3)$$

The individual voter does not know the true functional forms of $y_{it}(\eta)$ and $a_t(\eta)$. He carries out two tasks in order to deduce the expected values of y_{it} and a_t . First, he assesses the policy platform undertaken *per se* and, second, he considers the valence of the incumbent in order to correct this assessment. In the first task, the individual voter carries out a theoretical exercise. Assuming that the government is efficient, each voter uses his mental model in order to simulate the policy's impact on y_{it} and a_t and thus on U_i . Voter i 's mental model mm_i is a function that assigns certain policies vectors to

⁵ Alternatively, if we fixed $\delta = 0$, the voters would just take into account the first period.

⁶ A particular policy field will be represented in each row of the policy vector and the value in that row indicates the measured of the policy instrument applied. For example, one row can represent education and the value could be the expenditures on education or other.

estimates of y_i and a_i , that is, $mm_i: R^K \rightarrow R^2$. Estimated values are labeled with a “hat”, e.g., \hat{y}_{it} is the income in period t for voter i , estimated by his mental model:

$$\{\hat{y}_i(\eta), \hat{a}_i(\eta)\} = mm_i(\eta) \quad (4)$$

Voter i assesses the isolated effects of the policy platform on his utility, using these estimations: $\hat{U}_i = \hat{U}_i(\eta)$. The voter prefers the policy with a higher estimated value, *ceteris paribus*.

Then, the second task is to correct this assessment by considering the politician’s competence. The responsibility hypothesis is applied to the incumbent, as it is shown by the empirical literature. Therefore, macroeconomic performance in his previous term of government, $t - 1$, will determine the incumbent’s competence in term t . Each voter contrasts the observable indicator index a_{t-1} with his individual benchmark value labeled \bar{a}_i . Then, voter i will attribute the following valence to the politician:

$$\gamma_{it}(\eta) = a_{t-1} - \bar{a}_i \quad (5)$$

An incumbent is considered competent by voter i if $\gamma_{it}(\eta) > 0$, that is, if the observed result of the politician's policy is greater than voter i 's benchmark. Otherwise, voter i considers the politician to be not competent: $\gamma_{it}(\eta) < 0$. In the model, this evaluation of competence is as *perceived*: e.g., an unexpected good macroeconomic performance leads to the conclusion that the politician is competent.

Combining the results, voter i at the time of the reelection will take into account his estimated utility, given the policy; that is, voter i will consider both the quality of the policy as it stands, and the politician’s perceived valence. This utility is denoted by $\hat{U}_{it}^y = \hat{U}_{it}^y[\hat{U}_i(\eta), \gamma_{it}(\eta)]$

We reach the following expression for the probability of reelection given by q_i , using the concept of probabilistic voting (Enelow and Hinich 1984):

$$q_i = f_i(\hat{U}_{it}^y) = f_j(\gamma_{it}(\eta), \hat{U}_i(\eta)) \quad (6)$$

$$\frac{\partial q_i}{\partial \gamma_{it}} > 0 \text{ and } \frac{\partial q_i}{\partial \hat{U}_i^y} > 0 \quad (7)$$

We assume that probability q_i is additively separable in two probabilities, one results from the expected utility and the other from the valence perceived. The expected vote

share of the politician in the reelection is obtained by making the integral across all voters' probabilities:

$$Q(\gamma_{it}(\eta), \hat{U}_i(\eta)) = \int_{i=0}^1 q_i di = \Lambda(\eta) + V_t(\eta) \quad (8)$$

The platform-related and the valence related vote-share are represented by the integrals $\Lambda(\eta)$ and $V_t(\eta)$, respectively.

3.2 Policy Platforms

The elected politician will choose the policy platform that maximizes his payoff. He can undertake two pure policy platforms: a long term, good policy η_g and a short term, bad policy η_b . We assume that the mental models of the majority of voters assert $\hat{U}_i(\eta_b) > \hat{U}_i(\eta_g)$ in order to follow the idea of biased beliefs. However, the opposite ordering holds for the true utility values, as we will see below. Only a minority of voters applies mental models that yield the accurate preference ordering, so biased beliefs hold in the aggregate.

On the politician's side we assume that he has complete information. A bad policy generates a positive result in the first period with $y_{i1}(\eta_b) > 0$ and $a_1(\eta_b) > \bar{a}_i \forall i \in [0,1]$ and a negative in the second period $y_{i2}(\eta_b) < 0$ and $a_2(\eta_b) < 0 \forall i \in [0,1]$. A good policy generates a bad result in the first period with $y_{i1}(\eta_g) < y_{i1}(\eta_b)$ and $a_1(\eta_g) < \bar{a}_i \forall i \in [0,1]$ and a good result in the second period with $y_{i2}(\eta_g) > 0$ and $a_2(\eta_g) > 0 \forall i \in [0,1]$.^{7, 8}

The total utility to voter i from the bad and good policies is given by:

$$U_i(\eta_b) = U_{i1}(y_{i1}(\eta_b), a_1(\eta_b)) + \delta_i U_{i2}(y_{i2}(\eta_b), a_2(\eta_b)) \quad (9)$$

$$U_i(\eta_g) = U_{i1}(y_{i1}(\eta_g), a_1(\eta_g)) + \delta_i U_{i2}(y_{i2}(\eta_g), a_2(\eta_g)) \quad (10)$$

Moreover, we assume that:

$$U_i(\eta_g) > U_i(\eta_b) \forall i \in [0,1] \quad (11)$$

⁷ For a matter of simplification, it is assumed that this is applied to all voters. However, it can also be assumed that this applies to more than a half of the voters.

