A Structural Model with Explicit Distress*

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Abstract

We construct a model for valuing firms and corporate securities incorporating economic and financial distress, in which distress emerges when a firm needs external financing to cover its replacement investments (economic distress) and financing obligations (financial distress). Using standard parameter values, our results show that costs of economic distress are not significant, but costs of financial distress are significant even for low leverage ratios and are more important than direct costs of bankruptcy. The inclusion of financial distress produces lower estimates of optimal leverage but higher estimates of debt capacity and addresses different problems of structural models. Namely, underestimating (overestimating) spreads on safe (risky) bonds, relying on unrealistic estimates for direct costs of bankruptcy and being unable to explain the low debt/zero debt puzzle. Overall, and when compared with a classical endogenous default model, the distress model generates more realistic estimates of leverage ratios, credit spreads and recovery rates.

JEL Classification: G12, G13, G32, G33.

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The analysis of the capital structure of firms has long evolved since Modigliani and Miller (1958) developed the irrelevance proposition. Later work on the role of tax benefits (e.g. Modiglianni and Miller, 1963) and bankruptcy costs (e.g. Robichek and Myers, 1965) established the trade-off theory of capital structure as the dominant paradigm explaining optimal capital structure as the amount of leverage that balances the marginal benefits and the marginal costs associated with debt financing.

One family of models that took prominence builds on the developments of Fisher (1959) and estimates the price and returns of corporate securities as functions of firm specific financial and economic characteristics. This approach is commonly known as structural modeling and the first structural models date back more than thirty years (e.g. Merton, 1974, Black and Cox, 1976 and Geske, 1977, 1979). Despite their increasing complexity and sophistication, these models generated unrealistically high predictions for optimal leverage and unrealistically low predictions for the credit spreads (see Jones, Mason and Rosenfeld, 1983, 1984 and Ogden, 1987). Further, the generation of realistic estimates of leverage ratios and credit spreads, meant that these models had to rely on unrealistically high estimates of bankruptcy costs (see Miller, 1977)[1].

Although Robichek and Myers (1965) refer to 'disadvantages of leverage', the direct costs of bankruptcy took the dominant role within the overall deadweight costs of debt and bankruptcy costs represent the most common aspect of theoretical models of capital structure[2]. Such modeling trajectory resulted in limited theoretical attention given to the commonly defined indirect costs of bankruptcy. Even though Haugen and Senbet (1978) claim that the truly significant deadweight costs are associated with liquidation and that indirect costs should be insignificant as long as the different stakeholders of the firm behave rationally, empirical evidence on the indirect costs confirms their significance and higher importance compared to the direct costs (e.g. Altman, 1984)[3].

[1]An early empirical analysis of the size of bankruptcy costs in Warner (1977) estimated these costs to represent 5.3 % of firm value at the moment of filing the bankruptcy petition and 1% 7 years before bankruptcy.

[2]A different deadweight cost of debt that was latter addressed was the agency cost of debt. It was analyzed in terms of risk shifting incentives in Leland (1998), underinvestment incentives in Mauer and Ott (2000) and overinvestment incentives in Mauer and Sarkar (2005).

[3]Given the results of early empirical analyses, much research focused on addressing other aspects such as deviations from absolute priority rules (Leland, 1994, Longstaff and Schwartz, 1995), the possibility of debt renegotiations (Anderson and Sundaresan, 1996, Mella-Barral and Perraudin, 1997, Mella-Barral,

This paper explores the role of the (so called) indirect costs of bankruptcy by constructing a cash-flow based model of the firm which incorporates distress. We claim that the indirect costs should be defined more accurately as costs of financial distress, because as Brealey, Myers and Allen (2008) argue, it is not the event of bankruptcy that generates costs of financial distress, but the costs of financial distress that eventually lead the firm to bankruptcy. We assume that the onset of distress is triggered by cash flow shortage associated with the obligations of the firm to (i) make the appropriate reinvestments required to maintain production capacity and (ii) pay its coupons. The firm experiences economic distress when the cash flow shortfall is triggered by the need to realize productive reinvestments and financial distress when the cash flow shortfall is triggered by the obligations of the firm to pay its coupons. The inclusion of distress is shown to affect significantly the value of the firm and its corporate securities, optimal leverage, debt capacity and credit spreads and it is even able to explain the low debt / zero debt puzzle within a classical trade-off context.

This paper contributes to different strands of the literature on structural modeling. Firstly, it relates to the literature incorporating and modeling the different events troubled firms experience, that overwhelmingly focused on the downstream events that follow default (e.g. see literature on strategic debt service such as Anderson and Sundaresan, 1996, Mella-Barral and Perraudin, 1997, among others) by effectively separating default from bankruptcy and allowing for partial defaults. Reversing this trend, we focus on the upstream event of financial distress that precedes default. Possibly, the most common feature in the different analyses of troubled firms is the assumption that default marks the onset of distress (e.g. in the theoretical see Leland, 1994, Mella-Barral and Perraudin, 1997 and in the empirical literature see Dahiya, Saunders and Srinivasan 2003). However, not all authors consider that financial distress is triggered by default. Common text book definitions of financial distress do not imply the occurrence of default as presented in Bodie and Merton (2000) and Brealey, Myers and Allen (2008). Dahiya, Saunders and

1999, Hege and Mella-Barral, 2000), aspects of debt maturity (Leland and Toft, 1996), dynamic capital structure (Goldstein, Ju and Leland, 2001, Ju, Parrino, Poteshman and Weisbach, 2005) and different dynamics for the state variable such as process with jump components (Delianedis and Geske, 2001 and Cremers, Driessen and Maenhout, 2008) or additional state variables such as stochastic interest rates (Ronn and Verma, 1986, Kim, Ramaswamy and Sundaresan, 1993, Longstaff and Schwartz, 1995).

Srinivasan (2003) discuss how several public announcements such as declines in earnings and dividend cuts precede default. In their empirical analysis of financially troubled firms, Couderc and Renault (2005) discuss how the vast majority of defaults do not arise suddenly but represent the conclusion of a long lasting process; in the same line, Norden and Weber (2010) show that credit line usage, limit violations, and cash inflows exhibited abnormal patterns several months before default. The definition of troubled firms used in the empirical works of Franks and Torous (1994), Opler and Titman (1994), amongst others, also does not imply default.

Secondly, this paper contributes to the literature that analyses the different effects of additional deadweight costs of debt such as agency costs (Leland, 1998, Mauer and Ott, 2000 and Mauer and Sarkar, 2005) by considering the costs of financial distress. Our work shows that financial distress costs are possibly more important than the direct costs of bankruptcy and their inclusion lowers optimal leverage estimates and generates more realistic estimates of credit spreads than a classical endogenous default model with bankruptcy costs.

Finally, the paper also contributes to the literature that considers regime changes such as Mella-Barral and Perraudin (1997) and Guo, Miao and Morellec (2005), by modeling distress as a decrease in the growth rate of cash flows. Contrary to Mella-Barral and Perraudin (1997), we consider a change that is triggered by distress and not default and the change of regime affects the growth rate of cash flows and not prices and costs as in Mella-Barral and Perraudin (1997). Regarding Guo, Miao and Morellec (2005), we consider a change in dynamics triggered by the onset of financial distress and aimed at representing the costs of distress while Guo, Miao and Morellecs paper considers a change in dynamics representing macroeconomic cycles and they focus on analyzing its impact on the investment decisions of a firm.

This work is possibly closer to Mella-Barral and Perraudin (1997) and Basak and Shapiro (2005) in terms of modeling what we define as costs of financial distress. Important differences between this paper and Mella-Barral and Perraudin (1997) concern the onset and the resolution of distress. In our case default is a result of distress whereas in Mella-Barral and Perraudin (1997) distress is a result of default, and therefore the distress costs implied in both models never overlap. Regarding the resolution of distress, Mella-Barral and Perraudin (1997) focus on analyzing debt restructurings as a resolution of distress while we consider that the resolution of distress implies bankruptcy. There are many important differences between this paper and Basak and Shapiro (2005). Firstly, we work in continuous time and assume that the dynamics of cash flows are exogenously determined. Basak and Shapiro (2005) work in discrete time and their asset dynamics are endogenously determined. In addition, we mainly focus on different aspects of leverage decisions, while Basak and Shapiro (2005) ignores determinants of corporate borrowing and focuses on default and investment decisions and deviations from absolute priority rules that are significantly affected by an asset composition that includes intangible assets.

The remainder of this paper is structured as follows; Section I describes the benchmark and distress models, Section II presents the analyses of our results in terms of firm and security values, debt effects, credit spreads and a calibrating exercise. Section III concludes.

I The General Model and Main Assumptions

The assumptions developed in this section hold for the development of the two models constructed in this work. A benchmark model and the model with distress.

Consider a firm that generates a cash-flow x before taxes and operational investments, which is uncertain and follows a geometric Brownian motion process (gBm),

$$dx = \mu_i x dt + \sigma x dz \tag{1}$$

μ_i represents the drift rate of the cash-flows and in the benchmark model this drift rate is unique, σ represents the constant standard deviation of the cash-flows and dz is the increment of a standard Wiener process.

In order to restore its production capacity, the firm must realize continuous capital expenditures that translate in a continuous stream of reinvestment costs d[4]. Furthermore, the firm has perpetual debt issued, in which c represents the continuous coupon payment

[4]The flow x is similar to a cash flow version of EBITDA, therefore the cash flows that remunerate investors are net of the investments in productive capital required to maintain production capacity.

that remunerates lenders and pays corporate taxes at the rate θ. The tax system is assumed to be fully symmetrical allowing for tax carry-backs and carry-forwards, in which the government equally and proportionally participates in both the losses and profits of the firm, and in practice, it also contributes to the reinvestment costs through the operational tax shields (θd). Shareholders are not financially constrained and the benefits of debt financing are limited to the generation of interest tax shields (we do not explicitly consider any incentive driven benefits of debt financing in the style of Jensen, 1986).

Although the firm's cash-flow x cannot be negative (follows gBm), cash-flows to shareholders may be positive or negative, due to the fixed nature of reinvestment costs (d) and coupons (c). Positive cash flows are simply distributed as dividends (the firm does not accumulate cash nor holds cash reserves) and, negative cash flows will trigger a cash injection by shareholders in order to maintain production capacity and avoid default.

Equityholders are willing to make cash injections, as long as the cash injection does not exceed the market value of their shares. Whenever the firm experiences cash flow shortfalls and the equityholders stop injecting cash in the firm, the firm defaults. Default, immediately triggers formal bankruptcy procedures and alternative outcomes for default such as a liquidation or debt restructurings are not considered. Bankruptcy results in a transfer of ownership of the firm to its original lenders, and the financially restructured firm is not re-levered. At this moment, direct costs of bankruptcy are incurred, representing all legal and administrative expenses associated with the formal bankruptcy procedure, and following Weiss (1990) we assume that the direct costs of bankruptcy are variable in nature. A further decrease in the cash flows of the firm may lead the firm into abandonment in which case there is no capital recovery.

We next turn to the detailed description of the two models, a benchmark model of endogenous default without distress and a distress model.

A The Benchmark Model

Our first model is a standard structural model with endogenous default and without financial distress. This model is an adaptation of Leland (1994) with uncertain cash flows instead of asset values and with fixed reinvestment costs (d). It considers three possible events triggered with decreases of the cash flow x: (i) default, which is triggered when equityholders stop injecting funds in the firm to cover the cash flow shortfall and occurs when $x = x_b$ (ii) bankruptcy that occurs simultaneously with default and it implies the transfer of ownership of the firm from shareholders to debtholders and (iii) abandonment, which is associated with the unlevered firm. For low enough cash flows it is optimal to simply abandon a loss making business, and abandonment occurs when $x = x_a$.

The benchmark model is represented in the following proposition.

Proposition 1. *The value of an Unlevered firm ($V_u(x)$), a Levered firm ($V(x)$), its Equity ($E(x)$) and Debt ($D(x)$), in the absence of financial distress, and the default x_b and abandonment triggers x_a, and the expected times to default τ_b and abandonment τ_a are given by,*

$$V_u(x) = \begin{cases} \left(\frac{x}{\gamma_h} - \frac{d}{r}\right)(1-\theta) - \left(\frac{x_a}{\gamma_h} - \frac{d}{r}\right)(1-\theta)\left(\frac{x}{x_a}\right)^{\beta_{h2}} & \text{if } x > x_a \\ 0 & \text{if } x \leq x_a \end{cases} \tag{2}$$

$$V(x) = \begin{cases} \left(\frac{x}{\gamma_h} - \frac{d}{r}\right)(1-\theta) + \frac{c\theta}{r} - \left[\left(\frac{x_b}{\gamma_h} - \frac{d}{r}\right)(1-\theta)\delta + \frac{c\theta}{r}\right]\left(\frac{x}{x_b}\right)^{\beta_{h2}} - \\ \left(\frac{x_a}{\gamma_h} - \frac{d}{r}\right)(1-\theta)(1-\delta)\left(\frac{x}{x_a}\right)^{\beta_{h2}} & \text{if } x > x_b \\ \\ \left(\frac{x}{\gamma_h} - \frac{d}{r}\right)(1-\theta)(1-\delta) - \left(\frac{x_a}{\gamma_h} - \frac{d}{r}\right)(1-\theta)(1-\delta)\left(\frac{x}{x_a}\right)^{\beta_{h2}} & \text{if } x_a < x \leq x_b \\ \\ 0 & \text{if } x \leq x_a \end{cases} \tag{3}$$

$$E(x) = \begin{cases} \left(\frac{x}{\gamma_h} - \frac{d+c}{r}\right)(1-\theta) - \left(\frac{x_b}{\gamma_h} - \frac{d+c}{r}\right)(1-\theta)\left(\frac{x}{x_b}\right)^{\beta_{h2}} & \text{if } x > x_b \\ 0 & \text{if } x \leq x_b \end{cases} \tag{4}$$

$$D(x) = \begin{cases} \frac{c}{r} - \left[\frac{c}{r} - \left(\frac{x_b}{\gamma_h} - \frac{d}{r}\right)(1-\theta)(1-\delta)\right]\left(\frac{x}{x_b}\right)^{\beta_{h2}} & \\ -\left(\frac{x_a}{\gamma_h} - \frac{d}{r}\right)(1-\theta)(1-\delta)\left(\frac{x}{x_a}\right)^{\beta_{h2}} & if\ x > x_b \\ \left(\frac{x}{\gamma_h} - \frac{d}{r}\right)(1-\theta)(1-\delta) - \left(\frac{x_a}{\gamma_h} - \frac{d}{r}\right)(1-\theta)(1-\delta)\left(\frac{x}{x_a}\right)^{\beta_{h2}} & if\ x_a < x \leq x_b \\ 0 & if\ x \leq x_a \end{cases} \tag{5}$$

with $\gamma_h = r - \mu_h$, $\gamma_l = r - \mu_l$ and with the following default trigger (x_b) and expected time to default (τ_b),

$$x_b = \frac{\beta_{h2}}{\beta_{h2}-1}\frac{c+d}{r}\gamma_h, \tag{6}$$

$$\tau_b = \frac{1}{\mu_h - \frac{1}{2}\sigma^2} ln\left(\frac{x_b}{x}\right) = \frac{1}{\mu_h - \frac{1}{2}\sigma^2} ln\left(\frac{\beta_{h2}}{\beta_{h2}-1}\frac{c+d}{r}\frac{\gamma_h}{x}\right), \tag{7}$$

and the following abandonment trigger (x_a) and expected time to abandonment (τ_a)

$$x_a = \frac{\beta_{h2}}{\beta_{h2}-1}\frac{d}{r}\gamma_h \tag{8}$$

$$\tau_a = \frac{1}{\mu_h - \frac{1}{2}\sigma^2} ln\left(\frac{x_a}{x}\right) = \frac{1}{\mu_h - \frac{1}{2}\sigma^2} ln\left(\frac{\beta_{h2}}{\beta_{h2}-1}\frac{d}{r}\frac{\gamma_h}{x}\right). \tag{9}$$

The following plot represents the three stages considered in the benchmark model.

