Tactical Target Date Funds

Francisco Gomes[*] Alexander Michaelides[†] Yuxin Zhang[‡]

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[*]Department of Finance, London Business School, London NW1 4SA, UK. E-mail: fgomes@london.edu.

[†]Department of Finance, Imperial College London, South Kensington Campus, London SW7 2AZ, UK. E-mail: a.michaelides@imperial.ac.uk.

[‡]Hanqing Advanced Institute of Economics and Finance, RenMin University, Beijing, China. E-mail: yx.zhang@ruc.edu.cn.

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Abstract

We show that saving for retirement in target date funds (TDFs) modified to take advantage of predictability in excess returns driven by the variance risk premium generates economically large welfare gains. We call these funds tactical target date funds (TTDFs). To be easily implementable and communicated to investors, the portfolio rule followed by TTDFs is designed to be extremely simplified relative to the optimal policy rules. Despite this significant mis-specification, the significant welfare gains persist. Crucially, these gains remain economically important even after we introduce turnover restrictions that limit the annual turnover of TTDFs to be comparable to that of the average mutual fund, and after we take into account for potential increases in transaction costs. Finally we show that this predictability does not appear to be correlated with household risk.

JEL Classification: G11, D14, D15

Key Words: Target date funds, life cycle portfolio choice, retirement savings, variance risk premium, strategic asset allocation, tactical asset allocation, market timing.

1 Introduction

The conventional financial advice is that households should invest a larger proportion of their financial wealth in the stock market when young and gradually reduce the exposure to the stock market as they grow older. This advice is given by several financial planning consultants (for instance, Vanguard) who recommend target-date funds (TDFs) that reduce exposure to the stock market as retirement approaches. The long term investment horizon in these funds, and the slow decumulation of risky assets from the portfolio as retirement approaches, can be thought of as strategic asset allocation (see Campbell and Viceira, 2002), where a long term objective (financing retirement) is optimally satisfied through the TDF. This investment approach arises naturally in the academic literature in the presence of undiversifiable labor income risk (for example, Cocco, Gomes, and Maenhout (2005), Gomes and Michaelides (2005), Polkovnichenko (2007), and Dahlquist, Setty and Vestman (forthcoming)).[1] Moreover, the most recent empirical evidence shows that, even outside of these pension funds, households follow this life-cycle investment pattern (Fagereng, Gottlieb and Guiso (2017)).

In this paper we investigate whether exploiting time variation in expected returns can significantly enhance the strategic asset allocation perspective of a life cycle investor saving for retirement, through tactical asset allocation movements over a quarterly frequency.[2,3] More precisely we consider a recently proposed predictability factor, the variance risk premium (hereafter VRP) proposed by Bollerslev, Tauchen and Zhou (2009) and Bollerslev, Marrone, Xu, and Zhou (2014)). Crucially, we explore how the welfare gains from the optimal policies

[1] Benzoni, Collin-Duffresne, and Goldstein (2007), Lynch and Tan (2011) and Pastor and Stambaugh (2012) show that this conclusion can be reversed under certain conditions.

[2] In models without labor income Kim and Omberg (1996), Brennan, Schwartz and Lagnado (1997), Brandt (1999), Campbell and Viceira (1999), Balduzzi and Lynch (1999), Barberis (2000), Campbell et. al. (2001 and 2003), Wachter (2002), Liu (2007), Lettau, and Van Nieuwerburgh (2008), and Johannes, Korteweg and Polson (2014) among others, show that optimal stock market exposure varies substantially as a response to time variation in the equity risk premium.

[3] The portfolio choice literature is not limited to the papers studying time variation in the equity risk premium. For example, Munk and Sorensen (2010) and Koijen, Nijman, and Werker (2010) focus on time variation in interest rates and bond risk premia, while Brennan and Xia (2002) study the role of inflation. Chacko and Viceira (2005), Fleming, Kerby and Ostdiek (2001 and 2003) and Muir and Moreira (2017a and 2017b) consider time variation in volatility while Buraschi, Porchia and Trojani (2010) incorporate time-varying correlations.

can be replicated through simple strategies that can be easily implemented by improved target date funds, in the same spirit as the optimal life-cycle strategies are replicated by the current TDFs. Building on our initial discussion, we refer to those modified funds as Tactical Target Date Funds (hereafter TTDFs).

Our focus on the predictability driven by the VRP is motivated not only by its empirical success as a predictive factor but also by the high-frequency nature of this time variation in expected returns. More traditional predictive variables, such as CAY or the dividend-yield, capture more low frequency movements (both are more persistent than the VRP) and tend to be associated with bad economic conditions and/or discount rate shocks, both of which might affect households directly.[4] On the other hand, the VRP predictability is more likely driven by constraints on banks, pension funds and mutual funds (e.g. capital constraints or tracking error constraints). Such high frequency predictability is unlikely to be significantly correlated with household-level risks and in the paper we present evidence supporting this argument. As a result, households are in a prime position to "take the other side" and exploit this premium. Furthermore, in general equilibrium households naturally own the banks and the wealth invested in the pension/mutual funds and this actually adds a further motivation for taking the other side of the VRP. If those institutional investors are forced to scale down their risky positions because of exogenous constraints then household should be keen to offset this by increasing the risk exposure in their individual portfolios.

In that respect our paper differs from Michaelides and Zhang (2017) who incorporate stock market predictability through the dividend-yield in a life-cycle model of consumption and portfolio choice. More importantly, and differently from the previous literature on predictability, the focus of our paper is not on quantifying the welfare gains from following an optimal policy. Instead, we use the output of the model to design an approximate portfolio rule that can be easily implementable by an improved target date fund and thus be transparently communicated to investors. This is an important consideration since the individual investors are the ones who decide where to allocate their retirement savings, and several of them have limited financial literacy and might be skeptical about complex financial

[4] Bad economic conditions will tend to be associated with negative labor income shocks, and discount rate shocks might reflect increased risk aversion from households.

products.[5] Furthermore, we show that this approximate portfolio rule is able to capture a significant fraction of welfare gains implied by the optimal policy functions from the model.

Relative to an investor that assumes i.i.d. expected returns, the investor that exploits the predictability of the VRP (henceforth VRP investors) earns a significantly higher expected return. This result holds even in the presence of fully binding short-selling constraints which limit the ability of the VRP investor to exploit the time variation in the risk premium. Her expected return in such a model is still between 2.5% to 4% higher at each age. As a result the VRP investor accumulates substantially more wealth by age 65, with increases in excess of 200% across a wide range of preference parameters. These implied welfare gains are also quite large, with an age-65 certainty equivalent gain of 97% for the baseline value of relative risk aversion (5). The welfare gains are even larger as consider investors with risk aversion of 10.

Having documented large welfare gains from following the optimal decision rules derived in the model we turn to the main question that we wish to explore in our paper. Designing improved TDFs that are both transparent and easy to implement and yet can replicate, as much as possible, those welfare gains. Existing target date funds do not use the exact policy functions of individual households, they instead offer an approximation that can be implementable at low cost. For example, the exact policy functions imply different portfolio allocations for investors with different levels of wealth (relative to future labor income).[6] Furthermore, the optimal life-cycle asset allocation is actually a convex function of age as the investor approaches retirement, not a linear one. However, the approximate rule is easier to understand for investors that might have limited financial literacy, and they are the ones who decide where to allocate their retirement savings. Therefore, in the same spirit as current TDFs, we approximate the optimal asset allocations with simple linear rules that can be followed by a Tactical Target Date Fund. We estimate the best linear rule from regressions on our simulated data, where we include as explanatory factors not only age, but

[5] There is a growing literature documenting the low levels of financial literacy in the population at large. Lusardi and Mitchell (2014) provide an excellent survey. Guiso, Sapienza and Zingales (2008) show that trust is an important determinant of stock market participation decisions.

[6] In a similar spirit to ours, Dahlquist, Setty and Vestman (forthcoming) study simple adjustments to the portfolio rules of TDFs to take this into account.

also the predictive factor (i.e. the variance risk premium).[7] We further truncate the fitted linear rule by imposing fully binding short-sale constraints. It might be hard for funds taking short positions to be allowed in some pension plans, and even if that is not a concern, they might be a tough sell among investors saving for retirement that have (on average) limited financial education.

We find that this simple rule generates substantial increases in age-65 wealth accumulation and certainty equivalent welfare gains. In our analysis we take into account for a potential increase in transaction costs implied by the additional trading of VRP strategy. Even as we consider a 0.25% decrease in expected returns due to increased portfolio turnover the certainty equivalent gain from the TTDF versus the standard TDF is still 26% for our baseline calibration. The expected age-65 wealth accumulation is 131% higher. Consistent with the previous results, we find that the gains are particular higher for investors with moderate or high risk aversion. From this we can can conclude that, if the TTDFs are introduced, then those investors are the ones that would benefit the most from switching from standard TDFs into these new products.

Given that one drawback of the TTDF is that it implies significant turnover, we next consider versions of the fund were we explicitly restrict quarterly turnover to a maximum threshold. It is particularly interesting to discuss the case we set this threshold such that the average turnover of the constrained TTDF is comparable (even slightly lower) than the average turnover of the typical mutual fund (78% from Sialms, Starks and Zhang (2013)). Although the increases in expected wealth accumulation are now smaller, the turnover constraint also decreases the volatility of wealth/consumption. Therefore, even when we impose this constraint the certainty equivalent gains, although smaller, remain economically meaningful. For the baseline parameter values the certainty equivalent gain from the TTDF is still 4%.

We further show that different natural extensions to the proposed TTDF can lead to even larger welfare gains. Those extensions include relaxing the short-sale constraints, considering a portfolio rule where we allow the age effects to interact with the predictive factor, and

[7] We also explore more sophisticated rules which naturally deliver higher wealth accumulation and utility gains but, for reeasons just discussed, this one will be our baseline case.

extending the TTDF beyond age 65 by adding a linear portfolio rule for the retirement period also. Despite the improved results we believe that all of the above face non-trivial implementation problems relative to the simpler TTDF, which is why we only present them as extensions to our baseline case.

The paper is organized as follows. Section II outlines the theoretical life-cycle model, outlines the numerical solution algorithm and discusses the parameter choices for the calibration at a quarterly frequency level. Section III describes the data and the estimations used to calibrate the model. In Section IV we discuss the optimal portfolio strategy of the investor that uses the VRP model and compare it with that of an investor who assumes i.i.d. returns. Section V discusses the design of the proposed TTDFs and in Section VI we explore different extensions. In Section VII we provide evidence in support of the assumption that a higher realization of the VRP does not forecast increased household risk, and section VIII provides concluding remarks.

