

A Welfare Analysis of Fragmented Liquidity Markets*

Alexander Guembel[†] and Oren Sussman[‡]

3rd March 2014

Abstract

Leveraged investors may be subject to contagion when sales of repossessed collateral create a downward spiral in fire sales prices, increasing margin requirements and drying up the supply of liquidity. This raises the question whether market integration is desirable when the risk of contagion is significant. While a policy that erects barriers to the free flow of liquidity across countries (fragmentation) can mitigate the incidence of contagion, it creates another problem: Liquidity may remain idle in one country while its neighbour suffers a financial crisis. We conduct a welfare analysis to net out the two effects. We show that, by itself, fragmentation has a negative welfare effect: It can only fend off mild financial crises, at the cost of exposing the country to more severe ones. At the same time, since liquidity is under-provided in a competitive equilibrium, governments should inject more of it, which could involve fragmentation, in some cases. Nevertheless, in the absence of coordination, governments are likely to fragment over and above the social optimum. Such sub-optimal fragmentation is particularly likely when governments become massive suppliers of liquidity. It follows that the recent increase in fragmentation may reflect the implementation of beggar-thy-neighbour policies rather than the proper treatment of market failures.

JEL classification: G01, F32.

Keywords: Contagion, fire sales, market integration.

*We would like to thank Bruno Biais, Guillaume Plantin, Javier Suarez, Jean Tirole and seminar and conference participants at Bangor Business School, ESCP Paris, the University of Leicester Business School, the Luxembourg School of Finance, the Tinbergen Institute, Toulouse School of Economics, the Research Unit in Complexity and Economics conference, Lisbon and the 2013 SAET conference, Paris, for helpful comments. Guembel would like to thank the Institut Europlace de Finance and the Fédération des Banques Françaises for financial support. All remaining errors are ours.

[†]Toulouse School of Economics (CRM, IDEI).

[‡]University of Oxford, Said Business School.

1 Introduction

It has long been recognized that the failure of one high-leverage investor, such as a bank or a hedge fund, may create a negative spillover effect for other investors. Such contagion may occur for various reasons, for example, direct exposure (Allen and Gale, 2000), wealth effects (Kyle and Xiong, 2001), downward spirals in margin requirements and collateral prices (Kiyotaki and Moore, 1997, Suarez and Sussman, 1997), or portfolio constraints (Pavlova and Rigobon, 2008). The risk of contagion has led researchers (e.g., Stiglitz, 2010) and policy makers to question the benefits of the international integration of capital markets. It seems that the drive for a “fragmentation” policy strengthens in the wake of a financial crisis, and the crisis that started in 2007 is no exception. According to *The Economist* (2013, p.6) “cross-border banks were an important channel for transmitting the mortgage crisis in America... to other countries. To limit such spillovers... regulators around the world are seeking to ring-fence their banking systems.”

Apart from a need to evaluate specific regulatory measures, a deeper question remains poorly understood: Is market integration desirable in an environment that features significant risk of contagion? To put it differently, is the regulatory and policy drive towards less integration justified on welfare grounds, or is it an instance of coordination failure - the old “beggar-thy-neighbour” problem? And what is the relation between massive injections of public liquidity and financial market fragmentation? In this paper we evaluate fragmentation and liquidity supply policies via an exact welfare accounting that weighs the benefits of avoiding contagion against the costs of the inefficient use of funds due to fragmentation.

To that end, we construct a two-country model in which contagion results from a downward spiral in collateral value leading to tighter margin requirements and the drying up of liquidity (for a succinct description see Krishnamurthy, 2010). Liquidity is provided by speculators who profit from low fire sale prices in times of financial crisis due to “cash-in-the-market” pricing of assets (Allen and Gale, 1994). Fire sales of investment goods are a direct consequence of debt secured by collateral, as in Hart and Moore (1998). International contagion is an immediate effect of both countries sharing a market for liquidity. Although contracts are optimally negotiated between debtors and creditors (given market prices), neither the amount nor the price of repossessed investment goods is a reflection of their social value. As a result, competitive markets are grossly inefficient, liquidity is a public good and, as such, underprovided in a competitive equilibrium. Governments may provide extra liquidity subject to a capacity constraint that captures tax distortions. The macro shock is continuous and captures the fraction of investors in each country that

need access to external funding. Country shocks are uncorrelated, so that fragmentation may lead to one country suffering a financial crisis while liquidity stands idle with its neighbour.

Our first major result is that when the trade-off between contagion avoidance and the inefficiency due to idle liquidity is properly accounted for, the former is dominated by the latter. As already hinted above, fragmentation fends off financial crisis for some realizations of shocks, but for others, it prevents access to idle liquidity. We show that the former (positive) effect applies to mild shocks while the latter (negative) effect applies to more severe ones. Intuitively, each country by itself can fend off only relatively mild shocks with its limited resources. Fending off severe shocks would require its own liquidity *and* that of its neighbour, which the latter can provide in the lucky event that it suffers only a relatively mild shock. In other words, fragmentation tends to substitute mild financial crises for relatively severe ones. It follows that the net welfare effect of fragmentation is negative when the two sets of events have the same probabilities. We also show that when speculators are the only source of liquidity, competition keeps the crisis probability constant, with or without fragmentation. In this case fragmentation unambiguously reduces welfare.

A second result is that fragmentation raises concerns about policy coordination along the lines of the old beggar thy neighbour argument (Stiglitz, 1999). When a country restricts the free flow of liquidity to avoid contagion, it ignores the effect its own idle liquidity has on the welfare of its neighbour. While fragmentation is sometimes socially optimal (i.e. increases welfare in both countries), the propensity to fragment unilaterally is always (weakly) stronger than justified by joint welfare maximization. Hence, it is possible that the recent increase in market fragmentation is a coordination failure rather than rejection of a market integration dogma. As *The Economist* (2013, p.6) puts it,

Regulators around the world...have since [the crisis] tried to reduce the threat of a big bank collapse..., but many of these efforts have undermined banks' incentive and ability to do business across borders. For example, domestic regulators used to allow foreign banks to rely on capital, liquidity and regulatory oversight of the foreign parent. Now many of them are pressing units of foreign banks...to maintain sufficient liquidity and capital independent of the parent.

The same article concludes that “The best way to maintain financial globalization...would be increased cooperation among regulators” (p.8).

We also show that coordination problems are aggravated when massive injections of

government liquidity dominate the market. When the governments rely on profit-oriented speculators to provide liquidity, the effect of fragmentation is similar to that of a tax on liquidity, which drives away speculators. In contrast, when governments take over the market, fragmentation turns into a grab of liquidity for the domestic market at the expense of the neighboring country. Our model is thus consistent with the observation that increased fragmentation tends to coincide with post-crisis periods during which governments inject large amounts of public liquidity.

Moreover, when two countries differ in their tax capacity, a greater share of the benefits of pooling public liquidity is harvested by the country with the lower tax capacity. In the extreme case, it is even possible that a country with high tax capacity would lose from pooling funds while its neighbour gains. This is because for a rich country (one with a high tax capacity), the cost of idle liquidity in the poor country is low, while the risk of contagion is very high. This finding may help to explain why the issue of a banking union at the European level is so divisive.

A third major result is that although our analysis provides only limited support for fragmentation, it provides a strong support for a policy that injects liquidity over and above the competitive level. Liquidity is a public good and, as such, underprovided in a competitive equilibrium. It turns out that in some cases the injection of liquidity that is restricted for the exclusive use of the domestic market can boost the total amount of liquidity. This is because free-to-flow and domestic liquidity are imperfect substitutes. In these cases, the positive welfare effect is due to the increase in total liquidity rather than fragmentation. Put differently, fragmentation is undesirable by itself, but may enhance welfare when used to support an injection of liquidity.

Lastly, we extend the analysis to the case in which the governments can make fragmentation contingent on the realization of the macro shock. The derived optimal (ex post) rule is reminiscent of triage, the decision rule used in emergency medicine to prioritize treatment according to the severity of the injury. In some cases, the optimal action is for a country undergoing a mild shock to give all its liquidity to its more badly-injured neighbour, but if the neighbour's shock is so severe that it cannot be rescued all liquidity should be allocated to the less severely affected country. Clearly, such rules are practical only with a strong commitment mechanism, which calls for a role for international bodies such as the International Monetary Fund (IMF) or the European Central Bank (ECB). The triage analysis highlights another important property of our model: Social welfare increases with the scale of liquidity, because the more liquidity is supplied, the larger is the potential financial crisis and associated social cost avoided.

Our paper builds on the literature following Fisher (1933) modeling a financial ampli-

fier stemming from deleveraging and fire sales (e.g., Bernanke and Gertler, 1989; Shleifer and Vishny, 1992; Kiyotaki and Moore, 1997 and Suarez and Sussman, 1997). Caballero and Krishnamurthy (2001, 2003, 2004) apply this logic within an international finance context and show that emerging economies whose assets are not internationally pledgeable will tend to underinsure and have firms with excessive leverage. Mendoza (2010) shows that financial amplification may help explain the deep recessions experienced by emerging economies following “sudden stops” in international capital flows. Korinek (2010, 2011) extends this framework and shows that capital controls can act as a Pigouvian tax on excessive international borrowing. Unlike these papers, we focus on the international contagion of crisis, via a common and endogenously determined pool of liquidity.

Like us, Brusco and Castiglionesi (2007) conduct a welfare analysis of financial integration. They show that integration can reduce the capitalization of banks, leading them to take on more risk and therefore exposing them more to distress. Moreover, integration can lead to contagion because of cross-country exposure by banks (as in Allen and Gale, 2000). On the other hand, integration improves liquidity coinsurance across regions. Since depositors anticipate the effects of integration, they only choose to be exposed to other banks if the above trade-off is favourable. On balance, therefore, integration increases welfare. Our paper differs in that a key driving force for our welfare effects comprises the externalities stemming from the need to liquidate investments and the impact this has on fire sales prices (as in Bhattacharya and Gale, 1987, or Lorenzoni, 2008). Moreover, we analyse the welfare implications of public liquidity injections in addition to market fragmentation. Finally, in our paper, fragmenting a market can be a country’s unilateral policy choice, giving rise to coordination failures, which is not considered by Brusco and Castiglionesi (2007).

Castiglionesi, Feriozzi, and Lorenzoni (2009) look into the effect that international financial integration has on the equilibrium liquidity holdings of banks. Integration improves insurance opportunities and therefore may reduce the amount of liquidity held by banks. As a result, crises, if they occur, are more severe and interest rates more volatile. In contrast, the main focus of our paper is on the welfare analysis of integration. In addition, we allow for the provision of publicly subsidized liquidity, which is not considered by Castiglionesi et.al. Finally, the underlying mechanism for crisis is somewhat different between the two models. In Castiglionesi et.al. banks can co-insure each other across regions and a systemic crisis occurs only when both regions experience a severe shock and therefore cannot insure each other. By contrast, our focus is on the effect of contagion, triggered by the fact that two countries have access to a common pool of liquidity, supplied by profit-oriented speculators.

The rest of the paper is organized as follows: Section 2 presents the setting, Section 3 analyses the contract, and Section 4 develops a single-country benchmark. Section 5 presents the two-country ex post equilibrium and Section 6 presents the two-country ex ante equilibrium and the welfare analysis. Section 7 deals with the ex post optimal allocation of liquidity and Section 8 concludes.

2 Setting

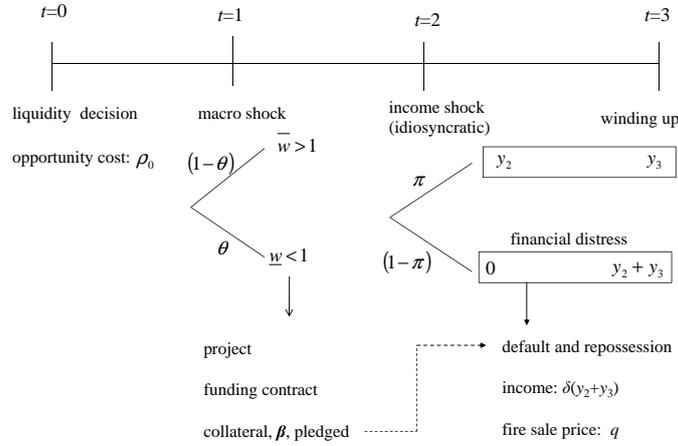
There are four dates $t = 0, \dots, 3$ (see Figure 1) and two countries $i = A, B$. Liquidity is a storable consumption good (the numeraire). Its opportunity cost is given by an alternative linear technology, which generates a gross income ρ_0 at $t = 3$ per unit of $t = 0$ investment. Liquidity is supplied by risk-neutral profit-oriented speculators and welfare-oriented governments. All liquidity decisions are taken at $t = 0$. The linear technology vanishes thereafter, the investment cannot be drawn back at $t = 1, 2$ and the income is generated only at $t = 3$, upon winding up. Note the similarity to Diamond and Dybvig (1983). Speculators are competitive and wealthy, so that their supply of liquidity at $t = 0$ (denoted by L_S) is perfectly elastic. Any additional liquidity L_G that the governments inject needs to be borrowed at the competitive rate ρ_0 and be repaid by raising taxes at $t = 3$. To capture the convex nature of tax distortions, suppose that lump sum taxes can fund a liquidity position up to T (for each government) but the distortions increase to infinity beyond that point.

