

# Collaborative Innovation and the Supply of Informed Capital\*

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## Abstract

I model an economy where heterogeneous firms collaborate in an attempt to produce a valuable innovation. The model assumes that it is efficient for firms to match up assortatively—i.e., high-quality firms should associate with one another—but a costly certification process is required for assortative matching to obtain. In this setting I show that there is a symbiotic interaction between the real and financial sides of the economy. On one hand, assortative matching allows financial intermediaries such as venture capitalists to learn about good investment opportunities through their current ventures. On the other hand, a critical supply of informed capital is sometimes a necessary condition to obtain large-scale assortative matching in the first place. In the main equilibrium of interest, informed and uninformed capital is available in the economy, and high types mostly finance using relatively inexpensive informed capital; inexpensive financing incentivizes high-quality firms to incur the costly certification process required in order to match with another high type. Two main factors determine whether a critical supply of informed capital is important for generating large-scale assortative matching: firms' internal funds need to be neither too low nor too high; firms with non-scarce technology need to be the ones bearing certification costs.

## PRELIMINARY

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# 1 Introduction

After the late 1970's a significant portion of innovative activity stems from collaborative endeavors encompassing several distinct organizations (Hagedoorn, 2002), as illustrated by the functioning of the biotechnology sector. For example, all ten top-selling biotechnology drugs in 2001 were developed by specialized biotech firms; but only five were marketed by biotech companies, and only four were marketed by the originating firm (Powell et al., 2005).<sup>1</sup> If innovation is collaborative, it is reasonable to conjecture that getting the “right” organizations to match/associate plays an important role in obtaining success.<sup>2</sup> However, efficient matching may be challenging in fast-paced sectors that are naturally plagued by high uncertainty and informational asymmetries. This paper claims that a minimum supply of informed capital, for example the number of venture capitalists catering to a particular technology cluster, is instrumental in enabling efficient matches on a large scale, where many high-quality organizations associate with one another. The mechanism proposed does not require venture-capital firms to possess very deep industry (technical) or managerial expertise, not even do they need to be extensively well-informed about the quality of many potential investments. All the mechanism requires is that venture-capital firms have a critical mass and are able to learn about the quality of their extant ventures.

The model I propose builds on a labor economics paper on job search and social connections (Montgomery, 1991). In this paper, employees exogenously display (positive) *assortative matching*, i.e. the tendency for high types to associate with other high types. As

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<sup>1</sup>Another example of collaborative innovation is the development of an animal model for Alzheimer's disease in 1995 (published in *Nature*). This discovery was the product of joint work between two new biotech companies, a large pharmaceutical, a university, a federal research laboratory, and a nonprofit research institute (Powell, Koput, and Smith-Doerr, 1996).

<sup>2</sup>Some papers on technological clusters and venture capital focus on matching (Inderst and Müller, 2004; Sorensen, 2007), but their focus is on the matching between venture capitalists and entrepreneurial companies.

employers learn about which current employees are high-type, they are able to make informed above-market wage offers to their employees' social connections. In my model, the social connection is replaced by a business association between two organizations that wish to conduct collaborative innovation. These organizations also differ with respect to type as the employees in Montgomery (1991), and under certain conditions financiers will be able to infer the quality of their current ventures' business associations. In such an informative equilibrium, financiers pursue good investment opportunities and high-type firms fund their operations at a cheaper cost of financing. The main difference with respect to Montgomery (1991) is that I do not take assortative matching as exogenous. In the model all firms internalize the advantages of being associated with high types and sometimes assortative matching will not obtain in equilibrium.

The model assumes at the outset that generalized assortative matching is desirable. By generalized assortative matching I mean that there is a large number of heterogeneous organizations operating in the economy; and that high (low) types tend to match with other high (low) types.<sup>3</sup> In order for high-quality firms to find a high-quality partner, they need to incur an exogenous cost to participate in a certified matching market. If firms—and/or entrepreneurs—are financially constrained, then having to incur this cost limits their ability to offer a high price to prospective partners. Since low-type organizations do not have an incentive to participate in the certified matching market in the first place, this has the effect of relaxing their financial constraints. The relaxed financial constraints allow low-type firms to make large transfers to their own partners, who are picked from a residual matching market at random. Given high prices in the residual matching market, high-quality firms may not

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<sup>3</sup>A similar assumption is made in Rhodes-Kropf and Robinson (2008) in a context of mergers. Many other papers on search and matching start with the assumption that match synergies exist; for a review of this literature see Smith (2011). For a discussion of assortative matching and efficiency, see Durlauf and Seshadri (2003).

receive comparatively good enough offers from prospective high-quality partners; in which case generalized assortative matching does not occur. It turns out that the financial sector plays an important role in offsetting the negative impact of individually-borne certification costs.