INSERT [FIGURE 1]

The straight line in Figure 1 represents the abandonment trigger and the dashed line represents the default and bankruptcy trigger. Increases in leverage do not affect the abandonment trigger (x_a), because, given the financial restructuring occurring during formal bankruptcy procedures, when the firm abandons it is always unlevered. The abandonment decision is then a purely operational decision associated with the level of reinvestment costs d. The endogenous decision to default is associated with the cash injections equityholders make to avoid default and therefore x_b (the dashed line in Figure 1) increases monotonically with increases in leverage. In region (i) we have a levered firm owned by shareholders, which may be generating positive cash flows to equityholders, whenever $x > c + d$, or negative cash flows, whenever $x < c + d$. In region (ii), we have an unlevered firm that has previously defaulted and it is currently owned by its original lenders. Once again, in region (ii), we may have positive cash flows to the 'new' equityholders, whenever $x > d$, and negative cash flows, whenever $x < d$. Finally, in region (iii), following a decrease in cash flows, the firm is abandoned.

B The Model with Financial Distress

Regarding the onset of distress, we assume a firm is in distress when it is unable to honor all its obligations with internally generated cash flows, but it is not in default. Namely, distress precedes default and the costs of distress may be incurred by firms even if they never end up defaulting on their obligations, a characteristic of distress reported in Altman (1984) and Basak and Shapiro (2005) among others. Several common definitions of distress also separate the onset of distress from the event of default. According to John (1993) the onset of distress represents the moment in which the liquid assets of the firm are insufficient to meet the liquidity requirements of its hard contracts; Bodie and Merton (2000) state that a firm in imminent danger of defaulting is said to be in distress and Brealey, Myers and Allen (2008) define distress as the failure to meet payments to creditors or when the payment is honored with difficulty.

Distress is not solely associated with debt financing, since even unlevered firms may experience distress whenever the operational cash flows are insufficient to realize the necessary investments to replace lost production capacity. The lack of empirical evidence on stakeholders' reactions to changes in the financial performance of firms prevents us from clearly identifying the onset of distress, and any definition of a distress boundary will arguably be somewhat arbitrary, because different aspects may raise or lower these boundaries[5]. Based on the previous definitions, and on arguments we present next we assume that the distress boundaries are defined by the cash flow shortages.

Whenever $x < d$, the firm is assumed to experience economic distress and the economic

[5]There are alternative aspects that may affect the distress boundaries that we do not take into account. The distress boundary will decrease whenever the firm has cash reserves that provide it with a breathing margin allowing it to avoid distress while experiencing short periods of cash flow shortages. On the other hand, it is possible that the stakeholders of the firm anticipate difficulties even before the firm experiences cash flow shortages by analyzing its decline in earnings or dividends as Dahiya, Saunders and Srinivasan (2003) point out.

distress boundary, defined as x_{ud}, is given by $x_{ud} = d$. This represents economic distress, because financing choices or the use of debt financing have not been at the origin of distress and distress was triggered by the economic 'obligations' of the firm. The use of debt financing increases the likelihood of distress by increasing the requirements to service its hard contracts. With financial distress, the financial obligations are at the origin of the distress situation and more specifically the use of debt financing (coupons must be contractually paid whereas dividends do not have to be paid unless the firm has the capacity to do so). Therefore, whenever $x < d + c$, the firm experiences financial distress and the financial distress boundary, defined as x_d, is given by $x_d = d + c$.

Distress does not imply default, therefore, during distress the firm obtains external financing that covers the cash flow shortfall allowing it to avoid default. The external financing is obtained from equityholders through cash injections realized for as long as they do not exceed the market value of equity[6].

According to Altman (1984) 'public awareness of a firm's financial difficulties and bankruptcy potential will negatively impact its subsequent performance' and Myers and Majluf (1984) argue that equity financing is likely to be perceived as signaling bad news by the different stakeholders of the firm. However, for distress to be able to negatively impact firm performance and the value of its securities, there must be mechanisms that explain how the different stakeholders of the firm become aware of the financial difficulties of the firm and its need for external financing. It is hard for the firm to conceal poor performance and cash injections by equityholders, since firms are audited once a year and many of them even present a summary of their financial statements on a quarterly basis. Nonetheless, even if the firm is able to camouflage its poor performance and its fresh equity financing, as Norden and Weber (2010) points out, it is very likely that there may be information spillovers between the different stakeholders of the firm regarding its financial situation, triggering adverse reactions.

The existing literature describes several ways through which the costs of distress (the so called indirect costs of bankruptcy) materialize such as decreases in sales, profits and

[6]Under more restrictive assumptions such as the existence of financially constrained equityholders as modeled in Kim, Ramaswamy and Sundaresan (1993), Ericsson (2000) or Titman, Tompaidis and Tsyplakov (2004), distress would obviously imply default and the whole problem would collapse to a simple analysis of the direct costs of bankruptcy.

reductions in the market value of inventories (Altman, 1984, Weiss, 1990 and Opler and Titman, 1994), decreases in market share resulting from aggressive responses by competitors (Opler and Titman, 1994), wasted managerial resources (Altman, 1984, John, 1993, Basak and Shapiro, 2005), unfavorable credit terms by suppliers such as cash on delivery (Titman, 1984, Weiss, 1990) or loss of key employees or increases in labor costs (Titman, 1984, Weiss, 1990)[7]. The costs of distress affect the operational value of the firm and this creates several challenges to researchers, because it makes it very hard to disentangle the causality between poor and unexpected earnings unrelated with distress and poor and unexpected earnings resulting from stakeholder awareness of distress. In turn, this makes it very difficult to determine when the onset of financial distress takes place and creates ambiguity in the estimation of the indirect costs empirically[8].

Although the estimation of distress costs presents obvious challenges, all the expressions of the costs of distress listed previously present a common and obvious factor; they all are purely operational in their nature and therefore they all affect negatively the operational cash flows of the firm. In the distress model, we assume that the onset of distress affects the dynamics of firm value by decreasing the drift rate μ_i, in which the subscript $i = h, l$ represents the drift rate if the firm is safe or in distress ($i = h$, high, if the firm is safe and $i = l$, low, if the firm is in distress, and naturally $\mu_h > \mu_l$). The decision to incorporate the effects of distress through changes in the drift rate of the cash flows is supported by previous research. Altman (1984) reports that firms with poor performance experience a significant decrease in the growth rate of earnings, relative to firms that experience normal performance, and, in their sample of firms experiencing poor performance, the growth rate of the earnings is negative. Similar results are presented in Andrade and Kaplan (1998) that report a significant decrease in the growth rate of the ratio EBITDA/Sales with the onset of distress and, when in distress, this growth rate is also negative. Furthermore, regarding the onset of distress, we follow a similar logic

[7]Other aspects commonly associated with financial distress are increases in the cost of capital (Altman, 1984 and Weiss, 1990) and difficulties in obtaining new financing (Altman, 1984 and Opler and Titman, 1994). However, and although these are financial costs, it is clear that increases in the cost of capital and credit constraints are unarguably the result and not the origin of distress.

[8]This frustration in dealing with the difficulties of the costs of distress is well expressed in Altman (1984) that argues that by nature the indirect costs of bankruptcy are illusive and difficult to specify and let alone measure empirically (similar arguments are presented in Opler and Titman, 1994).

to Andrade and Kaplan regarding one of their distress triggers, namely, that the firm enters distress whenever its EBITDA is lower than its interest expenses. Mella-Barral and Perraudin (1997) also consider a change in regimes affecting its earnings through a decrease in the the prices and an increase in the costs, however, the change in dynamics in Mella-Barral and Perraudin is triggered by default and not distress.

The following proposition summarizes the distress model in terms of firm and security values, distress, abandonment and default triggers.

Proposition 2. *The value of an Unlevered firm ($V_u(x)$), Levered firm ($V(x)$), its Equity ($E(x)$) and Debt ($D(x)$), when financial distress is explicitly considered, and the distress x_{ud} and x_d, default x_b and abandonment triggers x_a, are given by,*

$$V_u(x) = \begin{cases} \left(\frac{x}{\gamma_h} - \frac{d}{r}\right)(1-\theta) + \alpha_{h2}^{V_u} x^{\beta_{h2}} & \text{if } x > x_{ud} \\ \left(\frac{x}{\gamma_l} - \frac{d}{r}\right)(1-\theta) + \alpha_{l1}^{V_u} x^{\beta_{l1}} + \alpha_{l2}^{V_u} x^{\beta_{l2}} & \text{if } x_a < x \le x_{ud} \\ 0 & \text{if } x \le x_a \end{cases} \tag{10}$$

$$V(x) = \begin{cases} \left(\frac{x}{\gamma_h} - \frac{d}{r}\right)(1-\theta) + \frac{c\theta}{r} + \alpha_{h2}^{V} x^{\beta_{h2}} & \text{if } x > x_d \\ \left(\frac{x}{\gamma_l} - \frac{d}{r}\right)(1-\theta) + \frac{c\theta}{r} + \alpha_{l1}^{V} x^{\beta_{l1}} + \alpha_{l2}^{V} x^{\beta_{l2}} & \text{if } x_b < x \le x_d \\ V_u(x)(1-\delta) & \text{if } x_a < x \le x_b \\ 0 & \text{if } x \le x_a \end{cases} \tag{11}$$

$$E(x) = \begin{cases} \left(\frac{x}{\gamma_h} - \frac{d+c}{r}\right)(1-\theta) + \alpha_{h2}^{E} x^{\beta_{h2}} & \text{if } x > x_d \\ \left(\frac{x}{\gamma_l} - \frac{d+c}{r}\right)(1-\theta) + \alpha_{l1}^{E} x^{\beta_{l1}} + \alpha_{l2}^{E} x^{\beta_{l2}} & \text{if } x_b < x \le x_d \\ 0 & \text{if } x \le x_b \end{cases} \tag{12}$$

$$D(x) = \begin{cases} \frac{c}{r} + \alpha_{h2}^{D} x^{\beta_{h2}} & \text{if } x > x_d \\ \frac{c}{r} + \alpha_{l1}^{D} x^{\beta_{l1}} + \alpha_{l2}^{D} x^{\beta_{l2}} & \text{if } x_b < x \le x_d \\ V_u(x)(1-\delta) & \text{if } x_a < x \le x_b \\ 0 & \text{if } x \le x_a \end{cases} \tag{13}$$

with $\gamma_h = r - \mu_h$, $\gamma_l = r - \mu_l$ and in which the distress triggers (x_{du} and x_d) are given by,

$$x_{ud} = d \tag{14}$$

$$x_d = d + c \tag{15}$$

the abandonment (x_a) and default triggers are obtained from numerically solving the following implicit equations,

$$\frac{x_a}{\gamma_l}(1-\theta) + \alpha_{l1}^{V_u}\beta_{l1}x_a^{\beta_{l1}} + \alpha_{l2}^{V_u}\beta_{l2}x_a^{\beta_{l2}} = 0 \tag{16}$$

$$\frac{x_b}{\gamma_l}(1-\theta) + \alpha_{l1}^{E}\beta_{l1}x_b^{\beta_{l1}} + \alpha_{l2}^{E}\beta_{l2}x_b^{\beta_{l2}} = 0 \tag{17}$$

the constants α_{h2}, α_{l1} and α_{l2} for a general claim A are given by,

$$\begin{aligned}\alpha_{h2}^A \;=\; & \frac{a_c x_D\left(\frac{1}{\gamma_l}-\frac{1}{\gamma_h}\right)}{x_D^{\beta_{h2}}} \\ & + \frac{a_c x_D\left(\frac{1}{\gamma_h}-\frac{1}{\gamma_l}\right)(1-\beta_{h2}) + \left(a_c\frac{x_f}{\gamma_l}+\frac{b_c}{r}-L_c\right)\left(\frac{x_D}{x_f}\right)^{\beta_{l2}}(\beta_{l2}-\beta_{h2})}{x_D^{\beta_{h2}}\left[(\beta_{l1}-\beta_{h2})-(\beta_{l2}-\beta_{h2})\left(\frac{x_f}{x_D}\right)^{(\beta_{l1}-\beta_{l2})}\right]} \\ & + \frac{a_c x_D\left(\frac{1}{\gamma_h}-\frac{1}{\gamma_l}\right)(1-\beta_{h2}) + \left(a_c\frac{x_f}{\gamma_l}+\frac{b_c}{r}-L_c\right)\left(\frac{x_D}{x_f}\right)^{\beta_{l1}}(\beta_{l1}-\beta_{h2})}{x_D^{\beta_{h2}}\left[(\beta_{l2}-\beta_{h2})-(\beta_{l1}-\beta_{h2})\left(\frac{x_f}{x_D}\right)^{(\beta_{l2}-\beta_{l1})}\right]},\end{aligned} \tag{18}$$

$$\alpha_{l1}^A = \frac{a_c x_D\left(\frac{1}{\gamma_h}-\frac{1}{\gamma_l}\right)(1-\beta_{h2}) + \left(a_c\frac{x_f}{\gamma_l}+\frac{b_c}{r}-L_c\right)\left(\frac{x_D}{x_f}\right)^{\beta_{l2}}(\beta_{l2}-\beta_{h2})}{x_D^{\beta_{l1}}\left[(\beta_{l1}-\beta_{h2})-(\beta_{l2}-\beta_{h2})\left(\frac{x_f}{x_D}\right)^{(\beta_{l1}-\beta_{l2})}\right]}, \tag{19}$$

$$\alpha_{l2}^A = \frac{a_c x_D\left(\frac{1}{\gamma_h}-\frac{1}{\gamma_l}\right)(1-\beta_{h2}) + \left(a_c\frac{x_f}{\gamma_l}+\frac{b_c}{r}-L_c\right)\left(\frac{x_D}{x_f}\right)^{\beta_{l2}}(\beta_{l1}-\beta_{h2})}{x_D^{\beta_{l2}}\left[(\beta_{l2}-\beta_{h2})-(\beta_{l1}-\beta_{h2})\left(\frac{x_f}{x_D}\right)^{(\beta_{l2}-\beta_{l1})}\right]}. \tag{20}$$

allowing us to determine the appropriate α coefficients for $V_u(x)$, $V(x)$, $E(x)$ and $D(x)$ by replacing in (18), (19) and (20) the corresponding expressions for a_c, b_c and L_c. The parameter a_c is equal to $(1-\theta)$ for $V_u(x)$, $V(x)$ and $E(x)$ and equal to 0 for $D(x)$, b_c is equal to $-d(1-\theta)$ for $V_u(x)$, $-d(1-\theta)+c\theta$ for $V(x)$, $-(d+c)(1-\theta)$ for $E(x)$ and c for $D(x)$, finally, L_c is equal to 0 in the abandonment case for $V_u(x)$ and in the case of default it is equal to $V_u(x_b)(1-\delta)$ for $V(x)$ and $D(x)$ and equal to zero for $E(x)$. As before, from the dynamics of x we can derive the expected times to abandonment and *default as,*

$$\tau_j = \frac{1}{\mu_i - \frac{1}{2}\sigma^2} ln\left(\frac{x_j}{x}\right) \tag{21}$$

in which the subscript $j = du, d, a, b$ for the cases of distress by an unlevered firm (du), distress by a levered firm (d), abandonment (a) and default (b) respectively. The subscript $i = h, l$ and $i = h$ whenever $j = d, du$ and $i = l$ whenever $j = a, b$.