⁸ As in Gersbach (2003), we could also assume that the good policy has uncertain short-term consequences. However, the results will be qualitatively equal.

Thus, the good policy is better for the society considering the true utility values. This is especially important when short term results differ from long term results. For instances, some labor market reforms lead to higher unemployment in the short term, but can lead to higher employment in the long run. Higher investment in education leads to the growth in human capital in the future, but can lead to superior taxes in the short run. Another case that can produce negative results in the short term, but benefits in the long term is the transition of centrally planned economies to market economy.

Given the policies' results presented above, a politician that wins the first election and undertakes a bad policy η_b will be allocated a positive valence, and a politician that wins the first election and undertakes a good policy η_g will be allocated a negative valence. We define:

$$V_2(\eta = \eta_b) \equiv v \quad (12)$$

$$V_2(\eta = \eta_g) \equiv -v \quad (13)$$

Therefore, there is an advantage in applying the bad policy.

With respect to the platform-related part of the expected vote share, we define:

$$\Lambda(\eta = \eta_b) = \frac{1}{2} + f \quad (14)$$

$$\Lambda(\eta = \eta_g) = \frac{1}{2} - f \quad (15)$$

Parameter $f > 0$ measures the effect of biased beliefs: after the first election, if the politician undertakes η_b , he will increase his chance of reelection by f , while if he undertakes η_g , he will decrease his chance of reelection by f . Overall, the difference in platform related part of the expected vote share is $2f$ in favor of the bad policy. We restrict the function value to the interval $[0,1]$ and assume $\frac{1}{2} + f + v \leq 1$ (see equation (8)).

Therefore, the bad policy is relatively more advantageous in terms of reelection probability. However, it produces worse economic results than the good policy.

3.3 Politician's utility

As in Gersbach (2004a) and Mueller (2008), the elected politician cares about the economic results he produces while in office. This benefit from office can be interpreted as, either personal satisfaction from the macroeconomic performance, or privately

appropriated gains. In any case, this is a measure of private benefit (satisfaction or gains) from office, which is a function of the macroeconomic performance, or of the economic results of the policy. The private benefit is given by $\tau \cdot \max\{a_t, 0\}$ where τ is some number $0 < \tau < 1$. The parameter τ defines the degree to which the politician benefits from the macroeconomic performance.

The politician also gets utility from holding office, given by $Z > 0$. Z can be monetary (e.g. wages) as well as non-monetary benefits (e.g. status or the pleasure of being in power).⁹

We define as zero the politician's utility of outside options. Therefore, the net surplus of a politician not holding office is normalized to zero. We assume that the politician has discount factor ρ with $\rho \in [0,1)$. Θ denotes the expected utility of the politician after the first election and before deciding the policy to implement, as evaluated with reference to stage 1:

$$\Theta = (1 - m)Z + m\tau a_1 + \rho Q[(1 - m)Z + m\tau a_2] \quad (16)$$

The parameter m , with $0 < m < 1$, is the fraction the politician assigns to the results he gets once in office, while $1 - m$ is the fraction of benefits from holding office. The values m is exogenously given a priori to the politician. If m is close to 1, this means that the politician is mainly motivated by the policies he undertakes. A low value of m means that the politician is mainly concerned by the reelection; in other words, a politician with a high value is a "policy success-seeker", while a politician with a low value is more like an "office-seeker" and may thus be called populist.

As $Q > 0$, running for reelection is a weakly dominant strategy for a politician no matter the policy undertaken. Therefore, the case where the politician does not want to run for reelection is not taken into account.

4 POLITICAL EQUILIBRIUM

We assume the following tie-breaking rule in order to hold a pessimistic view: the politician will choose the populist platform η_b , if he is indifferent between the two platforms.

⁹ The non-monetary benefits are transformed in monetary value in order to calculate all utility elements in one utility function.

4.1 Nash Equilibria in mixed compromise platforms

The politician can choose a compromising platform that combines both populist and good policy elements, since we allow him to choose any linear combination of the two pure strategies. β defines the portion of elements from the populist platform η_b chosen by the politician after the first election. The strategy and the equilibrium are described by the share of populist elements in the politician's policy platform. Given an interpretation in terms of quality, a policy platform will have more quality, the smaller the share of populist elements in it.

Remember that we do not deal with the policy actions after the reelection; in fact, the results in the second stage are totally determined by the strategy choice in stage 1. We could consider a policy choice in stage two, but that it would be simple and redundant: the politician would choose the policy with best payoff in just that period.

