Information Management by Bank Regulators*

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PRELIMINARY

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Abstract

In the recent financial crisis and the current sovereign debt crisis, there have been large questions surrounding both the health of banks and regulators' ability to provide capital to bail out banks. In principle, regulators should be providing information to the public and market participants on both of these issues. Nevertheless, we demonstrate in a theoretical model that regulators may manipulate this uncertainty and may even try to block information flows. The benefits are clear: by hiding weakness they can prevent runs or costly interventions. Information management can also reduce moral hazard or excess fragility. Stress tests will be more informative when regulators have more capital or when market beliefs are negative.

Keywords: bank regulation, bailouts, reputation, financial crisis, sovereign debt crisis

JEL Codes: G01, G21, G28

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"If money isn't loosened up, this sucker could go down,"

- Statement by former President George W. Bush, quoted in the New York Times on September 26, 2008

1 Introduction

In the quote above, former President George W. Bush highlights the uncertainty in the U.S. over whether funds would be released to resolve the banking crisis in 2008. Uncertainty over the health of financial institutions *and* whether the regulator can act to stabilize those institutions were elements of both the subprime crisis and the ongoing European sovereign debt crisis. In this paper, we examine the role of uncertainty about a regulator's access to capital and how regulators manipulate information in response. We demonstrate that regulators may prefer less information to be revealed, even if markets interpret the lack of information rationally. This may explain the weakness of the European stress tests and the unwillingness of the European Union to bring enlargements and reforms of the European Financial Stability Facility (EFSF) to a vote. Bad news can provoke runs, while no news may reduce their likelihood. Even regulators with good news (in terms of funding capacity) may try to hide it to prevent subsequent moral hazard.

We build a theoretical model where a bank regulator has private information about the losses that each bank will incur in an adverse macroeconomic scenario. The regulator can then determine the capital each bank must receive (if necessary) to prevent default in the adverse scenario, and the regulator can either inject capital, liquidate the bank, or forbear. The regulator's preferences over actions will depend on how much capital she has available. The market is uncertain both about the health of the bank *and* the regulator's ability to bail out banks. Depositors run on the bank if it does not receive enough capital and is expected to be insolvent. Banks that experience a run or end up insolvent impose costs on all agents. In the model, depositors can learn about the regulator's type through her previous actions. This creates room for manipulating perceptions about the regulator's type through reputation management.

The regulator's actions will depend on whether it has the firepower to stop a run. Its actions serve two roles: to resolve the bank and to signal its type. This gives rise to several important results. First, a regulator may take advantage of the uncertainty about its type to stave off runs. Second, it may also use strong actions to build a reputation that will prevent future runs. This incentive may be magnified when there is fragility in the banking system. Third, it may want to diminish its reputation so as to minimize subsequent risk taking by banks. Lastly, credible stress tests are more likely to come from well funded regulators and a regulator with insufficient access to funds is more likely to perform a credible stress test when priors about the banking system are negative.

There is a theoretical literature that examines regulator decisionmaking around the closing and bailing out of banks. The closest paper to ours is Morrison and White (2011), who argue that a regulator may choose to forbear when she knows that a bank is in danger of failing, because liquidating the bank may lead to a poor reputation about the ability of the regulator to screen and trigger contagion in the banking system. We also have potential contagion through reputation, but examine the resources of a regulator rather than its skill for screening. In addition, we explicitly model bailouts and asymmetric information about the regulator's type. Boot and Thakor (1993) also find that bank closure policy may be inefficient due to reputation management by the regulator, but this is due to the regulator being self-interested rather than being worried about contagion. In their model, the regulator has private information about its screening ability and there are no bailouts. Cordella and Levy Yeyati (2003) focus on the moral hazard dimension, where a regulator must balance being tough and committing no bailouts (avoiding moral hazard) with allowing for bailouts (increasing long run bank value through insurance). Keister (2011) also discusses the difference between committing to no bailouts and allowing the regulator discretion to use bailouts in the context of bank runs and liquidity. He allows for the government to use its remaining funds on public goods, which drive his novel results. Mailath and Mester (1994) discuss credible bank closure policies in a model with full information and without bailouts. Acharya and Yorulmazer (2007, 2008) examine the idea of "too many to fail" and show that because a regulator will use bail outs when many banks are failing, banks will herd in their risk taking.

A previous literature has examined the need for regulators to disclose information about the health of banks. DeYoung et al. (1998) and Berger and Davies (1994) find empirical evidence suggesting that banks disclose good news but look to hide bad news, which is revealed because of bank exams by regulators. However, Prescott (2008) develops a model to argue that too much information disclosure by a bank regulator leads to less information that the regulator can gather on banks. Peristiani et al. (2010) show that markets had largely identified the distribution of weaker and stronger banks before the 2009 US stress test was conducted, but the stress test provide new information about the size of capital needs among the weaker banks. Hirtle et al. (2009) highlight that the 2009 US stress test was credible and stabilizing for the banking system because the standard microprudential process of analyzing individual bank loss exposures was combined with a macroprudential focus of the need for broad financial stability and the upfront commitment to provide capital to banks.

We proceed as follows. Section 2 sets up the model and the benchmarks. In Section 3, we examine reputation dynamics. In Section 4, we allow the bank equityholders to risk shift, relating the regulator's position on bailouts to ensuing moral hazard. In Section 5, regulators can announce information through stress tests. In Section 6, we allow depositors to run before the regulator has made any decision and look at the impact of uncertainty on fragility. In Section 7, we conclude. All proofs are in the appendix.

2 The Model

There are two dates, $T = 1, 2$ and three types of risk neutral agents: the regulator, banks, and depositors. We break the setup of the model into three parts: banks and depositors, the regulator, and benchmarks.

2.1 Banks and Depositors

A bank has one unit of an asset. At date 2, the aggregate state of the world is revealed to be either high returns where all bank assets pay off $\bar{R}$, or low returns, where the bank's assets pays off $\underline{R}_\theta$, where $\theta \in \{G, B\}$. From an ex-ante perspective, the high returns state occurs with probability q. All agents have a prior over the type of the banks, α, that represents the probability that a bank is type G (where $1 - \alpha$ represents the probability that the bank is type B).

There are a mass one of depositors in each bank, who have each deposited 1 unit. For a solvent bank, the exogenous return promised on deposits is $\tilde{R}$ if withdrawn at date 2. The promised return is 1 if deposits are withdrawn earlier (at date 1). We assume that the bank liquidates its long term asset at date 1 if necessary. The liquidated asset provides a return of 1.[1] If the bank is insolvent at any date, the asset return is equally divided among all withdrawing depositors at that date. The remaining value of each bank is paid to equityholders at date 2. We assume the following ordering on returns:

$$\bar{R} \geq \underline{R}_G \geq \tilde{R} \geq 1 > \underline{R}_B. \tag{A1}$$

The good bank can always pay depositors, while the bad bank won't be able to in the bad state. The return promised to depositors for keeping their money in the bank is larger than that for withdrawing it.

At date 1, if depositors of a bank expect not to get a return at least as much as their outside option of 1, there is a run and they withdraw their money from the bank immediately, leaving the bank insolvent. We assume that if depositors knew a bank was bad, meaning that in the low returns state it would have a bad shock, they would run at date 1:

$$q\tilde{R} + (1-q)\underline{R}_B < 1$$

We assume that there is no deposit insurance for simplicity. From condition A1, if depositors know that the bank is good, then they would not run.

In order to define the beliefs of depositors, it is useful first to define a benchmark. We denote α^* as the probability that a bank is good when depositors are indifferent between a run and keeping their money in the bank. Specifically, α^* is defined by:

$$q\tilde{R} + (1-q)(\alpha^*\tilde{R} + (1-\alpha^*)\underline{R}_B) = 1 \tag{1}$$

If there is no run but the bank cannot fully pay depositors at date 2, the bank is insolvent. We assume that there is an insolvency cost C to society per bank that is insolvent (or has been made insolvent by a run).[2] The insolvency cost may represent the loss of value from future intermediation the bank may perform, the cost to resolve the bank, the cost of contagion,

[1] We could allow this return to be lower than 1. In that case, there would be multiple equilibria where self-fulfilling bank runs occur, but we could get similar results by focusing on the equilibria which have fundamentals-based runs.