2 The Model

Time is discrete, but contrary to most of the life-cycle asset allocation literature we solve the model at a quarterly rather than an annual frequency. This is crucial to capture the higher-frequency predictability in expected returns documented by Bollerslev et al. (2009). Households start working life at age 20, retire at age 65, and live (potentially) up to age 100, for a total of 324 quarters. In the notation below we will use t to denote calendar time and a to denote age.

2.1 Assets and Returns

In the model there are two financial assets available to the investor. The first one is a riskless asset representing a savings account or a short-maturity T-bill. The second is a risky asset which corresponds to a diversified stock market index. The riskless asset yields a constant gross after tax real return, R_f, while the gross real return on the risky asset is denoted by R, and its expectation is potentially time varying. The time variation in expected returns is captured by a predictive factor (f_t) and following Campbell and Viceira (1999) and Pastor and Stambaugh (2012) we construct the following VAR,

$$r_{t+1} - r_f = \alpha + \beta f_t + z_{t+1}, \tag{1}$$

$$f_{t+1} = \mu + \phi(f_t - \mu) + \varepsilon_{t+1}, \tag{2}$$

where r_f and r_t denote the net risk free rate and the net stock market return, respectively. The two innovations $\{z_{t+1}, \varepsilon_{t+1}\}$ are i.i.d. Normal variables with mean equal to zero and variances σ_z^2 and σ_ε^2, respectively. The formulation allows for contemporaneous correlations between z_{t+1} and ε_{t+1}.[8]

Building on Bollerslev, Tauchen and Zhou (2009) and Bollerslev, Marrone, Xu, and Zhou (2014) we assume that the predictive factor (f_t) is the variance risk premium (VRP_t), defined as the difference between the option-implied variance of the stock market (IV_t) and its realized variance (RV_t),

$$f_t \equiv VRP_t \equiv IV_t - RV_t \tag{3}$$

We follow Bollerslev et al. (2009) in computing the two variables on the right hand side of equation (3).[9]

For comparison we will also be reporting results from a model with i.i.d. excess returns, in which case

$$r_{t+1} - r_f = \mu + z_{t+1} \tag{4}$$

. In order for the i.i.d. model to be comparable to the factor model, the first two unconditional moments of returns are set to be equal in both cases. We will also consider cases where additional transaction costs from more active trading negatively impact the expected return earned by the fund that exploits the VRP predictability. This will be implemented by adjusting appropriately the value of α in equation (1).

[8]Unlike most commonly used predictors of expected returns, the factor that we consider in this paper (the variance risk premium) is not very persistent. Nonetheless, for generality sake, in the numerical solution of the model we approximate this VAR using Floden (2008)'s variation of the Tauchen and Hussey (1991) procedure, designed to better handle the case of a very persistent AR(1) process.

[9]The details are provided in the Estimation and Calibration section.

2.2 Preferences and Budget Constraint

The household has recursive preferences defined over consumption of a single non-durable good (C_a), as in Epstein and Zin (1989) and Weil (1990),

$$V_a = \max\left\{(1-\beta)C_a^{1-1/\psi} + \beta\left(p_a E_a(V_a^{1-\gamma})\right)^{\frac{1-1/\psi}{1-\gamma}}\right\}^{\frac{1}{1-1/\psi}}, \tag{5}$$

where β is the time discount factor, ψ is the elasticity of intertemporal substitution (EIS) and γ is the coefficient of relative risk aversion. The probability of surviving from age a to age $a+1$, conditional on having survived until age a is given by p_{a+1}.

At age a, the agent enters the period with invested wealth W_a and receives labor income, Y_a. Following Gomes and Michaelides (2005) we assume that an exogenous (age-dependent) fraction h_a of labor income is spent on (un-modelled) housing expenditures. Letting α_a denote the fraction of wealth invested in stock at age a, the dynamic budget constraint is

$$W_{a+1} = [\alpha_a R_{t+1} + (1-\alpha_a)R_f](W_a - C_a) + (1-h_{a+1})Y_{a+1} \tag{6}$$

where R_t is the return realized that period (so when $t=a$).In the baseline specification we assume binding short sales constraints on both assets, more precisely

$$\alpha_a \in [0,1] \tag{7}$$

In practice it is expensive for households to short financial assets and relaxing these assumptions would require introducing a bankruptcy procedure in the model. In the context of the life cycle fund shorting will be cheaper, but still not costless, and this will still require making assumptions about the liquidation process in case of default. For these reasons the baseline model assumes fully binding short-selling constraints but we will also discuss results where we relax these.

2.3 Labor Income Process and Normalization

The labor income follows the standard specification in the literature (e.g. Cocco et al. (2005)), such that the labor income process before retirement is given by[10]

$$Y_a = \exp(g(a)) Y_a^p U_a, \tag{8}$$

$$Y_a^p = Y_{a-1}^p N_a \tag{9}$$

where $g(a)$ is a deterministic function of age and exogenous household characteristics (education and family size), Y_a^p is a permanent component with innovation N_a, and U_a a transitory component of labor income. The two shocks, $\ln U_a$ and $\ln N_a$, are independent and identically distributed with mean $\{-0.5 \times \sigma_u^2, -0.5 \times \sigma_n^2\}$, and variances σ_u^2 and σ_n^2, respectively. We allow for correlation between the permanent earnings innovation ($\ln N_a$) and the shocks to the expected and unexpected returns (ε_{a+1} and z_{a+1}, respectively).

As also common in the literature the retirement date is exogenous ($a = K$, corresponding to age 65) and income is modelled as a deterministic function of working-time permanent income

$$Y_a = \lambda Y_K^p \text{ for } a > K \tag{10}$$

where λ is the replacement ratio of the last working period permanent component of labor income.

The unit root process for labor income is convenient because it allows the normalization of the problem by the permanent component of labor income (Y_a^p). Letting lower case letters denote the normalized variables the dynamic budget constraint becomes

$$w_{a+1} = \frac{1}{N_{a+1}} [r_{t+1}\alpha_{ia} + r_f(1 - \alpha_{ia})](w_a - c_a) + (1 - h_{a+1}) \exp(g(a+1)) U_{ia+1}. \tag{11}$$

[10] We are assuming that the quarterly data generating process for labor income is the same as the one at the annual frequency. The calibration section discusses this in more detail.

3 Estimation and Calibration

3.1 VAR model for stock returns

The stock market data come from the Center for Research in Securities Prices (CRSP). We use the quarterly bond returns, the CPI growth rate to compute inflation, daily value-weighted cumulative returns and daily value-weighted returns of the CRSP US portfolio index from Jan. 1st, 1990 to Dec. 31st, 2015 to construct the relevant series. The quarterly cumulative and ex-dividend return are constructed from the monthly return of the value-weighted CRSP portfolio index.

From equation (3), to construct the variance risk premium we need both the implied variance from index options and the stock market realized variance. The data for the quarterly implied variance index (IV_t) is taken from the Federal Reserve Bank of St. Louis. We construct the quarterly realized variance as in Bollerslev et al. (2009),

$$RV_t \equiv \sum_{j=1}^{n} \left[p_{t-1+\frac{j}{n}} - p_{t-1+\frac{j-1}{n}(\Delta)} \right]^2, \tag{12}$$

where RV_t is the return variation between $t-1$ and t and p_t is the natural log of the daily stock price.

Table 1 contains the descriptive statistics from the data set. The quarterly mean real free rate is 0.18% and its standard deviation is very low, and we will therefore assume it to be constant. The stock market return has a quarterly mean of 1.98% with a standard deviation equal to 7.9%. Following the life-cycle portfolio choice literature we assume an unconditional equity premium below the historical average, namely 4% at an annual frequency.

Figure 1 shows the time series variation in implied variance (IV_t), realized variance (RV_t) and the variance risk premium (VRP_t). Figure 1 replicates and extends essentially the original Bollerslev, Tauchen and Zhou (2009) measure. Table 2 reports the estimation results for the VAR model (1 and 2). Our quantitative estimates are largely consistent with the ones in Bollerslev et al. (2009). The factor innovation is very smooth with a standard deviation (σ_ε) of 0.007.

Given these estimates, we can infer that the unconditional variance of unexpected stock market returns from

$$\sigma_z^2 = Var(r_t) - \beta^2 \sigma_f^2 \tag{13}$$

The correlation between the factor and the return innovation ($\rho_{z,\varepsilon}$) is an important parameter in determining the hedging demands. For most common predictors in the literature (e.g. dividend yield and CAY) this is a large negative number (see, for example, Campbell and Viceira (1999) and Pastor and Stambaugh (2012)). By contrast, when the predictive factor is the VRP, this correlation is estimated as slightly positive, suggesting that hedging demands are not particularly important in this context.[11]

3.2 Income process and housing expenditures

As previously discussed we consider the typical income process in the household finance literature and therefore for the most part we use the estimates in Cocco et al. (2005), which are based on the PSID. We take their estimated deterministic component of labor income ($g(a)$) and linearly interpolate in between years to derive the quarterly counterpart. Likewise we use their replacement ratio for retirement income ($\lambda = 0.68$). Cocco et al. (2005) estimate the variances of the idiosyncratic shocks around 0.1 for both σ_u and σ_n, at an annual frequency, Since we assume that the quarterly frequency model is identical to the annual frequency model it can then be shown that the transitory variance (σ_u^2) remains the same as in the annual model while the permanent variance (σ_n^2) should be divided by four.

Angerer and Lam (2009) note that the transitory correlation between stock returns and labor income shocks does not empirically affect portfolios and this is consistent with simulation results in life cycle models (Cocco, Gomes, and Maenhout (2005)). We therefore set the correlation between transitory labor income shocks and stock returns equal to zero. The baseline correlation between permanent labor income shocks and unexpected stock returns ($\rho_{n,z}$) is set equal to 0.15, consistent with the mean estimates in most empirical work (Campbell et. al. (2001), Davis, Kubler, and Willen (2006), Angerer and Lam (2009) and

[11] Indeed, if we set $\rho_{z,\varepsilon}$ equal to zero in our model the results are not significantly different. For that reason we do not explore the role of hedging demands in the paper, but those results are available upon request.