The weighted average of the pure platforms' impact gives the expected and actual impact of a mixed platform on economic outcomes:

$$a_t(\beta\eta_b + (1 - \beta)\eta_g) = \beta a_t(\eta_b) + (1 - \beta)a_t(\eta_g) \quad (17)$$

$$mm_i(\beta\eta_b + (1 - \beta)\eta_g) = \beta mm_i(\eta_b) + (1 - \beta)mm_i(\eta_g) \quad (18)$$

With respect to the impact of mixed policy platforms on vote shares, this implies:

$$\Lambda(\beta) = \frac{1}{2} + \beta f \quad (19)$$

Lastly, we assume that the valence effect of a mixed platform is the linear combination of the single effects:

$$V_2(\beta) = \beta \cdot v + (1 - \beta) \cdot (-v) = v(2\beta - 1) \quad (20)$$

Then, the politician's payoff reads:

$$\begin{aligned} \Theta(\beta) = & (1 - m)Z + m\tau\beta a_1(\eta_b) \\ & + \rho \left[\frac{1}{2} + \beta f + v(2\beta - 1) \right] [(1 - m)Z + m\tau(1 - \beta)a_2(\eta_g)] \quad (21) \end{aligned}$$

Before we present the proposition, it is important to explain some probabilities:

- $\frac{1}{2} + v + f$ is the maximum reelection probability that a politician can get. Only choosing a purely bad policy, he will get it. We will denote this probability by Q_b^{max}
- $f + 2v$ is the marginal reelection probability, that is, $\frac{\partial Q}{\partial \beta}$. We will denote this probability by Q_{mg}
- $\frac{1}{2} - v$ is the maximum reelection probability under purely good policy; this probability is Q_g^{max} .

Proposition 1

- (a) If $\rho m \tau a_2(\eta_g) (Q_g^{max} - Q_{mg}) \geq m \tau a_1(\eta_b) + \rho(1 - m) Z Q_{mg}$, $\beta = 0$
- (b) If $\rho m \tau a_2(\eta_g) Q_b^{max} \leq m \tau a_1(\eta_b) + \rho(1 - m) Z Q_{mg}$, $\beta = 1$
- (c) If $\frac{m \tau a_1(\eta_b) + \rho(1 - m) Z Q_{mg}}{Q_b^{max}} < \rho m \tau a_2(\eta_g) < \frac{m \tau a_1(\eta_b) + \rho(1 - m) Z Q_{mg}}{(Q_g^{max} - Q_{mg})}$, $\beta \in (0, 1)$

Proof: See Appendix.

The politician faces a clear trade-off: if he chooses a more populist policy he gets more private benefits in the first period and a higher reelection probability, but he gets lower private benefits in the second period if he is reelected. Therefore, when the politician increases his chances of reelection by being more populist, he is losing payoff in the second period.

A politician will choose the entire good policy (case (a)) if the private benefits from it in the second period, weighted by the difference between the reelection probability under it and the marginal reelection probability,¹⁰ are higher than the private benefits from the bad policy in the first period plus the benefits of holding office in the second period weighted the marginal reelection probability.¹¹

A politician will choose purely bad policies (case (b)) if the private benefits from the good policy in the second period weighted by the maximum reelection probability¹² are

¹⁰ In this case, the politician will be reelected with probability $\frac{1}{2} - v$. However, he does not choose any element of the bad policy, so will lose the marginal reelection probability.

¹¹ This benefit is constant whatever the policy. However, the politician has a better chance of getting it if the reelection probability is higher. Thus, the benefit will be favorable to the bad policy since it provides better chances of reelection given by the marginal reelection probability.

¹² In this case, the politician will not get any private benefit in the second period, but he will get the maximum reelection probability. Thus, $\rho m \tau a_2(\eta_g) (\frac{1}{2} + v + f)$ is a loss that must be compensated for.

lower than the private benefits from the bad policy in the first period plus the benefits of holding office in the second period weighted by the marginal reelection probability.

For intermediate values, the politician will chose a mixture of platforms.

If the politician is myopic and does not take into account the second period and the reelection (i.e. $\rho = 0$), only case (b) is feasible. Thus, a myopic politician always chooses purely populist platform, because he only considers the private benefits in the first period that are favorable to the bad policy.

Note that if $Q_g^{max} \leq Q_{mg}$, that is, $3v + f \geq \frac{1}{2}$ only case (b) is feasible. The politician will not have incentives to undertake a purely good policy if the probability of reelection under it is inferior to the marginal reelection probability. In this case, the politician loses too much reelection probability if he does not choose a purely bad policy.

The motivation that a politician has regarding the policy that he implements also has a role. This results in the following corollary:

Corollary 1. The higher m , the lower β , that is, the more a politician is “policy-success” seeker, the lower will be the fraction of bad policies applied.

Proof: See Appendix.

In our model, the more a politician is “policy success-seeker” the more good policies he chooses, since they give him higher private benefits. He values the policies’ outcomes, and the policy that gives better overall results is the good one.