[2] The insolvency cost may be different in period 1 versus period 2, as in period 1 it occurs because of a run or liquidation, while in period 2 it occurs because of a bad shock. To simplify matters, we maintain it is the same in both periods. Mailath and Mester (1994) have a similar cost.

or we may assume that this is a cost of liquidating the long term asset in the case of a run (for example, it could reduce the value of the asset for other agents holding it).

2.2 The Regulator

The regulator costlessly observes the type of a bank. The regulator then may take action based on its findings. It has three possible actions: to inject capital, liquidate the bank, or do nothing (forbearance). Injecting an amount of capital X costs λX, where λ is larger than 1 and represents the deadweight loss of raising government funds.[3] The regulator's objective function is to maximize the sum of the expected surplus of all agents minus the cost of insolvencies and potential capital injections.

We make two assumptions that will streamline our presentation. First, while the regulator may also have the ability to order the banks to raise outside capital, we will put this tool aside. Second, we will assume that regulators prefer to stop both runs and insolvency at a bank, rather than only stopping a run (and permitting a possible insolvency).[4] This reduces the number of cases to consider and streamlines the presentation, but does not affect the results. Assumption A2 formalizes this:

$$C > \frac{(\tilde{R}-1)(\lambda-1)}{(1-q)^2(1-\alpha)} \tag{A2}$$

This does not imply we are ruling out insolvency - an insolvency will be the best solution for the regulator if the cost of raising capital for the regulator is too high.

When a bank is good, there will be no runs or capital injections. The expected surplus of the regulator (S_G) for the good bank is equal to:

$$S_G = (q\bar{R} + (1-q)\underline{R}_G)$$

When a bank is bad, it may be subject to runs and the regulator may inject capital. How much capital does a bad bank need? In order to prevent both a run and an insolvency in period 2, it needs to inject $X_I = \tilde{R} - \underline{R}_B$.

[3] Laffont and Tirole (1993) label this the "shadow cost of public funds".

[4] If the insolvency is prevented, the run will also be prevented, but the reverse does not hold true.

The surplus to the regulator from preventing an insolvency by injecting X_I is:

$$S(X_I) = q\bar{R} + (1-q)\underline{R}_B - (\lambda - 1)(\tilde{R} - \underline{R}_B) \tag{2}$$

The surplus to the regulator if a bank is liquidated is equal to $1 - C$, the value of the liquidated asset minus the insolvency cost C. This is the same surplus as if the bank were subject to a run.[5] Given the current timing, we will interpret the payoff of $1 - C$ as the regulator liquidating rather than a run occurring. In the section on fragility, we will look at what happens when the value to depositors from a liquidation and a run diverge.

When there is no information, an outcome where a bad bank has no run, no capital injection, and is not liquidated may arise. This occurs if the regulator can effectively "hide" the type of the bad bank through forbearance, i.e. the regulator (i) does not pursue a course of action to prevent potential default of a bank that it knows may be bad and (ii) a run is not provoked. This gives the regulator a payoff of:

$$S_F = q\bar{R} + (1-q)\underline{R}_B - (1-q)C \tag{3}$$

We make the following assumption on the parameters throughout the paper:

$$S_F > 1 - C \tag{A3}$$

This assumption puts us in the interesting case where the regulator would prefer to hide the type of the bank rather than liquidate it. This is at the heart of the information problem.

We suppose that the regulator knows the type of the bank, but depositors do not. Actions taken by regulator takes are observable, and may provide the market with signals. Doing nothing also sends a signal to the market, with all of its consequent implications. In section 5, we will allow the regulator to communicate information about banks through stress tests.

We define the type i of the regulator in terms of how costly it is to raise funds λ_i. Some regulators may have easy access to funds, but some may face a deadlocked political system and find that the tap is dry. A regulator with

[5] As long as runs are fundamentals based (i.e. there is no sequential service constraint, yielding a benefit to being the first in the queue) they will be the same.

high costs is defined as a regulator whose cost of capital λ_H is large enough so that:

$$S_H(X_I) < 1 - C$$

The high cost regulator is restricted in its possible actions. It can't afford to inject capital, so it must either liquidate the bank or attempt to hide the type of the bad bank to prevent a run.

The medium cost regulator has a cost of capital λ_M which allows it to avoid liquidations, but still prefers to hide the type of the bank to injecting capital:

$$1 - C < S_M(X_I) < S_F$$

The low cost regulator, who has cost of capital λ_L, can afford to inject capital and strictly prefers to do so, preventing the possibility of a costly future bankruptcy:

$$S_L(X_I) > S_F$$

The timing of the basic game for resolving one bank is as follows:

Date 1:

1. After observing the type of the bank, the regulator may make a capital injection, liquidate the bank, or forbear.

2. Depositors decide whether to run or not. If they run, they take their cash and invest it in their outside option for a return of 1.

Date 2:

1. The aggregate state of the world is publicly revealed, assets pay off, and insolvencies costs may occur.

2. Depositors collect their returns, either from the bank (if there was no run) or from the outside option (if there was a run). Equityholders collect the remaining returns, if there are any.

There is no discounting between dates for now. When we have two banks, which are handled in sequential periods, we will allow for discounting between periods. In addition, in the section on fragility we will allow for discounting between dates.

We use the concept of Perfect Bayesian Equilibrium.

2.3 Benchmarks

In this subsection, we will assume the depositors know the type of regulator and only look at the resolution of one bank. In subsequent sections, we will allow for uncertainty over how much firepower the regulator has to resolve the crisis and look at the sequential resolution of two banks.

The low cost regulator has a dominant strategy to inject capital into a bad bank and leave the good bank untouched. As depositors understand this incentive, they decide not to run on either type of bank.

For the middle and high cost regulators, things are more complicated. We solve for the choice of the high cost regulator in the following lemma, applying the intuitive criterion of Cho and Kreps (1987).

Lemma 1 *For the high cost regulator, if:*

1. *Inequality $\alpha \geq \alpha^*$ holds: There is a pooling equilibrium where the regulators of both types of bank inject no money and do not liquidate the bank.*

2. *Inequality $\alpha < \alpha^*$ holds: Define X^* such that $S_F - (\lambda_H - 1)X^* = 1 - C$. There is a unique equilibrium that is separating, where the good bank gets a capital injection of X^* and the bad bank is liquidated.*

When the regulator's access to capital is limited, it's actions depend on the beliefs of the market. If the market has favorable beliefs, it will do nothing even if it knows the bank is likely to default. If the market has negative beliefs, the regulator will wind down the bad bank and inject money into the good bank as a show of faith (thereby separating from the bad bank).

In part 1 of the Lemma we focus on one particular pooling equilibrium. We show in the appendix that there are no separating equilibria and argue that this pooling equilibrium is the most 'reasonable', as it exists for all off-the-equilibrium-path beliefs. We will only consider this equilibrium for the rest of the paper.

If there is a medium cost regulator making the decision, we have a similar result to the above:

Corollary 1 *For the medium cost regulator, if:*

1. *Inequality $\alpha \geq \alpha^*$ holds: There is a pooling equilibrium where both types of regulator inject no money and do not liquidate the bank.*

2. *Inequality $\alpha < \alpha^*$ holds: Define X^{**} such that $S_F - (\lambda_M - 1)X^{**} = S_M(X_I)$. There is a unique equilibrium that is separating, where the good bank gets a capital injection of X^{**} and the bad bank receives a capital injection that prevents insolvency.*

The difference here is that the medium cost regulator can resort to a bailout (and prefers to do so) if the priors are unfavorable.

There are clear inefficiencies when the type of the bank is unknown and the regulator can only signal through taking steps to resolve a bank. The inefficiency is well defined. The high cost regulator has to inject X^* of capital into a good bank when $\alpha < \alpha^*$. This injection is needed to separate itself from the high cost regulator with a bad bank. Similarly, the medium cost regulator has to inject X^{**} of capital into a good bank when $\alpha < \alpha^*$.

Surprisingly, there are efficiencies from the type of the bank being unknown. This is because in the situation where priors are favorable ($\alpha \geq \alpha^*$) the regulator can forbear on the bad bank rather than liquidate or inject capital. Forbearing creates a larger surplus for the high and medium cost regulators (S_F).