Bonaparte, Korniotis, and Kumar (2014)). We set the correlation between the innovation in the factor predicting stock returns and the permanent idiosyncratic earnings shocks ($\rho_{n,\varepsilon}$) to zero.

Finally we take the fraction of yearly labor income allocated to housing from Gomes and Michaelides (2005). This process is estimated from Panel Study Income Dynamics (PSID) and includes both rental and mortgage expenditures. As before, to obtain an equivalent quarterly process we linearly interpolate across years.

4 Optimal strategies

We first document the optimal life-cycle portfolio allocations in the model with time-varying expected returns (henceforth VRP model) for a baseline value of preference parameters for the investor (henceforth VRP investor). Next we discuss the utility gains and differences in the implied distribution of wealth accumulation at retirement relative to the case where the household follows the decision rules from the i.i.d. model. We conclude this section by reporting some comparative statics for different values of risk aversion. These results will form the basis for the next section, were we propose the tactical target date funds (TTDFs).

4.1 Optimal portfolio allocation

In the VRP model the optimal asset allocation is determined by age, wealth and the realization of the predictive factor (the variance risk premium). In Figure 2 we plot the average share invested in stocks for the VRP investor when the factor is at its unconditional mean ($\alpha_a[E(f)]$), the mean share across all realizations of the factor ($E[\alpha_a(f)]$), and the one obtained under the i.i.d. model ($E[\alpha_a^{iid}]$). In all cases wealth accumulation is being computed optimally using the appropriate policy functions.

The portfolio share from the i.i.d. model follows the classical hump-shape pattern (e.g. Cocco, Gomes and Maenhout (2005)).[12] The optimal allocation of the VRP investor, for the average realization of the predictive factor ($\alpha_a[E(f)]$), shares a very similar pattern and,

[12] The increasing pattern early in life is barely noticeable because under our calibration the average optimal share at young ages is (already) close to one.

except for the period in which both are constrained at one, we have

$$\alpha_a[E(f)] < E[\alpha_a^{iid}] \tag{14}$$

Even though under the two scenarios the expected return on stocks is the same, Figure 2 shows that $\alpha_a[E(f)]$ is below one already before age 35 and from then onwards it is always below $E[\alpha_a^{iid}]$. The main driving force behind this result is the difference in wealth accumulation of the two investors. As we show below, the VRP investor is richer and therefore allocates a smaller fraction of her portfolio to risky assets.[13]

We next compare the optimal risky share for the average realization of the factor ($\alpha_a[E(f)]$), with the optimal average risky share across all factor realizations ($E[\alpha_a(f)]$). If the portfolio rule were a linear function of the factor the two curves should overlap exactly. However, Figure 2 shows that there is a substantial difference between the two, particularly early in life. At this early stage of the life-cycle, age below 45, we have

$$E[\alpha_a(f)] < \alpha_a[E(f)] \text{ for } a < 45 \tag{15}$$

This result arises from a combination of the short-selling constraints and the fact that $\alpha_a[E(f)]$ is (much) closer to one than to zero. Given the high average allocation to stocks early in life, for realizations of the factor above its unconditional mean the portfolio rules are almost always constrained at one. On the other hand, for lower realizations of the predictive factor the optimal allocation is "free" to decrease, hence it is lower than $\alpha_a[E(f)]$. As a result, optimal allocation of the VRP investor is sometimes far below $\alpha_a[E(f)]$ and never exceeds it by much.[14]

Building on the previous intuition, it is not surprising to find that the sign of inequality flips once the portfolio allocation at the mean factor realization ($\alpha_a[E(f)]$) falls below 50%, which takes place around age 45. Now the more binding constraint is the short-selling

[13] The two policy allocations also differ because the policy rules from the VRP model take into account the hedging demands, but that effect is quantitatively much less important.

[14] It is similar to averaging a truncated distribution where the tructation is mostly binding at the upper limit.

constraint on stocks so we have:

$$E[\alpha_a(f)] > \alpha_a[E(f)] \text{ for } a > 45 \tag{16}$$

This comparison suggests that the welfare gains from the VRP model are likely to be much higher if we relax the short-selling constrains, which motivates our discussion of this particular extension in Section 6.

Combining inequalities (14) and (15) it is easy to see that, until age 45, we have:

$$E[\alpha_a(f)] < E[\alpha_a^{iid}] \tag{17}$$

namely that the average portfolio allocation in the VRP model ($E[\alpha_a(f)]$) will be much lower than the one in the i.i.d. model ($E[\alpha_a^{iid}]$), and the intuition follows from the previous discussions. In fact, even after age 45, when (15) is replaced by (16), we see that, although the difference between the optimal allocation of the VRP and i.i.d. investors decreases, equation (17) still holds: inequality (14) dominates inequality (15).

4.2 Portfolio returns

In this section we study the differences in expected returns between the VRP and i.i.d. investors. To avoid repetition we ignore transaction costs in these calculations, since we will naturally consider them in the next section when we discuss the implementation of these portfolio rules in the context of the improved target-date funds. In Figure 3 we plot the (annualized) average expected portfolio returns at each age

$$E(R_{t+1}^P) = \alpha_a E_t[R_{t+1}] + (1 - \alpha_a) R_f,\ a = 1, ..., T \tag{18}$$

which are computed by averaging (at each age) across all simulations.

Since we are averaging across all possible realizations of the factor, for a constant portfolio allocation ($\overline{\alpha}$) this would be a flat line. For example, if $\overline{\alpha} = 1$, this would be equal to the average equity portfolio return, regardless of age. In the i.i.d. model this line essentially inherits the properties of the optimal $\{\alpha_a\}_{a=1}^{T}$. The (annualized) expected portfolio return is around 5% early in life, increases slightly in the first years and then decays gradually as the investor approaches retirement and thus shifts towards a more conservative portfolio. In the VRP model the same average life-cycle pattern is present but now, since the household increases (decreases) α_a when the expected risk premium is high (low), the line is shifted upwards. As a result, even though as shown in Figure 2 the VRP investor has on average a lower exposure to stocks than the i.i.d. investor, her expected return is actually higher.

The vertical difference between the two lines gives us a graphical representation of the additional expected excess return that is actually earned by the VRP investor, and to facilitate the exposition we also plot it as a separate line in the figure. From age 37 onwards this difference increases monotonically, as the lower average equity share makes the short-selling constraint less binding and thus the VRP investor is more able to exploit time-variation in the risk premium. As the two agents reach retirement, the difference in expected returns is almost 4%. This difference is therefore at its maximum exactly when these investors have the highest wealth accumulation.

4.3 Wealth accumulation and utility gains

Consistent with the focus of our paper, designing improved target date funds, the baseline welfare calculations are computed by keeping pre-retirement consumption constant and comparing age-65 certainty equivalents, following Dahlquist, Setty and Vestman (forthcoming). The differences in certainty equivalents therefore represent the increase or decrease in *risk-adjusted* consumption level that the agent will register during the retirement period. This procedure guarantees that we do report high welfare gains at retirement at the expense of welfare losses in the pre-retirement period, for example. In this first analysis we also present certainty equivalents computed at age 21.

Consistent with the differences in expected returns documented in Figure 3, the wealth of the VRP investor grows at a much faster rate than that of the i.i.d. investor, and as a result she accumulates 269% more wealth by retirement age. However the market timing strategy also implies an increase in the standard deviation of age-65 wealth and, as shown in the second row of Panel B, these increases are quite large.[15,16] This highlights the importance of measuring the gains in terms of certainty equivalent (hereafter CE) consumption, otherwise we would be over-estimating the benefit of the market timing rules. The implied welfare implications for retirees are extremely large, with an age-65 certainty equivalent gain of 97%. This is computed as the difference in the certainty equivalent consumption levels at retirement for the VRP investor and for an investor that ignores predictability. In other words, the VRP investor will expect a 97% higher risk-adjusted consumption level per year, from age 65 onwards. This extremely high gain is however obtained under the optimal policy functions from the model. It only serves as motivation for the next section where explore whether an improved target date fund can potentially capture some of these gains while also taking into account for potential additional transaction costs.

In this section we also compute certainty equivalent gains at age 21. We now allow the investor to adjust her consumption decision optimally before retirement as well, since we are capturing those potential changes in the welfare calculation. Under this calculation the increase in wealth has to finance consumption over many more years so the increases in each year should be much smaller. Furthermore, due to the presence of borrowing constraints the agent cannot increase consumption by much early in life, despite the expectation of much higher wealth accumulation later on, and the gains late in life are heavily discounted from the perspective of the age-21 investor. As a result of these the age-21 certainty equivalent gains are naturally much lower in the context of these models.

In our case the corresponding age-21 certainty equivalent gain is 8%. This number is much smaller than the age-65 certainty equivalent of 97% for the reasons we just discussed, but it is equally impressive. For comparison, in the context of a very similar model Cocco, Gomes and Maenhout (2005) report life-time certainty equivalent losses from not investing in equities at all are between 0.9% and 4.0%, for a wide range of parameter values. The point we want to make is that a life-time certainty equivalent gain of 8% or an age-65 certainty equivalent gain of 97% are both equally impressive numbers, and they are largely

[15] Later on we will present results for constrained versions of the market timing rule, for which the increases in the standard deviations of wealth are much lower, and in some cases even negative.

[16] We are reporting the percentage increases since we believe it makes it easier to compare numbers across the different cases.

equivalent. For the remainder of the paper we focus on the age-65 gains, because they are easier to interpret in our context of improved target date funds, just as in Dahlquist, Setty and Vestman (forthcoming).

4.4 Comparative statics

The results presented so far were obtained under our baseline calibration of the preference parameters. In order to have a more complete understanding of how the market timing strategy might impact different households we now consider alternative calibrations. In Table 3 we report the average risky share at different ages, age-65 wealth accumulation and corresponding certainty equivalent gain, for different values of risk aversion (2, 5 and 10).[17]

In Table 3, Panel A we report the average allocation to stocks at different ages over the working part of the life cycle and the standard deviation of the share of wealth in stocks. As we increase risk aversion the average allocation to stocks naturally falls. This result is less pronounced early in life, when the allocation for a large range of realizations of the predictive factor is constrained at 100%, as is the case for $\gamma = 2$ and $\gamma = 5$, hence giving an unconditional average asset allocation at around 70%. But the pattern becomes quite clear as the investor ages. The cross sectional standard deviation of the share of wealth in stocks is higher for lower risk aversion coefficients: 44% for the investor with risk aversion of 2, versus 40% and 37% respectively for risk aversion of 5 and 10. This reveals that the less risk-averse investors are more willing to explore time-variation in the risk premium.[18] Intuitively they care less about the additional portfolio volatility (and hence consumption volatility) that this activity generates.