By considering financial and economic distress, the distress model generates a much richer set of scenarios than the benchmark model, and the latter can be considered as a special case of the distress model whenever $\mu_l = \mu_h$.

The following plot represents the different stages considered in the distress model.

INSERT [FIGURE 2]

The straight line in Figure 1 is the abandonment trigger (x_a) that represents a function of the level of reinvestment costs d needed to restore production capacity. The dash and doted line is the economic distress trigger (x_{du}), representing the level of cash flows generated by the firm that exactly cover the reinvestment costs d. The short dashed line represents the financial distress trigger (x_d) which equals the economic distress trigger when the firm is unlevered and increases monotonically as leverage increases. A firm enters into distress earlier, whenever the level of its financial obligations increase (c increases). Finally, the long dashed line represents the default and bankruptcy trigger (x_b). Similarly to the benchmark model, the default trigger increases monotonically with increases in leverage, and the higher is leverage the earlier will the firm default.

In region (i) we have a firm owned by equityholders, in which the cash flows generated are sufficient to cover the reinvestment costs d, the debt coupons c and to distribute dividends $(x - d - c)(1 - \theta) > 0$. In region (ii), the firm is owned by equityholders, but it experiences financial distress. The cash flows generated are sufficient to cover the reinvestment costs ($x > d$) but there is a cash flow shortfall regarding the payment of coupons ($x < d + c$) that is met through an equity cash injection. In this region, distress is caused by the use of debt financing, because an equivalent unlevered firm would not be in distress. In region (iii), the firm is owned by equityholders, but it experiences economic distress, in this region the cash flows generated by the firm are insufficient to cover the reinvestment costs ($x < d$) and the distress situation is augmented by the use of debt financing, because the cash flow shortfall is higher in the amount of coupons paid. In region (iv) equityholders have already defaulted and the original debtholders are now the current owners of an unlevered firm that experiences no distress, as the cash flows generated are sufficient to cover the reinvestment costs (d) and to distribute dividends $(x - d)(1 - \theta) > 0$. In region (v) equityholders have already defaulted and the original debtholders are now the current owners of an unlevered firm that experiences economic distress, because the cash flows generated are insufficient to cover the reinvestment costs (d) and the 'new' equityholders are forced to make cash injections to cover the cash flow shortfall $x < d$. Regions (iv) and (v) are the two possible outcomes of the distress resolution and represent the two types of defaulting firms presented in Wruck (1990) and White (1996), firms that emerge from Chapter 11 economically healthy and firms that later enter Chapter 7. In region (iv) we have firms that are economically sound and recover under restructuring, because default was due to temporary distress triggered by overleverage. In region (v) we have firms that are economically frail, require additional external financing to keep them afloat and are more likely to be abandoned. Finally, in region (vi), following a further decrease in the cash flows, the firm is abandoned.

II Numerical Analysis

The numerical analyses performed rely on empirically observed parameter values to generate realistic estimates of security values, determinants of capital structure, leverage ratios and debt spreads.

The initial cash flows (x_0) are assumed to be 100 with a growth rate of 1.5% (when not in distress) and with a growth rate of -1% (when in distress)(Titman and Opler, 1994, report a growth rate for normal performance of 3.9% and for poor performance of -3.6%). The fixed reinvestment costs (d) are assumed to be 10 so that the total payout to investors for the base case and in both models approximates 5%. This figure is in line with the observed total payout of 4.83% reported in Eom, Helwege and Huang (2004) and the total payout of 5% typical of an average S&P firm reported in François and Morellec (2004) and Hackbarth, Miao and Morellec (2006). Similarly, for the volatility of the cash flows we assume 26.3% in line with the estimations of Eom, Helwege and Huang (2004), Hackbarth, Miao and Morellec (2006) and Schaefer and Strebulaev (2008).

The direct costs of bankruptcy include all direct legal and administrative costs of bankruptcy that present a wide variability in their empirical estimates. Warner (1977) reports average direct costs of bankruptcy of 5.3% of the market value just before the petition date. Altman (1984) estimates the direct bankruptcy costs to average 4% of the value of the firm just prior to bankruptcy and these costs range between 6.2% and 11.1%. Weiss (1990) reports direct costs averaging 3,1% of the sum of the book value of debt plus the market value of equity and ranging from 1% to 6%. In Andrade and Kaplan (1998) the estimates for the average direct costs of bankruptcy range from 10% to 20% of the pre-distressed market value and Lawless and Ferris (2000) estimate these costs to average 18% of the total value of the assets. At the same time, theoretical research often assumes much higher figures for the direct costs of bankruptcy such as 40% in Leland (1994) or 100% in Mello, Parsons and Triantis (1995) amongst many others. Given the wide variability of empirical estimates and theoretical figures used, we assume that the direct costs of bankruptcy represent 15% of the post restructured market value of the firm.

For the risk free rate we consider 6.5%, the yearly average yield on 10 year maturity US T-bonds during the period 1986-1997, the sampling period used in Eom, Helwege Huang (2004), for consistency reasons. The tax rate is assumed to be 25%, a figure lower than the 40% statutory corporate tax rate of the US, but closer to the average effective US tax rate given all the loopholes, tax credits, and tax deductions inscribed in the US tax code[9].

A Firm, security values, optimal leverage and debt capacity

Structural models are the basis of plenty of research focused on determining firm and security values, financing policies and even operational policies[10]. This section analyses

[9]For an estimation of the tax advantage of debt considering corporate and personal taxes, Hackbarth, Miao and Morellec (2006) report an even lower figure of 15%.

[10]The possibility of renegotiating the terms of debt contracts initially analyzed in Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997) was later shown to significantly influence investment decisions in Sundaresan and Wang (2007) and expansion decisions in Pawlina (2010).

how economic and financial distress affect the firm's operational and financing decisions as well as the firm and security values. The following plots present the values of the firm and of its securities, the optimal debt levels and the debt capacity of the firms in the benchmark and distress models.

INSERT [FIGURE 3]

INSERT [FIGURE 4]

The inclusion of economic distress affects the value of the unlevered firm by accelerating abandonment; an economically distressed firm waits less before abandoning operations, due to the reduced likelihood of recovery associated with a lower drift rate of the cash flows. The abandonment trigger is 0.4 lower with the benchmark model and the expected time to abandonment (τ_a) is 3.3% shorter with the distress model. However, in terms of value, the inclusion of the economic costs of distress only marginally affects ($-0,04\%$) the operational value of the firm (the unlevered firm value in the benchmark and distress models is $1,386.1$ and $1,385.6$ respectively). In both models, the operational value of the firm increases with increases in the initial cash flow level (x_0), in the drift rates (μ_h and also μ_l for the distress model) and in volatility (σ) and with decreases in the reinvestment costs (d), corporate tax rates (θ) and interest rates (r).

Similarly to the effects of economic distress on abandonment, financial distress accelerates default and for a leverage ratio of 50% the default trigger is 3.1 lower with the benchmark model and the expected time to default (τ_b) is 8% shorter with the distress model. For a constant debt level, both the abandonment and default triggers are unaffected by changes in x_0, in the costs of bankruptcy (δ) and in the corporate tax rates (θ). This last effect reflects the neutrality of a symmetrical corporate tax rate system on default and abandonment decisions. Regarding changes in the other parameters, we observe an acceleration of default and abandonment whenever a change in a parameter value reduces the operational value of the firm.

Contrary to economic distress, financial distress significantly affects the value of the levered firm. The value of the levered firms increase to $1,580.5$ for an optimal leverage ratio of 69.7% ($D*/V$) in the benchmark model and $1,482.6$ for an optimal leverage ratio of 44.9% ($D*/V$) in the distress model. These figures show a reduction of 6.2% of firm value due to financial distress. Across the different leverage possibilities (for $0 < D/V < 1$), the average values of the levered firm, equity and debt are always lower with the distress model relative to the benchmark model.

For a constant debt level, the change in the values of the levered firm, and its securities for changes in the base case parameter values are qualitatively similar to those for the unlevered firm. There are however some important differences for debt (and consequently for the levered firm) regarding changes in volatility (σ) and in the bankruptcy costs (δ). The values of the unlevered firm and equity both increase with increases in σ due to the value of options to abandon and to default, however, risky debt (and consequently the levered firm) decreases in value with increases in σ due to the limited upside potential of a debt claim. Only when close to default does risky debt benefit from increases in σ, the scenario commonly known as "betting on the upside" and initially documented in Leland (1994). Changes in δ do not affect the value of the unlevered firm, but affect the value of debt. Increases in δ reduce the value of risky debt by reducing the post-restructuring value of the firm and consequently the recovery rate of debtholders.

In terms of the financing policies, we analyze the effects of financial distress in terms of the optimal leverage and the debt capacity of the firms.

Financial distress reduces optimal leverage in 25 percentage points relative to the benchmark model and brings the leverage estimate closer to empirically observed leverage ratios (e.g 30.3% reported in Eom, Helwege and Huang, 2004)[11]. The lower optimal leverage ratio of the distress model is explained by the inclusion of financial distress and it becomes clear how the dynamics of firm value change ($\mu_h \longrightarrow \mu_l$) at a leverage ratio of 67.2% with the onset of financial distress. At this leverage ratio $x_0 = x_d = c + d$ and if the firm issues more than 963 of debt, it immediately enters financial distress and experiences a decrease in the growth rate of its cash flows.

Regarding the sensitivity of optimal leverage to changes in the base case parameter values, we observe that in both models optimal leverage increases with increases in θ and

[11] The difference between the distress model estimate and the empirical figure presented in Eom, Helwege and Huang (2004) is still significant indicating that other aspects such as liquidity (Geske and Delianedis, 2001) or interest rate uncertainty (Kim, Ramaswamy and Sundaresan, 1993 and Longstaff and Schwartz, 1995) may play an important role in defining the optimal capital structure.

with decreases in σ and δ, however there are significant differences between both models regarding the sensitivity of optimal leverage to changes in d, μ_h and r. The coefficient d represents the fixed reinvestment costs required to maintain production capacity and it is a proxy for operational leverage. DeAngelo and Masulis (1980) argue that optimal financial leverage should be negatively related with the level of available operational tax shields ($d\theta$), because if firms have high operational tax shields they rely less on interests to generate tax savings, furthermore the existence of fixed operating costs allied with the issue of debt makes it more likely that the firm might end up in default causing the present value of the interest tax shields to decline. The negative relation between operational and financial leverage is perfectly clear in the distress model, in which increases in d are associated with decreases in $D*/V$, however, the benchmark model shows a relative insensitivity of optimal financial leverage for changes in d. Increases in the growth rate are associated with an increase in $D*/V$ in the benchmark model, however in the distress model, although increases in μ_l are associated with increases in $D*/V$ (a lower penalty when in distress), increases in μ_h are associated with a significant decrease in $D*/V$. The decrease in $D*/V$ with increases in μ_h relates to the operational nature of the distress costs, and any increase in the operational value of the firm translates into an increase in the costs of financial distress when they are incurred. Therefore, if the operational value of the firm is high, firms have little incentives to generate additional cash flows in the form of interest tax shields, because they become exposed to significantly high costs of financial distress. A similar argument explains why the decrease in the optimal leverage ratio associated with a reduction in the risk free rate r is dramatic in the distress model but only moderate with the benchmark model.

The leverage ratio that maximizes the value of the debt claim, when debt is issued, is described in Stiglitz and Weiss (1981) as marking the beginning of credit rationing by lenders and is described in Leland (1994) as representing the debt capacity of a firm. Following Leland (1994) the debt capacity of the firm represents the maximum amount of debt the firm may issue and is defined as the leverage ratio $D**/V$. Paradoxically, we observe that although the optimal capital structure of the firm is severely affected by the inclusion of financial distress, its debt capacity is not. As expected, in absolute terms, the benchmark model generates higher debt capacity estimates $(1,362.6)$ than the distress model $(1,226.1)$. However, in relative terms, the debt capacity of the benchmark model is associated with a leverage ratio of 93.2% and the debt capacity of the distress model is associated with a leverage ratio 97.0%. The distress model presents a higher debt capacity, because financial distress forces equityholders into an early default and into a faster resolution of financial distress (τ_b is shorter with the distress model). An early default in an economically sound firm has the positive effect of releasing the firm from what Gilson (1997) describes as the "subtle costs of a 'locked-in' suboptimal capital structure". The earlier the firm defaults, the earlier will it become economically sound once again and this in turn, translates into a higher recovery rate for debtholders[12]. These positive effects associated with financial distress could even be more significant if we allowed for releveraging during the resolution of financial distress. As expected, we observe that the debt capacity of the firm is highest when the post-distress firm value is high (low r, δ, θ and high μ_h) and when there is a faster resolution of distress (high σ and low μ_l).

B Interest tax shields and deadweight costs of debt

Debt financing affects the value of the firm through the generation of interest tax shields and deadweight costs of debt, allowing us to present the value of the levered firm as a representation of the trade off theory of capital structure.

For the benchmark model the following relation applies,

$$V(x) = V_u(x) + T(x) - B(x), \tag{22}$$

stating that the value of a levered firm $V(x)$ equals its purely operational value represented by the value of an equivalent unlevered firm $V_u(x)$, plus the interest tax shields generated $T(x)$ and minus the bankruptcy costs $B(x)$.