4.2 Extension 1: biased valence evaluation and the incumbent bias

This extension is based on Bischoff and Siemers (2011). It is shown in the literature that there is a systematic pro-incumbent bias: if a certain party or candidate has won the election, his probability of reelection directly increases (e.g., Alesina and Rosenthal 1995). Thus, we introduce an incumbent bias $\omega > 0$. The voters who elected the incumbent will make a biased assessment of him, resulting in an incumbent bias. Let P be the share of votes that the elected politician had. Thus, a vote share $M(P)$ that the politician can anticipate in the reelection, conditional on having been voted into office will weigh the incumbent bias. Given that incumbency requires a vote share of $\frac{1}{2}$ or higher, we get $\frac{1}{2} \leq M(P) < 1$, with $\partial M(P)/\partial P \geq 0$ for all $0 < P < 1$.

$M(P)$ cannot be deduced analytically, but as Bischoff and Siemers (2011) show it can be found a nearly perfect approximation via simulations. The approximation is $M(P) = \max\left(\frac{1}{2}, P\right)$. In our model, the politician chooses the policy to implement after the election, so he already knows P and consequently $M(P)$. Thus, $M(P)$ is a fixed value at the time of the decision. Therefore, we can define $\Omega = \omega \cdot M(P)$, where $0 < \Omega < 1$.

The politician's payoff is:

$$\Theta(\beta) = (1 - m)Z + m\tau\beta a_1(\eta_b) + \rho \left[\frac{1}{2} + \beta f + v(2\beta - 1) + \Omega \right] [(1 - m)Z + m\tau(1 - \beta)a_2(\eta_g)] \quad (22)$$

Proposition 2 Suppose $0 < P < 1$, incumbent bias ω , and $M(P) = \max\left(\frac{1}{2}, P\right)$. Then, the policy in equilibrium becomes less populist the higher the incumbent bias is: $\frac{\partial \beta}{\partial \Omega} < 0$.

Proof: See Appendix.

In our model, and contrary to Bischoff and Siemers (2011), the incumbent bias allows the politician to have a higher probability of reelection without having to choose bad policies, since here, in our model, the bias is the same whatever the policy. Then, the politician can benefit more from the good policy private benefits in the second period without having to build up valence. Therefore, he chooses a higher fraction of good policies.

4.3 Extension 2: limited accountability

This extension is based on Bischoff and Siemers (2011): the government does not fully control the economic outcome. It is hard to define the part of the economic performance the incumbent is really accountable for. The literature on retrospective voting suggests that the incumbent is punished or rewarded independently of his responsibility. Here, we distinguish between two types of factors. First, exogenous shocks like strong turns of the global business cycle or natural disasters can affect the incumbent (e.g., Achen and Bartels 2004; Leigh 2009). Second, the economy in one country may depend on the policy choices made by others governments or international institutions. In federal countries like the U.S. or Germany, the federal policies heavily influence the economic results from states. In the EU, countries' economic performance is influence by each other and by the EU institutions. This pattern is intensified by the practice of excessive

fiscal equalization among regions in federal countries (e.g, Kiewiet 2000; Stehn and Fedelino 2009). Consequently, the politician cannot control some fraction of the economic outcome, since it depends on exogenous shocks or the policy choice of exogenous actors.

Therefore, the incentives to build up valence change for the incumbent. Independently of being caused or not by the incumbent, the overall economic results of the country, will define the voters' opinion of the incumbent's valence. Thus, we change the incumbent's valence as follows:

$$\begin{aligned} V_2(\beta, \beta^{ex}) &= k[\beta \cdot v + (1 - \beta) \cdot (-v)] + (1 - k)[\beta^{ex} \cdot v + (1 - \beta^{ex}) \cdot (-v)] + \varepsilon \\ &= v[k(2\beta - 1) + (1 - k)(2\beta^{ex} - 1)] + \varepsilon \quad (23) \end{aligned}$$

Variable k ($0 < k < 1$) is the fraction of the economic outcome the incumbent can control. It follows that $1 - k$ is defined by other actors. Let β^{ex} be the strategy of these exogenous actors. The term ε captures the stochastic effects of economic shocks such as rain and temperature. For this reason, ε covers all unpredictable events, with $E(\varepsilon) = 0$. The incumbent forms an expectation $0 \leq E(\beta^{ex}) \leq 1$, since the policies of the other actors are not totally predictable.

Then, the politician gets private benefits from the economic results, so his utility will take into account the final result from his and the others' policies.¹³ Then, the payoff reads:

$$\begin{aligned} \Theta(\beta) &= (1 - m)Z + m\tau\{k\beta a_1(\eta_b) + (1 - k)E(\beta^{ex})a_1(\eta_b)\} \\ &\quad + \rho \left[\frac{1}{2} + \beta f \right. \\ &\quad \left. + v[k(2\beta - 1) + (1 - k)(2E(\beta^{ex}) - 1)] \right] \left[(1 - m)Z \right. \\ &\quad \left. + m\tau[k(1 - \beta)a_2(\eta_g) + (1 - k)(1 - E(\beta^{ex}))a_2(\eta_g)] \right] \quad (24) \end{aligned}$$

¹³ Alternatively, the economic results can have different importance if they are obtained from the politician's policies or from outside policies. Regarding the fraction of control and the expected strategy of exogenous actors, the conclusions are equal to proposition 3 with more complexity. In addition, we can conclude that if the politician values more his own policies, he will select better ones.