In Table 3, Panel B we compare the VRP investor with an otherwise identical i.i.d. investor in terms of age-65 wealth accumulation, pre-retirement consumption and certainty equivalent gains. The first row documents that the increase in age-65 wealth is higher for the more risk-averse investors: 334%, 269% and 202% for γ of 10, 5, and 2, respectively. The wealth accumulation results are largely affected by the presence of the short-selling

[17] Comparative statics for the other preference parameters are available upon request.

[18] The standard deviation naturally also reflects fluctuations in the portfolio share due to changes in wealth accumulation and age effects, just as in the i.i.d. model.

constraints. These constraints limit the ability to exploit time variation in expected returns but their impact is more complex since they affect different investors differently, depending on their average portfolio allocation. Those with an average allocation of 50% are less affected than those with an average allocation of 75% (25%), for example. The second investor is less able to exploit states with high (low) expected returns.[19] From this intuition we see that this effect works particularly against the investors with both very low and very high risk aversion. Therefore, it is not clear ex-ante for the range that we are considering the investor with risk aversion of 10 will experience a more substantial increase in wealth accumulation. This result can only be obtained from solving the calibrated model as we have done.

As discussed above the less risk-averse investors are the ones that will be more keen to exploit this predictability which suggests that they might be the ones who would benefit the most from it. On the other hand, the more risk averse investors are both the ones who obtain the highest increase in retirement wealth. Furthermore they are the ones who accumulate more wealth in the first place, and therefore might benefit more from an increase in the expected return on those savings.[20] In Panel B we also report the welfare gains and find that these are increasing in risk aversion within the context of our calibrated model. The certainty equivalent gain of the investor with a risk aversion of 2 is 52%, versus 97% for a risk aversion of 5, and 134% for a risk aversion of 10.

The gains reported in table 3 are extremely large but, as already discussed, they are not the focus of our paper. These gains are obtained under the optimal policy functions from the model and they don't take into account for potential additional transaction costs from implementing the VRP trading strategy. The analysis in this section provides the context and motivation for the next ones where we use these results to design the tactical target date funds.

[19]The investors with the 75% (25%) average allocation can partially compensate for this by being able to fully exploit states with even lower (higher) expected returns but, by definition, those states have low probability.

[20]The results in table 3 report the *increase* in wealth accumulation from using the VRP model, but the wealth accumulation itself is also higher for the more risk-averse investors, as standard in these models (see, for example, Gomes and Michaelides (2005)).

5 Tactical Target-date Funds

In the previous section we have shown that exploiting the equity premium predictability from the variance risk premium generates significantly higher expected wealth accumulation at retirement and leads to very large utility gains. However, those gains were computed for an investor using the optimal policy functions from the model, which is not a feasible solution for a mutual fund. Target date funds do not use the exact policy functions of individual households, they instead offer an approximation that can be implementable at low cost. This approach benefits from the further advantage that such a simpler strategy can be more easily communicated to investors that might have limited financial literacy, and are the ones who decide where to allocate their retirement savings.

The current practice therefore is for the vast majority of target-date funds (TDFs) to approximate the optimal life-cycle risky share using a linear function of age. This is an approximation to the typical optimal solution for the i.i.d. model which follows a hump shape pattern early in life, although not very pronounced for low levels of risk aversion, and has a convex shape later on as the investor approaches retirement. However, as the exact patterns of optimal policy will vary across individuals based on their preferences and other important factors (e.g. labor income profile and wealth accumulation), the linear function is thus viewed as simple to explain and a reasonable approximation to an heterogeneous set of optimal life-cycle profiles.

In the same spirit, in the baseline specification we derive a relatively straightforward portfolio rule that can be implemented by an improved target date fund (the TTDF) and which will aim to capture a large fraction of the welfare gains previously described. More precisely we now derive optimal policy rules that consist of linear functions of age and of the predictive factor. If we design more complicated rules we could potentially increase the certainty equivalent gains, and in fact we also explore some alternative portfolio rules along these lines. Finally, in this section, both for the i.i.d. and for the VRP cases, we further constrain the estimated portfolio rules by forcing them to satisfy the short-selling constraints. Later on we discuss the results obtained when we relax this constraint.

5.1 Designing tactical target-date funds

5.1.1 Tactical TDF with the VRP as a regression covariate (TTDF)

The simplest extension of the traditional TDF portfolio that incorporates the predictability channel is obtained by adding the predictive factor as an additional explanatory variable in a linear regression. More precisely, we use the simulated output from the model to estimate

$$\alpha_{iat} = \theta_0 + \theta_1 * a + \theta_2 * f_t + \varepsilon_{iat}. \tag{19}$$

Relative to the optimal simulated profiles this regression is quite restrictive as, in addition to linearity, it implies that both the regression coefficient on age (θ_1) and the intercept (θ_0) are the same regardless of the realization of the factor state. However, as previously argued, this is simple to implement and easier to explain to investors..

Table 4, Panel A and Figure 4 report the regression results from these rules for the baseline case of relative risk aversion equal to 5 and, for comparison, the results for the i.i.d. model.[21] Panel B reports the fitted linear rules for other values of risk aversion (2 and 10). These would correspond to three different TTDFs, each targeted to investors with different levels of risk aversion.

The life-cycle asset allocations for both the i.i.d. and the VRP baseline model are reasonably well captured by a linear regression rule. Despite the higher complexity of the optimal portfolio rules in the VRP case, the R-squared of the fitted linear regression is actually higher: 74% versus 45%. This is due to the lower implied average allocation to stocks, as already documented in Figure 2, which makes the short-selling constraints less binding. In the regression specification age is expressed in quarters starting for quarter 1, as in the model. Therefore, the rule age pattern for the i.i.d. case is slightly steeper than the popular "100-age" rule followed by several existing target-date funds, but not far away from it. Similarly, the average age pattern of the VRP rule is slightly flatter than the 100-age rule but, likewise, not very different from it. Of course under the VRP rule (equation (19)) the allocation also changes with the predictive factor. For example, for sufficiently high (or

[21] These are regressions on data simulated from the model so the t-statistics are all extremely high almost by definition, and therefore are omitted from the table.

sufficiently low) values of this factor, the short-selling constraints can become binding. Later on, when evaluating these strategies, we discuss their implied turnover.

In the last two columns of Table 4 we report the regression results for different values of relative risk aversion. As risk aversion decreases the coefficient on the predictive factor increases (in absolute value), consistent with the discussion in the previous section. The less risk averse investor is more willing to take advantage of time variation in expected returns. However, as also previously discussed, given that the less risk averse investor has an average portfolio allocation that is much closer to 1, her ability to actually follow the optimal market timing strategy is more limited by the presence of the short-selling constraints. This is reflected in the significantly lower regression R^2: 58 percent versus 74 (73) percent for relative risk aversion equal to 5 (10).

5.1.2 Tactical TDF conditioning on the VRP (TTDF2)

As previously discussed, the portfolio rule based on equation (19) is very straightforward but quite restrictive. Therefore, we also consider an alternative formulation where we fit the simulated shares of wealth in stocks on age using separate regressions conditional on the different realizations of the predictive factor. So, we run the following series of regressions for each f_j in our discretization grid

$$\alpha_{iat} = I_{f_t=f_j}\theta_0^j + I_{f_t=f_j} * \theta_1^j * a + \varepsilon_{iat}^j, \text{ for each } f_j \tag{20}$$

where $I_{f_t=f_j}$ equals to 1 if $f_t = f_j$ and equals to 0 otherwise.

The results are shown in Table 5 and Figure 5. Panel A of Table 5 reports, for the baseline case of risk aversion equal to 5, the regression results for three different values of f_j : mean and plus and minus two standard deviations.[22] Panels B and C report the same results for risk aversions of 2 and 10, respectively. As we can see, realization of the factor at plus (minus) two standard deviations away from the mean already imply a 100% (0%) allocation to stocks regardless of age. This pattern is not captured by the more restrictive TTDF rule (equation (19)) and is reminiscent of the Brennan, Schwartz and Lagnado (1997) results of

[22] As before, we again include the results for the i.i.d. investor for comparison.

a bang-bang solution with the intermediate cases closer to the mean having a pronounced age effect due to the presence of undiversifiable labor income.

5.2 Utility gains

Having identified a feasible portfolio rule for the TTDF we now proceed to compute the corresponding certainty-equivalent utility gains. In these calculations, as previously mentioned, we also take into account a potential increase in transaction costs implied by the market timing strategy. More specifically, we consider that the TTDF might face an effectively lower expected equity return as a result of these costs. We then report the wealth accumulation at age 65 and certainty equivalent gains from investing in the TTDFs relative to the standard TDF that ignores the market timing information provided by the realization of the factor. Results are shown for different values of risk aversion and for different assumptions about the *additional* transaction costs (tc) faced by the former.[23]

In both cases, TTDF and standard TDF we assume the same asset allocation rules at retirement, more precisely we assume that the investor ignores predictability from age 65 onwards. In other words we are measuring the gains from changing the portfolio rule in TDF only. The gains would naturally be larger if we also allowed the investor to exploit time-variation in the risk premium during retirement as well, and we present results for this case in one of our extensions below. Finally we assume that each investor is able to identify the fund that matches her level of risk aversion, both for the TTDFs and the standard TDF. Finally, as discussed in section 4.3, the welfare gains are certainty equivalents for retirees, computed while holding pre-retirement consumption constant.

5.2.1 Tactical Target Date Fund 2 (TTDF2)

It is useful to start the discussion by computing the wealth and welfare changes when the more sophisticated TTDF2 rule (equations (20)) is used. This is the rule where the regressions are performed conditional on the factor realization, implying that the age effects are different

[23] The standard TDF will also face transaction costs but in our simulations we only explcitly introduce them for the enchanced fund, which is why we view them as additional costs, over and above those already faced by the standard TDF.

across factor realizations. The results are reported in Table 6.