3 Unobservable Costs and Dynamics

When considering the case of Europe, two facts are key. First, there is great uncertainty regarding the amount of capital that the euro-zone countries have been able to (and willing to) dedicate towards resolving their banking crisis. Second, the crisis has been protracted, and has already involved two formal stress tests.[6] These facts are also relevant to the U.S. case, especially as one looks ahead. Thus we construct a two period model where there is uncertainty about the regulator's cost of capital and in each period the regulator faces a new bank that may need to be resolved. In its final form, this will bear a similarity to models of reputation building, a la Kreps and Wilson (1982) and Milgrom and Roberts (1982). Here however, we have two sources of asymmetric information, the type of the regulator and the health of the bank. We also do not have a "behavioral" type player, as any regulator type will play rationally given their preferences. We will see that reputation acquisition can be costly.

[6]Another unannounced stress test was conducted by European regulators in October 2011 so that regulators could decide on the size of a recapitalization package.

Depositors have an ex-ante belief that with probability $1-z$, the regulator has easy access to funds (is type L). With probability z, the regulator is believed to have higher costs for capital. We will analyze the cases where this higher cost regulator is the medium cost regulator (type M) and where it is the high cost regulator (type H) as defined in the previous section. We will assume that the types of the regulator and the types of the bank are independent.[7]

We first generalize the one period model from the previous section to the one period case of uncertainty about regulator capital:

Corollary 2 *In the one period game with unobservable costs (where the ex-ante probability that the regulator is type L is $1-z$):*

1. *If the other regulator type is high cost (H):*
   - (a) *If $\frac{\alpha}{\alpha+z(1-\alpha)} \geq \alpha^*$: There is an equilibrium where the high cost regulators of both types of bank pool with the low cost regulator of the good bank and inject no money. The low cost regulator of the bad bank injects X_I.*
   - (b) *If $\frac{\alpha}{\alpha+z(1-\alpha)} < \alpha^*$: There is a unique equilibrium where both types of regulator of the good bank provide a capital injection of X^*, the high cost regulator of the bad bank liquidates it, and the low cost regulator of the bad bank injects X_I.*
2. *If the other regulator type is medium cost (M):*
   - (a) *If $\frac{\alpha}{\alpha+z(1-\alpha)} \geq \alpha^*$: There is an equilibrium where the medium cost regulators of both types of bank pool with the low cost regulator of the good bank and inject no money. The low cost regulator of the bad bank injects X_I.*
   - (b) *If $\frac{\alpha}{\alpha+z(1-\alpha)} < \alpha^*$: There is a unique equilibrium where both types of regulator of the good bank provide a capital injection of X^{**}, the medium cost and the low cost regulator of the bad bank injects X_I.*

[7] In reality, it may be the case that the type of the regulator and the type of the bank are correlated. The regulator's function outside of times of crisis is supervising and screening banks. If its ability to supervise and screen is related to its funding (or both are explained by institutional framework), then it can be the case that the quality of the banking system is related to its funding.

In the corollary, the higher cost regulator who has a good bank does whatever the low cost regulator with a good bank does. The low cost regulator with a bad bank injects enough money to prevent insolvency. The higher cost regulator with a bad bank will hide its type if priors are favorable, or take its preferred remaining action (inject capital or liquidate) if priors are unfavorable.

Compared to the model with just one type of regulator, the higher cost regulator strictly benefits, as it chooses the same actions, but there is a larger range for which beliefs about the type of bank are favorable, due to the pooling. The low cost regulator has lower surplus from this situation, as when beliefs are unfavorable, it must now inject X^* or X^{**} (as opposed to zero before).

These injections have a flavor of the initial TARP injections, where several banks (e.g. J.P. Morgan) received capital injections when they did not need it. The commonly held view is that by injecting all of the largest financial institutions with capital, the U.S. regulators were trying to hide which banks were bad. Our model provides a slightly different perspective on the capital injections. Here, the injections are not to hide bad banks, but to let the market know which banks will not fail.

As in the previous section, the equilibrium when priors are favorable is not unique, but is the most 'reasonable', as it exists for any off-the-equilibrium-path beliefs. We will treat it as unique, which is important for defining the dynamics of the model.

Now consider a two period model, where there is a new bank each period that needs to be dealt with. Each period consists of two dates, as before. There is a discount factor δ between periods. We assume that the regulator does not know the type of its period 2 bank in period 1 and that this type is independent of the period 1 type. We further assume that the ex-ante probability of having a good bank is α in both periods. As depositors are uncertain about the type of the regulator, the probability that the regulator is a higher cost type may vary between periods, depending on the inference of the depositors at the end of period 1. We will therefore define the depositors' beliefs as z_1 for period 1 and z_2 for period 2. The belief z_1 is of course the ex-ante belief. There is clearly room for building a good reputation here - the smaller the probability that the regulator is a higher cost regulator in period 2 (z_2), the more likely that the higher cost regulator can hide its bad bank from depositors.

By backward induction, the results of the second period are those of the one period model in corollary 2, where the belief that the second bank is good is $\frac{\alpha}{\alpha+z_2(1-\alpha)}$. In the following lemma we will demonstrate that the high cost regulator will not take advantage of this reputation building possibility, while the medium cost regulator may do so.

Proposition 1 *In the two period game with unobservable costs (where the ex-ante probability that the regulator is type L is $1 - z_1$):*

1. *If the other regulator type is high cost (H): the first period equilibrium is the same as in the static game.*
2. *If the other regulator type is medium cost (M):*
   - (a) *If $\frac{\alpha}{\alpha+z_1(1-\alpha)} \geq \alpha^*$ and $S_M(X_I)-S_F+\delta(1-q)(1-\Pr(\alpha \geq \alpha^*))(S_F-(1-C)) > 0$, the first period equilibrium has the M regulator pooling with the L regulator by injecting X_I into the bad bank and nothing into the good bank.*
   - (b) *Otherwise,the first period equilibrium is the same as in the static game.*

The high cost regulator finds that building a reputation is too costly. The way to build a reputation here would be to inject capital, something the high cost regulator wants to avoid at all costs. The medium cost regulator may find it worth it to build reputation. By injecting capital into a bad bank and saving it from insolvency in period 1, it pools with the low cost regulator, which may allow it to hide its bad bank in period 2.

In October 2011, it seemed like most European countries (including Germany) wanted to recapitalize their banks. This was likely because they either had the capital to inject into their banks or perhaps wanted to build their reputation for action. However, France protested against a coordinated action and recapitalizing in general.[8] They may not only have had larger costs of injecting capital for Greek debt losses into banks (there was some discussion of France losing its AAA rating), but they especially did not want to establish this as a precedent going forward because of their banks' exposure to Italy and Spain. In this sense, it seems like France couldn't afford to build it's reputation.

[8] The Economist ("Banks face new European stress tests", October 5, 2011) writes that, "The French government signalled it was uncomfortable with the accelerating talk of recapitalisation, insisting its banks did not need help...any state recapitalisation could threaten France's triple A soverign debt rating".

4 Moral Hazard

Moral hazard is a key risk discussed by policymakers when bailouts are considered.[9] The argument is that saving a bank today may imply that banks in the future will likely be saved, which will encourage those banks to take excess risks. In our model, bad banks are not necessarily saved; some are liquidated. This arises because high cost regulators can't afford to save them. How should a bank act knowing that it will not be saved? Will it reduce risk? In this case, the reputational effect discussed in the previous section would now be turned on its head - a low cost regulator may want to pretend to be a high cost regulator in order to reduce risktaking behavior by bad banks. Even though a low cost regulator may not be able to avoid the commitment problem associated with bailouts, it may use reputation management to mitigate it.

We use a simple model of moral hazard to demonstrate that there will be circumstances under which the low cost regulator would prefer to manage its reputation. We consider a three period model. In period 1, the regulator decides how to deal with a first bank as in the previous model. In period 1.5, the second bank arrives. If it is a bad bank, its equityholders can make an observable but non-verifiable choice[10] about their cash flows (described below). In period 2, the regulator decides how to deal with the second bank, whose cash flows may have been altered.

Suppose that the equityholders of a bad bank can risk shift, increasing expected returns in the good state while reducing expected returns in the bad state.[11] Specifically, they can increase $\bar{R}$ to $\bar{R}'$ while simultaneously reducing $\underline{R}_B$ to $\underline{R}'_B$.[12] We could allow this shift to be mean-preserving (i.e. set $\underline{R}'_B = \underline{R}_B - \frac{q(\bar{R}'-\bar{R})}{1-q}$), but do not place this restriction. We do however make this a discrete choice, as equityholders choose between $(\bar{R}, \underline{R}_B)$ and

[9] Keister (2011) summarizes and adds to the discussion.