At $t = 0$ governments make simultaneous decisions about the rules that regulate the flow of liquidity across countries. Specifically, each country can allow liquidity to flow freely or it can restrict liquidity (private or public) from flowing out.¹ Speculators' decisions are made with full knowledge of these rules and after having observed the amount of liquidity supplied by each government. The speculators can be thought of as foreigners who do not enter the government's welfare considerations. This assumption is immaterial, since speculators always earn the market return on liquidity, so their welfare remains the same across the various policies we analyze. Moreover, we assume that speculators' liquidity must be held within one of the two countries, such that it is subject to that government's flow restrictions.

Each country is populated with (and only with) a measure-one continuum of risk-neutral investors born at $t = 1$. (We can interpret investors as either financial or real

¹In principle, countries could block inflows of liquidity, but since it is not in their interest to do so, we do not develop this case here (for a case where blocking inflows can be desirable see Jeanne and Korinek, 2010, and Korinek, 2011).

Figure 1: Time line



operators.) Each investor is endowed with one project that needs one unit of consumption good to start up. Once invested, this consumption good is transformed into an investment good that might be traded later in the fire sale market. A certain fraction, θ_i , of investors are born poor with an endowment of $\underline{w} < 1$ - they have a weak balance sheet. The random variable θ_i is our macro shock, uniformly distributed over $[0, \Theta]$, with $\Theta < 1$.² The cross-country correlation of θ_A and θ_B is zero. To highlight the role of liquidity in our model, we assume that the aggregate amount of wealth in each country is fixed at one unit. Hence, for each realization of θ_i , there are $1 - \theta_i$ rich investors with an endowment of $\bar{w}(\theta_i) > 1$ and θ_i poor investors such that

$$\theta_i \underline{w} + (1 - \theta_i) \bar{w}(\theta_i) = 1. \quad (1)$$

The macro shock creates a financial market, in which investors affected by an adverse shock to their balance sheet seek external funding from investors with surplus funds. The demand for external funding increases with the realizations of θ_i , but so does the supply. Each country is thus endowed with sufficient resources to fund all its investment opportunities. Absent financial frictions, the macro shock is purely re-distributional, with no real effects. With frictions, shocks will have a real effect. We assume that investors with surplus funds can only invest domestically. This assumption is not important since the (potential) mobility of speculators' and governments' liquidity is enough to generate all the key effects of the model.

²Assuming a continuous distribution over the macro shock allows us to endogenize the crisis probability and to distinguish between crises of various intensities.

All projects are economically viable: They can, if carried to maturity, generate income $y_2 + y_3 > \rho_0$, to be strengthened by assumption (4), below. Though total (potential) income is fixed, its timing is subject to an idiosyncratic shock that is realized at $t = 2$. With probability $1 - \pi$ an investor's entire income is only realized at $t = 3$, which, as explained below, drives an externally-funded investor into financial distress. Otherwise, income is evenly distributed, with y_2 at $t = 2$ and y_3 at $t = 3$. Income flows are non-contractible³ resulting in an agency problem that closely resembles the treatment of Hart and Moore (1998). The advantage of this setting is that it generates optimal contracts that are easily interpreted as standard, collateralized debt contracts, a commonly perceived driver of the financial crisis.⁴ The reason for separating the macro from the idiosyncratic shock is to abstract from the issue of indexation. When the funding contract is signed (at $t = 1$), all macro uncertainty is already resolved, the future fire sale price is perfectly foreseen and can therefore be priced into the contract.

While income is not contractible, investment goods are. It immediately follows that no payment is enforceable at $t = 3$. Hence, the contract specifies a repayment r at $t = 2$, and a fraction β of the investment good that is pledged as collateral. If β is seized, the investor can continue operating on a smaller scale, $1 - \beta$. The threat of repossession is the only incentive investors have in repaying their loans. It follows that externally funded investors with no income at $t = 2$ are in financial distress and would see their collateral repossessed (in equilibrium). We adopt the common assumption that the initiating investor has a productivity advantage in managing the investment, so that repossession is *socially* (as well as privately) costly: Following repossession the investment good can generate only a fraction $\delta \in (0, 1)$ of its potential income. We make the further assumption that the lender cannot hold the collateral to maturity and instead has to convert it to liquidity (or "cash") at the prevailing fire sale price of $q \leq \delta(y_2 + y_3)$. This assumption of "no settlement in kind" may be justified if the lender itself is a financial intermediary subject to agency problems vis-à-vis its creditors (depositors in the case of a bank) that do not allow postponing payment to $t = 3$. This motivation is not modelled explicitly here. Note, also, that the assumption of "no settlement in kind" is similar to a cash in advance constraint and plays a similar role in the modelling. For simplicity, we exclude the income generated at $t = 2$ by non-distressed investors from the fire sale market. We assume that

³A complete contract alternative is suggested by Bolton and Scharfstein (1990), but the current formalization is chosen for its simplicity. In the spirit of complete contracts, we abstract from contract renegotiation, although this could easily be incorporated into the framework.

⁴See Holmstrom and Tirole (1998) for an alternative modelling approach.

the social cost of repossession is substantial:

$$\delta < \frac{1}{2}. \quad (2)$$

This assumption simplifies the exposition significantly and we discuss, below, in more detail, the role it plays.

The fire sale price and therefore the rate of return on hoarding liquidity for use in the fire sales market is, in equilibrium, perfectly foreseen at the time of contracting ($t = 1$). Moreover, speculators and rich investors are indifferent between funding poor investors and hoarding their liquidity. The risk-adjusted lending rate $\rho(q)$ at $t = 1$ is therefore determined by the arbitrage condition

$$\rho(q) = \frac{\delta(y_2 + y_3)}{q}. \quad (3)$$

We conclude with a few additional parametric assumptions. We assume the economic viability of projects after accounting for the deadweight loss of repossessions:

$$y_2 + y_3 - \rho_0 - (1 - \pi)(1 - \delta)(y_2 + y_3) > 0, \quad (4)$$

Moreover, assume

$$y_2 > y_3, \quad (5)$$

which simplifies some of the derivations below.

3 The contract

Consider a poor investor who needs to raise external finance, of $1 - \underline{w}$. He negotiates a contract (r, β) to maximize his expected final payoff (before tax),

$$\max_{r, \beta} \pi(y_2 + y_3 - r) + (1 - \pi)(1 - \beta)(y_2 + y_3), \quad (6)$$

subject to the following constraints. First, the repayment, r , must be incentive compatible, so that a non-distressed investor (with a realized income of y_2) prefers to repay r rather than default and lose the collateral and the corresponding future income, βy_3 :

$$r \leq \beta y_3. \quad (7)$$

Second, the contract must satisfy the lender's participation constraint. Due to competition, that constraint holds with equality:

$$\pi r + (1 - \pi) \beta q = \rho(q) (1 - \underline{w}), \quad (8)$$

Third, the investor's participation constraint must be satisfied so that he (weakly) prefers to invest in his own project rather than lend his wealth at the rate $\rho(q)$:

$$\pi (y_2 + y_3 - r) + (1 - \pi) (1 - \beta) (y_2 + y_3) \geq \rho(q) \underline{w}. \quad (9)$$

Finally, there are several feasibility constraints. The fraction of the investment good pledged as collateral must be non-negative and cannot exceed one:

$$\beta \in [0, 1]. \quad (10)$$

Repayment must be feasible, i.e., $r \in [0, y_2]$, which is implied by assumption (5) and the incentive compatibility constraint (7).

Since repossession destroys value, the optimal contract should satisfy the lender's participation constraint (8) by increasing the repayment, r , and by decreasing the collateral, β , as much as possible. Hence, subject to the feasibility constraint (10), the incentive constraint (7) should hold with equality. Consider the intersection of the (binding) incentive compatibility constraint, (7), and the lender's participation constraint, (8), in the (r, β) -space and let

$$b(q) = \frac{\rho(q) (1 - \underline{w})}{\pi y_3 + (1 - \pi) q} \quad (11)$$

map the fire sale price to the collateral, β , at that intersection point.

For a fire sale price such that $b(q) > 1$ the feasible set of the contract problem is empty and the investor is credit rationed. Otherwise, the investor can obtain credit at a riskless rate $\rho(q)$, pledging a share $b(q)$ of assets as collateral. Notice that $b'(q) < 0$: When the fire sale price drops, the repayment, r , needs to be increased to satisfy the lender's participation constraint. But since the incentive constraint (7) binds, that cannot be done without an increase in collateral, β . Further insight into the workings of the contract can be obtained by substituting the lender's participation constraint (8) into the investor's

objective function (6), which yields the before-tax consumption of the poor investor⁵

$$\underline{c}(q) = y_2 + y_3 - \rho(q) - l(q) + \rho(q) \underline{w}, \quad (12)$$

where

$$l(q) \equiv (1 - \pi) b(q) (y_2 + y_3 - q). \quad (13)$$

Intuitively, the poor investor's (before tax) consumption is the potential value of his project, net of the cost of funding, less the deadweight loss of external finance, $l(q)$, plus his own wealth, on which he earns market interest. Note that the dead-weight loss of external finance is positive and decreasing in the fire sale price: $l > 0$ and $l' < 0$.

As q drops, both the feasibility constraint (10) and the investor's participation constraint (9) tighten. We show that there is a non-empty set of structural parameters such that the feasibility constraint (10) is made redundant by the investor's participation constraint (9). In other words, we show that equilibria without credit rationing exist, which simplifies the analysis significantly. While an existence proof is postponed to the next section, we take a preliminary step here. Let q^n be the "fundamental" fire sale price

$$q^n = \delta (y_2 + y_3),$$

and let q^c be the price at which the investor's participation constraint (9) binds:

$$\underline{c}(q^c) = \rho(q^c) \underline{w}. \quad (14)$$

Then we have the following lemma.

Lemma 1. *i) q^c is uniquely defined. ii) $q^c < q^n$. iii) For any combination of structural parameters, y_2 , y_3 , δ and π , there exists a critical value $W(y_2, y_3, \delta, \pi)$ such that for a given $\underline{w} > W(y_2, y_3, \delta, \pi)$ and for any $q \in [q^c, q^n]$, the feasibility condition (10) is non-binding, that is $b(q) \leq 1$.*

Proof see Appendix.

The (before-tax) consumption of the rich investor is

$$\bar{c}(q) = y_2 + y_3 - \rho(q) + \rho(q) \bar{w}. \quad (15)$$

This expression is similar to that in equation (12) except the rich investor is internally

⁵We use the word *consumption* instead of income to avoid confusion with the project's income y_2 and y_3 . Actual consumption needs to account for taxation, which is done in the welfare accounting below.

funded and, thus, need not bear the deadweight loss of external finance.

At this point we add two additional parametric assumptions:

$$\underline{w} > W(y_2, y_3, \delta, \pi), \quad (16)$$

and

$$\rho^c \equiv \rho(q^c) > \rho_0. \quad (17)$$

Note that $\rho^n \equiv \rho(q^n) = 1$.

4 Single-country case

This section develops the competitive equilibrium and welfare properties of the one-country benchmark; the country index, i , is dropped for brevity.

Consider, first, the ex-post equilibrium, given a pre-determined supply of liquidity, L . We distinguish, below, between the case in which L is competitively supplied by speculators from that in which it is supplied by a welfare-oriented government.