Comparing the results in Table 6 with those in Table 3 we see that, with the TTDF2 rule and $tc = 0$, we captures approximately 60%-70% of the gains from the VRP model. The increases in wealth accumulation at age 65 are very similar. For the three different values of risk aversion these are now 201%, 260% and 337%. versus 202%, 269% and 334% in Table 3. Although the wealth increases are almost identical, the certainty equivalent gains are lower because the wealth levels are also lower. Under both the TTDF2 and TDF asset allocations the investors naturally accumulate less wealth than if they were following their optimal portfolio rule. Since they have less financial wealth, the fraction of consumption that is being financed out of those savings as opposed to retirement income, is also lower. Therefore the same percentage increase in financial wealth will lead to a lower percentage increase in consumption and ultimately to the lower certainty equivalent gains.

As we introduce differential transaction costs for the TTDF2 the increases in wealth accumulation are naturally smaller but, even for $tc = 0.25\%$, age-65 wealth is higher by more than 100% for all investors. As a result the utility gains are remain very large: 38.6% for the baseline risk aversion of 5, increasing (decreasing) to 78.9% (23.5%) for risk aversion of 2 (10). For the reasons that we previously discussed we do not view this rule as a very practical proposition for a TDF. However, these results suggest that individuals with high financial literacy would potentially be willing to invest in such funds if they were introduced, and could obtain very large CE gains from doing so.

5.2.2 Tactical Target Date Fund (TTDF)

We now study the results for the simpler TTDF rule (equation (19)). These are shown in Table 7, again for different values of risk aversion (γ) and different values of the additional transaction costs (tc).

When consider the case with $tc = 0.0$, which is directly comparable with the results in the previous section, the increases in age-65 wealth accumulation are 103%, 182% and 312%, for risk aversion of 2, 5 and 10,respectively. The associated CE gains are 20.3%, 40.5% and 80.3% corresponding to approximately 40% to 60% of those obtained under the optimal policy functions from the model (Table 3). These results show that, the simple rule proposed by equation (19) is able to capture a significant fraction of the gains that were identified in the previous section. This is particularly remarkable if we recall that, in this analysis, we are assuming that the investor does not exploit the predictability in expected returns at retirement.[24]

Importantly, the welfare gains remain economically large even as we introduce the additional transaction costs. For the baseline calibration of risk aversion (5), even with a 25 basis points increase in costs, relative to those of the standard TDF, age-65 wealth accumulation is still 131% higher under the TTDF and the certainty equivalent consumption gain is 26.2%. As before, these values are even higher for the less risk-tolerant investor (64.4%) and lower for the more risk-tolerant one (10.1%). One implication of these results is that it would be particularly beneficial to introduce the TTDFs in pension plans with investors with moderate or high risk aversion (5 or 10). Along these lines, if such funds are offered in parallel with standard target date funds, those investors are the ones that would benefit the most from switching away from the conventional product.

5.3 Introducing turnover restrictions

One potential concern with the TTDFs, as presented in the previous section, is that its implementation might imply very high portfolio turnover. The average (annualized) portfolio turnover of the standard TDF (i.e. the one that replicates the optimal allocation of the i.i.d. investor) is 23%. For the TTDF average turnover rises to 213% indicating that tactical asset allocation implies a more volatile asset allocation behavior over the life cycle. By comparison, the average turnover of the typical mutual fund is 78% (see Sialms, Starks and Zhang (2013)).

In the previous section we included in our analysis additional transaction costs that this high turnover might generate. In this section we follow a more direct approach where we explicitly restrict the turnover of the fund. The restriction limits the optimal rebalancing of portfolio share to a maximum threshold (k). More precisely, the portfolio rule is subject to

[24] In the previous section the investor was optimally using the VRP model at all ages, so part of those welfare gains were generated by the use of the enhanced portfolio strategy also after 65.

the additional constraint

$$\alpha_a = \begin{cases} \alpha_{a-1} + k & if\ \alpha_a^* > \alpha_{a-1} + k \\ \alpha_a^* & if\ |\alpha_a^* - \alpha_{a-1}| < k \\ \alpha_{a-1} - k & if\ \alpha_a^* < \alpha_{a-1} - k \end{cases} \tag{21}$$

where α_a^* is the optimal allocation in the absence of the constraint.

In our analysis we consider two thresholds, $k = 25\%$ and $k = 15\%$. We impose equation (21) ex-post on the previously estimated policy rules, instead solving the corresponding dynamic programming problem, for two reasons.[25] First, even though the optimal policy function would by definition satisfy constraint (21), that does not guarantee that the corresponding fitted linear rule estimated from the simulated data would as well.[26] Second, from an implementation perspective this again makes the rule more transparent and easy to follow. The asset allocation of the fund is given by the previous regression specification, which yields α_a^*, subject to this constraint. The results are shown in Table 8, for the case of an investor with risk aversion of 5.

With a maximum rebalancing limit of 25% the average turnover of the fund falls almost by half, to 107%. When the limit is even stricter, 15%, the average turnover is now only 69%, which is now even below that of the typical mutual fund (78% as mentioned above). High fund turnover was the motivation for including the additional transaction costs in the previous subsection. Therefore, since we are now limiting fund turnover directly, in these results we only consider the cases with $tc = 0.0$ and $tc = 0.10\%$.

The constraints naturally limit the fund's ability to exploit time-variation in the risk premium and this is reflected in lower expected wealth accumulation. For example, for $tc = 0$ the expected increase in age-65 wealth accumulation for the baseline case (risk aversion of 5) was 269% in the absence of the turnover constraints, but falls to 45% and 14% for $k = 25\%$ and $k = 15\%$, respectively. However, this is accompanied by an equally significant reduction in the impact on the standard deviation of (age-65) wealth. In the absence of turnover

[25] Any mis-specification of the optimal policy functions will only lead us to under-estimate the utility gains since the constraint is more binding for the TTDF than the standard TDF.

[26] This is the same issue we already had before with the short-selling constraints and with these also had to be imposed ex-post.

constraints this standard deviation had increased by 462% while now that 48% and 5%.

As we introduce these restrictions the extremely large welfare gains that we previously documented disappear, but we still obtain values that are economically quite meaningful and, we would argue, much more reasonable. With $tc = 0.0$ the certainty equivalent gains for the baseline case (risk aversion of 5) are 11.1% and 3.7%, for $k = 25\%$ and $k = 15\%$, respectively. Even with $tc = 0.1\%$ both of these still remain positive: 7.2% and 0.4% respectively.

As we consider investors with either higher or lower risk aversion we again find that the certainty equivalent gains are particularly larger for the former. Even with the tighter turnover restriction ($k = 15\%$) and $tc = 0.10\%$, the investor with risk aversion of 10 still accumulates 83% more wealth at age 65, on average, by using the TTDF. This corresponds to a certainty equivalent gain of 22%. Across all cases, the investment in the TTDF only leads to certainty equivalent loss for one them: the combination of the tighter turnover restriction *and* additional transaction costs for the investor with risk aversion 2. But as just discussed, even under this combination the investor with risk aversion of 10 still has a certainty equivalent gain of 22%.

Two of the $tc = 0.0$ cases are particularly interesting: the one for the investor with risk aversion of 2 and $k = 25\%$, and the one for the investor with risk aversion of 5 and $k = 15\%$. In both of these the change in the standard deviation of age-65 wealth is very small, -2% and 5% respectively, yet there are meaningful differences in wealth accumulation: 23% in the first case and 14% in the second. So for a very similar level of ex-ante risk the investor is obtaining a noticeable difference in average expected wealth. This is reflected in certainty equivalent gains of 4.9% and 3.7%.

Overall, the results in Table 8 confirm that it is possible to design a relative simple rule that exploits the risk premium predictability obtained from the VRP, only requires standard levels of turnover, and is able to generate economically large welfare gains for a wide range of investors.

6 Extensions

6.1 Relaxing the short-selling constraints

As shown in Figure 5, the optimal portfolio allocation implied by the VRP strategy is sometimes constrained at either 100% or 0%. These results suggest that the utility gains from the VRP strategy are likely to be higher if we relax the short-selling or borrowing constraints. In the life-cycle asset allocation literature it is common to impose fully binding short-selling and borrowing constraints since it is particularly hard or expensive for retail investors to engage in unsecured borrowing or short-selling. However, in our case the strategy will be implemented by a mutual fund and hence it should be much cheaper and feasible to take both borrowing and short positions. Nevertheless we have considered fully binding short-selling constraints as our baseline case. We did so because a mutual fund that takes leveraged positions might not be regarded as an acceptable choice by some pension plan providers.

In this section we investigate the case in which the TTDF can increase its allocation to stocks as far as 200% through borrowing at the same riskless rate, that is:

$$\alpha_a \in [0, 2] \tag{22}$$

For the range of parameter values that we consider the upper bound on this constraint becomes essentially non-binding since the optimal allocation rarely exceeds 200%. We could potentially also relax the short-selling constraint on the risky asset and the welfare gains would be even higher, but that particular constraint is less binding given that the average allocation to stocks is above 50%. Furthermore, short-selling the aggregate stock market is typically harder and more expensive to implement than borrowing to invest in stocks.

In the i.i.d. model the household borrows to invest in the stock market early in life and then the pronounced life cycle effect lowering the share of wealth in stocks takes over. We use this rule to construct the TDF for the i.i.d. model (the strategic asset allocation benchmark). In this model stock market turnover now rises to 113% relative to 23% in the benchmark analyzed earlier. We follow a similar strategy for the TTDF. Table 9 reports the differences in wealth accumulation and CE gains from taking advantage of the TTDF when we relax the short-selling constraint on the riskless asset for both funds.[27]

Comparing these results with those in Tables 7 and 8, where short-selling was completely ruled out, we find significant increases in certainty equivalent gains. Without any turnover restrictions (columns II to IV) the welfare gains more than double in size, increasing from 91.8% (40.5%) to 67.3% (26.2%) for $tc = 0$ (0.25%). The less tight short-selling constraint, equation (22), significantly increases the TTDF's ability to exploit the time-variation in the expected risk premium.

One potential concern here is that this strategy implies significantly higher portfolio turnover. In fact, we see that average fund turnover is now 360% as opposed to 213% for the case with fully binding short-selling constraints. To address this concern, in the other columns of Table 9 we report results for were we impose the exogenous constraint on trading (equation (21)). As we introduce the tighter version of constraint ($k = 15\%$) portfolio turnover drops significantly, to around 73%.