[10] It does not need to be observable, as depositors would rationally expect it, but this eliminates the possibility of multiple rational expecations equilibria.

[11] The equityholders of a good bank could potentially use this tactic as well, but as long as it does not impact regulatory decision making (driving the low return for a good bank $\underline{R}_G$ below $\tilde{R}$), it will not affect our results. Of course, this may be less likely to occur at a good bank because of better governance and monitoring in place.

[12] Similar results would arise if the bad bank's equity holders were able to extract cash payouts such as dividends. Dividend payouts from weak financial institutions were rampant in the early part of the recent financial crisis (see Acharya, Gurjal, Kulkarni, and Shin, 2011).

$(\bar{R}', \underline{R}'_B)$. Equityholders are rational, and maximize their risk-neutral payoff. Thus as their downside is limited, the only reason for them not to risk shift would be to avoid changing the behavior of the regulator; they can possibly persuade the regulator to save the bank by decreasing the cost of a bailout. We will also add another incentive for equityholders in the form of a payoff from preventing the bank from failing η, but the results will still hold if this is equal to zero. Of course, equityholders' actions can affect the behavior of depositors. We will describe this below.

Expected payoffs to equityholders and the regulator when there is risk shifting to $(\bar{R}', \underline{R}'_B)$ are summarized in the following table:

Regulator action	Payoff to Equityholders at Bad Bank	Surplus for type i Regulator
Bailout	$V_b = q(\bar{R}' - \tilde{R}) + \eta$	$S_i(X'_I)$
Liquidation	$V_l = 0$	$1 - C$
Forbearance	$V_F = q(\bar{R}' - \tilde{R}) + q\eta$	S_F

where $X'_I = X_I + (\underline{R}_B - \underline{R}'_B)$.

It is worth noting here that by our definition of surplus, the fact that equityholders risk shift alone does not affect surplus (it is just a transfer), except for the fact that it induces the regulator to wastefully pump in more money in a bailout.

The decision of equityholders to shift risk will impact the expectations of depositors. This implies that there is a different cutoff for when depositors decide to run. We denote the cutoff when equityholders risk shift as α'^*, which is defined by:

$$q\tilde{R} + (1-q)(\alpha'^* \tilde{R} + (1-\alpha'^*)\underline{R}'_B) = 1$$

It is obvious from the above that $\alpha'^* > \alpha^*$.

In this section, we are interested in how a low cost regulator may manage its reputation to avoid inducing risk shifting. We will assume that there are only two types of regulators, the low cost regulator (initial probability $1 - z_1$) and the medium cost regulator (initial probability z_1). The key condition for reputation managment by the low cost regulator is:

$$S_M(X'_I) < 1 - C \tag{4}$$

This condition says that diversion of cash flows will cause the medium cost regulator to prefer liquidation to a bailout. This means that the equityholders create a risk for themselves by risk shifting. It is important that the higher cost regulator be medium cost here. A high cost regulator would always liquidate irrespective of equityholders' actions. The medium cost regulator changes its behavior, creating a punishment for equityholders. The low cost regulator thus wants to mimic the medium cost regulator to create the perceived threat of punishment.

For the low cost regulator, we assume that:

$$S_L(X_I') > S_F \tag{5}$$

This implies that the low cost regulator will still bail out a bad bank even if risk shifting occurs.[13] Thus, what is necessary is that there are incentives for the equityholders to risk shift when there is a low cost regulator, but potentially not risk shift when there is a medium cost regulator. That creates the motivation for the low cost regulator to pretend to be the medium cost regulator. Given the definition of low cost and medium cost, we can find parameters which satisfies these conditions.

Proposition 2 *There are parameters for which an equilibrium exists where in the first period, the low cost regulator with a bad bank pools with the medium cost regulator of the bad bank and forbears when* $\frac{\alpha}{\alpha+z_1(1-\alpha)} \geq \alpha^*$*. Furthermore, the equilibrium where in the first period the low cost regulator with a bad bank separates when* $\frac{\alpha}{\alpha+z_1(1-\alpha)} \geq \alpha^*$ *(as in Proposition 1 part 2b) no longer exists under these parameters.*

In the proof, we first demonstrate that there can be a profitable deviation for the low cost regulator from the equilibrium strategy in Proposition 1, part 2b. We then demonstrate conditions under which there is a pooling equilibrium, i.e. the low cost regulator increases depositors' perceptions that it is medium cost.

The popular media has designated the position of the European leadership as trying to "muddle" through. While Germany seems like it could act, it repeatedly mentions legal restraints on itself, the Euro-zone, and the ECB. Could this tough talk be a play to reduce future moral hazard? The New York Times writes, "Mrs. Merkel and other German officials fear that giving

[13] We can, in fact, impose a weaker condition, which would be that the low cost regulator prefers to bail out a bad bank rather than liquidate when risk shifting occurs. We assume the stronger condition for presentation purposes as it reduces the number of cases.

in to the calls for collective bonds or using the European Central Bank as a lender of last resort will ease pressure on the debtor nations, allowing them to avert the drastic structural changes that Berlin says that they need to make to become competitive, while making Germany and other creditors liable for their debts."[14]

5 Stress Tests

During the financial crisis of 2007-09, the United States and the European Union conducted stress tests designed to measure potential bank losses. The results of stress tests in the US were believed to be credible, while those in Europe were not. One explanation is that under TARP, the US had funds available for banks if they were short of capital. This allowed US regulators to provide stress test results that would not trigger bank runs because of concerns over future insolvency. In contrast, Europe was seen as lacking the fiscal unity for regulators to be able to provide equity to banks that would be revealed to have large shortfalls.

In this section we offer an explanation that is supportive of the above view, but is more nuanced. Suppose that we take the basic model and add an initial stage where regulators may commit to doing stress tests. In the initial stage, we will assume the regulator (i) does not know the types of the banks and (ii) does not yet know the realization of α (the probability that a bank is good). A stress test, when performed, is costless and will perfectly reveal the type of the bank to the public. We will interpret this perfect revelation as an effective stress test and the lack of a stress test as either simply that or an ineffective stress test.[15]

Consider two types of regulators, low cost (L) and high cost (H). The low cost regulator has a dominant strategy to do the stress tests. Its per-period payoff from doing the stress test (irregardless of what the type H regulator does) is:

$$E(\alpha)S_G + (1 - E(\alpha))S_L(X_I) \tag{6}$$

[14] Steven Erlanger and Nicholas Kulish, "German Leader Rules Out Rapid Action on the Euro", The New York Times, November 24, 2011.

[15] While stress tests by their nature are inherently noisy, there is not much to be gained in this model by having a stress test that is not on the extreme ends of full or no revelation.

The low cost regulator's per-period payoff from not doing the stress test (when the high cost regulator also does not do the stress test[16]) is:

$$\Pr\left(\frac{\alpha}{\alpha+z(1-\alpha)} \geq \alpha^*\right)\{E(\alpha \mid \frac{\alpha}{\alpha+z(1-\alpha)} \geq \alpha^*)S_G \tag{7}$$

$$+(1-E(\alpha \mid \frac{\alpha}{\alpha+z(1-\alpha)} \geq \alpha^*))S_L(X_I)\} \tag{8}$$

$$+\left(1-\Pr\left(\frac{\alpha}{\alpha+z(1-\alpha)} \geq \alpha^*\right)\right)\{E(\alpha \mid \frac{\alpha}{\alpha+z(1-\alpha)} < \alpha^*)(S_G-(\lambda_L-1)X^*)$$

$$+(1-E(\alpha \mid \frac{\alpha}{\alpha+z(1-\alpha)} < \alpha^*))S_L(X_I)\} \tag{9}$$

The difference between the two equations is that when the L regulator does not do the stress test and priors about the bank are unfavorable, it will need to inject some capital into the good bank to separate from the H regulator with a bad bank. This injection, of course, is costly. By doing the stress test, the L regulator can choose its most preferred action in all states.