An ex post equilibrium is characterized by a fire sale price, q , a riskless rate $\rho(q)$ and the contract, $b(q)$, determined simultaneously following the realization of θ . When the participation constraint of poor investors, (9) binds, some of them might not invest in their own projects but, instead, may lend their wealth at the market rate. Let η denote the fraction of poor investors who invest in their own projects. Note that due to the arbitrage condition (3), the lending and fire sale markets are cleared by a single price, q . Since an excess liquidity supply is conceivable, we write the clearing condition as an inequality:

$$L + (1 - \theta)(\bar{w} - 1) + \theta(1 - \eta)\underline{w} \geq \theta\eta(1 - \underline{w}) + \theta\eta(1 - \pi)qb(q).$$

In other words, pre-determined liquidity plus investors' surplus wealth not used for internal funding must weakly exceed the funding shortage of poor investors plus the liquidity needed to absorb fire sales. Using the aggregate wealth constraint (1), we simplify the above to

$$L \geq \theta\eta(1 - \pi)qb(q) - \theta(1 - \eta). \quad (18)$$

We can now show the following proposition:

Proposition 1. *Given L , there exists an ex post equilibrium, with*

$$q(\theta) = \begin{cases} q^n & \text{if } \theta \leq \theta^*, \\ q^c & \text{if } \theta > \theta^*, \end{cases} \quad (19)$$

where

$$\theta^* = \frac{L}{(1 - \pi) q^n b(q^n)}. \quad (20)$$

Proof. When the inequality in (18) is strict, all poor investors receive funding and $\eta = 1$. The return of liquidity drops to the rate of return on storage, namely, one. It follows that $q(\theta) = q^n$, as speculators bid up the fire sale price to the fundamental value of repossessed investment goods. From (18), such an equilibrium exists for $\theta \leq \theta^*$. Otherwise, a financial crisis occurs and fire sale prices drop to q^c . Since $qb(q)$ is decreasing in q , the clearing condition (18) holds with equality and η equates supply and demand. \square

Figure 2 plots the two sides of equation (18) for $\eta = 1$, which we interpret as supply and demand for liquidity in the fire sale market. Evidently, multiple equilibria may exist for some interim realizations of θ , supported by self-fulfilling expectations. (Once a low fire sale price is anticipated, a larger collateral is pledged, and the greater supply of fire sales would push the price downwards.) It can be shown, however, that in these cases the governments can coordinate expectations towards the Pareto dominating equilibrium at no fiscal expense, similar to the analysis of deposit insurance of Diamond and Dybvig (1983). We therefore ignore the Pareto dominated equilibria in the remainder of the analysis.

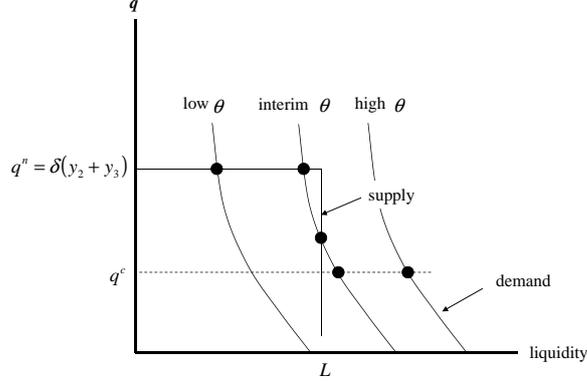
Note that contagion exists even in the single-country case: A realization ε above θ^* will drive the whole economy into financial crisis with downwards-spiralling fire sale prices. As a result, all poor investors have to pledge a larger proportion of their investments as collateral. Indeed, the multiplier effect at the threshold realization, θ^* , tends to infinity. Moreover, times of crisis are characterized by liquidity hoarding and a higher cost of external finance (as in Diamond and Rajan, 2011).

4.1 Competitive equilibrium

Consider, next, the determination of speculators' ex-ante choice of liquidity L_S when there is no government intervention. Let

$$\psi(\theta^*) \equiv \int_{\theta^*}^{\Theta} \frac{1}{\Theta} d\theta$$

Figure 2: Supply and demand for liquidity in the fire sale market



be the probability of crisis (for a generic crisis threshold θ^*). Then, in a competitive equilibrium, expected trading profits must be zero and the return on liquidity therefore equals its opportunity cost, namely, the return on the linear technology, ρ_0 . Hence, the crisis threshold θ_S^* in a competitive equilibrium is determined by

$$[1 - \psi(\theta_S^*)] + \psi(\theta_S^*) \rho^c = \rho_0$$

and the liquidity supply is implicitly given by (20). We thus obtain the following lemma.

Lemma 2. *The competitive probability of financial crisis is strictly positive at*

$$\psi^e = \frac{\rho_0 - 1}{\rho^c - 1}. \quad (21)$$

Proof. Immediate. □

Speculators lose money unless there is a financial crisis because the rate of return on storage is not sufficient to cover the cost of providing liquidity. In that respect, financial crises are part of the normal functioning of a competitive economy (see also Gorton and Huang, 2004, or Acharya, Shin and Yorulmazer, 2013).

4.2 Welfare

Suppose now that the government is the sole supplier of liquidity, $L_G \leq T$. As in the previous section, we map these quantities into crisis thresholds:

$$\theta_G^* \equiv \frac{L_G}{(1 - \pi) q^n b(q^n)}, \quad \theta_T^* \equiv \frac{T}{(1 - \pi) q^n b(q^n)}. \quad (22)$$

The government operates like a welfare-oriented speculator, using its liquidity to purchase investment goods on the fire sale market and provide funding to poor investors, at market prices. It follows that the government, as any speculator, makes “trading profits” in times of financial crisis. This policy resembles, for example, the U.S. treasury’s purchases of mortgage-backed securities under the Troubled Asset Relief Programme (TARP) starting in 2008. The estimated profit, net of financing costs, of this programme was approximately US\$12 billion (see U.S. Department of Treasury, 2013). In addition, by early 2014 the Fed had direct holdings of \$1.5 trillion of mortgage-backed securities, about 9% of the U.S. GNP (see Fed Statistics, 2014). In principle, the government could employ its liquidity in other ways, for example, by directly injecting equity into the financial sector, or by assuming part of its liabilities. In a companion paper (Guembel and Sussman, 2010), we analysed such alternative policies. We show that the basic mechanism whereby supplying liquidity can increase the crisis threshold, does not change, which justifies the focus of this paper.

Trading profits net out against the cost of funding the provision of liquidity. Hence, the budget surplus z ,

$$z(\theta) = [\rho(q(\theta)) - \rho_0] L_G,$$

is distributed back via lump-sum taxes. Let the expected surplus, as a function of the crisis threshold, be $Z(\theta_G^*)$. It can be decomposed into trading profits and the cost of funding liquidity:

$$Z(\theta_G^*) = \psi(\theta_G^*)(\rho^c - 1)L_G - (\rho_0 - 1)L_G. \quad (23)$$

Next, we calculate the investor’s expected before-tax consumption, conditional on θ (taking conditional expectations over $c \in \{\underline{c}(q), \bar{c}(q)\}$ given by (12) and (15)):

$$E(c|\theta) = y_2 + y_3 - \theta l(q(\theta)).$$

Taking expectations over θ yields expected (pre-tax) consumption, denoted by the function $C(\theta_G^*)$:

$$C(\theta_G^*) = y_2 + y_3 - \int_0^{\theta_G^*} \theta l(q^n) \frac{d\theta}{\Theta} - \int_{\theta_G^*}^{\Theta} \theta l(q^c) \frac{d\theta}{\Theta}.$$

Differentiating with respect to liquidity (expressed in terms of the crisis threshold, θ_G^*) we

obtain

$$\frac{dC(\theta_G^*)}{d\theta_G^*} = \frac{\theta_G^*}{\Theta} \Delta, \quad \Delta \equiv l(q^c) - l(q^n) \geq 0.$$

Intuitively, by providing extra liquidity, the government can shift the crisis threshold rightwards and raise the $\theta_G^* + \varepsilon$ realization (with density $1/\Theta$) out of crisis. There are $\theta_G^* + \varepsilon$ poor investors in that realization, whose deadweight losses from external finance fall from $l(q^c)$ to $l(q^n)$. Notice that the higher fire sale price would relieve *all* poor investors (not just the extra ε) from higher collateral and the greater deadweight loss of external finance. Clearly, expected pre-tax consumption has increasing marginal returns in liquidity supply: The more liquidity provided, the more severe the potential financial crisis that is avoided at the margin. This is a generic property of the market for liquidity due to the downward spiral in fire sales prices and collateral requirements triggered by a small shock. It is also a key driver behind the main results in this paper.

Our analysis is complicated by the fact that, while the function $C(\theta_G^*)$ is convex in θ_G^* , trading profits, $\psi(\theta_G^*)(\rho^c - 1)L_G$, are concave in θ_G^* . Indeed, trading profits are hump shaped: They are zero when either $L_G = 0$ or $\theta_G^* = \Theta$ and, therefore, the probability of crisis, $\psi(\theta_G^*)$, drops to zero. It turns out that the former effect (convexity) dominates the latter. The proof relies on the following technical result:

Lemma 3. *It follows from assumption (2), ($\delta < \frac{1}{2}$), that*

$$\Delta - 2(\rho^c - 1)(1 - \pi)q^n b(q^n) > 0. \quad (24)$$

Proof see Appendix.

We can now define the social welfare function as the sum of pre-tax consumption and the distributed budget surplus,

$$SW(\theta_G^*) \equiv C(\theta_G^*) + Z(\theta_G^*),$$

whose first derivative is

$$\begin{aligned} SW'(\theta_G^*) = & \frac{\theta_G^*}{\Theta} \Delta \\ & + \psi'(\theta_G^*)(\rho^c - 1)L_G + \psi(\theta_G^*)(\rho^c - 1) \frac{dL_G}{d\theta_G^*} \\ & - (\rho_0 - 1) \frac{dL_G}{d\theta_G^*}. \end{aligned} \quad (25)$$

The first line in (25) captures the marginal effect of liquidity on *private* welfare, as explained above. The marginal social value of liquidity is derived by adding the change in trading profits (second line) and the increase in the cost of funding liquidity (third line).

Before deriving the government's optimal policy, consider how public and private liquidity interact. From (21), it is clear that private liquidity supply will adjust to maintain a constant crisis probability. It follows that when $\theta_G^* < \theta_S^*$, any public liquidity injection fully crowds out a corresponding amount of private liquidity (for a related result see Acharya, Shin and Yorulmazer, 2011). Public liquidity therefore has no welfare effect up to the point that private liquidity is fully crowded out and the crisis probability drops below ψ^e . In that case, the government becomes the sole liquidity provider.

Proposition 2 below demonstrates that (using Lemma 3) the cost of liquidity provision does not reverse its positive welfare effect.

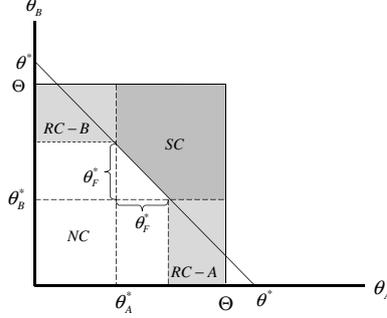
Proposition 2. *If $\theta_T^* > \theta_S^*$, the government's optimal policy is to provide public liquidity up to the corner solution, such that the shock that can be withstood is θ_T^* or Θ . If $\theta_T^* \leq \theta_S^*$, the injection of public liquidity fully crowds out (one for one) private liquidity so that total liquidity is not affected and $L_G = 0$ is (weakly) optimal.*

If a country's tax capacity is below competitive liquidity, then public policy is impotent. If the tax capacity is high (such that $T > L_S$), then the government should inject liquidity to raise total liquidity above the competitive level. Note that this result does not rely on assumption (2); the competitive supply of liquidity is always sub-optimal, even without that assumption. The contribution of assumption (2) is to guarantee that $SW' > 0$ globally so that the optimal policy is to inject liquidity up to the corner of either T or Θ , whichever binds first (the probability of crisis drops to zero in the latter case). Since the optimal policy is always to inject some liquidity, one should consider assumption (2) as a simplification that saves the effort of dealing separately with both cases above. That simplification is particularly helpful in the two-country case.

5 Two countries: ex-post equilibrium

In a two-country setting, policy is two dimensional: Governments have to decide how much public liquidity to inject and what restrictions, if any, to impose on the free flow of liquidity, both public and private. The two dimensions are independent; we do not rule out the possibility that a government injects liquidity that is allowed to flow freely across borders, nor do we rule out the possibility that a government imposes (at $t = 0$) rules and regulations that restrict the flow of private liquidity (at $t = 1$) out of the country, where it is "parked". (As noted in Section 2, liquidity must be held inside a country, so any outflow is subject to the rules and regulations of the hosting country.) The incentives

Figure 3: Equilibrium regimes



of governments to implement such fragmentation policies and their equilibrium effect on private liquidity are the focus of the following sections.