Proposition 3 Suppose the politician faces limited control

- i. Politicians become less populist if the fraction of control increases, unless the effect of the long term policy gains is negative and higher in absolute value than the effect of short term policy gains, that is,

$$\left\{ \begin{array}{l} \frac{\partial \beta}{\partial k} \geq 0 \text{ if } C > 0 \wedge |Cm\tau\rho a_2(\eta_g)| \geq A\rho(1-m)Z + Bm\tau a_1(\eta_b) \\ \frac{\partial \beta}{\partial k} < 0 \text{ otherwise} \end{array} \right.$$

With $A < 0$; $B < 0$; $C \leq 0$

- ii. If the expected action beyond the control of the government becomes qualitatively better, that is, $E(\beta^{ex})$ decreases, the politician becomes more populist: $\frac{\partial \beta}{\partial E(\beta^{ex})} < 0$, for all $0 < k < 1$ and $0 \leq E(\beta^{ex}) \leq 1$

Proof: See Appendix.

Regarding the effect of the fraction of control, an increase in means a higher share of the payoff under control by the politician and a smaller share of the uncontrolled part. The uncontrolled payoff, as well as the controlled one, impinges on private benefits in both periods and on the probability of reelection.

The effects of the bad policy gains (given by $A(1-m)Z + Bm\tau a_1(\eta_b)$) will lead to a decrease in populism, since the politician can reduce it and still benefit, receiving a larger part of the controlled payoff. The reduction of uncontrolled payoff with respect to bad policy gains is not taken into account because it does not affect the controlled payoff.

The effect of the private returns from the good policy in the second period can either be negative or positive. There are two effects in opposition: (i) the increase in the control of the payoff in the second period and the reduction of the uncontrolled probability of reelection; (ii) the increase in the control of the probability of reelection and the reduction of uncontrolled payoff in the second period.¹⁴ If the second effect dominates, the politician will choose a lower β in order to get more payoff in the second period from the private returns of the good policy without affecting the probability of reelection that he controls ($C < 0$). If the first effect dominates, the politician will

¹⁴ In this case, the uncontrolled payoff affects the controlled payoff, so the uncontrolled probability of reelection and the uncontrolled payoff in the second period will influence the effect of the private returns from the good policy in the second period.

choose a higher β in order to amplify his chances of reelection without affecting the second period payoff that he controls ($C > 0$).

The overall effect will depend on which effect dominates: the one from the short term policy gains or the one from the long term policy private returns.

Similar to Bischoff and Siemers (2011), the quality improves if politicians are more pessimistic about the quality of policy choices by the external actors, that is, if they think that the outside agents will choose bad policies they have to compensate by choosing good policies.

4.4 Extension 3: Penalization due to deviations from the campaign promises

Politicians seeking office make promises. They do it in the conviction that voters' beliefs about the policies the politician will implement and about the capabilities of the politician will change. However, these promises may later be harmful to an office holder seeking re-election. If the politician deviates from his campaign promises, his chances of reelection will decrease. In the literature there are some papers that study this: Austen-Smith and Banks (1989), Wittman (1990) and Aragonés et al. (2000). In this extension, we do not derive campaign promises and we just consider this as an exogenous effect. For this purpose, we extend the model changing stage 1:

Stage 1: An elected politician undertakes a policy project known to the voters. The politician will take into account that before the first election, he had promised to the voters that he would implement a given β^* fraction of elements of the bad policy, with $0 \leq \beta^* \leq 1$. The reelection probability decreases by g times the absolute value of the deviation, that is, $g|\beta - \beta^*|$ with $g > 0$. Thus, the politician's payoff reads:

$$\begin{aligned} \Theta(\beta) = & (1 - m)Z + m\tau\beta a_1(\eta_b) \\ & + \rho \left[\frac{1}{2} + \beta f + v(2\beta - 1) \right. \\ & \left. - g|\beta - \beta^*| \right] [(1 - m)Z + m\tau(1 - \beta)a_2(\eta_g)] \quad (25) \end{aligned}$$

In the spirit of the paper, we will also assume that an increase in β leads to an increase in the reelection probability, so $f + 2v \pm g > 0$.

Proposition 4

- i. When the politician is less populist than what he promised, an increase in the costs of deviating from the policy platform promised will lead to a more populist platform applied: $\frac{\partial \beta}{\partial g} > 0$ if $\beta < \beta^*$
- ii. When the politician is more populist than what he promised, an increase in the costs of deviation from the policy platform promised will lead to a more populist platform if the private benefits from the short term policy are higher than the private benefits from the long term policy, that is $\frac{\partial \beta}{\partial g} > 0$ if $m\tau a_1(\eta_b) > \rho m\tau a_2(\eta_g) [Q_g^{max} + \beta^* Q_{mg}] \wedge \beta > \beta^*$, or lead to a less populist platform if the private benefits from the short term policy are lower than the private benefits from the long term policy, that is: $\frac{\partial \beta}{\partial g} < 0$ if $m\tau a_1(\eta_b) < \rho m\tau a_2(\eta_g) [Q_g^{max} + \beta^* Q_{mg}] \wedge \beta > \beta^*$.

Proof: See Appendix.

When the politician is less populist than what he promised, an increase in the costs of deviating from the policy platform promised will lead to a more populist platform applied. The politician will become more populist in order to decrease its costs of deviating.