The welfare gains naturally decrease substantially but, as before, remain economically significant. As we compare them with the ones in Table 8 we find that they are very similar but still higher. For example, for $tc = 0.0$, the certainty equivalents are now 13.2% and 5.2% for $k = 25\%$ and $k = 15\%$, respectively, compared with 11.1% and 3.7% in Table 8. These results show that, by relaxing the short-selling constraint on the riskless asset can increase the welfare gains from investing in the TTDF, even if we restrict the fund's turnover to reasonable levels.

6.2 Adding VRP strategies during retirement

In the previous section the investor only exploited time variation in expected returns before retirement, through the TTDF. The goal was to isolate the role of the TTDF and thus show how introducing these market timing strategies in a target date fund alone could improve welfare. In this section we consider the benefits of trying to capture the VRP strategy throughout the life-cycle. For this purpose we consider a combination of the simple TTDF

[27] We maintain all other assumptions as in the baseline case, namely relative risk aversion of 5. Results for other values of risk aversion are available upon request.

with an otherwise equally designed fund for the retirement period. More precisely, we also run the regression given by equation (19) for age greater than 65. From this we obtain a linear portfolio rule for the retirement period which complements the TTDF.

The results are shown in Table 10, for the baseline case of risk aversion 5 and with turnover restrictions to keep trading volume consistent with that of typical mutual funds. As we expected the welfare gains are now even larger. For the tighter turnover restrictions the certainty equivalent gains are between 12.3% and 20.1% which compares with 0.4% to 3.7% in table 8.

7 Variance risk premium and household risk

The previously results documented that a high value of the VRP forecasts high expected returns next quarter, consistent with the findings of Bollerslev et al. (2009). However, the optimality of increasing the allocation to stocks when the VRP is high will be overstated if the high expected returns next quarter are accompanied by an increase in risk for households. Therefore an implicit assumption in our analysis is that this is not the case, and in this section we provide evidence in support of this assumption.

7.1 VRP and stock return volatility

In our first analysis we measure return risk directly. If the high expected returns forecasted by a high VRP are accompanied by an increase in return volatility then the former might just be a compensation for the later and there is nothing to be gained by adjusting one's portfolio in response to changes in the VRP.

In Panel A of Table 11 we report the correlation between VRP and the realized volatility of stock returns, both at the monthly and quarterly frequency. We can see that this correlation is statistically zero in both cases (p-values of 0.95 and 0.38) with point estimates of 0.0037 and 0.0851. So that a high value of the VRP predicts higher expected returns without an associated change in stock return volatility. Therefore, from the distribution of returns alone, there is no evidence of an increase in risk in these periods.

7.2 VRP and consumption risk

Although return volatility does not increase (on average) following periods of high VRP it could be that these events are otherwise associated with an increase in household risk through other channels. Ultimately those risk only matter if they affect the marginal utility in that state so we can measure this by estimating the correlation between VRP and different measures of (household) consumption risk.

Building on this, in our second test we compute the correlation between the (quarterly) VRP and the growth rate of non-durable aggregate consumption over the next quarter. The aggregate consumption data is taken from the seasonally-adjusted NIPA tables as provided by the Bureau of Economic Analysis. As documented in panel B of Table 11 this correlation is -0.16 and only marginally significant, i.e. at the 10% confidence level. As we consider consumption growth beyond the next quarter none of correlations is statistically significant indicating that this effect dies out quickly.

In the final series of tests we explore the possibility that the VRP might be associated with a future increase in cross-sectional consumption risk. Using data from the Consumer Expenditure Survey (CEX) we construct the standard deviation, skewness and kurtosis of cross-sectional consumption growth.[28] To control for outliers we also consider an alternative specification where compute the moments after first windsorizing the distribution at its 10th and 90th percentiles. Data from the CEX measures consumption at a monthly frequency so we correlate it with a monthly VRP measure.[29] The results are shown in Panel C of Table 11.

When we don't windsorize the consumption growth data the correlations are all very small and none of them is statistically different from zero. When we consider the windsorized data the correlations of VRP with both future skewness and future kurtosis are still negligible and not statistically significant. Higher VRP is only associated with a small increase in

[28] We have also considered the variance, instead of the standard deviation, and the results were qualitatively the same.

[29] The CEX is a quarterly survey but individuals are asked about consumption in each of the previous three months seperately. Therefore we prefer to keep the monthly unit of observation since it avoids time aggregation considerations and increases the number of data points. If we use the quarterly VRP measure instead the point estimates are very similar, but we have even lower statistical significance which might be due to the smaller sample size.

next-period's cross-sectional standard deviation of consumption growth, but the correlation is just 0.16. To have a better sense of the economic magnitude of this value we compute the average monthly standard of cross-sectional consumption growth in the month after the VRP is in either the upper quartile or the lower quartile of its own distribution. We find that when the VRP is above its 75th percentile the average cross-sectional standard deviation of consumption growth in the next month is 22% versus 20% when the VRP is below its 25th percentile. This confirms that, although there is a correlation between the two variables, the effect is economically very weak.

7.3 Discussion

From the evidence presented in this section we can conclude that a high value of the VRP does not seem to be associated with an economically meaningful increase in household consumption risk. It is important to clarify that we are not arguing that the changes in expected returns forecasted by the VRP do not reflect risk, as such a discussion is beyond the scope of our paper. We are merely stating that, if it is indeed risk, this risk appears to be faced by other agents in the economy and not by households directly. For example, institutional investors such as mutual funds or banks face constraints that might lead them to reduce their risk baring capacity in these periods.[30]. Not being directly exposed to this risk it is therefore natural for households to increase their allocation to stocks in these periods and thus earn the additional premium by effectively taking the other side of this trade. Naturally if we take the alternative view that a high value of the VRP does not represent an increase in risk then the same conclusion applies: households should exploit this predictable variation in the risk premium. From a general equilibrium perspective, since households naturally own the banks and the wealth invested in the pension/mutual funds, this adds a further motivation for taking the other side of the VRP. If institutional investors are forced to scale down their risky positions then household should be keen to offset this by increasing the risk in their individual portfolios.

[30] For example, tracking error constraints for mutual funds or VAR constraints for banks.

8 Conclusion

We have exploited how target date funds can combine the long term strategic asset allocation perspective of a life cycle investor with the short term market information that gives rise to tactical asset allocation. Further we have shown how these enhanced funds, which we call Tactical Target Date Funds (TTDFs), can be designed in a parsimonious way and can deliver substantial welfare gains. These gain are substantial and remain economically large even after we include transactio costs and restrict the turnover of the TTDF. In unreported experiments we have extended the analysis to a wider set of preference parameter configurations and different models of investor behavior during retirement. We also think that there are potentially larger welfare gains that could be even obtained by allowing for a negative position in the stock market. Looking further into the design and commercialization of the proposed TTDFs, and the potential complications, that may arise in such implementations is an interesting topic for future research.

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Table 1: Descriptive Statistics for Predictive Variables

Table 1 presents descriptive statistics of quarterly data from 1990Q1 to 2016Q3: r denotes the real return on the S&P 500 index (deflating using the consumer price index (CPI)), IV denotes the quarterly "model free" implied variance or VIX index, and RV is the quarterly "model free" realized variance. Inflation (π) is derived from CPI and r_f is the real 90-day T-bill rate. These two series and the S&P 500 index are from the Center for Research in Security Prices (CRSP).

Panel A. Summary Statistics

1990Q1 –2016Q3	r	IV	RV	$IV - RV$	π	r_f
Mean (%)	1.98	1.11	0.62	0.49	0.06	0.16
SD (%)	7.84	0.94	0.98	0.75	0.08	0.09
Kurtosis	3.24	8.16	54.23	31.83	9.64	5
Skewness	-0.4	2.25	6.45	-3.24	-1.39	0.32
AR(1)	0.0	0.41	0.47	-0.17	0.88	0.09

Panel B. Correlation Matrix

1990Q1 –2016Q3	r	IV	RV	$IV - RV$	π	r_f
r	1.00	-0.52	-0.42	-0.096	-0.11	0.096
IV	–	1.00	0.7	0.34	-0.18	0.12
RV	–	–	1.00	-0.43	-0.46	0.3
$IV - RV$	–	–	–	1.00	0.38	-0.24
π	–	–	–	–	1.00	-0.76
r_f	–	–	–	–	–	1.00

Table 2: Predictive Regressions

Table 2 presents predictive regressions based on quarterly data from the first quarter of 1990 to the third quarter of 2016. The parameters related to the predictive regression using VRP as a predictor are estimated from the following restricted VAR:

$$\begin{bmatrix} VRP_{t+1} \\ r_{t+1} - r_f \end{bmatrix} = \begin{bmatrix} Const \\ \alpha \end{bmatrix} + \begin{bmatrix} \phi & 0 \\ \beta & 0 \end{bmatrix} \begin{bmatrix} VRP_t \\ r_t - r_f \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ z_{t+1} \end{bmatrix}$$

Newey-West t-statistics are reported in parentheses (α is set to zero).

1990Q1 –2016Q3	VRP
Constant	0.0058 (6.72)
α	0.0
β	3.6 (4.48)
ϕ	-0.18 (-1.84)
$\rho_{z,\varepsilon}$	-0.04
σ_ε	0.0074
σ_z	0.0746
σ_r	0.079
Adj. R^2 (%)	15

Table 3: VRP relative to i.i.d. model and effect of preference heterogeneity.
Table 3 presents percentage changes in the share of wealth in stocks between VRP and i.i.d. model. Percentage changes reported.

Panel A: Effect of Preference Heterogeneity			
γ	2	5	10
ψ	0.5	0.5	0.5
δ	0.9875	0.9875	0.9875
α_{20-29}	70	71	48
α_{30-39}	73	64	36
α_{40-49}	72	51	33
α_{50-65}	72	45	20
$std(\alpha_{20-65})$	44	40	37
Panel B: Relative to I.I.D. Model			
W_{65} (% inc.)	202	269	339
$Std(W_{65})$ (% inc.)	283	462	484
Age-65 CE Gain	52.3	96.9	133.6

Table 4: TDF with constant age effects across risk aversion parameters
Table 4 presents the regression of simulated portfolios on age and factor realizations across different relative risk aversion coefficients (2, 5, 10).