The distress model encompasses as deadweight costs of debt the costs of financial

[12] Other positive aspects of financial distress are discussed in Jensen (1989), Wruck (1990) and Titman and Opler (1994). Jensen (1989) and Wruck (1990) argue that financial distress forces an earlier adoption of value-creating decisions and Titman and Opler (1994) discusses how financial distress increases the bargaining power of the firm with its different stakeholders.

distress, and therefore the following relation applies,

$$V(x) = V_u(x) + T(x) - B(x) - W(x), \tag{23}$$

stating that the value of a levered firm $V(x)$ equals the value of a similar unlevered and non distressed firm $V_u(x)$ plus the interest tax shields $T(x)$, minus the bankruptcy costs $B(x)$ and the costs of financial distress $W(x)$. In this representation we isolate the costs of financial distress, however, since these costs are purely operational, we could have allow them to directly affect the purely operational value of the distressed firm $V_{ud}(x)$ in which case we have the following relationship,

$$V(x) = V_{ud}(x) + T(x) - B(x). \tag{24}$$

The following proposition presents the value of the interest tax shields and bankruptcy costs with the benchmark and distress models.

Proposition 3. *The value of the interest tax shields ($T(x)$) and the bankruptcy costs ($B(x)$), for the benchmark model, and for $x > x_b$ are given by,*

$$T(x) = \frac{c\theta}{r} - \frac{c\theta}{r}\left(\frac{x}{x_b}\right)^{\beta_{h2}}, \tag{25}$$

$$B(x) = V_u(x_b)\delta\left(\frac{x}{x_b}\right)^{\beta_{h2}}. \tag{26}$$

For the model with financial distress, and for $x > x_b$, the values of $T(x)$ and $B(x)$ are given by,

$$T(x) = \begin{cases} \frac{c\theta}{r} + \alpha_{h1}^T x^{\beta_{h1}} + \alpha_{h2}^T x^{\beta_{h2}} & \textit{if } x > x_d \\ \frac{c\theta}{r} + \alpha_{l1}^T x^{\beta_{l1}} + \alpha_{l2}^T x^{\beta_{l2}} & \textit{if } x_b < x \leq x_d \end{cases} \tag{27}$$

$$B(x) = \begin{cases} \alpha_{h1}^B x^{\beta_{h1}} + \alpha_{h2}^B x^{\beta_{h2}} & \textit{if } x > x_d \\ \alpha_{l1}^B x^{\beta_{l1}} + \alpha_{l2}^B x^{\beta_{l2}} & \textit{if } x_b < x \leq x_d \end{cases} \tag{28}$$

in which α_{h1}^T and α_{h1}^B are equal to zero and the coefficients α_{h2}, α_{l1} and α_{l2} are obtained by replacing, in expressions (18), (19) and (20), a_c with 0 for $T(x)$ and $B(x)$, b_c with $c\theta$ for $T(x)$ and with 0 for $B(x)$ and L_c with 0 for $T(x)$ and with $V_u(x_b)\delta$ for $B(x)$.

As it becomes clear from the analysis of the previous value functions, financial distress indirectly affects interest tax shields and bankruptcy costs, but most importantly it directly generates costs of financial distress.

The costs of financial distress are commonly described as pure operational costs materializing in loss of businesses, tightening of business conditions, deteriorating stakeholder relationships and loss of managerial focus. Therefore, the costs of financial distress $W(x)$ may be simply expressed as follows,

$$W(x) = V_u(x) - V_{ud}(x), \tag{29}$$

in which $V_u(x)$ (the unlevered firm value) represents the purely operational value of a non distressed firm and $V_{ud}(x)$ represents the purely operational value of the firm exposed to financial distress. It is important to notice that $W(x)$ is not affected by economic distress, because the value effects of economic distress are independent of the financing structure and affect in an equal measure $V_u(x)$ and $V_{ud}(x)$.

The following proposition presents the operational value of the firm $V_{ud}(x)$ when exposed to financial distress.

Proposition 4. *The operational value of a firm exposed to financial distress that has not defaulted yet (for $x > x_b$) is given by,*

$$V_{ud}(x) = \begin{cases} \left(\frac{x}{\gamma_h} - \frac{d}{r}\right)(1-\theta) + \alpha_{h1}^{V_{ud}} x^{\beta_{h1}} + \alpha_{h2}^{V_{ud}} x^{\beta_{h2}} & \text{if } x > x_d \\ \left(\frac{x}{\gamma_h} - \frac{d}{r}\right)(1-\theta) + \alpha_{l1}^{V_{ud}} x^{\beta_{l1}} + \alpha_{l2}^{V_{ud}} x^{\beta_{l2}} & \text{if } x_b < x \leq x_d \end{cases} \tag{30}$$

in which $\alpha_{h1}^{V_{ud}}$ is equal to zero and and $\alpha_{h2}^{V_{ud}}$, $\alpha_{l1}^{V_{ud}}$ and $\alpha_{l2}^{V_{ud}}$ are obtained by replacing, in expressions (18), (19) and (20), a_c with $(1-\theta)$, b_c with $-d(1-\theta)$ and L_c with $V_u(x_b)$.

The following plot represents the values of the unlevered $V_u(x)$ and levered firms $V(x)$, interest tax shields $T(x)$ and bankruptcy costs $B(x)$ for the benchmark model and for different leverage ratios.

INSERT [FIGURE 5]

Figure 5 shows how the effects of interest tax shields clearly dominate the bankruptcy costs for reasonable amounts of debt financing. As bankruptcy approaches ($x \longrightarrow x_b$ as

$D/V \longrightarrow 1$), we observe that the interest tax shields tend towards zero (loss of interest tax shields is commonly presented as the first cost of bankruptcy) and the bankruptcy costs tend towards $-V_u(x_b)(1-\delta)$. The limit for the interest tax shields as bankruptcy approaches relates to the assumption that during the financial restructuring following default the firm is not relevered. With the transfer of ownership from equityholders to debtholders, the firm no longer has to pay interests and therefore the value of the interest tax shields is zero. If we allowed for releveraging following bankruptcy the present value of the interest tax shields would converge to the new present value of the interest tax shields as a function of the new optimal capital structure.

The effect of the bankruptcy costs only surpasses the effect of the interest tax shields for leverage ratios above 97% and the resulting optimal capital structure is quite high (69.7%) when compared with observed leverage ratios (e.g. 30.3% from Eom, Helwege and Huang, 2004). A high estimate of optimal leverage ratios is a common feature of this type of models and therefore that is why it is necessary to assume unrealistically high costs of bankruptcy to obtain reasonable estimates of leverage ratios and credit spreads.

The interest tax shields are a function of the corporate tax rate and are generated as long as the firm stays solvent, therefore they increase when there is an increase in θ and when a change in a parameter value increases the expected time to default such as increase in μ and decreases in d, σ and r. Regarding the bankruptcy costs, we observe an increase in the bankruptcy costs when the costs of bankruptcy increase or when the likelihood of default increases. An increase in the bankruptcy cost occurs when δ increase or when the operating value of the firm increases with decreases in d, θ and r. An increase in the likelihood of default is associated with a decrease in μ and with an increase in σ.

The following plot represents the values of the unlevered firm when it is not exposed to financial distress $V_u(x)$ and when it is exposed to financial distress $V_{ud}(x)$, and the values of the levered firm $V(x)$, the interest tax shields $T(x)$, the bankruptcy costs $B(x)$ and the costs of financial distress $W(x)$ for the distress model and for different leverage ratios.

INSERT [FIGURE 6]

Figure 6 shows how the deadweight costs of debt $(W(x)+B(x))$ outweigh interest tax shields for leverage ratios higher than 77.6%, a significantly lower figure than in the benchmark model case. It is interesting to observe the importance of the costs of financial distress at low levels of debt, when the onset of distress only occurs a leverage ratio of 67.2%.

As bankruptcy and the resolution of financial distress approaches ($x \longrightarrow x_b$ as $D/V \longrightarrow 1$) for high leverage ratios, we can clearly observe the positive effects of financial distress as an increase in the operational value of the firm exposed to financial distress ($V_{ud}(x)$). The increase in the operational value of the firm is also reflected in the costs of financial distress ($W(x)$) that tend to zero as bankruptcy approaches. During bankruptcy, the firm is financially restructured with the transfer of ownership from equityholders to debtholders, alleviating its cash flows from the burden of servicing the coupon payments. During bankruptcy, the different stakeholders immediately perceive the change in the financial situation of the firm (remember there is a single cash flow trigger for entering and exiting distress) and resume normal business relations, allowing an economic healthy firm to emerge from bankruptcy. If the cash-flow level at which the firm emerges from distress differed from the cash flow level that triggers the onset of distress the behavior of the costs of financial distress costs would naturally be different, and they would not converge to zero as $x \longrightarrow x_b$. Regarding the interest tax shields ($T(x)$) and the bankruptcy costs ($B(x)$) we observe that $T(x)$ tends to zero and $B(x)$ tends to $-V_u(x_b)(1-\delta)$. The interest tax shields and bankruptcy costs follows a similar logic as described in the analysis of the benchmark model.

The direct effects of distress are quite significant, but do not reveal the whole range of implications. The indirect effects are also found to play an important role in the way financial distress affects the present value of interest tax shields and bankruptcy costs. For all possible leverage ratios, we observe that the interest tax shields in the distress model are always lower than in the benchmark model and the bankruptcy costs are always higher. The present value of the interest tax shields is a function of the amount of interests paid and of the expected time to bankruptcy. For the same leverage ratio, the bankruptcy trigger x_b is always higher in the distress model and the expected time to bankruptcy τ_b is always shorter, this in turn reduces the expected amount of time the firm will be generating interest tax shields and their present value is naturally lower. Regarding bankruptcy, there are two cumulative effects explaining why the present value of bankruptcy costs is higher with the distress model. Firstly, an earlier bankruptcy (higher x_b) translates into a higher firm value at the time of bankruptcy and since bankruptcy costs are essentially proportional to the value of the firm this translates into higher bankruptcy costs. Secondly, an earlier bankruptcy means a shorter discounting period and a higher present value of bankruptcy costs.

Regarding the deadweight costs of debt, the costs of financial distress dominate the direct costs of bankruptcy for low and moderate levels of debt and only when the leverage ratio is significantly high ($D/V > 88\%$) and default is imminent, do the bankruptcy costs become dominant. This means that for our choice of base case parameters, the optimal capital structure is directly influenced by the costs of financial distress and only indirectly influenced by the direct costs of bankruptcy.

The direct and indirect effects analyzed are associated with our motivating intuition that it is not the event of bankruptcy that generates significant costs, but the significant costs of financial distress that leads the firm to bankruptcy. Financial distress accelerates default, therefore reducing the expected time through which the firm generates interest tax shields and increases the likelihood of default and subsequent bankruptcy.

It is interesting to observe how empirical studies on the indirect costs of bankruptcy present a similar pattern for the deadweight costs of debt. Andrade and Kaplan (1998) show a decrease in operating performance (decline in operating and net cash flow margins) from the onset of distress up until the resolution of financial distress. Following the resolution of distress, Andrade and Kaplan register a rebound in the financial performance of the previously distressed firms. Altman (1984) analyzes the direct and indirect costs of bankruptcy and the indirect costs are measured using a regression procedure and security analysts forecasts. The results of their regression procedure show that the indirect costs are more important than the direct costs and that the the direct costs increase as bankruptcy approaches while the indirect costs decrease with the exception of the year of default where they experience a significant increase. The results with the security analysts forecasts approach, show a steady decrease of the indirect costs of bankruptcy as bankruptcy approaches. One aspect that differs from the results of Altman to ours, is that, at bankruptcy, the indirect costs experience an important increase when considering the regression procedure. It is not easy to reconcile these differences in the results, but there are a number of possible explanations. Firstly, we assume an immediate resolution of financial distress, however, the period spent in bankruptcy may last for several years (e.g. see Franks and Torous, 1989 and Weiss, 1990)[13] and during this period firms may still bear in some cases the costs of financial distress as Altman points out with the example of AM International and with its analysis of the indirect costs of bankruptcy for the subsequent fiscal year following the bankruptcy petition date. Secondly, in our model, there is a clear resolution of distress through financial restructuring and distress is purely financial and not economic, which is not the case in the sample of firms of Altman (1984).

The operational nature of the costs of financial distress creates a natural association between the purely operational value of a firm ($V_u(x)$) and the importance of the costs of financial distress. In simple words, the higher is the purely operational value of a firm, the more exposed it will be to the costs of financial distress. The nature of the distress costs may help solve the low debt/zero debt puzzle initially reported in Graham (2000), regarding the inexplicable conservatism in the use of debt financing for a large number of firms[14]. Considering our base case parameter values and a growth rate when not is distress (μ_h) of 6% the distress model estimates an optimal leverage ratio of 0.5%. Several arguments may explain the low debt/zero debt puzzle such as those produced by the pecking order theory, by agency conflicts between managers and equityholders or by financial flexibility[15]. Our analysis provides an additional explanation for this

[13]Franks and Torous (1989) report that the average period spent in Ch. 11 for their sample of distressed firms is 4 years, however it may vary significantly from 37 days to 13.3 years. Weiss (1990) reports that the average period from petition to bankruptcy resolution on their sample was 2.5 years with a standard deviation of 1.4 years.

[14]According to Strebulaev and Yang (2012), from 1962 to 2009 an average of 22% of large public non-financial U.S. firms have a book leverage ratio lower than 5% and an average of 10.2% have no debt at all.

[15]According to the pecking order theory (Myers and Majluf, 1984) these valuable firms dispense debt financing, because of the significant internal financing they generate (operational cash flows). Regarding agency costs, managers are naturally risk averse, because they are undiversified by nature and are exposed to significant reputational costs with bankruptcy, therefore, they are reluctant to issue debt financing even if it benefited equityholders. Furthermore, by using internal financing, managers are able to use these funds discretionarily and evade market monitoring, but expose shareholders to the agency costs of free cash flows (see Jensen, 1986). Regarding financial flexibility, preserving financial flexibility is presented as the main debt policy factor in the CFO survey analysis performed by Graham and Harvey (2001).

puzzle situated perfectly within a classical trade off theory framework, not requiring the existence of information asymmetries or agency conflicts[16]. Valuable, fast growing and profitable firms choose to forgo interest tax shields because of the important costs of financial distress. To exploit what are arguably potentially important tax savings firms would end up jeopardizing their significant operational value by exposing themselves to costs of financial distress.

We now calibrate both models to match the empirical figures of Eom, Helwege and Huang (2004) in terms of leverage ratio, credit spread, bankruptcy costs and recovery rates. We calculate the leverage ratio by calibrating the spread with adjustments to the debt coupons and perform the opposite calculations for the credit spread. For the bankruptcy costs and recovery rates we simultaneously calibrate the leverage ratio and spread by adjusting the debt coupons and bankruptcy costs. The results of these calculations are presented in Table 1.