When the politician is more populist than what he promised, an increase in the costs of deviation from the policy platform promised will lead to a more populist platform if the private benefits from the short term policy are higher than the private benefits from the long term policy. The politician prefers to receive more in the first period, since the private benefits in that period are higher and chooses a more populist platform, which compensates the higher costs of deviating and consequently the lower probability of reelection. On the other hand, an increase in the costs of deviation from the policy platform promised can also lead to a less populist platform if the private benefits from the short term policy are lower than the private benefits from the long term policy. The private benefits in the second period are higher, so the politician decreases the populism in order to save costs from deviating and consequently to increase his reelection probability; besides, it also allows him to get higher private benefits in the second period.

5 CONCLUSION

The model deals with the choice of policy by an incumbent and intends to include in the analysis the down-up problem with biased and retrospective voting. There are several conclusions possible. First, we are able to show that politicians can choose good long term policy platforms even when those policies have bad short term results. Second, a policy success-seeker politician tends to implement a good policy. Third, the fact that incumbency gives an advantage for reelection will induce politicians to choose better policies. Fourth, if the economic results cannot be fully controlled by the incumbent and voters cannot distinguish responsibilities, policy outcomes can either become worse or better; however, a high expected level of populism from external agents improves the quality of policies chosen by the incumbent. Fifth, with higher costs of deviating from campaign promises, the policies actually applied become worse if these are below the campaign promise and can become either better or worse, if above. The main conclusion of the paper is that the implementation of a good policy does not depend on external incentives or some self-correction mechanism, but on the benefits and motivation to apply that policy.

Let us now relate our conclusions with the literature of reference. On one hand, in line with Bischoff and Siemers (2011), our model demonstrates that some crises can be prevented if the outcome of the good policy in the long term is higher enough. And otherwise, the worst outcomes will be prevented, since the choice of mixed policy platforms by the politician will lead to mediocre, but not bad, results.

On the other hand, the economic outcomes can be poor if the positive effects of good, long term policies do not become evident within one term, since the incentives to offer them are weak. This can be applied to important issues as high public debt, weak financial market regulation or global warming. One possible solution might be incentive contracts as in Gersbach (2003), Gersbach (2004a) and Mueller (2008) since they could internalize the future economic outcomes in the politician's payoff. Actually, incentive contracts and other possible external mechanisms could be a potential extension of the paper. However in this setting, this proposal of contracting may be seen as theoretically flawed. On one hand, contracting in the political process involves many dimensions and variable combination of degrees of such dimensions. On the other, on each side of any interesting contract that could be thought of, the number of elements in the contracting parties would require explicit mandates for negotiation of contracts. As Dixit (1996) and

others explain, the solution is not contracting, the solution of the political process is the democratic set up, as we know it or how we could imagine it. The political set up of a democracy is the way to resolve the problems of the difficulty of contracting. Contracting is a mistake, not a solution by any means¹⁵.

Apart from the discussion on the role of incentives in private benefits, another aspect is the role of wages as an element of compensation from holding office (Z). According to our model, the wage will not have a role in the first period. However, in the second period, a wage will make the incentives from holding office higher, so it will induce the politician to apply bad policies in order to build up valence and get reelected. In any case, the model is not immediately amenable to study how the magnitude of wages affects the quality of candidates or their motivations; this can be, however, a topic for further research.

Another topic for further research concerns federalist countries and federations. We could extend the scope of the model in order to include not only the expected action beyond the control of the government, but also the action itself. If the expectation would not be accurate, we could think of the government making mid-term changes in its policies, which might have consequences on the reelection.

A final topic for further research is deriving the campaign promises. We just consider this as an exogenous effect, however, the campaign promises can be derived in a model of competition between politicians to get elected to office in an earlier stage of the game.

REFERENCES

- Achen, C. H., and Bartels, L. M. (2004), "Blind retrospection: electoral responses to droughts, flu, and shark attacks", Working Paper n° 2004/199, UCLA, Los Angeles
- Alesina, A., and Rosenthal, H. (1995), *Partisan politics, divided government, and the economy*. Cambridge: Cambridge University Press.

¹⁵ See Dixit (1996). In fact, if we agree that the amount of contingencies in the political process tend to abound ("... the transaction-cost politics view leads me to argue that the political process should be viewed as indeed a process - taking place in real time, governed and constrained by history, and containing surprises for all parties." p. xv), thereby we can conclude that transaction costs are part of the givens of the political problem and that mechanisms involving high transaction costs are accordingly hardly part of the solution ("Many features and outcomes of the political process can be better understood and related to each other by thinking of them as the results of various transactions costs and of the strategies of the participants to cope with these costs." p. xiv-xv).