In the model there are two types of financiers. One group corresponds to *informed capital*, which stands for investors like venture-capital firms. The second group is a competitive uninformed financial market, which operates as long as adverse selection is not too severe. In an equilibrium with assortative matching, low-type firms are mostly financed by the uninformed market, and face higher costs of financing than high-quality firms. Moreover, *as the supply of informed capital expands*, the wedge between costs of financing across types increases. A big enough finance-induced wealth differential between high and low types incentivizes high types to engage in the costly process that leads to efficient generalized assortative matching. The supply of informed capital operates through two channels. First, a higher number of venture capitalists makes it more likely that high types find informed financing, which is cheaper. Second, with a higher number of venture capitalists in the economy, the proportion of high types facing the uninformed financial market decreases. The higher adverse selection generates higher (uninformed) financing costs for low types.

For a minimum supply of informed capital to be instrumental in generating generalized assortative matching, two conditions regarding internal funds need to be met. On one hand internal funds cannot be too high. This is intuitive, since financially unconstrained agents much more easily implement the efficient outcome in equilibrium. On the other hand, internal funds cannot be too low either. This condition is more subtle and directly related to the mechanism behind the main result. With too little internal funds, high-quality firms participating in the certified matching market tend to overbid for potential partners; since

with low internal funds these high bids come at the expense of outside financiers, especially the uninformed financial market. Investors anticipate this behavior and thus uninformed finance is no longer available. Without uninformed finance, the wedge in costs of financing across types ceases to exist and the proposed mechanism no longer works. In addition to internal funds lying in an intermediate region, another necessary condition for the proposed mechanism to work is that the firms incurring the certified-matching-market participation cost are not scarce. The intuition for this result is that scarce firms naturally appropriate enough of the surplus such that they can (individually) afford certified-matching costs.

This paper is related to several strands of literature. First, the topic of innovation and venture-capital financing is connected to the literature on industrial and technological clusters. Dating back at least to Marshall (1920), economists have noted that informational spillovers may be important determinants of co-location. These determinants are thought to be especially relevant for R&D-intensive industries (Audretsch and Feldman, 1996) such as computers and biotech (Swann and Prevezer, 1996; Powell et al., 2002), which are the settings of interest for my paper. In particular, co-location can be thought of as a way to reduce the certification costs in my model, which for simplicity I take as given.

Venture-capital financing is a salient aspect of the functioning of technological clusters;<sup>4</sup> and venture-capital firms frequently operate in close proximity to their ventures (Zook, 2002). The location choice of venture-capital firms presumably enhances their ability to collect information, especially in settings such as technological clusters, where social networks are dense (Saxenian, 1994; Castilla et al., 2000) and where much information on investment opportunities flows locally via interpersonal contacts (Sorenson and Stuart, 2001), including referrals from previously financed entrepreneurs (Fried and Hisrich, 1994). Collecting infor-

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<sup>4</sup>For a recent review of venture-capital literature focusing on economics and finance, see Da Rin, Hellman, and Puri (2011).

mation seems of paramount importance for venture-capital firms, especially in light of some value-enhancing activities these financiers are believed to undertake, namely screening and monitoring.<sup>5</sup> My model provides a rationale for the pervasiveness of venture-capital firms in technological clusters that is admittedly more indirect than screening and monitoring; but perhaps less stringent than traditional arguments in terms of what is required, ability-wise, of venture-capital firms.<sup>6</sup> Furthermore, my argument does not rely on the advantages of information flowing through venture-capital networks. As pointed out by Hochberg, Ljungqvist, and Lu (2010), these networks could actually be used to limit economic efficiency; which would be an argument, at the margin, for not observing successful technological clusters systematically relying on venture-capital funding. My model also provides an argument for industry specialization, since it is basically by staying within the same sector that venture-capital firms capitalize on the private information that is being gathered as uncertainty about extant ventures resolves. This channel would also contribute to the clustering of venture-capital firms.

There is much empirical literature studying the role of venture capital in promoting innovation and higher productivity. Kortum and Lerner (2000) and Hirukawa and Ueda (2008a) show that venture-capital financing correlates with contemporaneous patent production. Hirukawa and Ueda (2008b) and Chemmanur, Krishnan, and Nandy (2011) find that backing by venture-capital firms predicts growth in total factor productivity. These findings are consistent with the predictions from the model.

The mechanism I propose is also consistent with the documented persistence in venture-capital returns (Kaplan and Schoar, 2005): learning that an initial venture is a high type

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<sup>5</sup>There is also evidence that venture capitalists provide value-added services in terms of management (Lerner, 1995; Hellman and Puri, 2002).

<sup>6</sup>I note that screening and monitoring are probably complementary to the explanation I suggest.

is simultaneously associated with a high contemporaneous return and the access to good subsequent investment opportunities (the current venture’s partners).

Finally, my model is also related to the general topic of finance and economic growth. Levine (2005) provides an overview of this literature, a subfield of which focuses on the role of financial intermediaries in alleviating informational frictions. For example, De la Fuente and Marín (1996) develop a model where intermediaries monitor innovative activities and improve credit allocation; Lee (1996) models a process of financial development that is driven by information accumulation; Keuschnigg (2004) proposes an endogenous growth model with venture capital. Notwithstanding my paper being related to this strand of the macroeconomics’ literature, the mechanism I propose—where a critical mass of financial intermediaries enables efficient matching—is novel.