In a two-country setting, a financial crisis may be either regional or systemic. In the former case the fire sale price in, say, country A drops to q^c , while the fire sale price in country B remains at q^n . In the latter case, the fire sale price in both countries drops to q^c . Obviously, it is also possible that there will be no crisis in either country. Denote by L_i the amount of *domestic* liquidity in country i (i.e. liquidity restricted from flowing out of country i) and by L_F the amount of *free-to-flow* liquidity (whether public or private). Like in the single-country case, it is convenient to characterize the equilibrium in terms of crisis thresholds:

$$\begin{aligned}\theta_i^* &\equiv \frac{L_i}{(1-\pi)q^n b(q^n)}, \\ \theta_F^* &\equiv \frac{L_F}{(1-\pi)q^n b(q^n)}, \\ \theta^* &\equiv \theta_A^* + \theta_B^* + \theta_F^*.\end{aligned}$$

Figure 3 provides a geometrical representation of the thresholds in the space $[0, \Theta] \times [0, \Theta]$.

Each country has exclusive access to its own domestic liquidity. Hence, country A , say, is immune from financial crisis as long as its own realization, θ_A , is smaller than the shock θ_A^* that can be withstood using only domestic liquidity (θ_A is to the left of the vertical θ_A^* line in Figure 3). Potentially, country A can avoid a crisis as long as the residual demand of both countries, after exhausting all domestic liquidity, $\Sigma(\theta_i - \theta_i^*)$, is

smaller than the shock θ_F^* that the free-to-flow liquidity can absorb ($\theta_A + \theta_B$ is below the downward-sloping θ^* -diagonal in Figure 3). Nevertheless, country A suffers a financial crisis if $\theta_A > \theta_A^* + \theta_F^*$, such that A 's liquidity demand exceeds its own domestic liquidity plus all the free-to-flow liquidity. A crisis in A occurs even if $\Sigma(\theta_i - \theta_i^*) > \theta_F^*$, that is, country B has enough liquidity to cover the entire shortage in country A but B 's domestic liquidity is not accessible to country A . Hence, the area below the θ^* diagonal and to the right of the $(\theta_A^* + \theta_F^*)$ vertical is where the *idle liquidity* problem arises. A symmetric characterization applies to country B . To summarize, we have the following proposition.

Proposition 3. *The following are equilibrium prices as function of the aggregate shocks:*

(NC) *There is no crisis in either country ($q_A = q_B = q^n$) if*

$$\begin{aligned}\theta_A + \theta_B &\leq \theta_A^* + \theta_B^* + \theta_F^*, \\ \theta_i &\leq \theta_i^* + \theta_F^*, \quad i = A, B.\end{aligned}$$

(RC- i) *There is a regional crisis in country i ($q_i = q^c, q_j = q^n, j \neq i$) if*

$$\begin{aligned}\theta_i &> \theta_i^* + \theta_F^*, \\ \theta_j &\leq \theta_j^*, \quad j \neq i.\end{aligned}$$

(SC) *There is a systemic crisis ($q_A = q_B = q^c$) otherwise.*

Against the problem of idle liquidity one has to weigh the problem of cross-country contagion. Consider a realization (θ_A, θ_B) in the NC region, just below the θ^* -diagonal and just to the right of the θ_A^* -vertical in Figure 3. Now suppose that the country- B shock increases enough to push both countries into the systemic-crisis area, SC . Country A would now suffer a crisis, even though its domestic shock is the same; A is affected by contagion from B . Note that country A could have fended off the crisis by making some of the free-to-flow liquidity domestic and thereby preventing it from flowing towards country B . This corresponds to a rightward shift of the vertical θ_A^* line, keeping all else equal. This, of course, exacerbates the idle liquidity problem for country B . Clearly, an exact welfare evaluation is required in order to determine which effect dominates.

In the single-country case, the probability of financial crisis conditional on the realization of θ is either zero or one. This is no longer the case in the two-country setting because, conditional on the realization of θ_A , the incidence of crisis still depends on the realization of θ_B , such that, conditional on θ_A alone, the probability of crisis for country A may be between zero and one. This conditional probability plays a central role in our welfare analysis. Let $\boldsymbol{\theta} \equiv (\theta_A^*, \theta_B^*, \theta^*)$ be the allocation of domestic and free-to-flow liq-

uidity in both countries (note that θ_F^* is defined as a residual). Then, using Proposition 3, the probability of a country- i crisis conditional on its own realization of θ_i is⁶

$$\psi_i(\boldsymbol{\theta}|\theta_i) \equiv Pr(q_i = q^c|\theta_i) = \begin{cases} 0 & \text{for } \theta_i \in [0, \max\{\theta_i^*, \theta^* - \Theta\}), \\ 1 - \frac{\theta_i^* - \theta_i}{\Theta} & \text{for } \theta_i \in [\max\{\theta_i^*, \theta^* - \Theta\}, \min\{\Theta, \theta^* - \theta_j^*\}], \\ 1 & \text{for } \theta_i \in (\min\{\Theta, \theta^* - \theta_j^*\}, \Theta]. \end{cases} \quad (26)$$

We also define the unconditional probability of crisis in each country as

$$\psi_i(\boldsymbol{\theta}) \equiv Pr(q_i = q^c) = \int_0^\Theta \psi_i(\boldsymbol{\theta}|\theta_i) \frac{d\theta_i}{\Theta}.$$

Let $\psi_{AVB}(\boldsymbol{\theta})$ be the probability of financial crisis in either country A or country B (or both). From Figure 3 it is clear that $\psi_{AVB}(\boldsymbol{\theta})$ can be written as

$$\psi_{AVB}(\boldsymbol{\theta}) = \frac{\theta_A^*}{\Theta} \psi_i(\boldsymbol{\theta}|\theta_A^*) + \psi_A(\boldsymbol{\theta}). \quad (27)$$

It is obvious that $\psi_{AVB}(\boldsymbol{\theta}) \geq \psi_A(\boldsymbol{\theta})$, with the following implication.

Fact 1. *The ex ante expected return on domestic liquidity is lower than the ex ante expected return on free-to-flow liquidity, because the latter can profit from crisis in both countries, while the former can profit only from domestic crisis.*

6 Ex ante equilibrium

In our model, governments only consider the well-being of their own citizens, which raises the possibility of coordination failure, where governments ignore the negative effect of fragmentation on their neighbour - a beggar-thy-neighbour problem. As explained in Section 2, the non-cooperative game between both countries is played as follows (within $t = 0$): First, both governments make a simultaneous decision about (i) the amount of public liquidity that each supplies, (ii) the allocation of that liquidity to free-to-flow and domestic liquidity, and (iii) the rules that apply to private liquidity, whether free to flow or domestic. Speculators then decide how much private liquidity, if any, they supply. At

⁶Note that, strictly speaking, the probability of crisis at the threshold $\theta_i = \theta_i^*$ is equal to zero. In the treatment below, the increase in the probability of crisis, moving just across the θ_i^* realization, will play an important role. It is therefore notationally more convenient to define the function $\psi_i(\boldsymbol{\theta}|\theta_i)$ according to (26). Since the imprecision concerns an atomistic point, the integration and thus the overall probability of crisis are completely unaffected.

this point, θ is determined. We analyse Nash equilibria in this game. We also analyse the socially optimal policy, that is that policy which maximizes the joint welfare of both countries. A Nash equilibrium is deemed a coordination failure if it deviates from the social optimum.

Using the conditional probability function, $\psi_i(\theta|\theta_i)$, we can derive the expected pre-tax consumption, $C_i(\theta)$, similarly to the single-country case:

$$C_i(\theta) = y_2 + y_3 - \int_0^\Theta \{[1 - \psi_i(\theta|\theta_i)]\theta_i l(q^n) + \psi_i(\theta|\theta_i)\theta_i l(q^c)\} \frac{d\theta_i}{\Theta}. \quad (28)$$

We also calculate the expected budget surplus $Z_i(\theta)$ as in (23), using the liquidity injections by the domestic government, $L_{G,i}$:

$$Z_i(\theta) = [\psi_R(\theta)(\rho^c - 1) - (\rho_0 - 1)] L_{G,i}, \quad (29)$$

where $R \in \{i, i \vee j\}$ denotes the region of deployment for public liquidity. Total welfare is just

$$SW_i(\theta) = C_i(\theta) + Z_i(\theta). \quad (30)$$

6.1 No capacity for public liquidity, $T_i = 0$

We start with the simple case in which the governments have no capacity to supply public liquidity so that the only decisions that they make is whether to allow private liquidity to flow freely across borders. It follows from Fact 1 that the decision is binary: If some private liquidity is allowed to flow freely and the rest is restricted, the restricted part would earn a lower rate of return and attract no funds. Each country's decision thus reduces to either restricting or allowing free flow of the entire supply of private liquidity. We start the analysis by demonstrating that coordination is not an issue in the present case.

Proposition 4. *When governments have no capacity to provide liquidity (i.e. $T_i = 0$), the socially optimal policy (whether fragmentation or free to flow) is a weakly dominant strategy.*

Proof. Without loss of generality, we analyse the best response of country A to a country B policy. If country B allows the free flow of liquidity, country A is indifferent between fragmentation and free flow, since both options lead to the same equilibrium allocation of $\theta^{SF} \equiv (0, 0, \theta^{*SF})$. SF indicates that the liquidity allocation is one where all liquidity

is supplied competitively by speculators (hence S) and is free-to-flow (hence F). Clearly,

$$\psi_{AVB}(\boldsymbol{\theta}^{SF}) = \psi^e.$$

Trivially, this is the allocation if country A chooses a free-to-flow policy. If country A responds with fragmentation, speculators will have to decide whether to park their liquidity in country B , from where it can flow out to country A , or in country A , out of which it cannot flow. By Fact 1, they would prefer the former option, leaving country A with zero domestic liquidity and parking θ^{*SF} in country B , free to flow.

Alternatively, consider the case in which country B imposes restrictions on liquidity outflows. If country A responds with a free-to-flow policy, the allocation, again, is $\boldsymbol{\theta}^{SF}$. If, however, country A responds with a fragmentation policy, the equilibrium allocation would be $\boldsymbol{\theta}^{SR} \equiv (\theta_d^{*SR}, \theta_d^{*SR}, 2\theta_d^{*SR})$, where SR indicates competitive liquidity supply with restrictions on flows. Like before,

$$\psi_i(\boldsymbol{\theta}^{SR}) = \psi^e.$$

Due to the symmetry of both $\boldsymbol{\theta}^{SF}$ and $\boldsymbol{\theta}^{SR}$, whatever policy maximizes country A 's welfare also maximizes country- B 's welfare. \square

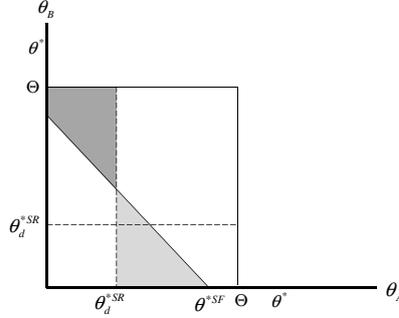
The next step is to find the optimally coordinated policy through an exact welfare accounting.

Proposition 5. *When governments have no capacity to provide liquidity ($T_i = 0$), the socially optimal policy is to allow the free flow of liquidity across borders.*

Proof see Appendix.

Figure 4 develops the intuition behind Proposition 5 by comparing equilibria with and without (bilateral) fragmentation (when $\psi^e > 1/2$), again focusing on the welfare of country A . With fragmentation, country A suffers from a financial crisis if its own realization is to the right of the θ_d^{*SR} vertical. Without fragmentation, country A suffers from a crisis if the (θ_A, θ_B) realization is above the θ^{*SF} diagonal. Hence, the net welfare effect of fragmentation is derived by subtracting the welfare valuation of the realizations in the light-shaded triangle from the welfare valuation of the realizations in the dark-shaded trapezoid. Namely, the downside of fragmentation due to the idle liquidity problem needs to be weighed against the upside of fragmentation due to the prevention of cross-country contagion. Since $\psi_{AVB}(\boldsymbol{\theta}^{SF}) = \psi_A(\boldsymbol{\theta}^{SR})$ (see the analysis in Proposition 4) the areas

Figure 4: The effect of fragmentation when $T_i = 0$ and $\psi^e > \frac{1}{2}$



of the two shapes are equal.⁷ Fragmentation has no effect on the probability of financial crisis. Nevertheless, fragmentation shifts the incidence of the crisis from realizations to the left of the θ_d^{*SR} vertical to realizations to its right; namely, it trades off mild financial crises for more severe ones. The reason is simple: Shocks that a country can fend off using only its own liquidity are smaller than those it can fend off by pooling its own liquidity with that of its neighbour.