- Aragonès, E., Palfrey, T., and Postlewaite, A. (2007), “Political reputations and campaign promises”, *Journal of the European Economic Association*, 5(4): 846–884.
- Austen-Smith, D., and Banks J. (1989), “Electoral Accountability and Incumbency”, In *Models of Strategic Choice in Politics*, ed. Peter Ordeshook. Ann Arbor: University of Michigan Press, pp. 121-148
- Beilharz, H.-J., and Gersbach, H. (2004), “General equilibrium effects and voting into a crisis”. Discussion Paper n° 4454, CEPR, London.
- Bischoff, I., and Siemers, L.-H.R. (2011), “Biased beliefs and retrospective voting: why democracies choose mediocre policies”, *Public Choice*, First Online, DOI: 10.1007/s11127-011-9889-5
- Canovan, M. (1981), *Populism*. New York: Harcourt Brace Jovanovich.
- Caplan, B. (2007), *The myth of the rational voter. Why democracies choose bad policies*. Princeton: Princeton University Press.
- Dixit, Avinash K. (1996), *The Making of Economic Policy: A transaction-Cost Politics Perspective*. Cambridge: Cambridge University Press.
- Enelow, J.M., and Hinich, M. J. (1984), *The spatial theory of voting*. Cambridge: Cambridge University Press.
- Green, D. W., McManus, I. C., and Derrick, B. J. (1998), “Cognitive structural models of unemployment and employment”, *British Journal of Social Psychology*, 37: 415–438.
- Gersbach, H. (2003), “Incentive contracts and elections for politicians and the down-up problem”. In *Advances in economic design*, ed. Sertel and S. Koray. MBerlin: Springer-Verlag, pp. 65-76.
- Gersbach, H. (2004a), “Competition of politicians for incentive contracts and elections”, *Public Choice*, 121: 157–177.
- Gersbach, H. (2004b), “The Paradox of Competence”. Discussion Paper n° 4362, CEPR, London.
- Johnson-Laird, P. N. (1983), *Mental models*. Cambridge: Cambridge University Press.
- Kiewiet, D. R. (2000), “Economic retrospective voting and incentives for policy-making”, *Electoral Studies*, 19(2–3): 427–444.

- Legrenzi, P., and Girotto, V. (1996), “Mental models in reasoning and decision making processes”, In *Mental models in cognitive science*, ed. J. Oakhill and A. Garnham. Hove: Psychology Press, pp. 95–118
- Leigh, A. (2009), “Does the world economy swing national elections?”, *Oxford Bulletin of Economics and Statistics*, 71(2): 163–181.
- Leiser, D., and Drori, S. (2005), “Naive understanding of inflation”, *Journal of Socio-Economics*, 34: 179–198.
- Morton, R., and Cameron, C (1992), “Elections and the theory of campaign contributions: a survey and critical analysis”, *Economic and Politics*, 4(1): 79-108
- Mueller, D. (1989), *Public choice III*. Cambridge: Cambridge University Press.
- Müller, M. (2007), “Motivation of politicians and long-term policies”, *Public Choice*, 132(3–4): 273–289.
- Paldam, M. (2004), “Are vote and popularity functions economically correct?”, In *The Encyclopedia of Public Choice 1*, ed. C. K. Rowley and F. Schneider. Dordrecht: Kluwer Academic, pp. 49–59
- Persson, T. and Svensson, L.E.O. (1989), “Why a Stubborn Conservative would Run a Deficit: Policy with Time-Inconsistent Preferences”, *Quarterly Journal of Economics*, 104 (2): 325-345.
- Smart, M., and Sturm, D. (2004), “Term limits and electoral accountability”, Discussion Paper n° 4272, CEPR, London
- Stehn, S. J., and Fedelino, A. (2009), “Fiscal incentive effects of the German equalization system”, IMF Working Paper n° 09/124, IMF, Washington
- Tetlock, P. E. (1989), “Structure and function in political belief systems”, In *Attitude structure and function*, ed. A. R. Pratkanis, S. J. Breckler, and A. G. Greenwald. Hillsdale: Erlbaum, pp. 129–151.
- Tetlock, P. E. (1999), “Theory-driven reasoning about plausible pasts and probable futures in world politics: are we prisoners of our preconceptions?”, *American Journal of Political Science*, 43(2): 335–366.

Wittman, Donald (1990), "Spatial Strategies when Candidates have Policy Preferences", In *Advances in the Spatial Theory of Voting*, ed. J. Enelow and M. Hinich. Cambridge: Cambridge University Press, pp. 66-98.

APPENDIX

Proof of Proposition 1: Based in (21) the first order condition for the optimal choice of β is given by:

$$\begin{aligned} \frac{\partial \theta}{\partial \beta} = 0 &\Leftrightarrow \beta \\ &= \frac{m\tau a_1(\eta_b) + \rho \left\{ -m\tau a_2(\eta_g) \left(\frac{1}{2} - v \right) + [(1-m)Z + m\tau a_2(\eta_g)] [f + 2v] \right\}}{2m\tau a_2(\eta_g) [f + 2v]} \end{aligned} \quad (26)$$

Therefore, we have $\beta = 0$ if $\rho m\tau a_2(\eta_g) \left(\frac{1}{2} - 3v - f \right) > m\tau a_1(\eta_b) + \rho(1-m)Z[f + 2v]$ and $\beta = 1$ if $\rho m\tau a_2(\eta_g) \left(\frac{1}{2} + v + f \right) < m\tau a_1(\eta_b) + \rho(1-m)Z[f + v]$. Using the definitions for the probabilities we reach proposition 1