	VRP	$i.i.d.$	$\gamma = 2$	$\gamma = 10$
Constant	0.51	1.06	0.46	0.26
Age	−0.00191	−0.00308	−0.000312	−0.00128
Factor	45.6		45.1	43.0
R^2	74%	45%	58%	73%

Table 5: Age regressions conditional on factor realizations

Table 5 presents the regression of simulated portfolios on age conditional on each factor realization, that is, age coefficients are different across factors. The experiments are shown for different relative risk aversion coefficients (2, 5, 10).

Panel A ($\gamma = 5$)				
	VRP	VRP	VRP	$i.i.d.$
Factor	2 s.d. above mean	Mean	2 s.d. below mean	
Constant	1.00	1.06	0.0	1.06
Age	0.00	−0.0046	0.0	−0.00308
R^2	0%	79%	0%	45%
Panel B ($\gamma = 2$)				
Factor	2 s.d. above mean	Mean	2 s.d. below mean	$i.i.d.$
Constant	0.96	0.96	0.0	0.89
Age	0.0003	0.0002	0.0	0.0008
R^2	3%	1%	0%	7%
Panel C ($\gamma = 10$)				
Factor	2 s.d. above mean	Mean	2 s.d. below mean	$i.i.d.$
Constant	1.0	0.43	0.0	0.45
Age	0.0	−0.002	0.0	−0.002
R^2	0%	41%	0%	39%

Table 6: TTDF conditioning on Factor (TTDF2)

Table 6 presents results from comparing the TTDF2 with the standard TDF for different relative risk aversion coefficients and additional transaction costs from trading the TTDF2. In the standard TDF the portfolio allocation rule is a linear function of age only. Under the TTDF2 the portfolio allocation also depends on the variance risk premium (VRP), by considering different linear functions of the age for each realization of the VRP. The results are reported in percentages.

γ	2	2	2	5	5	5	10	10	10
tc (inc.)	0.00	0.10	0.25	0.00	0.10	0.25	0.00	0.10	0.25
W_{65} (% inc.)	201	172	134	260	234	198	377	358	331
$Std(W_{65})$ (% inc.)	289	254	205	363	336	297	570	561	546
Age-65 CE Gain	37.8	31.6	23.5	55.3	48.2	38.6	95.0	88.4	78.9

Table 7: TDF with factor as regressor (TTDF)

Table 7 presents results from comparing the TTDF with the standard TDF for different relative risk aversion coefficients and additional transaction costs from trading the TTDF. In the standard TDF the portfolio allocation rule is a linear function of age only. Under the TTDF the portfolio allocation also depends on the variance risk premium (VRP), which enters as an additional variable in the linear regression. The results are reported in percentages.

γ	2	2	2	5	5	5	10	10	10
tc (inc.)	0.00	0.10	0.25	0.00	0.10	0.25	0.00	0.10	0.25
W_{65} (% inc.)	103	83	57	182	161	131	312	293	265
$Std(W_{65})$ (% inc.)	97	77	50	248	223	187	506	491	466
Age-65 CE Gain	20.3	15.9	10.1	40.5	34.4	26.2	80.3	73.7	64.4

Table 8: Results with turnover restrictions

Table 8 presents results from comparing the TTDF with the standard TDF for different rebalancing restrictions and transaction costs. In the standard TDF the portfolio allocation rule is a linear function of age only. Under the TTDF the portfolio allocation also depends on the variance risk premium (VRP), which enters as an additional variable in the linear regression. The results are reported in percentages.

Risk Aversion	2	2	2	2	5	5	5	5	10	10	10	10
Max Rebalancing	25	25	15	15	25	25	15	15	25	25	15	15
Mean Turnover	108	108	72	72	106	106	69	69	100	100	66	66
tc (inc.)	0.00	0.10	0.00	0.10	0.00	0.10	0.00	0.10	0.00	0.10	0.00	0.10
W_{65} (% inc.)	23	10	0.6	-11	45	32	14	3.4	141	128	94	83
$Sd(W_{65})$ (% inc.)	-2	-14	-27	-36	48	33	5	-5.5	275	255	190	173
Age-65 CE Gain	4.9	1.8	0.24	-2.5	11.1	7.2	3.7	0.35	38.5	34.0	26.2	22.3

Table 9: Results with less-tight short selling constraints

Table 9 presents results from comparing the TTDF with the standard TDF when both funds are allowed to invest up to 200% in the risky asset. Results are show for different rebalancing restrictions and transaction costs. In the standard TDF the portfolio allocation rule is a linear function of age only. Under the TTDF the portfolio allocation also depends on the variance risk premium (VRP), which enters as an additional variable in the linear regression. These results are for the case of the investor with risk aversion of 5 (for both funds). The results are reported in percentages.

Summary Statistics

Maximum Rebalancing	100	100	100	25	25	25	15	15	15
Average Turnover	360	360	360	114	114	116	73	73	74
tc (inc.)	0.00	0.10	0.25	0.00	0.10	0.25	0.00	0.10	0.25
W_{65} (% inc.)	572	523	452	65	44	16	24	7	-15
$Std(W_{65})$ (% inc.)	615	599	569	108	82	49	41	21	-2.5
Age-65 CE Gain	91.8	81.5	67.3	13.5	8.1	1.1	5.2	0.6	-5.0

Table 10: Results for exploiting predictability both during working life and retirement

Table 10 presents summary statistics comparing results between the VRP model and the i.i.d. model for the baseline model for different rebalancing restrictions and transaction costs. The portfolio allocations of both the iid and the VRP investors are given by the corresponding funds both during working life, TDF and TTDF respectively, and during retirement. The asset allocations of the retirement funds are constructed following the same procedure as for the pre-retirement funds. Percentage changes reported.

Maximum Rebalancing	25	25	15	15
Average Turnover	108	108	69	69
tc (inc.)	0.00	0.10	0.00	0.10
W_{65} (% inc.)	63	41	34	13
$Std(W_{65})$ (% inc.)	107	71	55	22
Age-65 CE Gain	30.0	21.6	20.1	12.3

Table 11: Volatility risk premium and risk

Table 11 presents correlations between the VRP and different measures of risk. Panel A reports the correlation with with the standard deviation of stock returns, both at the quarterly and monthly frequencies. Panel B reports the correlation with seasonally-adjusted aggregate non-durable consumption growth at the horizons of one to four quarters. Finally panel C presents the correlations with three moments from the cross-sectional distribution of consumption growth at a monthly frequency: standard deviation, skewness and kurtosis. The aggregate consumption data is taken from NIPA while the cross-sectional data is from the Consumer Expenditure Survey.

Panel A: Stock Returns				
	Frequency			
	Monthly		Quarterly	
Realized Std. Dev. at t+1	0.0037	(0.95)	0.0851	(0.38)
Panel B: Aggregate Consumption Growth				
	$s=1$	$s=2$	$s=3$	$s=4$
at $t+s$	-0.167	-0.041	-0.064	-0.054
	(0.08)	(0.67)	(0.52)	(0.58)
Panel C: Cross-Sectional Consumption Growth				
	Windsorized			
	No		Yes	
Std. Dev. at t+1	-0.007	(0.92)	0.158	(0.02)
Skewness at t+1	-0.030	(0.64)	0.023	(0.71)
Kurtosis at t+1	-0.015	(0.82)	0.005	(0.94)

Figure 1: Implied volatility (IV), realized volatility (RV) constructed from daily US CRSP returns stock market data and the variance risk premium (VRP) as the difference between the two series. All data are quarterly between 1990 and 2016.

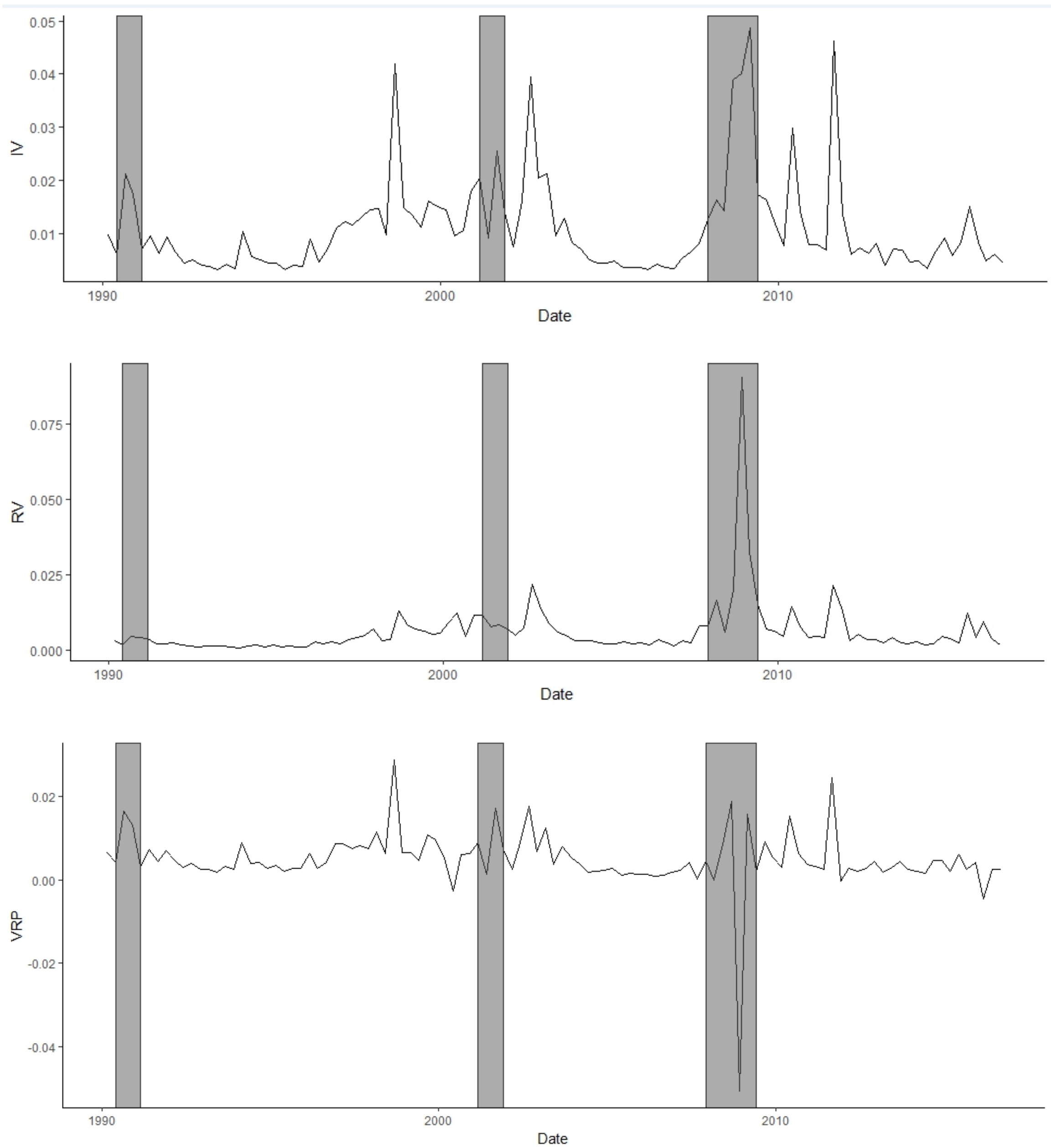

Figure 2 shows over the working part of the life cycle the share of wealth in stocks when the factor is at its median factor realization (factor = 0.49%) in the VRP model, the mean share of wealth in stocks in the VRP model and the mean share of wealth in stocks in the i.i.d. model. The baseline preferences are Epstein-Zin with a risk aversion of 5 and an elasticity of substitution equal to 0.5 and a quarterly discount factor equal to 0.99. VRP is the variance risk premium model and the decision frequency is quarterly.