Since, in the $T_i = 0$ case, fragmentation has no effect on the probability of financial crisis, we interpret the result here as highlighting the “pure” effect of fragmentation. Moreover, from the discussion in Section 4.2, we know that welfare is decreasing in the scale of the shocks that triggered the crises, a property that carries over to the two-country case. The pure fragmentation effect is therefore unambiguously negative.

Taken together, Propositions 4 and 5 lead to another insight: Fragmentation is not even unilaterally attractive when governments have no capacity to provide their own liquidity. In such a case, governments rely on private speculators to supply liquidity and have no incentive to impose regulations on them that would diminish the profitability of the liquidity-provision business. The result is reversed in the next section.

⁷Hence, the point $(\theta_d^{*SR}, \theta_d^{*SR})$ lies below the diagonal (above the diagonal in the $\psi^e \leq 1/2$ case). Had it been on the diagonal (and in the middle, as implied by symmetry), the triangle would have been smaller in area.

6.2 High capacity of public liquidity

At the other extreme is the case in which T_i is sufficiently high to allow governments to crowd out completely any private liquidity⁸ and become the sole provider of liquidity. Although the argument for a binary decision, presented in Section 6.1, is no longer valid (since the government may wish to keep part of its liquidity domestically and allow the rest to flow freely, the former part bearing a lower rate of return relative to the latter), we stay with the binary case. This is both for clarity of exposition and because the propensity for a corner solution is implied by Proposition 9 below. We first focus on the case in which both countries have the same tax capacity $T \equiv T_A = T_B$. The following Proposition demonstrates a complete reversal of Proposition 4 above.

Proposition 6. *When private liquidity is fully crowded out so that the only available liquidity is supplied by the governments, fragmentation is a dominant strategy and a unique Nash equilibrium.*

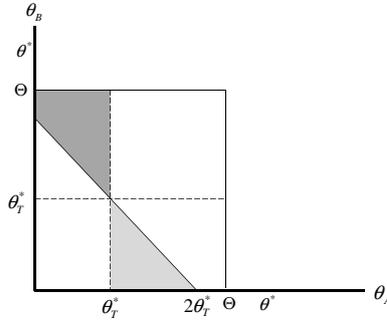
Proof see Appendix.

Figure 5 develops the intuition behind Proposition 6. Suppose country B adopts a free-to-flow policy. If country A responds with a free-to-flow policy, then the resulting allocation is $\theta^{TF} \equiv (0, 0, 2\theta_T^*)$, where T stands for tax capacity and F stands for free to flow. Alternatively, if country A responds with a fragmentation policy, the resulting allocation is $\theta^{RF} \equiv (\theta_T^*, 0, 2\theta_T^*)$. As a result, country A can avoid a financial crisis for any realization $\theta_A \leq \theta_T^*$ to the left of the θ_T^* vertical. Moreover, since country B still allows liquidity outflows, country A can avoid crisis whenever $\theta_A + \theta_B \leq 2\theta_T^*$, below the $2\theta_T^*$ diagonal. Namely, the probability of crisis falls by the entire dark-shaded trapezoid, without exposing the realization in the light-shaded triangle to crisis. If, on the other hand, country B adopts a fragmentation policy and country A responds with a free-to-flow policy, realizations within the dark-shaded trapezoid expose country A to contagion from B . It follows that, regardless of country B 's decision, a fragmentation policy by country A reduces its probability of crisis. The formal proof requires a full welfare accounting that takes into consideration the drop in the government's trading profits due to the lower probability of crisis. The latter effect is, however, dominated.

The potential for coordination failure is clear: The unilateral incentive for each country is to beggar its neighbour by keeping all the liquidity that it supplies domestically, ignoring the idle liquidity problem that it imposes on its neighbour. However, in equilibrium, when country B also implements the dominating fragmentation strategy, the allocation

⁸Note that full crowding out may take place even at very low levels of T if ψ^e is sufficiently close to one.

Figure 5: The effect of fragmentation for the case of full crowding out where $\psi^e > \frac{1}{2}$



of liquidity is not θ^{TRF} but, rather, $\theta^{TR} = (\theta_T^*, \theta_T^*, 2\theta_T^*)$, with zero free-to-flow liquidity. As a result, country A can no longer benefit from an inflow of liquidity from country B , particularly for realizations within the light-shaded triangle, where liquidity remains idle in country B . To pin down that failure, we demonstrate that fragmentation is only a socially optimal policy for low levels of θ_T^* . Since a free-to-flow policy is otherwise socially optimal and fragmentation the unique equilibrium, there is coordination failure.

Proposition 7. *When private liquidity is fully crowded out, the socially optimal policy is to allow (public) liquidity to flow freely for any $\frac{\theta_T^*}{\Theta} \geq \frac{3}{8}$. Otherwise, fragmentation is socially optimal for small values of θ_T^* (below a certain threshold).*

Proof see Appendix

Two points deserve some elaboration. First, fragmentation is an optimal policy only for low levels of liquidity. At low values of θ_T^* , a systemic crisis is the likely outcome, with or without fragmentation. Fragmentation makes a difference only for low country- A realizations. Conditional on a low country- A realization, fragmentation can avoid contagion even for high country- B realizations (a high probability event), but would induce an idle-liquidity problem only for low country- B realizations (a low probability event); see the dark-shaded trapezoid and the light shaded triangle of Figure 5 once again. So fragmentation is socially optimal. Second, comparing the results in this section to those in the previous one, the beggar-thy-neighbour problem arises when liquidity is publicly provided. Then, fragmentation serves as a means of grabbing more liquidity to service the domestic market. Since the total amount of liquidity is given, this grabbing game may end with more idle liquidity, which is socially sub-optimal for high levels of

θ_T^* . In contrast, when liquidity is provided privately, fragmentation directly decreases the profitability of liquidity provision, which is either ineffective because speculators can find a way around it or it decreases the supply of liquidity to the fragmented economy.

It is interesting to extend the above welfare analysis to the case in which countries do not have a symmetric tax capacity. We thus distinguish between country-specific thresholds $\theta_{T_A}^*$ and $\theta_{T_B}^*$, defined by (22), using country-specific levels of tax capacity, T_A and T_B . Suppose total public liquidity is high enough so that private liquidity is fully crowded out. We can then demonstrate the following result.

Proposition 8. *Suppose the sum of both countries' tax capacities $T_A + T_B$ is constant. Country A's benefit from coordinating to pool liquidity with country B is decreasing in A's tax capacity. Moreover, there are parameters such that when A's tax capacity is sufficiently high, it is better off not coordinating in pooling liquidity, even though doing so would maximize joint welfare.*

Proof see Appendix.

This proposition shows that the benefits to pooling do not accrue irrespective of tax capacity. A country with high tax capacity benefits relatively little from pooling with a country with low tax capacity; the “richer” country suffers only a mild idle liquidity problem, since the country with little ability to inject public funds will rarely be able to help. On the other hand, when liquidity is pooled, the richer country suffers severely from contagion, since the poor country will frequently need to draw on its liquidity. Even though *joint* welfare may be improved from pooling, the richer country may not benefit from it. Getting it to agree to pool liquidity may therefore require a lump sum transfer from the poor country.

The above result relates to the recent debate on creating a European banking union. Such a union has as its aim (among other things) to pool resources to insure deposits at a European level rather than at a national level. One reason for the drive towards such a union is to ensure that a country's banking system can be rescued, even if locally available funds are insufficient. This issue has come up in the recent European debt crisis, where countries such as Ireland and Cyprus did not have sufficient (sovereign) debt capacity to guarantee the debts of their local banks. This situation is not unlike the case in our model, where tax capacity only allows a country to withstand a shock up to a certain boundary. The process of implementing a banking union has encountered resistance from the richer EU countries, notably Germany. Our model provides a rationale for the difference in attitudes towards the pooling of resources. A country with a high tax capacity (say Germany) would significantly increase its risk of being subject to contagion

from a country with lower tax capacity (e.g. Spain) if both countries agree to pool funds. The upside for Germany would be generated in those states of the world where Spain's tax capacity would be able to rescue German banks, an event that is perceived to be unlikely, given the two countries' relative tax capacities.

This discussion takes for granted that the pooled funds are allocated by a market mechanism, that is, funds flow to where fire sales prices are lowest. Although pooled funds may not literally be allocated by a market mechanism in the above example, the outcome would be the same if countries employ the simple ex post allocation rule by which a country experiencing a liquidity shortage has priority access to pooled funds. We return to the question of the optimal ex post allocation of funds in Section 7.

6.3 A mix of private and public liquidity

Consider now the case where the governments can provide some liquidity (i.e. $\theta_T^* > 0$), but not enough to fully crowd out private speculators. Note that if public liquidity were free to flow, it would directly compete with private liquidity and crowd out the latter, one for one. When tax capacity is insufficient to replace all private liquidity, public free-to-flow liquidity is useless. We therefore focus on the case of domestic public liquidity. Since domestic liquidity earns a lower expected rate of return relative to free-to-flow liquidity, public and private liquidity become imperfect substitutes. This raises the possibility that injections of public liquidity crowd out private liquidity only partially, leading to an increase in the total liquidity.

To see why, consider Figure 3 again, except that we now add the information that θ_A^* and θ_B^* are public and θ_F^* is private. The amount of the latter is determined from $\psi_{A \vee B}(\boldsymbol{\theta}) = \psi^e$. Let country A inject some additional domestic public liquidity, increasing θ_A^* . Suppose, by way of contradiction, that one-for-one crowding out took place. Since the extra liquidity of country- A is not available to country B , the $RC - B$ region would expand, increasing the probability of crisis in either country so that $\psi_{A \vee B}(\boldsymbol{\theta}) > \psi^e$. This cannot be an equilibrium. Instead, the private liquidity supply would increase, bringing its rate of return back to the competitive level. Hence, an increase in θ_A^* would increase θ^* . Note, however, that one-to-one crowding out is possible if the $RC - B$ region is empty (i.e. when $\theta_A^* > \theta^* - \Theta$). Obviously, public liquidity injections would then be useless. From now on, we therefore focus on the case in which regional crisis may occur and partial crowding-out is possible. Hence, let $\boldsymbol{\theta}^{TS}$ be an allocation of liquidity, with

both government and speculative liquidity and non-empty regional crisis areas:

$$\boldsymbol{\theta}^{TS} \in \{ \boldsymbol{\theta} \mid \theta_T^* \geq \theta_i^* > 0, \theta^* - \theta_A^* - \theta_B^* > 0, \theta_i^* > \theta^* - \Theta, \psi_{AVB}(\boldsymbol{\theta}^{TS}) = \psi^e \}.$$

We can now formally calculate the exact effect of domestic liquidity on total liquidity, using (27) and $\psi_{AVB}(\boldsymbol{\theta}^{TS}) = \psi^e$:

$$\left. \frac{\partial \theta^*}{\partial \theta_A^*} \right|_{\boldsymbol{\theta} \in \boldsymbol{\theta}^{TS}} = \frac{\theta_A^*}{\theta^*} > 0. \quad (31)$$

Welfare in country i is given by (28), (29) and (30). The budget surplus (29) can be rewritten using the fact that free-to-flow liquidity is supplied competitively and that all public liquidity is domestic and therefore $L_{G,i} = \theta_i^* (1 - \pi) q^n b (q^n)$:

$$Z_i(\boldsymbol{\theta}) = -[\psi_{AVB}(\boldsymbol{\theta}) - \psi_i(\boldsymbol{\theta})] (\rho^c - 1) \theta_i^* (1 - \pi) q^n b (q^n).$$

To analyse the incentive of, say, country A to inject domestic liquidity, we calculate the derivative

$$\frac{dSW_A(\boldsymbol{\theta}^{TS})}{d\theta_A^*} = \frac{\partial C_A(\boldsymbol{\theta}^{TS})}{\partial \theta_A^*} + \frac{\partial Z_A(\boldsymbol{\theta}^{TS})}{\partial \theta_A^*} + \left[\frac{\partial C_A(\boldsymbol{\theta}^{TS})}{\partial \theta^*} + \frac{\partial Z_A(\boldsymbol{\theta}^{TS})}{\partial \theta^*} \right] \frac{\partial \theta^*}{\partial \theta_A^*}.$$

The first two terms capture the effect of a change in θ_A^* (holding θ^* constant) and can be interpreted as a pure fragmentation effect. Using (28), we can derive

$$\frac{\partial C_A(\boldsymbol{\theta}^{TS})}{\partial \theta_A^*} = \frac{\theta_A^*}{\Theta} \psi_A(\boldsymbol{\theta}^{TS} \mid \theta_A^*) \Delta > 0. \quad (32)$$

The derivative has a simple geometric interpretation: By injecting domestic liquidity, the government can decrease the probability of financial crisis for the marginal state, θ_A^* (with density $1/\Theta$) from $\psi_A(\boldsymbol{\theta}^{TS} \mid \theta_A^*)$ to zero, with a proportional decrease in the deadweight cost of external finance, Δ . This effect is always positive, which generalizes the result of Proposition 6 for the case of interim levels of fragmentation.