INSERT [TABLE 1]

The results of the calibration exercise reported in Table 1 show how the distress model better approximates the empirical estimates of the leverage ratio and credit spread (Eom, Helwege and Huang, 2004), the recovery rate (Franks and Torous, 1994, and Keenan, Shtogrin and Sobehart, 1999) and the bankruptcy costs (Warner, 1977, Altman, 1984, Weiss, 1990 and Andrade and Kaplan, 1998) relative to the benchmark model. The distress model underestimates the leverage ratio by 1 percentage point and overestimates the credit spread by 3 basis points. The benchmark model overestimates the leverage ratio by 6 percentage points and underestimates the credit spread by 17 basis points. Regarding the recovery rate, and although both models poorly approximate the empirical estimates of Franks and Torous (1994) and Keenan, Shtogrin and Sobehart (1999) it is clear that the error of the distress model is much smaller than that of the benchmark model. Finally, regarding the bankruptcy costs we observe that the estimate of the distress model fall within the empirical estimates for the direct costs of bankruptcy. The benchmark model however, requires unreasonably high bankruptcy costs to match the leverage and spread

[16] Ju et al. (2005) also generates significantly low predictions for optimal leverage (aprox. 15%) in a trade-off theory model, however the model of Ju et al. incorporates a dynamic capital structure while we incorporate costs of financial distress.

estimates of Eom, Helwege and Huang (2004). Notice that the estimates of the benchmark model are even higher than the empirical estimates for the costs of liquidation of Alderson and Betker (1995) and Gilson (1997)[17].

C Leverage ratio and debt spreads

Previous sections reported that the costs of distress affect firm value even at low leverage ratios, showed that the debt capacity of a firm in relative terms is higher when exposed to financial distress and argued how this effect could be explained by a higher recovery rate at bankruptcy. This section analyzes how the inclusion of financial distress affects the credit spreads of debt and if the credit spreads reflect these aforementioned effects.

Figure 7 presents the credit spreads for the benchmark and the distress models across the full range of financing possibilities.

INSERT [FIGURE 7]

Figure 7 shows how the spread predictions of the distress model are higher than the benchmark model for low and moderately high levels of debt financing. For very high levels of debt financing ($D/V > 95\%$) the spread predictions of the benchmark model are significantly higher than the predictions of the distress model. This result is in line with the observations that the distress costs are significant even for low leverage ratios and that the debt capacity of the firm is higher for the distress model. Furthermore these results show that the distress model addresses a common problem for structural models discussed in Jones, Mason and Rosenfeld (1983) and in Eom, Helwege and Huang (2004), namely, that these models tend to overstate the spread predictions for high leverage ratios (or high coupon bonds) while understating the same predictions for lower leverage ratios. For a leverage ratio of 93.3% the spread estimates produced by the benchmark and distress models are the same, however this equality not always produces at such high leverege ratios. We observe that the leverage ratio at which both models generate a similar spread estimate reduces whenever a change in a parameter value is associated

[17]For the median firm in their samples, and measuring the liquidation costs as a percentage of going concern value, Alderson and Betker (1995) reports liquidation costs of 36.5% and Gilson (1997) reports liquidation costs of 45.5%.

with an increase in the purely operational value of the firm ($V_u(x)$). From the previous section, we know that an increase in the operational value of the firm is associated with an increase in the costs of financial distress, even for very low levels of debt, and with an earlier default that implies a higher recovery rate for debtholders explaining why the distress model in these cases generates higher spreads for low risk debt and lower spreads for high risk debt relative to the benchmark model.

Financial distress affects the value of debt and the credit spreads in different ways, so, in order to have a clearer picture of these different effects and better understand the role of the recovery rate, we decompose the credit spread into a positive spread component (s_c), associated with the loss of coupons at default, and a negative spread component (s_r), associated with capital recovery at default. The following proposition summarizes this decomposition.

Proposition 5. *For any given promised coupon c, the default spread for perpetual debt such as a console bond with present value $D(x)$ has two components: a component associated with the loss of coupons defined as $s_c(x)$ that increases the spread and a component associated with capital recovery at default defined as $s_r(x)$ that reduces the spread. The values of these components are given by,*

$$s_c(x) = r_D \frac{C(x)}{D_F(x)}, \tag{31}$$

$$s_r(x) = r_D \frac{R(x)}{D_F(x)}, \tag{32}$$

in which r_D represents the required return on the risky bond ($r_D = c/D(x)$), $D_F(x)$ represents the present value an equivalent risk free bond ($D_F(x) = c/r$), $C(x)$ represents the present value of the loss of coupons at default and $R(x)$ represents the present value of the capital recovery at default. For the benchmark model, and for $x > x_b$, the values of $C(x)$ and $R(x)$ are given by,

$$C(x) = -\frac{c}{r}\left(\frac{x}{x_b}\right)^{\beta_{h2}}, \tag{33}$$

$$R(x) = V_u(x_b)(1-\delta)\left(\frac{x}{x_b}\right)^{\beta_{h2}}. \tag{34}$$

For the model with financial distress, and for $x > x_b$, the values of $C(x)$ and $R(x)$ are given by,

$$C(x) = \begin{cases} \alpha_{h1}^C x^{\beta_{h1}} + \alpha_{h2}^C x^{\beta_{h2}} & \text{if } x > x_d \\ \alpha_{l1}^C x^{\beta_{l1}} + \alpha_{l2}^C x^{\beta_{l2}} & \text{if } x_b < x \leq x_d \end{cases} \tag{35}$$

$$R(x) = \begin{cases} \alpha_{h1}^R x^{\beta_{h1}} + \alpha_{h2}^R x^{\beta_{h2}} & \text{if } x > x_d \\ \alpha_{l1}^R x^{\beta_{l1}} + \alpha_{l2}^R x^{\beta_{l2}} & \text{if } x_b < x \leq x_d \end{cases} \tag{36}$$

in which α_{h1}^C and α_{h1}^R are equal to zero and the coefficients α_{h2}, α_{l1} and α_{l2} are obtained by replacing, in expressions (18), (19) and (20), a_c and b_c with 0 for $C(x)$ and $R(x)$, and L_c with c/r for $C(x)$ and with $V_u(x_b)(1-\delta)$ for $R(x)$.

Table 2 presents the decomposition of the credit spreads between loss of coupons ($s_c(x)$) and capital recovery ($s_r(x)$) for leverage ratios ranging between 5% to 100%.

INSERT [TABLE 2]

Table 2 shows that both the loss of coupons component and the capital recovery component of the credit spread are always higher in the distress model. Financial distress accelerates default, therefore reducing the expected period during which the firm will be paying coupons, but the accelerated default simultaneously increases the recovery rate of debtholders.

Regarding the implications of accelerated default considering different leverage ratios, we observe that in the distress model (i) the relative weight on the spread of the component associated with the loss of coupons s_c is more important for low levels of debt and that (ii) the relative weight on the spread of the component associated with capital recovery s_r is more important for high levels of debt. With increases in leverage, the decrease in s_r is stronger than the increase in s_c and this explains the lower credit spread estimates of the distress model relative to the benchmark model for very high leverage ratios. These results are in line with the observation that the costs of financial distress are significant even for low levels of debt and the relative debt capacity of a firm increases with the exposure to financial distress. The accelerated default induced by financial distress, increases the present value of the loss of coupons even when the leverage ratio is low, but an accelerated default also increases the capital recovery at default, because debtholders became the shareholders of a more valuable firm.

In both models we observe that changes in the credit spreads are usually associated with changes in the operational value of the firm ($V_u(x)$). The operational value of the firm serves as collateral to safeguard the value of the debt claim, as so, when a change in a parameter value decreases the value of $V_u(x)$ the post restructuring value of the firm reduces and s_r increases. These effects explain the increase in the credit spreads associated with increases in d and θ and decreases in μ. Increases in δ directly reduce the recovery rate of debtholders and therefore increase s_r and increase the spread. The credit spreads increase with increases in the loss given default, and also with increases in the likelihood of default. In terms of costs for debtholders an earlier default implies an increase in the present value of the coupons lost (s_c increases). Default occurs earlier whenever d, σ and r increase and whenever μ decreases. These cases are associated with a significant increase in the credit spreads in virtue of the increase in s_c. As expected, s_c is almost insensitive to changes in θ and in δ.

C.1 Model estimates and observed spreads

This section analyzes the performance of the benchmark and distress models in explaining observed credit spreads and leverage ratios of bonds with different agency ratings. We use the results of Huang and Huang (2002) (henceforth H-H) for 6 different agency ratings considering 10 year maturity bonds and covering the period of 1973-1993. The rating classification reported in H-H is similar to the S&P rating classification and assumes a one-to-one matching to Moodys credit ratings (e.g. $AAA = Aaa, AA = Aa$, etc.). We use H-H average historical leverage ratio and yield spreads, and the implied asset volatility obtained from their simulations performed with Leland and Toft (1996) assuming perpetual maturity. Our analysis of the spread, implies adjusting the coupons to match historical leverage ratios, and vice versa for the analysis of the estimated leverage ratios.

The scope of leverage ratios considered ranges from 13.1% to 65.7%, which is somewhat limited, because it prevents performing a comprehensive analysis of the overestimation of spreads for high leverage and underestimation of spreads for low leverage as reported in Jones, Mason and Rosenfeld (1983) and Eom, Helwege and Huang (2004). Regarding the payout ratio to investors, we reduced the base case parameter value of capital investments d to 5 to better approximate the average total payout ratio of 6% reported in H-H. In terms of other parameters, we are unable to fully calibrate the models, since we lack information regarding the the level of operational leverage, growth rates of earnings and other important parameter values[18].

Table 3 presents the spreads and leverage ratios empirically estimated by Huang and Huang (2002) and our own estimates using the benchmark and the distress models.

[INSERT TABLE 3]

Our estimates confirm the results of Jones, Mason and Roselfeld (1984) and Huang and Huang (2002) regarding the higher predictive power of structure models for non investment bonds. The average absolute differences between the leverage and spread estimates of both models and the figures of H-H for the four classes of investment grade bonds are significantly higher than the same estimates for the two classes of non investment grade bonds. The lower predictive power of constant interest rate models such as our benchmark and distress models for investment grade bonds, is explained by the higher sensitivity of this class of bonds to interest rate risk and their lower sensitivity to credit risk[19].

The average absolute differences for the estimates of the two models across all bond classes are always lower with the distress model. Regarding leverage, the average absolute differences are 11.6 percentage points for the benchmark model *vs* 7.8 percentage points for the distress model. In terms of the spreads, the average absolute differences are 52 basis points for the benchmark model *vs* 47 basis points for the distress model. Finally, regarding recovery rates, the average absolute differences are 10.0 percentage points for the benchmark model *vs* 5.9 percentage points for the distress model. Across individual

[18]This lack of further parameter values explains why some individual class estimates are not very reasonable such as a higher estimation of the credit spreads for the AAA class when compared with the AA class. It also shows the importance of the volatility parameter in driving the estimated results.

[19]For investment grade bonds, structural models with stochastic interest rates such as Kim, Ramaswamy and Sundaresan (1993) and Longstaff and Schwartz (1995) are expected to perform better by addressing the interest rate risk component associated with fixed income investments.

bond classes, the distress model also outperforms the benchmark model in terms of leverage and credit spread estimates in five out of six bond classes, and regarding recovery rates, the distress model outperforms the benchmark model in all bond classes.

III Conclusions

This paper presents a structural model incorporating costs of economic and financial distress that serves to value firms and corporate securities, and to estimate leverage ratios, credit spreads and recovery rates. Our results show that distress accelerates the decisions to default and latter to abandon firm operations. In terms of the value impact, economic distress is less significant than financial distress. The inclusion of financial distress significantly affects the value of levered firm, reduces its optimal leverage but it increases its debt capacity.

In terms of the costs of financial distress (the commonly defined indirect costs of bankruptcy) our results show a dominance of the distress costs relative to the bankruptcy costs in explaining capital structure decisions. These costs outweigh the direct costs of bankruptcy for low and moderate leverage ratios and for high leverage they decrease as the resolution of distress approaches. This explains the higher debt capacity of the firm exposed to financial distress and the higher recovery rates of debt. Similar effects are present in the credit spreads, and the distress model actually generates higher spreads for low leverage and lower spreads for high leverage addressing a common problem that afflicts the spread estimates of classical structural models. The operational nature of the costs of distress create a natural association between the operational value of a firm and its exposure to the costs of distress. This association allows us to naturally explain the low debt / zero debt puzzle within the context of the trade off theory of capital structure.

The results for the effect of distress on the operational decisions and on the deadweight costs of debt, confirm previous insights into the relation between financial distress and bankruptcy. Our results indicate that the costs of financial distress eventually may lead the firm into bankruptcy and that it is not the event of bankruptcy that generates the significant deadweight costs of debt.

In simple calibrating exercises, our results show that the distress model produces more realistic estimates than the benchmark model (a classical endogenous default model) for leverage ratios, credit spreads, recovery rates and direct costs of bankruptcy. For the benchmark model to produce similar estimates to the distress model, it has to assume unrealistically high costs of bankruptcy.

In this paper, distress was modeled in the most parsimonious way, reflecting previous theoretical developments and existing empirical evidence. Better empirical evidence on the onset and resolution of distress will allow further developments of this model and further extensions are possible building on previous research on alternative resolutions of distress or on different characteristics of the debt contracts.

A Appendix

Consider a general claim on cash flows x, defined as $A(x)$. This general claim may represents the value of any type of claim to cash flow x. It may represent firm values ($V(x)$ levered or unlevered $V_u(x)$), corporate securities (Equity $E(x)$ and Debt $D(x)$), loss of coupons ($R(x)$) or recovery rates ($R(x)$). The differences between the different cash flows and default values for each claim are presented in the following table, in which $a_c + b_c = \pi_A$ representing the cash flows accruing to each claim and L_c represents the value when default (x_b levered firm) or abandonment (x_a unlevered firm) take place (the trigger x_f):

[INSERT TABLE 4]

The values of each claim are affected by financial distress, as so we have different value functions when financial distress is disregarded (Proposition 1) and when financial distress is considered (Proposition 2).