Given that the L regulator proceeds with the test, how does the H regulator react? It's per-period payoff from doing the stress test is:

$$E(\alpha)S_G + (1-E(\alpha))(1-C) \tag{10}$$

When the high cost regulator does not do the stress test, the market realizes that it is the high cost regulator, i.e. it is a separating equilibrium. The high cost regulator's per-period payoff from not doing the stress test therefore is:

$$\Pr(\alpha \geq \alpha^*)\{E(\alpha \mid \alpha \geq \alpha^*)S_G + (1-E(\alpha \mid \alpha \geq \alpha^*))S_F\} \tag{11}$$

$$+(1-\Pr(\alpha \geq \alpha^*))\{E(\alpha \mid \alpha < \alpha^*)(S_G-(\lambda_H-1)X^*) + (1-E(\alpha \mid \alpha < \alpha^*))(1-C)\}$$

Subtracting equation 11 from equation 10 we get:

$$\Pr(\alpha \geq \alpha^*)(1-E(\alpha \mid \alpha \geq \alpha^*))\{(1-C)-S_F\} \tag{12}$$

$$+(1-\Pr(\alpha \geq \alpha^*))E(\alpha \mid \alpha < \alpha^*)(\lambda_H-1)X^*$$

[16]If the high cost regulator did the stress test while the low cost regulator did not, it would perfectly identify the types of regulators. It is straightforward to see that it has no incentive to do so.

We can rewrite this expression using the definition of X^*:

$$(S_F-(1-C))\{(1-\Pr(\alpha \geq \alpha^*))E(\alpha \mid \alpha < \alpha^*)-\Pr(\alpha \geq \alpha^*)(1-E(\alpha \mid \alpha \geq \alpha^*))\} \tag{13}$$

This expression may be positive or negative.

The first thing to notice is that a high cost regulator is less likely to enact stress tests than a low cost regulator. The second thing to notice is that the type of the high cost regulator is revealed irrespective of what it does. This is a type of unraveling result as the low cost regulator has a dominant strategy. Nevertheless, there is a tradeoff for the high cost regulator for putting stress tests in place. When priors are favorable, the stress test does not allow the high cost regulator to hide the bad bank, and it is forced to liquidate the bank. When priors are unfavorable, the stress test saves the regulator the injection of X^* to separate the good bank from the bad bank. Therefore the tradeoff depends on the expectations of what the likelihood is that the bank will be good. In this case, better expectations mean a stress test is less likely.

6 Fragility

Uncertainty about a regulator's funding and hence its decisions is likely to lead to volatility in the financial sector. Up until now, we have restricted the role of bank runs by assuming that depositors get the same return in a liquidation as if they ran and not allowing them to run until the regulator makes a decision. In this section, we will relax these assumptions. The threat of runs will affect reputation concerns: the high cost regulator, who previously found it too costly to improve its reputation, now may decide to do so.

We give bank runs more bite by making the following changes. First, depositors are given two chances to run: after the regulator's choice of how to resolve the bank, as before, and *before* the regulator's choice. Second, should the regulator liquidate, the depositors value their 1 unit of money liquidated by the regulator at rate $\beta < 1$. We may think of this as a discount factor - by withdrawing immediately the depositor gets cash, but by waiting for a liquidation, the depositor faces uncertainty over the date when it will have access to its claim.[17] Therefore if the depositor knew for sure that the

[17] We already defined δ as the between-period discount factor. Therefore β would be the

bank was bad and the regulator could not bail it out, she would run before the regulator makes a decision. This creates a window for which the depositor would run if it (i) places a high likelihood that the bank is bad and (ii) thinks that the regulator response will be a liquidation.

Consider first the case of one period and just one regulator who has high costs of funding. If the depositors don't run at the initial stage, the continuation game is given by Lemma 1. The depositors would clearly not run initially if $\alpha \geq \alpha^*$, as there will not be a liquidation. When $\alpha < \alpha^*$, the depositors have a payoff of 1 from running and $\alpha\tilde{R} + (1-\alpha)\beta$ if they don't run. Define α^R as the probability such these two are equal. Therefore, for $\alpha < \alpha^R$, there will be a run. We will assume that:

$$\beta > q\tilde{R} + (1-q)\underline{R}_B \tag{14}$$

This assumption tells us that runs will not always occur if priors are unfavorable. From equations 14 and 1 then, we know that $\alpha^R < \alpha^*$. As β decreases, α^R increases, and a run is more likely.

Now consider the two period model where the regulator types can be H or L. Our previous result (Proposition 3) was that without the initial run stage, the one period equilibrium will be played in each period. We will now allow for a stage at the beginning of each period where the depositors can run.

We will work backwards. If there is no run in the initial stage of period 2, then Corollary 2 describes subsequent equilibrium play. Thus if $\frac{\alpha}{\alpha+z_2(1-\alpha)} \geq \alpha^*$, there would be no initial run. If $\frac{\alpha}{\alpha+z_2(1-\alpha)} < \alpha^*$, there is an initial run when:

$$\alpha\tilde{R} + (1-\alpha)(z_2\beta + (1-z_2)\tilde{R}) < 1$$

or, rewriting

$$\alpha < 1 - \frac{\tilde{R}-1}{z_2(\tilde{R}-\beta)}$$

Define this cutoff as $\alpha^R(z_2)$. Note that this cutoff is increasing in z_2, i.e. when the type of the regulator is more likely to be high cost, there is a higher chance of an initial run. When the cutoff is evaluated at $z_2 = 0$, it is equal to

between-date (or within-period) discount factor.

negative infinity and at $z_2 = 1$ it is positive, implying that there are ranges of z_2 for which there will never be a run.

The other relevant cutoff is our previous condition of unfavorable priors, $\frac{\alpha}{\alpha+z_2(1-\alpha)} < \alpha^*$. We can rewrite this cutoff as:

$$\alpha < \frac{\alpha^* z_2}{1 - \alpha^*(1 - z_2)}$$

This cutoff, which we will denote as $\alpha^C(z_2)$, is also increasing in z_2. There will be an initial run in the second period if $\alpha < \min[\alpha^R(z_2), \alpha^C(z_2)]$. As decreasing z_2 may avoid costly runs, there will be scope for reputation management by the regulator here. In the following proposition, we detail the conditions under which the high cost regulator is willing to deviate from the equilibrium in Proposition 1 in order to improve its reputation.

Proposition 3 *1. If $\alpha \geq \alpha^C(z_1)$ and $S_H(X_I) - S_F + \delta(1-q)\{(\Pr(\alpha < \alpha^C(1)))(S_F - (1-C)) + \Pr(\alpha < \min[\alpha^R(1), \alpha^C(1)])\alpha(S_G - S_F)\} > 0$, then the high cost regulator with a bad bank injects X_I in period 1 and pools with the low cost regulator of a bad bank. Otherwise, the equilibrium in Proposition 1, part 1 holds in period 1.*

2. If $\alpha^R(z_1) \leq \alpha < \alpha^C(z_1)$,there is no reputation building, i.e. the equilibrium in Proposition 1, part 1 holds in period 1.

3. If $\alpha < \min[\alpha^R(z_1), \alpha^C(z_1)]$, there is a run in both periods.

The proposition states that the high cost regulator will attempt to manage its reputation by pooling with the low cost regulator and bailing out a bad bank if priors about the bank are favorable and if C is large. This condition is fairly restrictive as α must be larger than $\alpha^C(z_1)$ but smaller than $\alpha^C(1)$ for it to potentially hold.

7 Conclusion

Bank runs are often tied to uncertainty about the health of the bank in question and the regulator's response to perceived weakness. We model the uncertainty about the regulator to be about its ability to bail out banks. This is particularly relevant in the current sovereign debt crisis, where there has been uncertainty both about where the money for bailouts would be coming from (Germany, the ECB, the IMF, China, and so on) and whether the EU would actually allow money to be used for bailouts. We demonstrate that regulators can take advantage of this uncertainty. The benefits are clear: by hiding their weakness they can prevent runs. Further reputation management could prevent future runs, moral hazard, or excess fragility.

It would be interesting to extend the model to allow for a richer set of instruments available to the regulator such as forcing banks to raise outside equity or merge. Elaborating on the political economy of the regulator's decision process and allowing for correlation between regulator funding and bank quality would also be worth pursuing.