Next, we analyse the welfare implications of increasing the total liquidity supply through partial crowding out:

$$\begin{aligned} \frac{\partial C_A(\boldsymbol{\theta}^{TS})}{\partial \theta^*} = & \left\{ \left(\frac{\theta^* - \theta_A^* - \theta_B^*}{\Theta} \right) \left(\frac{\theta^* + \theta_A^* - \theta_B^*}{2} \right) \right. \\ & \left. + [1 - \psi_A(\boldsymbol{\theta}^{TS} \mid \theta^* - \theta_B^*)] (\theta^* - \theta_B^*) \right\} \frac{1}{\Theta} \Delta > 0. \end{aligned} \quad (33)$$

This derivative also has a simple geometric interpretation: When the θ^* diagonal shifts

rightwards by $d\theta^*$, the conditional probability of crisis for any realization within the set $(\theta_A^*, \theta^* - \theta_B^*)$ falls by $\frac{d\theta^*}{\Theta}$; $\left(\frac{\theta^* - \theta_A^* - \theta_B^*}{\Theta}\right)$ is the probability measure of that set and $\left(\frac{\theta^* + \theta_A^* - \theta_B^*}{2}\right)$ is the average magnitude of a realization within that set. Also, when the θ^* diagonal shifts rightwards, the NC region expands rightwards, so the conditional probability of crisis for a marginal realization $\theta^* - \theta_B^* + \varepsilon$ falls from one to $\psi_A(\boldsymbol{\theta}^{TS} | \theta^* - \theta_B^*)(\theta^* - \theta_B^*)$. Δ has the usual interpretation.

These positive effects have to be weighed against the increased cost of supplying domestic liquidity. This is done in the following proposition.

Proposition 9. *i) When there is a mix of public and private liquidity and when the regional crisis areas are non-empty, the injection of public domestic liquidity is a dominant strategy, namely*

$$\frac{dSW_A(\boldsymbol{\theta}^{TS})}{d\theta_A^*} > 0. \quad (34)$$

(ii) Partial crowding out strengthens the incentive to supply public domestic liquidity, namely

$$\frac{\partial C_A(\boldsymbol{\theta}^{TS})}{\partial \theta^*} + \frac{\partial Z_A(\boldsymbol{\theta}^{TS})}{\partial \theta^*} > 0.$$

Proof see Appendix.

It follows that, in absence of coordination, each country would use its tax capacity to the limit and inject domestic liquidity all the way up to the θ_T^* constraint. Such a policy has two effects: First, it increases the degree of fragmentation. Second, it increases the total supply of liquidity due to the fact that public liquidity is subsidized and only partially crowds out private liquidity. It is clear from the previous discussion that fragmentation has a negative spillover effect on the neighbour country. However, fragmentation is also a by-product if a country with limited tax capacity wishes to increase the total liquidity supply. The latter constitutes a positive spillover effect. The next proposition shows that, on balance, an uncoordinated equilibrium is (weakly) a beggar-thy-neighbour policy:

Proposition 10. *An uncoordinated level of fragmentation is weakly suboptimal: the governments allocate (weakly) too little free-to-flow liquidity.*

Proof. Since the uncoordinated equilibrium is, in general, fragmentation to the limit, the uncoordinated equilibrium cannot fail by providing “too little fragmentation” (i.e. too much free-to-flow public liquidity). Clearly, full fragmentation may be a socially-optimal policy (see Proposition 7) but it can fail by providing too much fragmentation (i.e. too little free-to-flow public liquidity). For example, consider a situation where the socially optimal policy for $\theta^* = 2\theta_T^*$ (and no private liquidity) is zero fragmentation

(see Proposition 7). Clearly, there exist a ψ^e such that, with uncoordinated governments fragmenting to the θ_T^* limit, private liquidity is positive, $\theta^* = 2\theta_T^* + \varepsilon$, but small enough to be fully crowded out if the governments coordinate to avoid fragmentation. Namely, liquidity injection is effective (see Proposition 2). Due to continuity, there exists an ε small enough to make such a coordination a Pareto improvement. \square

It follows that, although we have shown that the “pure” welfare effect of fragmentation is negative (with a given probability of crisis, fragmentation trades of a mild financial crisis for a severe one; see Proposition 5), governments may still have a role in injecting liquidity in the market. Liquidity is a public good, under-provided in a competitive equilibrium (see Proposition 2). Moreover, in some cases, injecting liquidity exclusively for domestic use is the optimal policy. The case for such policy is enhanced by the partial crowding out effect (see result *ii*) in Proposition 9). Yet, in the absence of coordination, governments would implement “too much fragmentation”, leading to beggar thy neighbor liquidity policies.

7 Ex post contingent allocation of liquidity

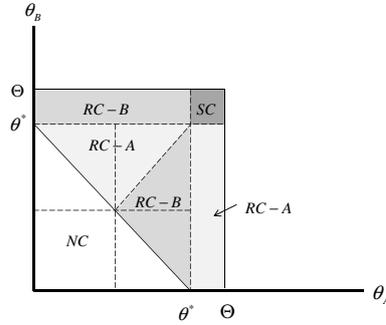
So far, coordination has meant that both countries reach an ex-ante agreement regarding liquidity injections and liquidity flow restrictions. By assumption, they stick to that agreement ex post. In this section we adopt an alternative assumption, that the two countries can reach an ex ante enforceable agreement about the ex post allocation of liquidity contingent on the realization (θ_A, θ_B) . We maintain the assumption that total liquidity, θ^* , is predetermined (due to the non-reversible nature of the investment in the alternative linear technology). The analysis is presented for the sake of completeness, but also because it highlights the pivotal role that increasing returns to liquidity play in our analysis: It is optimal to direct liquidity to treat the country that is suffering the higher realization of poor investors (provided that it can be rescued). The results bear a similarity to the triage rules of emergency medicine: Prioritize treatment to the more badly injured, provided that there are sufficient resources to save them. However, if they cannot be saved, one should leave the badly injured to their fate and treat the less badly injured instead. More accurately, we provide the following proposition.

Proposition 11. *Under the ex post optimal allocation of liquidity, both countries should be rescued if possible, that is if*

$$\theta_A + \theta_B \leq \theta^*.$$

Otherwise, if $\theta_i + \theta_j > \theta^$, country i should be rescued from crisis (by the injection of an amount of liquidity equal to $\theta_A(1 - \pi)q^n b(q^n)$, exactly) while country j should be allocated*

Figure 6: Crisis regimes under the ex post optimal allocation of liquidity



the remaining liquidity (which is insufficient to fend off the crisis) if the rescue of country i is feasible, $\theta_i \leq \theta^*$, and either the rescue of country j is infeasible

$$\theta_j \geq \theta^*,$$

or the rescue of country j is feasible, $\theta_j \leq \theta^*$, but

$$\theta_i \geq \theta_j.$$

A systemic crisis is unavoidable if for both $i = A, B$, $\theta_i > \theta^*$.

Proof see Appendix.

The various crisis regimes are described in Figure 6.

Two points deserve a brief elaboration. First, in the absence of strong commitment, it is hard to see how the ex post optimal allocation of liquidity can be implemented, which is why (to the best of our knowledge) it is not implemented in reality. This is because it requires, in some cases, a country that has enough liquidity to fend off a crisis to “sacrifice itself” (from an ex post point of view) to save its badly injured neighbour. The difficulty in implementation justifies the attention given to the case analyzed in Section 6, where countries may coordinate ex ante on whether or not to pool funds but will ex post allocate the pooled funds with priority to that country experiencing a liquidity shortage. Perhaps the result can provide a rationale for an international crisis management institution (such as the IMF or the ECB) that centralizes the allocation of liquidity ex post. Second, the result vividly illustrates why the standard prescription of allowing goods to flow across

markets until the prices are equal across countries does not apply in our model: Under increasing returns to scale, such a rule does not implement economic efficiency.

8 Conclusions

The conclusion that fragmentation is a “good policy” seems straightforward from the ex post point of view of a country that suffered or is about to suffer a financial crisis due to contagion from abroad. We argue that things are much less clear-cut when fragmentation is evaluated from the point of view of international coordination, say, from the point of view of the IMF or the ECB. For then, the straightforward advantage of unilateral fragmentation may turn into coordination failure, just as in the old beggar-thy-neighbour analysis. The reason why the coordinated analysis differs from the unilateral one is that it considers not just the event of contagion, but also the event where a country can be rescued from crisis if it can access the idle liquidity of its neighbor, that is when the market is not fragmented. We show that the net welfare effect of “pure” fragmentation is negative because it trades off relatively mild financial crises for more severe ones. We emphasize that speculators who provide market liquidity do not internalize its full social value and, as a result, liquidity is underprovided in a competitive equilibrium, just like any other public good. Yet, that the market fails in providing sufficient liquidity does not imply that other aspects of market allocation, such as the tendency of goods to flow towards the highest bidder, are necessarily sub optimal. Liquidity policy has two dimensions: how much liquidity to provide and whether to impose restrictions on free flow. In some cases, the optimal policy is to inject as much liquidity as possible and to allow it to flow freely. Unfortunately, when the liquidity market is dominated by public players, the temptation to fragment unilaterally is also the strongest, which is when the role of coordination bodies such as the IMF or ECB is so critical.

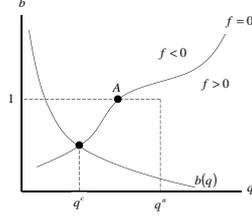
Appendix

Proof of Lemma 1

Rewrite the participation constraint (14) as the function

$$f(b, q) = (y_2 + y_3) - \frac{\delta(y_2 + y_3)}{q} - (1 - \pi)b(y_2 + y_3 - q),$$

Figure 7: proof of Lemma 1



where q^c is given by the solution to $f(b, q) = 0$. For a given b , f is a convex function in q with one positive solution. Let the $f = 0$ graph in Figure 8 provide that solution against b . Since $f_b < 0$ and $f_q > 0$ the graph is upwards sloping. The figure also plots the graph of the $b(q)$ function. Clearly, q^c is determined by the intersection of these graphs. Since $b' < 0$, it is unique. Now, condition 4 guarantees that $f(1, q^n) > 0$ and hence lies below the $f = 0$ graph. Notice that the location the $b(q)$ curve depends on \underline{w} . Let $W(y_2, y_3, \delta, \pi)$ be the \underline{w} such that the $b(q)$ passes through point A . Since an increase \underline{w} shifts the graph of the $b(q)$ function leftwards, $q^c < q^n$ for $\underline{w} > W(y_2, y_3, \delta, \pi)$ and the feasibility constraint (10) does not bind. Since $b' < 0$, it does not bind for any $q \in [q^c, q^n]$.

Proof of Lemma 3

Using the definition of $b(q)$ in (11) it can be shown that

$$\frac{b(q^c)}{b(q^n)} > \rho^c.$$

Hence, and using the definition the $l(q)$ we derive:

$$\begin{aligned} & \Delta - 2(\rho^c - 1)(1 - \pi)q^n b(q^n) \\ &= (1 - \pi)b(q^n) \left[\frac{b(q^c)}{b(q^n)}(y_2 + y_3 - q^c) - (y_2 + y_3 - q^n) - 2(\rho^c - 1)q^n \right] \\ &> (1 - \pi)b(q^n) [\rho^c(y_2 + y_3 - q^c) - (y_2 + y_3 - q^n) - 2(\rho^c - 1)q^n] \\ &> (1 - \pi)b(q^n)(\rho^c - 1) [\rho^c(y_2 + y_3) - 2\delta(y_2 + y_3)] > 0 \end{aligned}$$

under the assumption that $\delta < \frac{1}{2}$.