Proof of Proposition 1

The value of any general claim when distress is disregarded satisfies the following Partial Diferencial Equation (PDE),

$$0.5\sigma^2 x^2 A_{xx} + \mu_h x A_x - rA + \pi_A = 0. \tag{A.1}$$

that has a general solution of the following type,

$$A(x) = \begin{cases} \frac{a_c x}{\gamma_h} + \frac{b_c}{r} + \alpha_{h1}^A x^{\beta_{h1}} + \alpha_{h2}^A x^{\beta_{h2}} & \text{if } x > x_f \\ L_c & \text{if } x \leq x_f \end{cases} \tag{A.2}$$

in which $\gamma_i = r - \mu_i$ represents the return shortfall, α_{h1}^A and α_{h2}^A are constants to be be determined given appropriate boundary conditions and β_{i1} and β_{i2} (when financial distress is disregarded $i = h$) are solutions to the following quadratic equation,

$$0.5\sigma_i^2 \beta_i^2(\sigma) + \left(\mu_i - \frac{\sigma^2}{2}\right) \beta_i(\sigma) - r = 0, \tag{A.3}$$

and therefore,

$$\beta_{i1} = 0.5 - \frac{\mu_i}{\sigma^2} + \sqrt{\left(\frac{\mu_i}{\sigma^2} - 0.5\right)^2 + \frac{2r}{\sigma_i^2}} > 1, \tag{A.4}$$

$$\beta_{i2} = 0.5 - \frac{\mu_i}{\sigma^2} - \sqrt{\left(\frac{\mu_i}{\sigma^2} - 0.5\right)^2 + \frac{2r}{\sigma^2}} < 0. \tag{A.5}$$

As $x \to \infty$ the likelihood of default (or abandonment in the unlevered case) becomes negligible implying that $\alpha_{h1}^A = 0$ and the constant α_{h2}^A is determined from the following value matching condition,

$$A(x_f) = L_c. \tag{A.6}$$

Equation (A.15) states that, when $x_f = x_a$ for the unlevered firm case, the abandonment value is equal to zero. For the levered case, when $x_f = x_b$, following default, the debtholders become the sole owners of an unlevered firm that suffers direct bankruptcy costs δ and the value at default for equityholders is zero enforcing limited liability and absolute priority rules.

Replacing in (A.2) $\alpha_{h2}^A = 0$ and the solution for α_{h2}^A obtained from (A.15)we get the particular solution for the value of a general claim $A(x)$,

$$A(x) = \begin{cases} \frac{a_c x}{\gamma_h} + \frac{b_c}{r} - \left(\frac{a_c x_f}{\gamma_h} + \frac{b_c}{r} - L_c\right) \left(\frac{x}{x_f}\right)^{\beta_{h2}} & \text{if } x > x_f \\ L_c & \text{if } x \leq x_f \end{cases} \tag{A.7}$$

The values of individual claims are obtained by replacing the appropriate values of a_c, b_c and L_c from Table 4.

The abandonment trigger for the unlevered firm (x_a) and the default trigger for the levered firm (x_b) are determined by the following smooth pasting equations,

$$\left.\frac{\partial V_u}{\partial x}\right|_{x=x_a} = 0, \tag{A.8}$$

$$\left.\frac{\partial E}{\partial x}\right|_{x=x_b} = 0, \tag{A.9}$$

yielding the following abandonment (x_a) and default (x_b) triggers,

$$x_a = \frac{\beta_{h2}}{\beta_{h2}-1}\frac{d}{r}\gamma_h, \tag{A.10}$$

$$x_b = \frac{\beta_{h2}}{\beta_{h2}-1}\frac{c+d}{r}\gamma_h. \tag{A.11}$$

Proof of Proposition 2

The value of any general claim, when distress is considered, satisfies the following pair of PDEs that describe the diffusion of a general claim $A(x)$ in a normal state $x > x_D$ and when in distress $x \leq x_D$,

$$0.5\sigma^2 x^2 A_{xx} + \mu_h x A_x - rA + \pi_A = 0 \quad x > x_D, \tag{A.12}$$

$$0.5\sigma^2 x^2 A_{xx} + \mu_l x A_x - rA + \pi_A = 0 \quad x \leq x_D. \tag{A.13}$$

that has a general solution of the following type,

$$A(x) = \begin{cases} \frac{a_c x}{\gamma_h} + \frac{b_c}{r} + \alpha_{h1}^A x^{\beta_{h1}} + \alpha_{h2}^A x^{\beta_{h2}} & \text{if } x > x_D \\ \frac{a_c x}{\gamma_l} + \frac{b_c}{r} + \alpha_{l1}^A x^{\beta_{l1}} + \alpha_{l2}^A x^{\beta_{l2}} & \text{if } x_f < x \leq x_D \\ L_c & \text{if } x \leq x_f \end{cases} \tag{A.14}$$

in which, x_D represents the distress trigger ($x_D = x_d$ in the levered firm case and $x_D = x_{du}$ in the unlevered firm case), x_f is the bankruptcy or abandonment trigger ($x_f = x_b$ in the levered firm case and $x_f = x_a$ in the unlevered firm case), β_{h1} and β_{l1} are given by expression (A.4), β_{h2} and β_{l2} are given by expression (A.5) ($i = h$ in a normal state and $i = l$ when in distress). As $x \to \infty$ the likelihood of distress becomes negligible implying that $\alpha_{h1}^A = 0$, the constants α_{h2}^A, α_{l1}^A and α_{l2}^A are determined with the following boundary conditions,

$$\lim_{x \uparrow x_D} A(x) = \lim_{x \downarrow x_D} A(x), \tag{A.15}$$

$$A(x_f) = L_c, \tag{A.16}$$

$$\lim_{x \uparrow x_D} \frac{\partial A}{\partial x} = \lim_{x \downarrow x_D} \frac{\partial A}{\partial x}, \tag{A.17}$$

yielding,

$$\begin{aligned} \alpha_{h2}^A = & \frac{a_c x_D \left(\frac{1}{\gamma_l} - \frac{1}{\gamma_h} \right)}{x_D^{\beta_{h2}}} \\ & + \frac{a_c x_D \left(\frac{1}{\gamma_h} - \frac{1}{\gamma_l} \right) (1 - \beta_{h2}) + \left(a_c \frac{x_f}{\gamma_l} + \frac{b_c}{r} - L_c \right) \left(\frac{x_D}{x_f} \right)^{\beta_{l2}} (\beta_{l2} - \beta_{h2})}{x_D^{\beta_{h2}} \left[(\beta_{l1} - \beta_{h2}) - (\beta_{l2} - \beta_{h2}) \left(\frac{x_f}{x_D} \right)^{(\beta_{l1} - \beta_{l2})} \right]} \\ & + \frac{a_c x_D \left(\frac{1}{\gamma_h} - \frac{1}{\gamma_l} \right) (1 - \beta_{h2}) + \left(a_c \frac{x_f}{\gamma_l} + \frac{b_c}{r} - L_c \right) \left(\frac{x_D}{x_f} \right)^{\beta_{l1}} (\beta_{l1} - \beta_{h2})}{x_D^{\beta_{h2}} \left[(\beta_{l2} - \beta_{h2}) - (\beta_{l1} - \beta_{h2}) \left(\frac{x_f}{x_D} \right)^{(\beta_{l2} - \beta_{l1})} \right]}, \end{aligned} \tag{A.18}$$

$$\alpha_{l1}^A = \frac{a_c x_D \left(\frac{1}{\gamma_h} - \frac{1}{\gamma_l} \right) (1 - \beta_{h2}) + \left(a_c \frac{x_f}{\gamma_l} + \frac{b_c}{r} - L_c \right) \left(\frac{x_D}{x_f} \right)^{\beta_{l2}} (\beta_{l2} - \beta_{h2})}{x_D^{\beta_{l1}} \left[(\beta_{l1} - \beta_{h2}) - (\beta_{l2} - \beta_{h2}) \left(\frac{x_f}{x_D} \right)^{(\beta_{l1} - \beta_{l2})} \right]}, \tag{A.19}$$

$$\alpha_{l2}^A = \frac{a_c x_D \left(\frac{1}{\gamma_h} - \frac{1}{\gamma_l} \right) (1 - \beta_{h2}) + \left(a_c \frac{x_f}{\gamma_l} + \frac{b_c}{r} - L_c \right) \left(\frac{x_D}{x_f} \right)^{\beta_{l1}} (\beta_{l1} - \beta_{h2})}{x_D^{\beta_{l2}} \left[(\beta_{l2} - \beta_{h2}) - (\beta_{l1} - \beta_{h2}) \left(\frac{x_f}{x_D} \right)^{(\beta_{l2} - \beta_{l1})} \right]}. \tag{A.20}$$

Once again, the values of individual claims are obtained by replacing the appropriate values of a_c, b_c and L_c from Table 4.

The distress triggers for the unlevered firm x_{ud} and for the levered firm x_d are determined by the following equations,

$$\pi_{V_u} = 0 \quad \longrightarrow \quad (x_{ud} - d)(1 - \theta) = 0, \tag{A.21}$$

$$\pi_E = 0 \quad \longrightarrow \quad (x_d - d - c)(1 - \theta) = 0, \tag{A.22}$$

yielding,

$$x_{ud} = d, \tag{A.23}$$

$$x_d = d + c. \tag{A.24}$$

The abandonment trigger for the unlevered firm (x_a) and the default trigger for the levered firm (x_b) are determined by the following super contact conditions,

$$\left.\frac{\partial^2 V_u}{\partial x^2}\right|_{x=x_a} = 0, \tag{A.25}$$

$$\left.\frac{\partial^2 E}{\partial x^2}\right|_{x=x_b} = 0, \tag{A.26}$$

yielding the following implicit equations that can be numerically solved for x_a and x_b,

$$\frac{x_a}{\gamma_l}(1 - \theta) + \alpha_{l1}^{V_u}\beta_{l1}x_a^{\beta_{l1}} + \alpha_{l2}^{V_u}\beta_{l2}x_a^{\beta_{l2}} = 0 \tag{A.27}$$

$$\frac{x_b}{\gamma_l}(1 - \theta) + \alpha_{l1}^{E}\beta_{l1}x_b^{\beta_{l1}} + \alpha_{l2}^{E}\beta_{l2}x_b^{\beta_{l2}} = 0 \tag{A.28}$$

Regarding the time to default and to abandon we have and the following expected time to default

$$\tau_b = \frac{1}{\mu_h - \frac{1}{2}\sigma^2} ln\left(\frac{x_b}{x}\right) = \frac{1}{\mu_h - \frac{1}{2}\sigma^2} ln\left(\frac{\beta_{h2}}{\beta_{h2} - 1}\frac{c + d}{r}\frac{\gamma_h}{x}\right) \tag{A.29}$$

and the following expected time to abandonment,

$$\tau_a = \frac{1}{\mu_h - \frac{1}{2}\sigma^2} ln\left(\frac{x_a}{x}\right) = \frac{1}{\mu_h - \frac{1}{2}\sigma^2} ln\left(\frac{\beta_{h2}}{\beta_{h2} - 1}\frac{d}{r}\frac{\gamma_h}{x}\right). \tag{A.30}$$

Proof of Proposition 3

From propositions 1 and 2 we can easily determine $T(x)$ and $B(x)$. By taking equations (A.7) and (A.14) with a_c equal to zero, b_c equal to $c\theta$ and L_c equal to zero we obtain the expressions for $T(x)$. By taking equations (A.7) and (A.14) with a_c and b_c equal to zero and L_c equal to the bankruptcy value of the firm $V_u(x_b)\delta$ we obtain the expressions for $B(x)$.

Proof of Proposition 4

The value of the unlevered firm in financial distress is affected by the lower drift rate μ_l, the emergence of distress with leverage occurs earlier ($x_d > x_{du}$) reducing the operational value of the firm simply due to the use of debt financing, therefore the designation of financial distress. On the other hand, upon default of the levered firm, the operational value of the firm is released from the effects of financial distress, translated in the following boundary $V_{ud}(x_b) = V_u(x_b)$. Notice that this value matching does not take into account the direct costs of bankruptcy because these are taken into account and classified as costs of bankruptcy.

Similarly to the derivation of previous claims, the value of $V_{ud}(x)$ is obtained by taking equation(A.14) with a_c equal to $(1-\theta)$, b_c equal to $-d(1-\theta)$ and L_c equal to $V_u(x_b)$ we obtain the expression for the value of $V_{ud}(x)$.

Proof of Proposition 5

The default spread on a risky bond represents the difference between the required return on the bond defined as r_D and the risk-free rate r. The required return of the bond r_D is simply the ratio between the promised coupon c and the present value of the bond $D(x)$ ($r_D = c/D(x)$). The value of a risky bond is expressed in propositions 1 and 2, however we may simply express it as a function of an equivalent risk free bond $D_F(x)$, in which $D_F(x) = c/r$, the present value of the loss of coupons at default $C(x)$ and the present value of the capital recovery at default. In this case we have that $D(x) = D_F(x) - C(x) + R(x)$ and similarly $D_F(x) = D(x) + C(x) - R(x)$. With a simple manipulation of the formulas we can expand the formula of the default spread,

$$r_D - r = r_D \frac{D_F(x)}{D_F(x)} - r, \tag{A.31}$$

$$r_D - r = r_D \frac{D(x)}{D_F(x)} + r_D \frac{C(x)}{D_F(x)} - r_D \frac{R(x)}{D_F(x)} - r, \tag{A.32}$$

replacing $D(x)$ with c/r_D and $D_F(x)$ with c/r in the first term on the right hand side of the previous expression and simplifying, we get,

$$r_D - r = r_D \frac{C(x)}{D_F(x)} - r_D \frac{R(x)}{D_F(x)}, \tag{A.33}$$

$$r_D - r = s_c(x) - s_r(x). \tag{A.34}$$

The first component on the right hand side s_c represents the negative part of the spread associated with the loss of coupons at default and the second component on the right hand side s_r represents the positive part of the spread associated with capital recovery at default.

From propositions 1 and 2 we can easily determine $C(x)$ and $R(x)$. By taking equation (A.7) with a_c and b_c equal to 0 if we replace L_c with c/r we get the expression for $C(x)$, if we replace L_c with $V_u(x)(1-\delta)$ we get the expression for $R(x)$. Following the same procedure for the model with financial distress and using equation (A.14) we similarly get the expressions for $C(x)$ and $R(x)$.

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Table 1: Leverage ratios, spreads, recovery rates and bankruptcy costs

	Total Payout (a)	Leverage Ratio (a)	Credit Spread (a)	Recovery Rate (b)	Bankruptcy Costs (c)
Empirical figures	4.83%	30.3%	93.5bp	51.3%	5% − 20%
Benchmark model	4.56%	36.5%	76.2bp	13.4%	62.0%
Distress model	4.60%	29.4%	96.7bp	35.1%	14.0%

Notes: This table presents the implied figures for bankruptcy costs and recovery rates following a calibration of the model to match the total payout, leverage ratio and credit spreads (a) of Eom, Helwege and Huang (2004). The empirical figure for the recovery rate (b) is from Franks and Torous (1994) and Keenan, Shtogrin and Sobehart (1999) and the range for the bankruptcy costs (c) comes from the empirical estimates of Warner (1977), Altman (1984), Weiss (1990) and Andrade and Kaplan (1998). We approximated the payout ratio of 4.83% by assuming a value of 10 for the reinvestment costs d, we calibrated the leverage (spread) figures by adjusting the coupon rate to match the spread (leverage) and we calculated the bankruptcy costs by simultaneously calibrating the spread and leverage by minimizing the following error expression: $Error = 0.5 \times ABS(D/V(c,\delta) - D/V(EHH)) + 0.5 \times ABS(spread(c,\delta) - spread(EHH)) \times 20$.

Table 2: Credit spreads for different leverage ratios.