Proof of Corollary 1: Using (26), we get:

$$\frac{\partial \beta}{\partial m} = \frac{2m\tau a_2(\eta_g) [f + 2v]^2}{\left\{ 2m\tau a_2(\eta_g) [f + 2v] \right\}^2} < 0 \quad (27)$$

Proof of Proposition 2: Using (22) we get:

$$\frac{\partial \beta}{\partial \Omega} = \frac{-m\tau a_2(\eta_g)}{2m\tau a_2(\eta_g) [f + 2v]} < 0 \quad (28)$$

Proof of Proposition 3: Using (24), we get

$$\frac{\partial \beta}{\partial k} = \frac{A(1-m)\rho Z + Bm\tau a_1(\eta_b) + Cm\tau a_2(\eta_g)}{2k^2m\tau a_2(\eta_g) (f + 2vk)^2} \quad (29)$$

$$\text{With } A = -[f^2 + 4fkv + 4k^2v^2] < 0$$

$$B = -2vk^2 < 0$$

$$C = 2E(\beta^{ex})k^2v[v + f] + k^2v - (1 - E(\beta^{ex})) [f^2 + 4fkv + 6k^2v^2] \geq 0$$

Thus, we have:

$$\begin{cases} \frac{\partial \beta}{\partial k} \geq 0 \text{ if } C > 0 \wedge |Cm\tau\rho a_2(\eta_g)| \geq A(1-m)\rho Z + Bm\tau a_1(\eta_b) \\ \frac{\partial \beta}{\partial k} < 0 \text{ otherwise} \end{cases}$$

We can also deduce:

$$\frac{\partial \beta}{\partial E(\beta^{ex})} = \frac{-(1-k)[f+4vk]}{2k(f+2vk)} < 0, \text{ for all } 0 < k < 1 \quad (30)$$

Proof of Proposition 4: Using (25), we get:

$$\frac{\partial \beta}{\partial \beta^*} = \begin{cases} \frac{g}{2(f+2v+g)} > 0 \text{ if } \beta < \beta^* \\ \frac{-g}{2(f+2v-g)} < 0 \text{ if } \beta > \beta^* \end{cases} \quad (31)$$

We can also deduce:

$$\frac{\partial \beta}{\partial g} = \begin{cases} \frac{-m\tau a_1(\eta_b) + \rho m\tau a_2(\eta_g) \left[\frac{1}{2} - v + \beta^*[f+2v] \right]}{2m\tau\rho a_2(\eta_g)[f+2v+g]^2} > 0 \text{ if } \beta < \beta^* \\ \frac{m\tau a_1(\eta_b) - \rho m\tau a_2(\eta_g) \left[\frac{1}{2} - v + \beta^*[f+2v] \right]}{2m\tau\rho a_2(\eta_g)[f+2v+g]^2} \geq 0 \text{ if } \beta > \beta^* \end{cases} \quad (32)$$

When, $\beta > \beta^*$, $\frac{\partial \beta}{\partial g} > 0$ if $m\tau a_1(\eta_b) > \rho m\tau a_2(\eta_g) \left[\frac{1}{2} - v + \beta^*[f+2v] \right]$ and $\frac{\partial \beta}{\partial g} < 0$ if $m\tau a_1(\eta_b) < \rho m\tau a_2(\eta_g) \left[\frac{1}{2} - v + \beta^*[f+2v] \right]$.

When, $\beta < \beta^*$, we only have $\rho m\tau a_2(\eta_g) \left[\frac{1}{2} - v + \beta^*[f+2v] \right] > m\tau a_1(\eta_b)$, thus $\frac{\partial \beta}{\partial g} > 0$. To prove this, we know from (25) that:

$$\begin{aligned} & \beta \\ & = \frac{m\tau a_1(\eta_b) + \rho(1-m)Z(f+2v+g) - \rho m\tau a_2(\eta_g) \left(\frac{1}{2} - f - 3v - g(1+\beta^*) \right)}{2m\tau\rho a_2(\eta_g)(f+2v+g)} \quad (33) \end{aligned}$$

We also know that $\beta < \beta^*$ (34). Therefore, replacing (33) in (34), we get:

$$\begin{aligned} \rho m \tau a_2(\eta_g) \left[\frac{1}{2} - v + \beta^*(f + 2v) - (1 - \beta^*)(f + 2v + g) \right] \\ > m \tau a_1(\eta_b) + \rho(1 - m)Z(f + 2v + g) \quad (34) \end{aligned}$$

So, since $\rho m \tau a_2(\eta_g) \left[\frac{1}{2} - v + \beta^*[f + 2v] \right] > \rho m \tau a_2(\eta_g) \left[\frac{1}{2} - v + \beta^*(f + 2v) - (1 - \beta^*)(f + 2v + g) \right]$ and $m \tau a_1(\eta_b) + \rho(1 - m)Z(f + 2v + g) > m \tau a_1(\eta_b)$, we can conclude that $\rho m \tau a_2(\eta_g) \left[\frac{1}{2} + \beta^*[f + v] \right] > m \tau a_1(\eta_b)$.

Then, using the definitions for the probabilities we reach proposition 4.