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8 Appendix

8.1 Proof of Lemma 1

Proof. In Part 1, there is a pooling equilibrium where both types get no capital injections and are not liquidated. This exists for any beliefs off the equilibrium path. For all other potential equilibria, we will consider off the equilibrium path beliefs where the probability that a bank is good is the ex-ante belief.[18] Consider other pooling equilibria with capital injections of size X. Each type of regulator would deviate to a zero capital injection. Any possible separating equilibrium where the bad bank has a positive capital injection has a profitable deviation for the regulator to a zero capital injection. Any possible separating equilibrium with the regulator of the bad bank giving a zero capital injection would provoke a run, meaning that the regulator would deviate to a positive capital injection. Therefore there are no separating equilibria.

Define X^* such that $S_F-(\lambda_H-1)X^* = 1-C$. Given that $1-C > S_H(X_I)$, the injection X^* is smaller than X_I, i.e. it is not large enough to prevent insolvency.

For Part 2b, in any potential pooling equilibrium with positive capital injections, depositors believe the bank is bad and the regulator of the bad bank would deviate to zero capital injection. In a potential pooling equilibrium with zero capital injections, the regulator of the bad bank would not deviate, which using the intuitive criteria places zero weight on the bad bank having a capital injection and implies a deviation of the regulator of the good bank to a cash injection of ε. Therefore there are no pooling equilibria. The separating equilibrium where the good bank gets a capital injection of X^* and the bad bank is liquidated has no profitable deviations. ∎

8.2 Proof of Proposition 1

Proof. Consider part 1. From the first period perspective, the high cost regulator with a good bank would not deviate from the static game strategy, as she is pooling with the low cost regulator and can't improve her reputation. The low cost regulator with a good bank would not deviate, as she is doing the minimum possible to be recognized as having a good bank (zero injection if $\frac{\alpha}{\alpha+z_1(1-\alpha)} > \alpha^*$ and injection of X^* if $\frac{\alpha}{\alpha+z_1(1-\alpha)} < \alpha^*$). The low cost regulator with a bad bank has a dominant strategy to inject X_I and is recognized as low cost for doing so, meaning that there is no deviation that could improve its reputation.

[18] Note that if we consider beliefs off the equilibrium path that the bank is bad ($\alpha < \alpha^*$), there are other equilibria. The pooling equilibrium that we found still will exist (it exists for any beliefs) and is the most realistic of the equilibria.

The high cost regulator with a bad bank is the only type who might deviate. If $\frac{\alpha}{\alpha+z_1(1-\alpha)} > \alpha^*$ and the bad bank had no injection and was not liquidated (the static strategy), the bank would go insolvent with probability $1-q$. If it goes insolvent, the depositors realize that the regulator has high costs with probability $z_2 = 1$. Otherwise, $z_2 = z_1(\frac{\alpha+(1-\alpha)q}{\alpha+z_1(1-\alpha)q})$, which is clearly greater than z_1. We will define $\hat{z}_2 \equiv z_1(\frac{\alpha+(1-\alpha)q}{\alpha+z_1(1-\alpha)q})$. Therefore its payoff from using the static strategy in period 1 is:

$$\begin{aligned} S_F + \delta\{[q(\Pr(\frac{\alpha}{\alpha+\hat{z}_2(1-\alpha)} &> \alpha^*) + (1-q)(\Pr(\alpha \geq \alpha^*)](\alpha S_G + (1-\alpha)S_F) \\ +[q(1-\Pr(\frac{\alpha}{\alpha+\hat{z}_2(1-\alpha)} &> \alpha^*)) + (1-q)(1-\Pr(\alpha \geq \alpha^*))](\alpha(S_G-(\lambda-1)X^*) \\ &+(1-\alpha)(1-C))\} \end{aligned} \tag{15}$$

where δ is the discount factor, and $\Pr(\alpha \geq \alpha^*)$ is the probability that the bank in period 2 is good ($\frac{\alpha}{\alpha+z_1(1-\alpha)} > \alpha^*$ does not imply that $\alpha \geq \alpha^*$ or $\frac{\alpha}{\alpha+\hat{z}_2(1-\alpha)} > \alpha^*$). Note that we assume that if the second period prior is above α^*, the equilibrium described in Corollary 2 part 1a is selected[19]. This amounts to assuming a coordination device or that beliefs off the equilibrium path are always the same as the prior. As we noted in the corollary, the equilibrium described is particularly robust as it exists for all beliefs off the equilibrium path.

The only deviation of interest to the regulator would be to imitate the low cost regulator with a bad bank and inject enough capital to prevent insolvency. In this case, her payoff would be:

$$S_H(X_I) + \delta(\alpha S_G + (1-\alpha)S_F) \tag{16}$$

It is straightforward to prove equation 15 is larger than equation 16. First, subtracting equation 16 from equation 15 and simplifying yields:

$$S_F - S_H(X_I) + \delta[1-q(\Pr(\frac{\alpha}{\alpha+\hat{z}_2(1-\alpha)} > \alpha^*) - (1-q)(\Pr(\alpha \geq \alpha^*)]\{(1-C)-S_F\} \tag{17}$$

where we use the fact that $S_F - (\lambda-1)X^* = 1-C$.

One can see that the second period term, all multiplied by δ, is negative by assumption A3. Therefore to make it as unlikely as possible that the

[19] When we discuss the case below with a type M firm, we will naturally assume that the equilibrium in Corollary 2 part 2a is selected for the second period.

expression is positive, set $\delta = 1$. At $\delta = 1$, the expression can be simplified to:

$$\begin{aligned}(q(\Pr(\frac{\alpha}{\alpha+\hat{z}_2(1-\alpha)} &> \alpha^*) + (1-q)(\Pr(\alpha \geq \alpha^*))(S_F - S_H(X_I)) + \\ (1-q(\Pr(\frac{\alpha}{\alpha+\hat{z}_2(1-\alpha)} &> \alpha^*) - (1-q)(\Pr(\alpha \geq \alpha^*))((1-C) - S_H(X_I))\end{aligned} \tag{18}$$

which is positive by the definition of $S_H(X_I)$.

In contrast, if $\frac{\alpha}{\alpha+z_1(1-\alpha)} < \alpha^*$, the high cost regulator with the bad bank is identified perfectly by depositors using the static strategy. Her two period payoff is therefore:

$$(1-C) + \delta(\alpha(S_G - (\lambda-1)X^*) + (1-\alpha)(1-C))$$

since $\frac{\alpha}{\alpha+z_1(1-\alpha)} < \alpha^*$ implies $\alpha < \alpha^*$. The capital injection X^* is still enough to prevent her from deviating as the second period payoff would be the same since $\Pr(\frac{\alpha}{\alpha+\hat{z}_2(1-\alpha)} < \alpha^*) = 1$. Similarly, a deviation to X_I would not be profitable. Thus the static strategy is still an equilibrium strategy here as well.

Consider part 2. Using the same logic from part 1, if $\frac{\alpha}{\alpha+z_1(1-\alpha)} \geq \alpha^*$, there may be a profitable deviation for the medium cost regulator with the bad bank. This can be seen from equation 18, replacing $S_H(X_I)$ with $S_M(X_I)$. If equation 18, after replacing $S_H(X_I)$ with $S_M(X_I)$, is negative, there is a range of δ between some cutoff $\tilde{\delta}$ and 1 for which the static game equilibrium does not hold.

Now posit that there is an equilibrium in the first period when $\frac{\alpha}{\alpha+z_1(1-\alpha)} \geq \alpha^*$ where the medium cost regulator injects X_I into the bad bank and nothing into the good bank and the low cost regulator does the same. The payoff for the medium cost regulator with the bad bank is:

$$S_M(X_I) + \delta(\alpha S_G + (1-\alpha)S_F) \tag{19}$$

Since it is given that $\frac{\alpha}{\alpha+z_1(1-\alpha)} \geq \alpha^*$ and z_2 will equal z_1.