Leverage	Benchmark model			Distress model		
Ratio	Spread	s_c	s_r	Spread	s_c	s_r
5%	$26bp$	$29bp$	$-3bp$	$33bp$	$38bp$	$-4bp$
10%	$34bp$	$41bp$	$-7bp$	$43bp$	$53bp$	$-10bp$
15%	$42bp$	$54bp$	$-11bp$	$53bp$	$70bp$	$-17bp$
20%	$52bp$	$68bp$	$-16bp$	$65bp$	$89bp$	$-24bp$
25%	$63bp$	$84bp$	$-21bp$	$78bp$	$110bp$	$-32bp$
30%	$75bp$	$101bp$	$-27bp$	$93bp$	$134bp$	$-40bp$
35%	$88bp$	$121bp$	$-33bp$	$110bp$	$160bp$	$-50bp$
40%	$103bp$	$144bp$	$-40bp$	$129bp$	$190bp$	$-61bp$
45%	$120bp$	$169bp$	$-48bp$	$150bp$	$224bp$	$-73bp$
50%	$140bp$	$198bp$	$-58bp$	$175bp$	$262bp$	$-87bp$
55%	$162bp$	$230bp$	$-69bp$	$202bp$	$305bp$	$-103bp$
60%	$188bp$	$269bp$	$-81bp$	$232bp$	$353bp$	$-120bp$
65%	$218bp$	$314bp$	$-96bp$	$267bp$	$408bp$	$-140bp$
70%	$255bp$	$368bp$	$-113bp$	$307bp$	$470bp$	$-163bp$
75%	$299bp$	$435bp$	$-135bp$	$352bp$	$541bp$	$-189bp$
80%	$355bp$	$517bp$	$-162bp$	$405bp$	$624bp$	$-219bp$
85%	$429bp$	$627bp$	$-198bp$	$470bp$	$727bp$	$-257bp$
90%	$534bp$	$783bp$	$-249bp$	$559bp$	$867bp$	$-308bp$
95%	$706bp$	$1,039bp$	$-333bp$	$692bp$	$1,078bp$	$-385bp$
100%	$1,258bp$	$1,860bp$	$-602bp$	$1,078bp$	$1,686bp$	$-608bp$

Notes: This figure presents the credit spreads for the benchmark model and distress models and their decomposition between loss of coupons ($s_c(x)$) and capital recovery ($s_c(x)$). The base-case parameter values are as follows: the initial cash flow level x_0 is 100, the growth rate of cash flows (μ_h) is 1.5% when the firm is not in distress and the growth rate of cash flows (μ_l) is -1% when the firm is in distress, the volatility of cash flows (σ) is 26.3%, the reinvestment costs (d) are 50, the risk free rate (r) is 6.5%, the bankruptcy costs (δ) are 15% and the corporate tax rate (θ) is 25%.

Table 3: Agency ratings, leverage ratios, credit spreads and recovery rates

Agency rating	AAA	AA	A	BBB	BB	B
Volatility (a)	32.1%	28.4%	25.6%	25.8%	32.4%	39.5%
Total Payout (b)	6.0%	6.0%	6.0%	6.0%	6.0%	6.0%
Benchmark	6.2%	6.0%	5.8%	5.7%	5.7%	5.7%
Distress	6.2%	6.1%	6.0%	5.9%	6.0%	6.2%
Leverage (b)	13.1%	21.0%	32.0%	43.3%	53.5%	65.7%
Benchmark	16.1%	34.6%	51.3%	64.3%	64.8%	64.5%
Distress	13.2%	29.5%	44.2%	56.8%	58.2%	58.2%
Spread (b)	63*bp*	91*bp*	123*bp*	194*bp*	320*bp*	470*bp*
Benchmark	55*bp*	52*bp*	59*bp*	93*bp*	234*bp*	486*bp*
Distress	65*bp*	63*bp*	74*bp*	120*bp*	283*bp*	565*bp*
Rec. Rate (c)	51.3%	51.3%	51.3%	51.3%	51.3%	51.3%
Benchmark	28.7%	37.1%	43.5%	47.0%	45.5%	45.8%
Distress	31.2%	41.0%	48.0%	52.1%	50.7%	51.0%

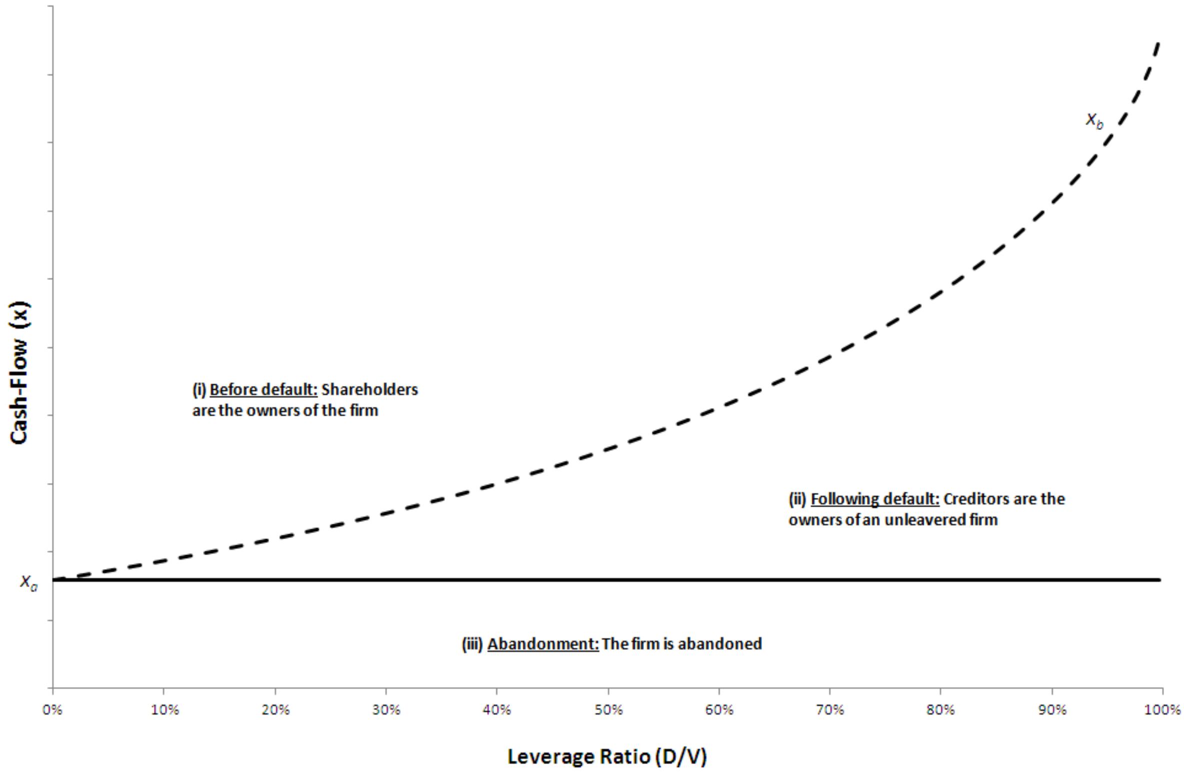
Notes: This table presents the implied figures for the total payout, leverage ratios, spreads and recovery rates following a calibration of the model. Total payout is determined by adjusting the reinvestment costs d, leverage is determined by adjusting the spread and the spread is determined by adjusting the leverage ratio. The figures for the total payout and recovery rate represent the averages of the two calibrating exercises of leverage ratio and spread. The volatility figures are from Huang and Huang (2002) and represent the implied volatility (a) determined by the Leland and Toft (1996) model assuming perpetual debt. The total payout, leverage and spread figures (b) refer to the results of Huang and Huang (2002) for the 10 year maturity bonds. The average recovery rate figures (c) are from Franks and Torous (1994) and Keenan, Shtogrin and Sobehart (1999).

Table 4: Cash flows, default and abandonment values for the different claims on x.

	C	x_f	x_D	a_c	b_c	L_c
Unlevered firm	V_u	x_a	x_{du}	$(1-\theta)$	$-d(1-\theta)$	0
Levered firm	V	x_b	x_d	$(1-\theta)$	$-d(1-\theta)+c\theta$	$V_u(x_b)(1-\delta)$
Equity	E	x_b	x_d	$(1-\theta)$	$-(d+c)(1-\theta)$	0
Debt	D	x_b	x_d	0	c	$V_u(x_b)(1-\delta)$
Loss of coupons at default	C	x_b	x_d	0	0	c/r
Capital recovery at default	R	x_b	x_d	0	0	$V_u(x_b)(1-\delta)$
Interest tax shields	T	x_b	x_d	0	$c\theta$	0
Bankruptcy costs	B	x_b	x_d	0	0	$V_u(x_b)\delta$
Operational value with financial distress	V_{ud}	x_b	x_d	$(1-\theta)$	$-d(1-\theta)$	$V_u(x_b)$

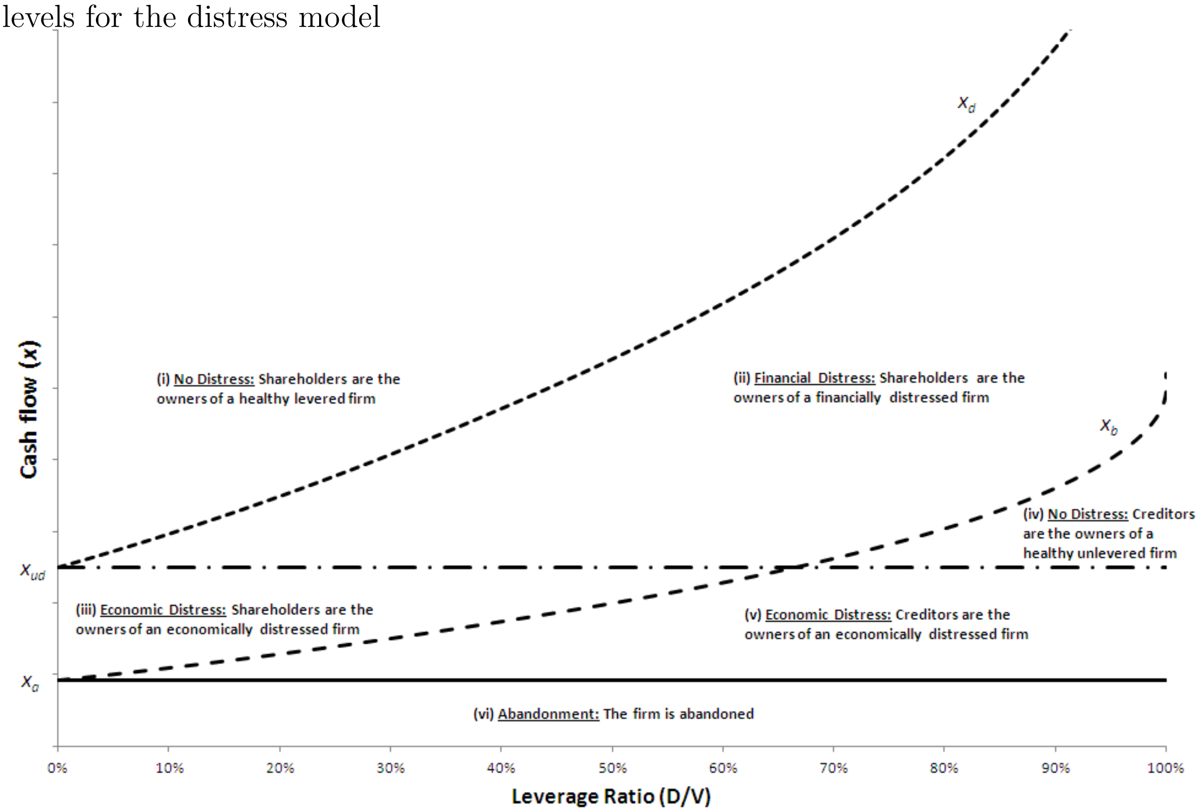
Note: the abandonment value L_c applies for $V_u(x)$ at abandonment, when $x_f = x_a$. For all other claims the value L_c applies at default, when $x_f = x_b$.

Figure 1: Abandonment and default for different leverage ratios and cash flow levels for the benchmark model



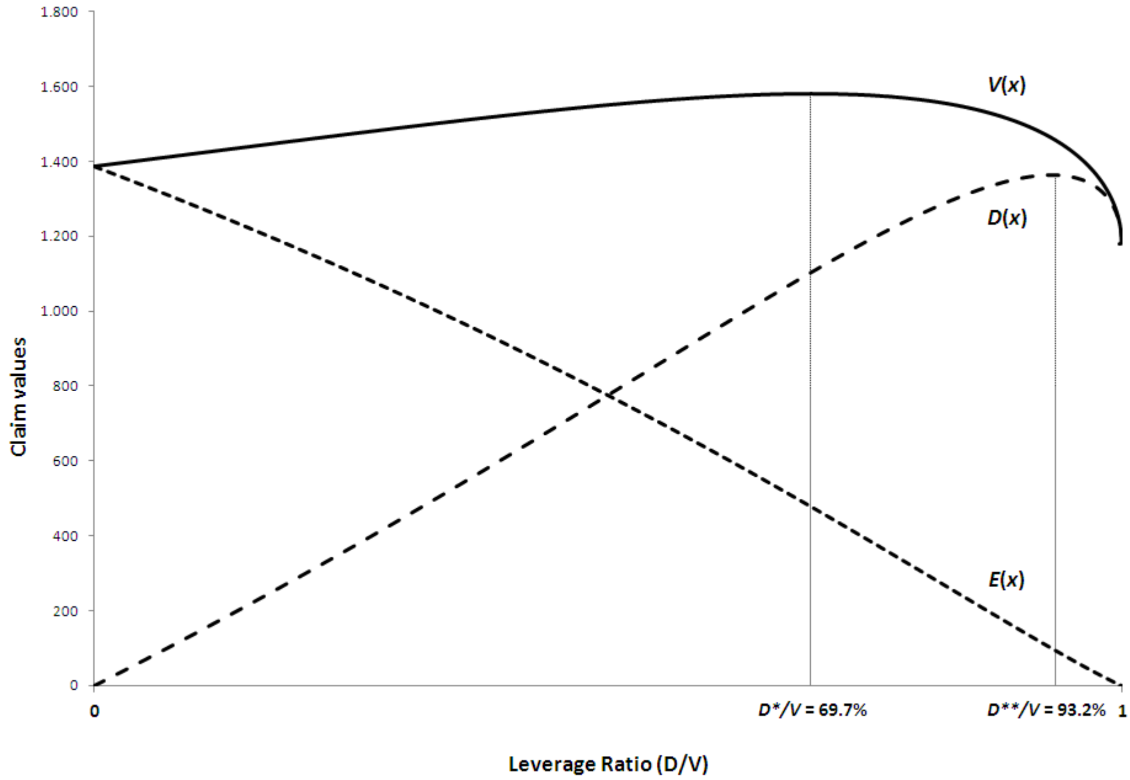
Notes: This figure presents the default (x_b) and abandonment (x_a) triggers for different leverage ratios.