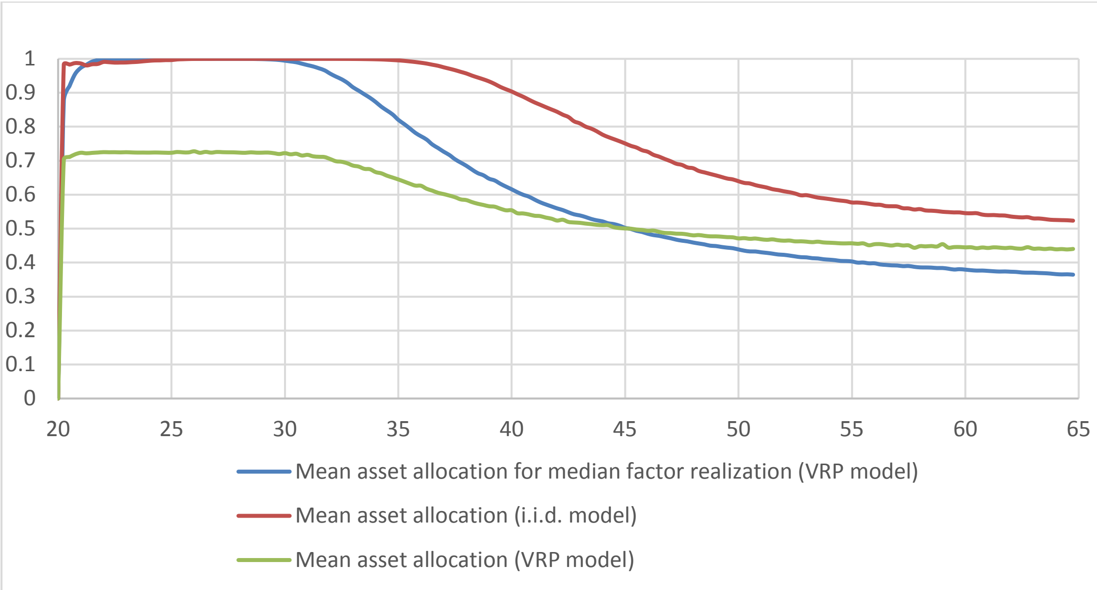

Figure 3 shows the expected portfolio return between the VRP model and i.i.d. model and their difference. The baseline preferences are Epstein-Zin with a risk aversion of 5 and an elasticity of substitution equal to 0.5 and a quarterly discount factor equal to 0.99. VRP is the variance risk premium model and the decision frequency is quarterly.

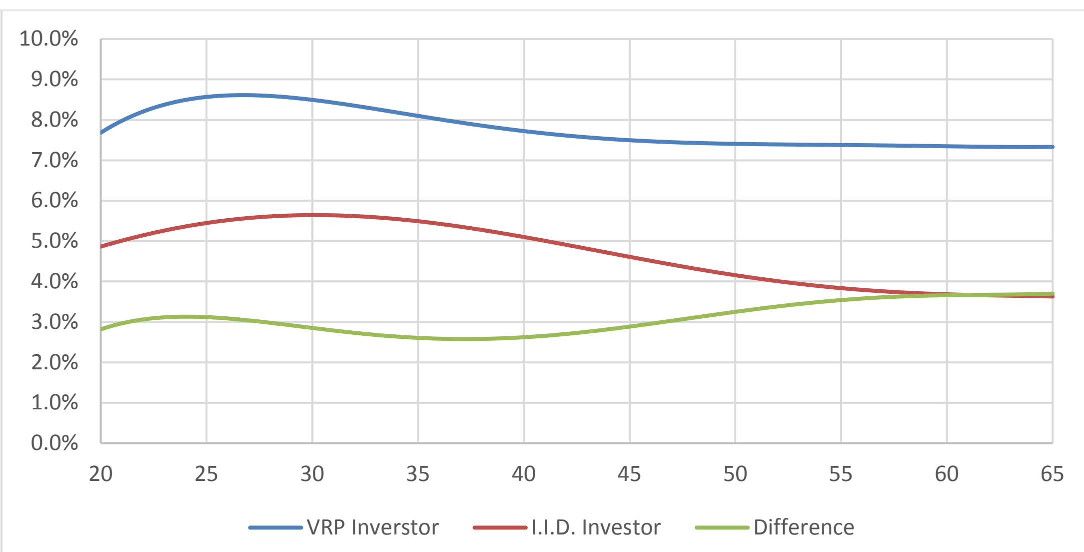

Figure 4 shows the mean share of wealth in stocks for the VRP and i.i.d. models and the target date funds (TDFs) that are constructed based on simulated shares of wealth in stocks and a multivariate regression on age and factor. In the i.i.d. model the factor state is irrelevant (as it should be). The data generating process (DGP) for stock returns in the simulation is the VRP for either model. The baseline preferences are Epstein-Zin with a risk aversion of 5 and an elasticity of substitution equal to 0.5 and a quarterly discount factor equal to 0.99. VRP is the variance risk premium model and the decision frequency is quarterly.

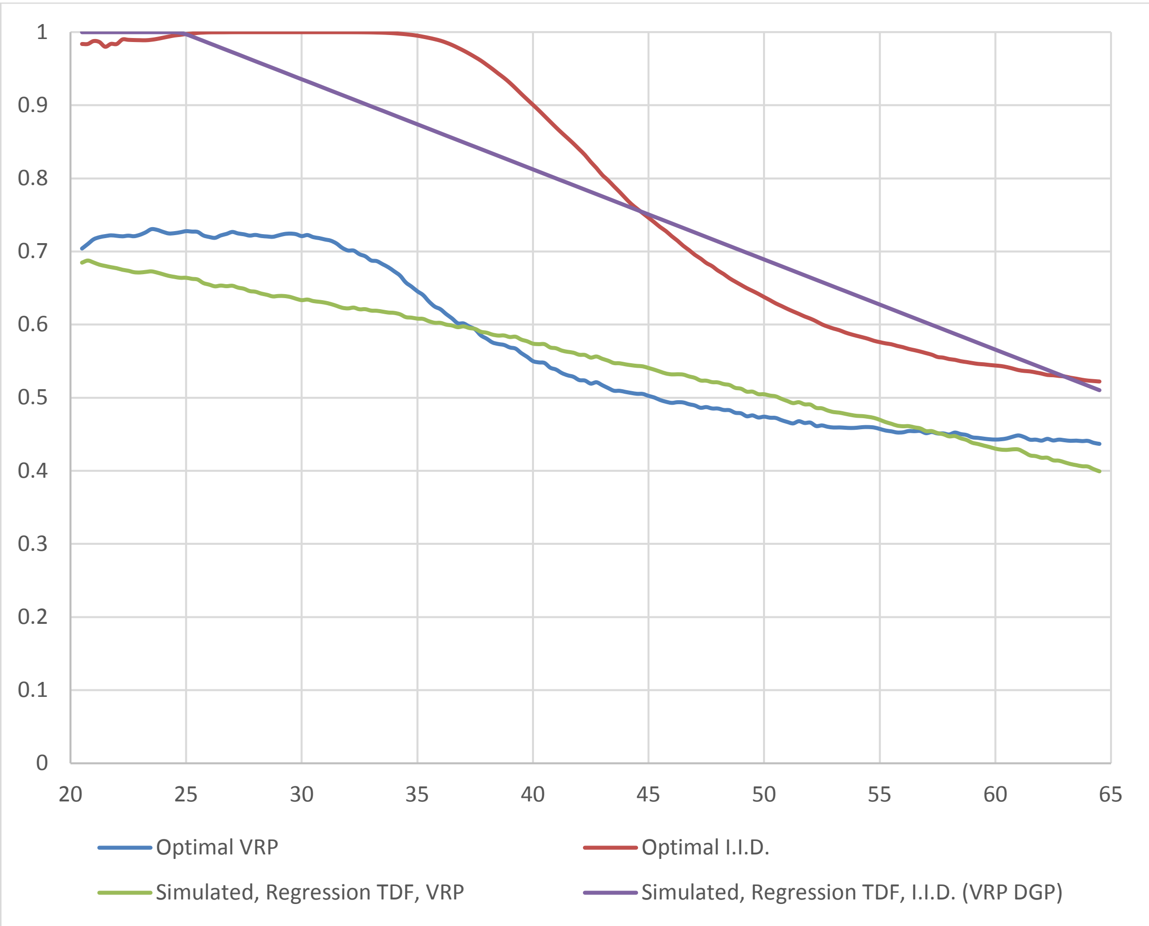

Figure 5 shows the share of wealth in stocks for the target date funds (TDFs) based on different factor realizations and the mean share of wealth in stocks for the VRP model. The data generating process (DGP) for stock returns in the simulation generating the simulated shares of wealth on which the TDF regressions are based is the VRP baseline model. The baseline preferences are Epstein-Zin with a risk aversion of 5 and an elasticity of substitution equal to 0.5 and a quarterly discount factor equal to 0.99. VRP is the variance risk premium model and the decision frequency is quarterly.