Proof of Proposition 2

Using the competitive zero-profit condition (21) to evaluate the derivative (25) at the point $\theta_G^* = \theta_S^*$ we get

$$SW'(\theta_S^*) = \frac{\theta_S^*}{\Theta} \Delta + \psi'(\theta_S^*) (\rho^c - 1) L_S,$$

which is positive under Lemma 3. It follows that social welfare is increasing in liquidity at the competitive equilibrium, so that the government should inject extra liquidity if the θ_T^* constraint allows. Taking a second derivative of SW and using Lemma 3 again we can verify that social welfare is convex in liquidity. It follows that the government should inject liquidity to the corner solution θ_T^* , or Θ . Notice, however, that if $\theta_T^* < \theta_S^*$ so that the speculators are still active in the market, their zero-profit condition must be satisfied and the probability of crisis is ψ^e , independently of θ_G^* : there is one-to-one crowding out of private liquidity by public liquidity.

Proof of Proposition 5

Since, in this case, the governments have no fiscal activity, $Z_i(\boldsymbol{\theta}) = 0$ and welfare reduces to expected consumption (28). A free-to-flow policy strictly dominates fragmentation if and only if (regardless of $\psi^e \gtrless 1/2$)

$$SW_i(\boldsymbol{\theta}^{SF}) > y_2 + y_3 + \int_0^{\theta_d^{*SR}} \theta_{Al}(q^n) \frac{d\theta_A}{\Theta} + \int_{\theta_d^{*SR}}^{\Theta} \theta_{Al}(q^c) \frac{d\theta_A}{\Theta};$$

We can substitute the following into the right-hand side of the inequality:

$$\theta_{Al}(q^n) = \theta_A \psi_A(\boldsymbol{\theta}^{SF} | \theta_A) l(q^n) + \theta_A [1 - \psi_A(\boldsymbol{\theta}^{SF} | \theta_A)] l(q^n).$$

Doing the same decomposition for $\theta_{Al}(q^c)$, and canceling out against equal terms in $SW_i(\boldsymbol{\theta}^{SF})$ as specified in equation (28) we get a simplified condition for the dominance of the free-to-flow policy:

$$\int_{\theta_d^{*SR}}^{\Theta} [1 - \psi_A(\boldsymbol{\theta}^{SF} | \theta_A)] \theta_A \frac{d\theta_A}{\Theta} > \int_0^{\theta_d^{*SR}} \psi_A(\boldsymbol{\theta}^{SF} | \theta_A) \theta_A \frac{d\theta_A}{\Theta}.$$

The above is implied by (i) the fact that all θ_A s on the LHS of the integration are greater than θ_A s on the RHS, together with (ii) the equality of the two shaded areas in Figure 4, which corresponds to

$$\int_{\theta_d^{*SR}}^{\Theta} [1 - \psi_A(\boldsymbol{\theta}^{SF} | \theta_A)] \frac{d\theta_A}{\Theta} = \int_0^{\theta_d^{*SR}} \psi_A(\boldsymbol{\theta}^{SF} | \theta_A) \frac{d\theta_A}{\Theta}.$$

Proof of Proposition 6

Note that the cost of funding only depends on the amount of liquidity supply which is the same regardless of whether the governments fragment or not. Let $\boldsymbol{\theta}^{TF} = (0, 0, 2\theta_T^*)$ and $\boldsymbol{\theta}^{TRF} = (\theta_T^*, 0, 2\theta_T^*)$ be the allocation when country B adopts a free-to-flow policy and country A responds with a free-to-flow and fragmentation policies, respectively. To derive the difference in country- A welfare between the two responses we define $D_A^{TRF}(\theta_T^*) \equiv SW_A(\boldsymbol{\theta}^{TRF}) - SW_A(\boldsymbol{\theta}^{TF})$. Using equation (28), and adding the (negative) change in trading profits from (29) using $L_{G,A} = \theta_T^*(1 - \pi)q^n b(q^n)$, we can simplify along the lines of the proof of Proposition 5.

$$D_A^{TRF}(\theta_T^*) = \int_0^{\theta_T^*} \psi(\boldsymbol{\theta}^{TF} | \theta_A) [\theta_A \Delta - \theta_T^*(\rho^c - 1)(1 - \pi)q^n b(q^n)] \frac{d\theta_A}{\Theta}.$$

Solving the integral yields two different expressions depending on whether or not $2\theta_T^* \leq \Theta$.

$$D_A^{TRF}(\theta_T^* | 2\theta_T^* \leq \Theta) = \Theta \left\{ \left[\frac{1}{2} \left(\frac{\theta_T^*}{\Theta} \right)^2 - \frac{2}{3} \left(\frac{\theta_T^*}{\Theta} \right)^3 \right] \Delta - \left[\left(\frac{\theta_T^*}{\Theta} \right)^2 - \frac{3}{2} \left(\frac{\theta_T^*}{\Theta} \right)^3 \right] (\rho^c - 1)(1 - \pi)q^n b(q^n) \right\}$$

and

$$D_A^{TRF}(\theta_T^* | 2\theta_T^* > \Theta) = \Theta \left\{ \left[-\frac{1}{6} + \left(\frac{\theta_T^*}{\Theta} \right) - \frac{3}{2} \left(\frac{\theta_T^*}{\Theta} \right)^2 + \frac{2}{3} \left(\frac{\theta_T^*}{\Theta} \right)^3 \right] \Delta - \left[\frac{1}{2} \left(\frac{\theta_T^*}{\Theta} \right) - \left(\frac{\theta_T^*}{\Theta} \right)^2 + \frac{1}{2} \left(\frac{\theta_T^*}{\Theta} \right)^3 \right] (\rho^c - 1)(1 - \pi)q^n b(q^n) \right\}$$

Notice that the coefficient of Δ is positive. Moreover, given the assumption $\delta < 1/2$, we know that $\Delta - 2(\rho^c - 1)(1 - \pi)q^n b(q^n) > 0$. Since, for both $2\theta_T^* \gtrless \Theta$, twice the coefficient of Δ exceeds the coefficient of $(\rho^c - 1)(1 - \pi)q^n b(q^n)$, in the expressions for it $D_A^{TRF}(\theta_T^*)$ it follows that $D_A^{TRF}(\theta_T^*) > 0$.

Proof of Proposition 7

Following similar steps as in the proof of Proposition 6 we derive the difference in country-A welfare between the θ^{TR} allocation and the θ^{TF} , i.e., $D_A^T(\theta_T^*) \equiv SW_A(\theta^{TR}) - SW_A(\theta^{TF})$:

$$D_A^T(\theta_T^*) = \int_0^{\theta_T^*} \psi(\theta^T | \theta_A) [\theta_A \Delta - \theta_T^* (\rho^c - 1) (1 - \pi) q^n b(q^n)] \frac{d\theta_A}{\Theta} - \int_{\theta_T^*}^{2\theta_T^*} [1 - \psi(\theta^T | \theta_A)] [\theta_A \Delta - \theta_T^* (\rho^c - 1) (1 - \pi) q^c b(q^c)] \frac{d\theta_A}{\Theta}.$$

Solving the integral for the $2\theta_T^* \leq \Theta$ case we get:

$$D_A^T(\theta_T^* |_{2\theta_T^* \leq \Theta}) = \Theta \left\{ \left[\frac{1}{2} \left(\frac{\theta_T^*}{\Theta} \right)^2 - \frac{4}{3} \left(\frac{\theta_T^*}{\Theta} \right)^3 \right] \Delta - \left[\left(\frac{\theta_T^*}{\Theta} \right)^2 - 2 \left(\frac{\theta_T^*}{\Theta} \right)^3 \right] (\rho^c - 1) (1 - \pi) q^n b(q^n) \right\}.$$

Notice that the coefficient of Δ is negative for $\frac{\theta_T^*}{\Theta} > \frac{3}{8}$ while $\left(\frac{\theta_T^*}{\Theta} \right)^2 - 2 \left(\frac{\theta_T^*}{\Theta} \right)^3$ is (weakly) positive for the entire $\theta_T^* \in [0, \frac{1}{2}]$ range, which guarantees that $D_A^T < 0$ to the right of $\frac{3}{8}$. Otherwise, rewrite the integral as

$$D_A^T(\theta_T^* |_{2\theta_T^* \leq \Theta}) = \Theta \left(\frac{\theta_T^*}{\Theta} \right)^2 \left[1 - 2 \left(\frac{\theta_T^*}{\Theta} \right) \right] \left\{ \frac{\frac{1}{2} - \frac{4}{3} \left(\frac{\theta_T^*}{\Theta} \right)}{1 - 2 \left(\frac{\theta_T^*}{\Theta} \right)} \Delta - (\rho^c - 1) (1 - \pi) q^n b(q^n) \right\},$$

and notice that the coefficient of Δ increases to $\frac{1}{2}$ as $\left(\frac{\theta_T^*}{\Theta} \right)$ falls zero. Since the assumption $\delta < \frac{1}{2}$ implies $\frac{1}{2} \Delta - (\rho^c - 1) (1 - \pi) q^n b(q^n) > 0$, it follows that there exists a $\frac{\theta_T^*}{\Theta}$ small enough such that $D_A^T(\theta_T^* |_{2\theta_T^* \leq \Theta}) > 0$.

Solving the integral for the case where $2\theta_T^* > \Theta$

$$D_A^T(\theta_T^* |_{2\theta_T^* > \Theta}) = -\Theta \left\{ \left[-\frac{1}{6} + \frac{3}{2} \left(\frac{\theta_T^*}{\Theta} \right)^2 - \frac{4}{3} \left(\frac{\theta_T^*}{\Theta} \right)^3 \right] \Delta - \left[\left(\frac{\theta_T^*}{\Theta} \right) - 3 \left(\frac{\theta_T^*}{\Theta} \right)^2 + 2 \left(\frac{\theta_T^*}{\Theta} \right)^3 \right] (\rho^c - 1) (1 - \pi) q^n b(q^n) \right\}$$

which is negative by a similar argument to the one in Proposition 6.

Proof of Proposition 8

The proof proceeds by showing that the slope of the difference in welfare is negative in threshold $\theta_A^* = \theta_{TA}^*$. Solving the welfare integral for the general case yields

$$D_A^T(\theta_A^* | \theta^* \leq \Theta) = \left\{ \left[\frac{1}{2} \left(\frac{\theta_A^*}{\Theta} \right)^2 - \frac{1}{6} \left(\frac{\theta^*}{\Theta} \right)^3 \right] \Delta - \left[\left(\frac{\theta_A^*}{\Theta} \right)^2 - \frac{1}{2} \frac{\theta_A^*}{\Theta} \left(\frac{\theta^*}{\Theta} \right)^2 \right] (\rho^c - 1) (1 - \pi) q^n b(q^n) \right\} \Theta$$

and

$$D_A^T(\theta_A^* | \theta^* > \Theta) = \left\{ \left[\frac{1}{3} + \frac{1}{2} \frac{\theta^*}{\Theta} + \frac{1}{2} \left(\frac{\theta_A^*}{\Theta} \right)^2 + \frac{1}{6} \left(\frac{\theta^*}{\Theta} - 1 \right)^3 \right] \Delta - \frac{\theta_A^*}{\Theta} \left[1 - 2 \frac{\theta^*}{\Theta} + \frac{\theta_A^*}{\Theta} + \frac{1}{2} \left(\frac{\theta^*}{\Theta} \right)^2 \right] (\rho^c - 1) (1 - \pi) q^n b(q^n) \right\} \Theta$$

Taking the derivative yields

$$\frac{\partial D_A^T(\theta_A^* | \theta^* \leq \Theta)}{\partial \theta_A^*} = \frac{\theta_A^*}{\Theta} \left\{ \Delta - 2 (\rho^c - 1) (1 - \pi) q^n b(q^n) \right\} + \frac{1}{2} \left(\frac{\theta^*}{\Theta} \right)^2 (\rho^c - 1) (1 - \pi) q^n b(q^n),$$

which by assumption 2 is positive. Similarly,

$$\frac{\partial D_A^T(\theta_A^* | \theta^* > \Theta)}{\partial \theta_A^*} = \frac{\theta_A^*}{\Theta} \left\{ \Delta - 2 (\rho^c - 1) (1 - \pi) q^n b(q^n) \right\} - \left\{ \left[1 - \frac{\theta^*}{\Theta} - \frac{\theta^*}{\Theta} \left(1 - \frac{1}{2} \frac{\theta^*}{\Theta} \right) \right] (\rho^c - 1) (1 - \pi) q^n b(q^n) \right\},$$

which is also positive. Hence, pooling becomes less desirable as θ_A^* increases.