Figure 2: Abandonment, distress and default for different leverage ratios and cash flow levels for the distress model



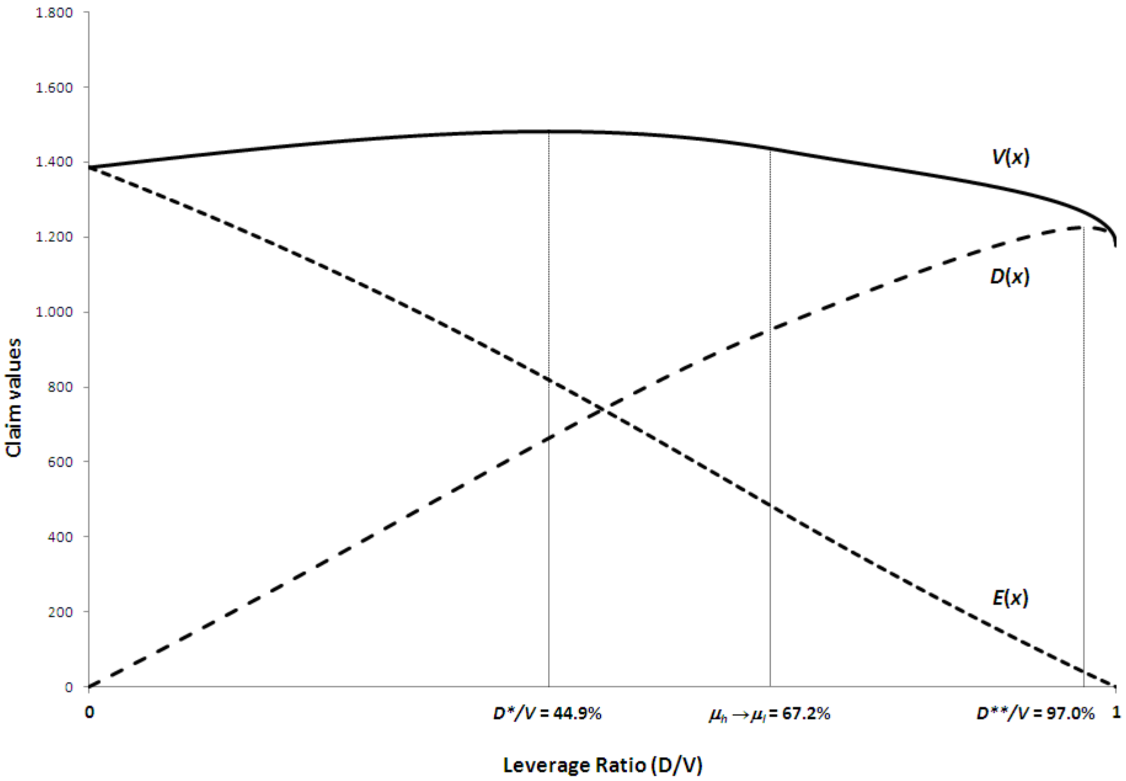
Notes: This figure presents the default (x_b), distress (x_d and x_{du}) and abandonment (x_a) triggers for different leverage ratios.

Figure 3: Firm, security values and optimal capital structure for the benchmark model



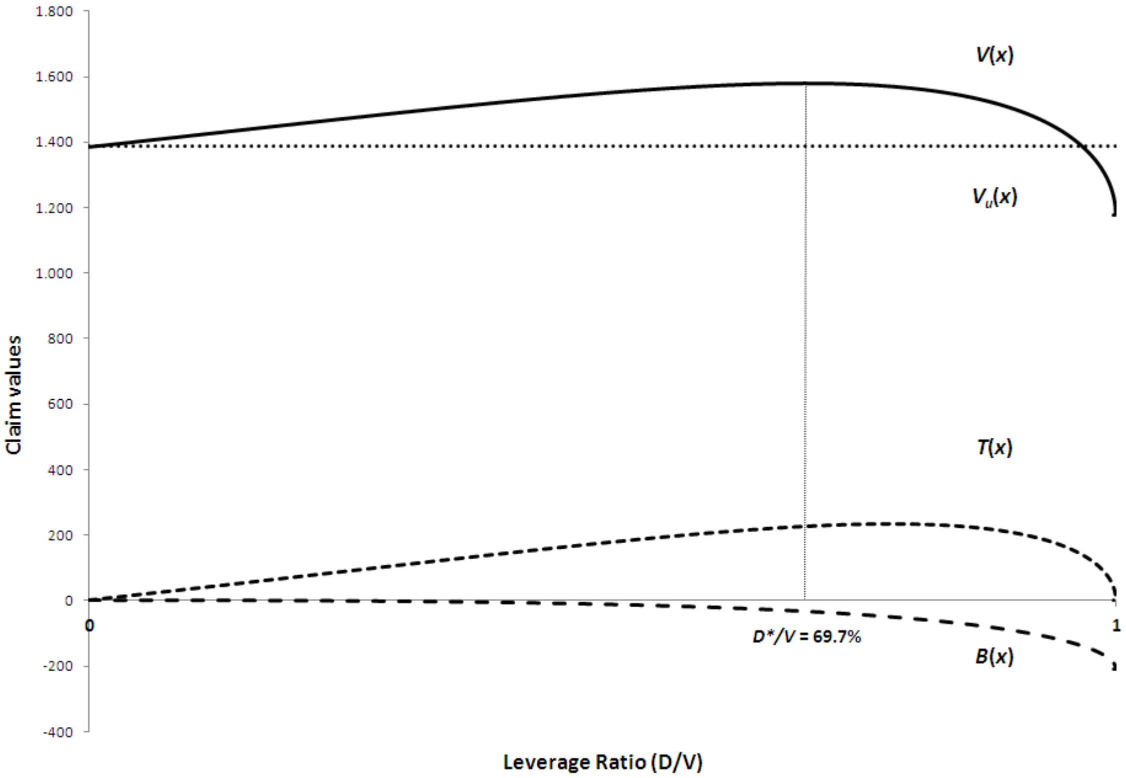
Notes: Notes: This figure presents the levered firm ($V(x)$), equity ($E(x)$) and debt ($D(x)$) values, the optimal capital structure ($D*/V$) and debt capacity ($D**/V$) for the benchmark model, considering all possible leverage ratios. The base-case parameter values are as follows: the initial cash flow level x_0 is 100, the growth rate of cash flows (μ_h) is 1.5%, the volatility of cash flows (σ) is 26.3%, the reinvestment costs (d) are 50, the risk free rate (r) is 6.5%, the bankruptcy costs (δ) are 15% and the corporate tax rate (θ) is 25%.

Figure 4: Firm, security values and optimal capital structure for the distress model



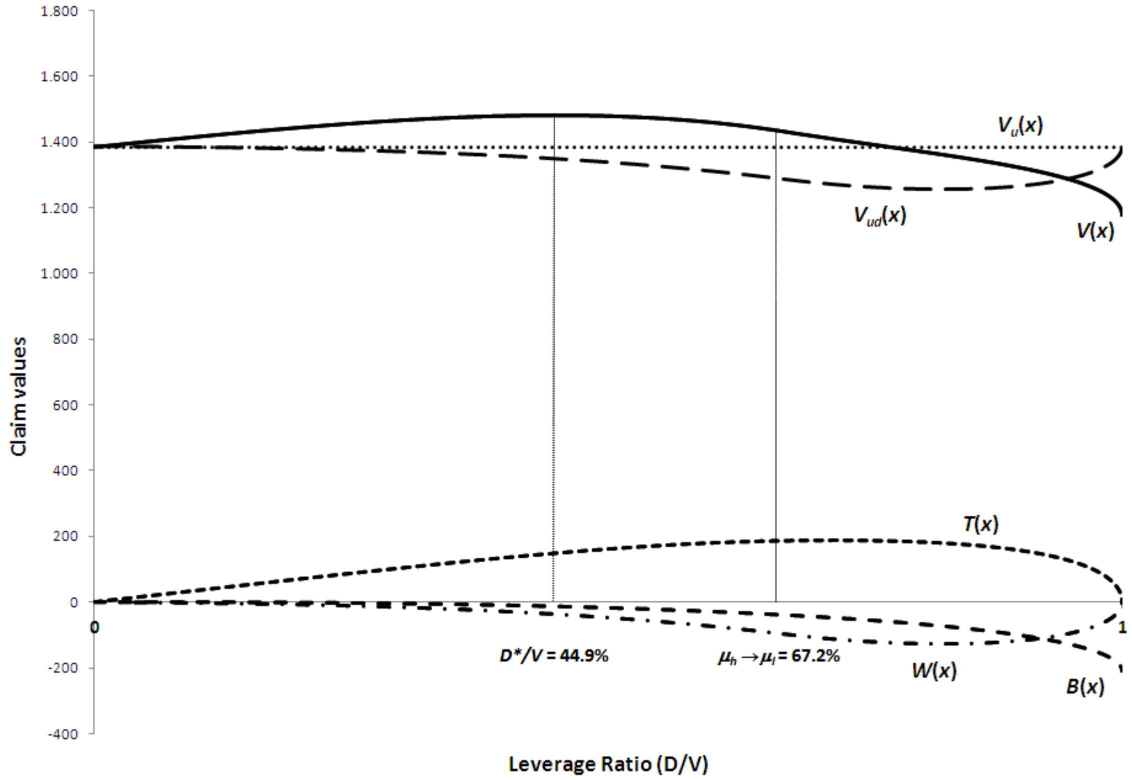
Notes: This figure presents the levered firm ($V(x)$), equity ($E(x)$) and debt ($D(x)$) values, the optimal capital structure ($D*/V$) and debt capacity ($D**/V$) for the distress model, considering all possible leverage ratios. The base-case parameter values are as follows: the initial cash flow level x_0 is 100, the growth rate of cash flows (μ_h) is 1.5% when the firm is not in distress and the growth rate of cash flows (μ_l) is -1% when the firm is in distress, the volatility of cash flows (σ) is 26.3%, the reinvestment costs (d) are 50, the risk free rate (r) is 6.5%, the bankruptcy costs (δ) are 15% and the corporate tax rate (θ) is 25%.

Figure 5: Firm values, interest tax shields and deadweight costs of debt for the benchmark model



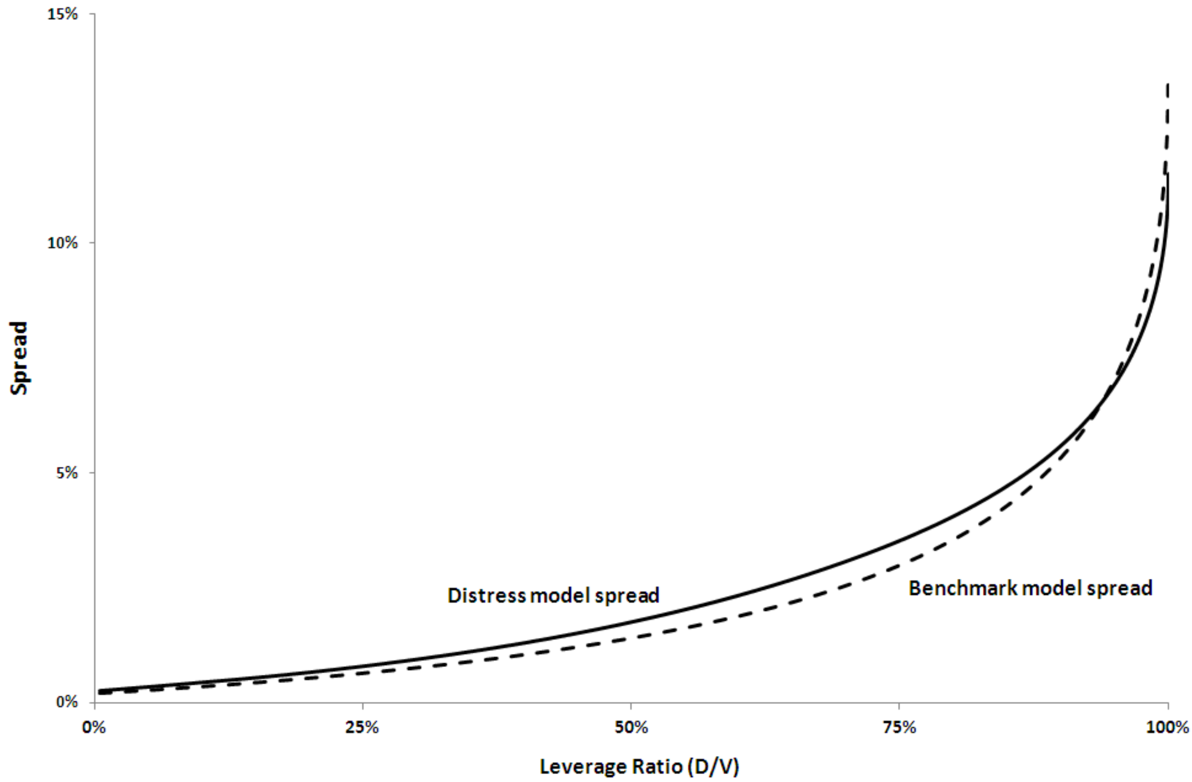
Notes: This figure presents the unlevered firm ($V_u(x)$), levered firm ($V(x)$), interest tax shields ($T(x)$) and bankruptcy costs ($B(x)$) for different leverage ratios for the benchmark model. The base-case parameter values are as follows: the initial cash flow level x_0 is 100, the growth rate of cash flows (μ_h) is 1.5%, the volatility of cash flows (σ) is 26.3%, the reinvestment costs (d) are 50, the risk free rate (r) is 6.5%, the bankruptcy costs (δ) are 15% and the corporate tax rate (θ) is 25%.

Figure 6: Firm values, interest tax shields and deadweight costs of debt for the distress model



Notes: This figure presents the unlevered ($V_u(x)$) and levered ($V(x)$) firm values, interest tax shields ($T(x)$), bankruptcy costs ($B(x)$) and financial distress costs ($W(x)$) for different leverage ratios for the benchmark model. The base-case parameter values are as follows: the initial cash flow level x_0 is 100, the growth rate of cash flows (μ_h) is 1.5% when the firm is not in distress and the growth rate of cash flows (μ_l) is -1% when the firm is in distress, the volatility of cash flows (σ) is 26.3%, the reinvestment costs (d) are 50, the risk free rate (r) is 6.5%, the bankruptcy costs (δ) are 15% and the corporate tax rate (θ) is 25%.

Figure 7: Financial leverage and credit spreads for the benchmark and distress models



Notes: This figure presents the spread for the benchmark model (the straight line) and the distress model (the dashed line) for different leverage ratios. The base-case parameter values are as follows: the initial cash flow level x_0 is 100, the growth rate of cash flows (μ_h) is 1.5% when the firm is not in distress and the growth rate of cash flows (μ_l) is -1% when the firm is in distress, the volatility of cash flows (σ) is 26.3%, the reinvestment costs (d) are 50, the risk free rate (r) is 6.5%, the bankruptcy costs (δ) are 15% and the corporate tax rate (θ) is 25%.