The best deviation would be if the medium cost regulator did nothing. It would then have a larger current payoff (S_F). The future payoff would be equally good if there was no default. If there was a default, depositors would realize that the regulator is medium cost with probability 1 (using the intuitive criterion, the low cost regulator would never have deviated to a zero injection). The payoff therefore is:

$$\begin{aligned} S_F + \delta\{[q + (1-q)(\Pr(\alpha \quad \geq \quad \alpha^*)](\alpha S_G + (1-\alpha)S_F) \\ +[(1-q)(1-\Pr(\alpha \quad \geq \quad \alpha^*))](\alpha(S_G - (\lambda-1)X^*) + (1-\alpha)(1-C))\} \end{aligned} \tag{20}$$

Subtracting equation 20 from equation 19 and rewriting, we get:

$$S_M(X_I) - S_F + \delta(1-q)(1-\Pr(\alpha \geq \alpha^*))(S_F - (1-C)) \tag{21}$$

where we use the fact that $S_F - (\lambda-1)X^* = 1 - C$. We can see that when $\delta = 1$, this can be positive, meaning this can be an equilibrium for δ sufficiently high. Compare this condition to the condition for there being a profitable deviation from the static equilibrium in the first period:

$$S_M(X_I) - S_F + \delta[1 - q(\Pr(\frac{\alpha}{\alpha + \hat{z}_2(1-\alpha)} > \alpha^*) - (1-q)(\Pr(\alpha \geq \alpha^*)](S_F - (1-C)) \tag{22}$$

It is clear that equation 21 is smaller. Therefore, if it is positive, the equilibrium where the medium cost regulator with the bad bank pools exists and the static equilibrium does not.

If $\frac{\alpha}{\alpha + z_1(1-\alpha)} < \alpha^*$, the medium cost regulator with the bad bank has a static strategy of preventing insolvency. This pools her with the low cost regulator of the bad bank, and maintains depositors' ex-ante beliefs ($z_2 = z_1$). A deviation to imitating a regulator with a good bank would not improve depositors' beliefs and therefore not be profitable. ■

8.3 Proof of Proposition 2

Proof. Period 2 (Resolving the second bank): We fix the probability of the regulator being medium cost after the events of period 1 as z_2. When the equityholders don't risk shift, both types of regulators behave as in Corollary 2. When the equityholders do risk shift, the regulators' behavior is the same as the first part of Corollary 2 if $\frac{\alpha}{\alpha + z_2(1-\alpha)} > \alpha'^*$. If $\frac{\alpha}{\alpha + z_2(1-\alpha)} < \alpha'^*$, the equilibrium strategies change. Define X_M^* such that $S_F - (\lambda_M - 1)X_M^* = 1 - C$. There is a unique equilibrium where both types of regulator of the good bank provide a capital injection of X_M^*, the medium cost regulator liquidates the bad bank, and the low cost regulator of the bad bank injects X_I'. This change in strategies is a direct result of equation 4.

Period 1.5 (Equityholder behavior at the bad bank): Suppose that $\frac{\alpha}{\alpha+z_2(1-\alpha)} > \alpha'^*$. In this case, there are two possible choices for equityholders in the bad bank, to do nothing or risk shift. We define the payoffs below:

$$\begin{aligned} \text{no change} &: (1-z_2)(q(\bar{R}-\tilde{R})+\eta)+z_2(q(\bar{R}-\tilde{R})+q\eta) \\ \text{risk shift} &: (1-z_2)(q(\bar{R}'-\tilde{R})+\eta)+z_2(q(\bar{R}'-\tilde{R})+\eta) \end{aligned} \tag{23}$$

Choosing risk shifting clearly dominates.

Now suppose that $\frac{\alpha}{\alpha+z_2(1-\alpha)} \in [\alpha^*, \alpha'^*]$. The payoffs are:

$$\begin{aligned} \text{no change} &: (1-z_2)(q(\bar{R}-\tilde{R})+\eta)+z_2(q(\bar{R}-\tilde{R})+q\eta) \\ \text{risk shift} &: (1-z_2)(q(\bar{R}'-\tilde{R})+\eta) \end{aligned} \tag{24}$$

The equityholders would prefer no change if $z_2 > \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}+\eta}$ and risk shift otherwise. This makes sense as equityholders are only willing to risk shift if there is a high probability that the regulator is low cost.

Lastly, suppose that $\frac{\alpha}{\alpha+z_2(1-\alpha)} < \alpha^*$. The payoffs are:

$$\begin{aligned} \text{no change} &: (1-z_2)(q(\bar{R}-\tilde{R})+\eta)+z_2(q(\bar{R}-\tilde{R})+\eta) \\ \text{risk shift} &: (1-z_2)(q(\bar{R}'-\tilde{R})+\eta) \end{aligned} \tag{25}$$

The equityholders would prefer no change if $z_2 > \frac{q(\bar{R}'-\bar{R})}{q(\bar{R}'-\tilde{R})+\eta}$ and risk shift otherwise. Compared to the previous case, it is less likely for the equityholder to risk shift, as the medium cost regulator now bails out bad banks with low returns of $\underline{R}_B$ (rather than forbearing).

Period 1(Resolving the first bank): We now can calculate expected continuation benefits to each type of regulator conditional on the perception z_2. Define the following probabilities:

$$\begin{aligned} p_1(z_2) &= \Pr(\frac{\alpha}{\alpha+z_2(1-\alpha)} > \alpha'^*) \\ p_2(z_2) &= \Pr(\frac{\alpha}{\alpha+z_2(1-\alpha)} \in [\alpha^*, \alpha'^*]) \end{aligned}$$

And the following indicator functions:

$$\begin{aligned} q_1(z_2) &= I_{z_2 > \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}+\eta}} \\ q_2(z_2) &= I_{z_2 > \frac{q(\bar{R}'-\bar{R})}{q(\bar{R}'-\tilde{R})+\eta}} \end{aligned}$$

Obviously, $q_1(z_2) = 1$ implies $q_2(z_2) = 1$ (but not the converse).

The expected continuation surplus for a low cost regulator for a given reputation of z_2 is thus:

$$\begin{aligned} ES_L(z_2) &= p_1(z_2)(\alpha S_G + (1-\alpha)S_L(X_I')) + p_2(z_2)\{q_1(z_2)(\alpha S_G + (1-\alpha)S_L(X_I)) \\ &\quad +(1-q_1(z_2))(\alpha(S_G - (\lambda_L - 1)X_M^*) + (1-\alpha)S_L(X_I'))\} \\ &\quad +(1-p_1(z_2)-p_2(z_2))\{q_2(z_2)(\alpha(S_G - (\lambda_L - 1)X^{**}) + (1-\alpha)S_L(X_I)) \\ &\quad +(1-q_2(z_2))(\alpha(S_G - (\lambda_L - 1)X_M^*) + (1-\alpha)S_L(X_I'))\} \end{aligned}$$

We need to look at the low cost regulator with the bad bank in period 1 when $\frac{\alpha}{\alpha+z_1(1-\alpha)} \geq \alpha^*$ (all other low cost regulator situations are pooling).Will it deviate from injecting X_I and bailing out the bad bank to forbearing on the bad bank? This will occur if the following holds:[20]

$$S_L(X_I) + \delta ES_L(0) < S_F + \delta ES_L(\tilde{z}) \tag{26}$$

Where $\tilde{z} = \frac{z_1}{z_1+(1-z_1)\alpha}$. For simplicity, assume that $z_1 > \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}+\eta}$. Then

$$\begin{aligned} ES_L(0) - ES_L(\tilde{z}) &= (1-p_1(\tilde{z}))(\alpha S_G + (1-\alpha)S_L(X_I')) - p_2(\tilde{z})(\alpha S_G + (1-\alpha)S_L(X_I)) \\ &\quad -(1-p_1(\tilde{z})-p_2(\tilde{z}))(\alpha(S_G - (\lambda_L - 1)X^{**}) + (1-\alpha)S_L(X_I)) \\ &= (1-p_1(\tilde{z})-p_2(\tilde{z}))(\lambda_L - 1)\alpha X^{**} - (1-p_1(\tilde{z}))(\lambda_L - 1)(1-\alpha)(\underline{R}_B - \underline{R}_B') \end{aligned} \tag{27}$$

given $p_1(0) = 1$ and $p_2(0) = 0$, which are true for interior α^* and α'^*.

This expression is guaranteed to be negative when α is small, when X^{**} is small (which will occur when $(1-q)C$ is close to $(\lambda_M - 1)(\tilde{R} - \underline{R}_B)$), $\underline{R}_B - \underline{R}_B'$ is large, or $\Pr(\frac{\alpha}{\alpha+z_2(1-\alpha)} < \alpha^*(0))$ is small. This only demonstrates that equation 26 can possibly hold. To prove there are parameters for which equation 26 does hold, note that we can choose $(1-q)C$ is close to $(\lambda_L - 1)(\tilde{R} - \underline{R}_B)$ such that $S_L(X_I) - S_F$ is small enough while setting δ close to 1 and α small (to guarantee $\delta(ES_L(0) - ES_L(\tilde{z}))$ is negative).