Next, it is easy to see that at the upper bound

$$D_A^T(\theta_A^* = \theta^* | \theta^* \leq \Theta) = \Theta \left(\frac{\theta^*}{\Theta} \right)^2 \left\{ \left[\frac{1}{2} - \frac{1}{6} \left(\frac{\theta^*}{\Theta} \right) \right] \Delta - \left[1 - \frac{1}{2} \frac{\theta^*}{\Theta} \right] (\rho^c - 1) (1 - \pi) q^n b(q^n) \right\}.$$

By assumption 2 the expression is positive if

$$\frac{1 - \frac{1}{2} \frac{\theta^*}{\Theta}}{\frac{1}{2} - \frac{1}{6} \left(\frac{\theta^*}{\Theta} \right)} \leq 2,$$

which is always true. Moreover, at the other extreme $\theta_A^* = 0$

$$D_A^T(\theta_A^* = 0 | \theta^* \leq \Theta) = \Theta \left\{ -\frac{1}{6} \left(\frac{\theta^*}{\Theta} \right)^3 \Delta \right\} < 0.$$

Hence, by continuity, a country with close to zero tax capacity strictly prefers fragmentation, while the other country strictly prefers pooling. Note also that for high values of ψ^e , even a small T is sufficient to fully crowd out private speculators. Take now the parameter constellation such that under symmetric tax capacity both countries prefer pooling. In that case, we can redistribute tax capacity to the point where one country loses marginally from pooling (from before we know that such a point exists). But since the gains from pooling have strictly increased for the other country, it follows that joint welfare would still be maximized under pooling.

Proof of Proposition 9

We account for the effect of the liquidity injections on the government's budget surplus:

$$\begin{aligned} \frac{\partial Z_A(\boldsymbol{\theta}^{TS})}{\partial \theta_A^*} &= (\rho^c - 1)(1 - \pi)q^n b(q^n) \left[\psi_A(\boldsymbol{\theta}^{TS}) - \frac{\theta_A^*}{\Theta} \psi_A(\boldsymbol{\theta}^{TS} | \theta_A^*) \right] \\ &- (\rho_0 - 1)(1 - \pi)q^n b(q^n). \end{aligned} \quad (35)$$

The top line accounts for the change in trading profits while the bottom line accounts for the increase in the cost of funding. Likewise, we calculate

$$\frac{\partial Z_A(\boldsymbol{\theta}^{TS})}{\partial \theta^*} = -(\rho^c - 1)(1 - \pi)q^n b(q^n) \frac{\theta_A^*}{\Theta} \left\{ \left(\frac{\theta^* - \theta_A^* - \theta_B^*}{\Theta} \right) + [1 - \psi_A(\boldsymbol{\theta}^{TS} | \theta^* - \theta_B^*)] \right\}. \quad (36)$$

The expression within the curly brackets has a geometric interpretation similar to the one in equation (33).

We first prove point *ii*), namely the welfare effect of partial crowding out. Given $\theta^* - \theta_B^* \geq \frac{\theta^* + \theta_A^* - \theta_B^*}{2}$, it follows that

$$\begin{aligned} &\frac{\partial C_A(\boldsymbol{\theta}^{TS})}{\partial \theta^*} + \frac{\partial Z_A(\boldsymbol{\theta}^{TS})}{\partial \theta^*} \\ &\geq \left[\left(\frac{\theta^* - \theta_A^* - \theta_B^*}{\Theta} \right) + 1 - \psi_A(\boldsymbol{\theta}^{TS} | \theta^* - \theta_B^*) \right] \left[\frac{\theta^* + \theta_A^* - \theta_B^*}{2\Theta} \Delta - \frac{\theta_A^*}{\Theta} (\rho^c - 1)(1 - \pi)q^n b(q^n) \right] > 0 \end{aligned}$$

because of $\frac{\theta^* + \theta_A^* - \theta_B^*}{2} > \theta_A^*$ (since free-to-flow liquidity, $\theta^* - \theta_A^* - \theta_B^*$ is non-negative) and Lemma 3.

Next, we show that the direct effect of fragmentation is also positive, namely

$$\frac{\partial C_A(\boldsymbol{\theta}^{TS})}{\partial \theta_A^*} + \frac{\partial Z_A(\boldsymbol{\theta}^{TS})}{\partial \theta_A^*} > 0.$$

Using the speculators zero-profit condition, $\psi_{A \vee B}(\boldsymbol{\theta}^{TS}) = \psi^e$ and substituting in equation (27) for $\psi_{A \vee B}(\boldsymbol{\theta}^{TS})$ we can simplify equation (35) to

$$\frac{\partial Z_A(\boldsymbol{\theta}^{TS})}{\partial \theta_A^*} = -2(\rho^c - 1)(1 - \pi)q^n b(q^n) \frac{\theta_A^*}{\Theta} \psi_A(\boldsymbol{\theta}^{TS} | \theta_A^*).$$

Using Lemma 3 we complete the proof.

Proof of Proposition 11

The cases where both countries can be rescued is straight forward. So is the case where only the rescue of country i is feasible. The interesting case is when the rescue of either country, but not both, is feasible. Suppose, without loss of generality, that $\theta_A, \theta_B > \theta^*$, $\theta_A + \theta_B < \theta^*$ but $\theta_A > \theta_B$. Due to symmetry, if joint welfare is maximized for each ex-post realization, each country's ex-ante expected welfare is maximized and equally allocated. Hence, joint welfare generated by the rescue of country A only is

$$\begin{aligned} SW_{resqA} = & 2(y_2 + y_3) - \theta_A l(q^n) - \theta_B l(q^c) \\ & + (\theta^* - \theta_A)(\rho^c - 1)(1 - \pi)q^n b(q^n) - (\rho_0 - 1)(1 - \pi)q^n b(q^n). \end{aligned}$$

Likewise, the joint value of the rescue of country B only is

$$\begin{aligned} SW_{resqB} = & 2(y_2 + y_3) - \theta_A l(q^c) - \theta_B l(q^n) \\ & + (\theta^* - \theta_B)(\rho^c - 1)(1 - \pi)q^n b(q^n) - (\rho_0 - 1)(1 - \pi)q^n b(q^n). \end{aligned}$$

By Lemma 3,

$$SW_{resqA} - SW_{resqB} = (\theta_A - \theta_B)[\Delta - (\rho^c - 1)(1 - \pi)q^n b(q^n)] > 0.$$

References

- [1] Acharya, V.V., H.S. Shin and T. Yorulmazer, 2013 "A Theory of Arbitrage Capital," *The Review of Corporate Finance Studies*, 2, 62-97..

- [2] Acharya, V.V., H.S. Shin and T. Yorulmazer, 2011, "Crisis Resolution and Bank Liquidity," *Review of Financial Studies*, 24, 2166-2205.
- [3] Allen, F. and D. Gale, 2000, "Financial Contagion," *Journal of Political Economy*, 108, 1-33.
- [4] Allen, F. and D. Gale, 1994, "Limited Market Participation and Volatility of Asset Prices," *American Economic Review*, 84, 933-955.
- [5] Bernanke, B. and M. Gertler, 1989, "Agency Costs, Net Worth and Business Fluctuations," *American Economic Review*, 79(1), 14-31.
- [6] Bhattacharya, S. and D. Gale, 1987, "Preference Shocks, Liquidity and Central Bank Policy," in W. Barnett and K. Singleton (eds.) *New approaches to monetary economics*, Cambridge University Press.
- [7] Bolton, P. and D. Scharfstein, 1990, "A Theory of Predation Based on Agency Problems in Financial Contracting," *American Economic Review*, 80(1), 93-106.
- [8] Brusco, S. and F. Castiglionesi, 2007, "Liquidity Coinsurance, Moral Hazard and Financial Contagion," *Journal of Finance*, 62, 2275-2302.
- [9] Caballero, R. and A. Krishnamurthy, 2001, "International and Domestic Collateral Constraints in a Model of Emerging Market Crisis," *Journal of Monetary Economics*, 48, 513-548.
- [10] Caballero, R. and A. Krishnamurthy, 2003, "Excessive Dollar Debt: Financial Development and Underinsurance," *Journal of Finance*, 58, 867-894.
- [11] Caballero, R. and A. Krishnamurthy, 2004, "Smoothing Sudden Stops," *Journal of Economic Theory*, 119, 104-127.
- [12] Castiglionesi, F., F. Feriozzi and G. Lorenzoni, 2009, "Financial Integration and Liquidity Crises," unpublished manuscript.
- [13] Diamond, D. and P. Dybvig, 1983, "Bank Runs, Deposit Insurance and Liquidity," *Journal of Political Economy*, 91(3), 401-419.
- [14] Diamond, D. W. and R. G. Rajan, 2011, "Fear of Fire Sales, Illiquidity Seeking, and Credit Freezes," *Quarterly Journal of Economics*, 126, 557-591.

- [15] The Economist, 2013, “Financial Fragmentation – Too much of a good thing,” Special Report “The Gated Globe” Oct 12th.
- [16] The FED, 2014, “Federal Reserve Statistical Release, February 6, 2014.
- [17] Fisher, I., 1933, “The Debt-Deflation Theory of Great Depressions,” *Econometrica*, 1(4), pp. 337-357.
- [18] Gorton, G., and L. Huang, 2004, “Liquidity, Efficiency, and Bank Bailouts,” *American Economic Review*, 94(3), pp. 455-483.
- [19] Guembel, A. and O. Sussman, 2010, “Liquidity, Contagion and Financial Crisis,” IDEI Working Paper #664.
- [20] Hart, O. and J. Moore, 1998, “Default and Renegotiation: A Dynamic Model of Debt,” *Quarterly Journal of Economics*, 113(1), pp. 1-41.
- [21] Holmström, B. and J. Tirole, 1998, “Private and Public Supply of Liquidity,” *Journal of Political Economy*, 106(1), pp. 1-40.
- [22] Holmström, B. and J. Tirole, 2011, *Inside and Outside Liquidity*, Boston: MIT Press.
- [23] Jeanne, O. and A. Korinek, 2010, “Excessive Volatility in Capital Flows: A Pigouvian Taxation Approach,” *American Economic Review Papers and Proceedings*, 100(2), 403-407.
- [24] Kiyotaki, N. and J. H. Moore, 1997, “Credit Cycles,” *Journal of Political Economy*, 105(2), pp. 211-248.
- [25] Korinek, A., 2010, “Regulating Capital Flows to Emerging Markets: An Externality View,” unpublished manuscript, University of Maryland.
- [26] Korinek, A., 2011, “Hot Money and Serial Financial Crisis,” *IMF Economic Review* 59(2), pp. 306-339.
- [27] Krishnamurthy, A., 2010, “Amplification Mechanisms in Liquidity Crisis,” *American Economic Journal: Macroeconomics*, 2(3), 1-30.
- [28] Kyle, A. and W. Xiong, 2001, “Contagion as a Wealth Effect,” *Journal of Finance* 56, 1401-1440.
- [29] Lorenzoni, G., 2008, “Inefficient Credit Booms,” *Review of Economic Studies*, 75(3), 809-833.

- [30] Mendoza, E.G., 2010, "Sudden Stops, Financial Crises and Leverage," *American Economic Review*, 100(5), pp. 1941-1965.
- [31] Pavlova, A. and R. Rigobon, 2008, "The Role of Portfolio Constraints in the International Propagation of Shocks," *Review of Economic Studies*, 75, 1215-1256.
- [32] Shleifer, A., and R. Vishny, 1992, "Liquidation Values and Debt Capacity: A Market Equilibrium Approach," *Journal of Finance*, 47, 1343-1366.
- [33] Stiglitz, J.E., 1999, "Beggars-Thyself versus Beggar-Thy-Neighbor Policies: The Dangers of Intellectual Incoherence in Addressing the Global Financial Crisis," *Southern Economic Journal*, 66, 1-38.
- [34] Stiglitz, J.E., 2010, "Risk and Global Economic Architecture: why Full Financial Integration may be Undesirable," NBER Working Paper 15718.
- [35] Suarez, J. and O. Sussman, 1997, "Endogenous Cycles in a Stiglitz-Weiss Economy," *Journal of Economic Theory*, 76, pp. 47-71.
- [36] U.S. Department of Treasury, 2013, "The Financial Crisis Five Years Later: Response, Reform and Progress," available at http://www.treasury.gov/connect/blog/Documents/FinancialCrisis5Yr_vFINAL.pdf