Given that there is a profitable deviation, we now ask if there is an equilibrium where both types of regulator with both types of bank pool by doing nothing (forbearing) when $\alpha \geq \alpha^*$ in the first period. There are two things we need to check. First, we will check if the low cost regulator with a bad

[20] Note that this does not tell us what the equilibrium is, just that the previous equilibrium does not exist.

bank will not deviate. Second, we will check that the medium cost regulator with a bad bank will not deviate.

We begin by seeing whether the low cost regulator with a bad bank would prefer to deviate to bailing out the bank. We use beliefs off the equilibrium path that the regulator is low cost with probability one. The condition for it *not* wanting to deviate is:

$$S_L(X_I) + \delta ES_L(0) < S_F + \delta ES_L(z_1) \tag{28}$$

This condition is quite similar to condition 26, the difference being the beliefs of depositors after observing pooling at forbearance. Depositors now retain their ex-ante beliefs z_1 (instead of $\tilde{z}$). Since we are looking at the situation where in the first period $\alpha \geq \alpha^*$, this implies in the second period $\frac{\alpha}{\alpha + z_1(1-\alpha)} \geq \alpha^*$. That means that $1 - p_1(z_1) - p_2(z_1) = 0$. We can adapt condition 27 for this case to be:

$$ES_L(0) - ES_L(z_1) = -p_2(z_1)(\lambda_L - 1)(1-\alpha)(\underline{R}_B - \underline{R}'_B)$$

which is negative as long as $p_2(z_1) > 0$. Furthermore, similar conditions to those listed above ($S_L(X_I) - S_F$ small, setting δ close to 1 and $p_2(z_1) > 0$) will guarantee that condition 28 will hold.

Now let us ask if it is possible that the medium cost regulator with a bad bank when $\alpha \geq \alpha^*$ in period 1 would not deviate from a situation where it pools at forbearing with the low cost regulator. We again use beliefs off the equilibrium path that the regulator is low cost with probability one. As forbearing is the best first period choice for an M regulator with a bad bank, any deviation gives less utility. The M regulator's best deviation is then to $S_M(X_I)$. The M regulator with the bad bank will not deviate if:

$$S_M(X_I) + \delta ES_M(0) < S_F + \delta ES_M(z_1) \tag{29}$$

where $ES_M(z_2)$ is defined by:

$$\begin{aligned} ES_M(z_2) \;=\; & p_1(z_2)(\alpha S_G + (1-\alpha)S_F) + p_2(z_2)\{q_1(z_2)(\alpha S_G + (1-\alpha)S_F) \\ & +(1-q_1(z_2))(\alpha(S_G - (\lambda_M - 1)X_M^*) + (1-\alpha)(1-C))\} \\ & +(1-p_1(z_2)-p_2(z_2))\{q_2(z_2)(\alpha(S_G - (\lambda_M - 1)X^{**}) + (1-\alpha)S_M(X_I)) \\ & +(1-q_2(z_2))(\alpha(S_G - (\lambda_M - 1)X_M^*) + (1-\alpha)(1-C))\} \end{aligned}$$

Once again we assume that $z_1 > \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R} + \eta}$. Note that since $\alpha \geq \alpha^*$, it must be the case that $\frac{\alpha}{\alpha + z_1(1-\alpha)} > \alpha^*$, which implies that $1 - p_1(z_1) - p_2(z_1) = 0$.

We can then write:

$$\begin{aligned} ES_M(0) - ES_M(z_1) &= (1 - p_1(z_1))(\alpha S_G + (1-\alpha)S_F) \\ &\quad - p_2(z_1)(\alpha S_G + (1-\alpha)S_F) \\ &= 0 \end{aligned}$$

Equation 29 is therefore satisfied. ■

8.4 Proof of Proposition 1

Proof. Consider part 1. To reduce notation, define $\varpi_1(z) \equiv \Pr(\alpha < \min[\alpha^R(z), \alpha^C(z)])$ and $\varpi_2(z) \equiv \Pr(\alpha^R(z) \leq \alpha < \alpha^C(z))$. We will check whether there is a beneficial deviation for the high cost regulator with a bad bank from the static strategy in Proposition 3, part 1. The payoff from the static strategy is:

$$\begin{aligned} &S_F + \delta\{\varpi_1(\hat{z}_2)(1-C) + \varpi_2(\hat{z}_2)(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C)) \\ &+(1 - \varpi_1(\hat{z}_2) - \varpi_2(\hat{z}_2))(\alpha S_G + (1-\alpha)S_F)\} \end{aligned} \tag{30}$$

If the regulator were to deviate, it would imitate the low cost regulator with the bad bank and inject capital X_I. This would give the depositors the belief that it was the low cost regulator for sure, i.e. $z_2 = 0$. Its payoff would be:

$$S_H(X_I) + \delta(\alpha S_G + (1-\alpha)S_F) \tag{31}$$

Then subtracting equation 31 from 30 and rewriting (using the definition of X^*) gives us:

$$S_F - S_H(X_I) + \delta\{(\varpi_1(\hat{z}_2) + \varpi_2(\hat{z}_2))(1 - C - S_F) - \varpi_1(\hat{z}_2)\alpha(S_G - S_F)\} \tag{32}$$

The current payoff $S_F - S_H(X_I)$ to maintaining the equilibrium strategy is positive, while the future payoff (the part in curly brackets) is negative. As in Proposition 3, we will set $\delta = 1$ to make the expression as negative as possible. Rewriting:

$$(1-\varpi_1(\hat{z}_2)-\varpi_2(\hat{z}_2))(S_F - S_H(X_I)) + (\varpi_1(\hat{z}_2)+\varpi_2(\hat{z}_2))(1-C-S_H(X_I)) - \varpi_1(\hat{z}_2)\alpha(S_G - S_F))$$

Clearly, if ϖ_1, α, or $S_G - S_F$ are small, there are no profitable deviations.

Now let's check if having the high cost regulator with the bad bank inject X_I can be an equilibrium. Since $z_2 = z_1$ in this case and we know $\frac{\alpha}{\alpha + z_1(1-\alpha)} >$

α^*, the equilibrium payoff is the same as the payoff in equation 31. The best deviation would be if the regulator did nothing. It would then have a larger current payoff (S_F). If there was no default, the future payoff would be the same as if it had injected X_I, since $z_2 = z_1$ and $\frac{\alpha}{\alpha + z_1(1-\alpha)} > \alpha^*$. If there was a default, depositors would realize that the regulator is high cost with probability 1 (using the intuitive criterion, the low cost regulator would never have deviated to a zero injection). The payoff therefore is:

$$\begin{aligned} &S_F + \delta\{(1-q)[\varpi_1(1)(1-C) + \varpi_2(1)(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))] \\ &+[q + (1-q)(1 - \varpi_1(1) - \varpi_2(1))](\alpha S_G + (1-\alpha)S_F)\} \end{aligned} \tag{33}$$

Subtracting equation 33 from equation 31 and rearranging:

$$S_H(X_I) - S_F + \delta(1-q)\{(\varpi_1(1) + \varpi_2(1))(S_F - (1-C)) + \varpi_1(1)\alpha(S_G - S_F)\}$$

This condition is tighter than the one for a deviation (equation 32), i.e. if this is positive, the equilibrium will have the medium cost regulator will pool and inject X_I, and the static equilibrium will no longer exist.

Consider part 2. The high cost regulator with the bad bank is identified perfectly by depositors using the equilibrium strategy. Her two period payoff is:

$$(1-C) + \delta(\alpha(S_G - (\lambda - 1)X^*) + (1-\alpha)(1-C))$$

since $\alpha < \alpha^C(z_1)$ implies $\alpha < \alpha^*$. The capital injection X^* is still enough to prevent her from deviating as the second period payoff would be the same since $z_2 = z_1$ and $\alpha < \alpha^C(z_1)$. Similarly, a deviation to X_I would not be profitable. Thus the static strategy is still an equilibrium strategy here as well.

For part 3, if there is a run in the first period, priors don't change, so $z_2 = z_1$ and there is a run in the second period. ■