The remainder of the paper is organized as follows. Section 2 contains the setup and section 3 the main results. Section 4 presents additional analyses and extensions. Section 5 concludes. All proofs are presented in the appendix.

## 2 Model

### 2.1 Description of the economy

The economy comprises firms with two technology types, that need to pair up and collaborate sequentially in order to create a successful innovation. I will use the terms “upstream sector” and “downstream sector” as shorthand for the two technologies. There is an exogenous risk-free rate of zero and all agents are risk-neutral.

The upstream sector comprises a large even number of firms  $F$ , each potentially pro-

ducing one unit of the “primary good”, which represents the first stage of the collaborative innovation process. There are two types of upstream firms, characterized by  $\alpha_i = 0$  (low type) or  $\alpha_i = 1$  (high type); the proportion of high types is normalized to  $1/2$ . Production is riskless and costs one unit upfront, and firms have internal funds  $\delta$ ; this includes an entrepreneur’s own funds for startups. I require  $\delta < 1$ , i.e. firms are financially constrained.

There are  $F$  downstream firms attempting to deliver one unit of the “final good”, each using exactly one unit of the input produced upstream. The input needs to be tailored to the specific production needs of the downstream firm, and so an upstream-downstream partnership needs to be in place before upstream production takes place. There are also two types of downstream firms; the number of high types is  $(F/2 + 1)/F$ , so approximately  $1/2$  for large  $F$ , but it is always true that upstream high types are scarce. Production of the final good—i.e., successfully completing collaborative innovation—is risky and yields revenues  $R$  if success is attained, and zero otherwise. The probability of success depends on the quality of both partners:<sup>7</sup>

$$\text{Probab. of success} = \begin{cases} 1 & ,\text{if } \alpha_i \alpha_{P(i)} = 1 \\ \gamma & ,\text{if } \alpha_i \alpha_{P(i)} = 0, \end{cases} \quad (1)$$

$$(2)$$

where  $P(i)$  returns  $i$ ’s upstream partner and  $\gamma \in [0, 1]$ . Upfront production costs corresponds to equilibrium input price plus  $\eta$  units. Downstream firms have internal funds  $\delta\eta$ .

There are  $\mu \times F$  venture capitalists (VC’s, or informed capital), with  $\mu \leq 1$ , and a competitive financial market (uninformed capital). Each VC can service at most one firm at a time and each firm can only have one VC.<sup>8</sup> It is an implicit assumption of my paper

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<sup>7</sup>The model and results are very similar if production is non-stochastic and the low-type combination generates revenues  $\gamma R$ . The approach I use simplifies the modeling of the downstream uninformed financial markets.

<sup>8</sup>Although it would be natural to consider multiple VC’s per venture—since syndication is common—,

that there is plausible exogenous variation in the supply of informed capital in the economy. This is consistent with the notion that it is not trivial to have the institutional framework in place that is necessary for the venture capital industry to function appropriately (Lerner, 1998).

The composition of the economy aims at being a simple representation of a setting where firms need to closely coordinate their activities in order to be successful; and where this occurs in a setting of high uncertainty about the quality of the agents involved. These characteristics seems to be present in the early stages of high-tech sectors where many important innovations are produced, as argued in the introduction. Next we turn to the sequence of production and financing events, described below.

$t = 1$  (**Upstream financing.**) VC's are randomly assigned to upstream firms and offer the firm a debt contract.<sup>9</sup> If feasible, a competitive uninformed debt market for upstream firms also opens. Surplus from informed financing is split according to Nash bargaining.

$t = 2$  (**Type realization.**) Upstream- and downstream-firms' types are realized but not observable to VC's or other firms.

$t = 3$  (**Upstream-downstream matching market.**) Downstream firms choose whether to participate in a *certified* matching market or in a *residual* matching market.<sup>10</sup> In the certified matching market, downstream firms attempt to find a high-type upstream partner; this entails incurring a cost  $\phi$ ,<sup>11</sup> and also submitting a bid  $X_i$

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this would significantly complicate the analysis. One may also interpret a VC in the model as a syndicate.

<sup>9</sup>In the model's setting, debt is equivalent to equity; modeling debt makes exposition easier.

<sup>10</sup>The setup of the matching markets is very close to the marriage market of Becker (1973), except there is a participation cost for one of the markets.

<sup>11</sup>The cost  $\phi$  is reminiscent of search costs in the dynamic search-theoretic labor literature (Diamond, 1993; Mortensen and Pissarides, 1994).

(the input price), to be transferred later to the upstream partner. The amount  $\phi + X_i$  is paid upon successful matching and successful financing only.<sup>12</sup> Tentative matches are made for the top  $F/2$  bids.<sup>13</sup> If there are more than  $F/2$  top bids, excess bids are taken out of the market randomly. Upstream firms are allowed to ultimately reject their tentative match. Firms not matched via the certified matching market participate in a residual random matching market.

$t = 4$  (**Upstream production.**) Upstream firms that obtained financing produce; those with VC relationships reveal their type to the financier. The identity of upstream firms' partners becomes common knowledge.

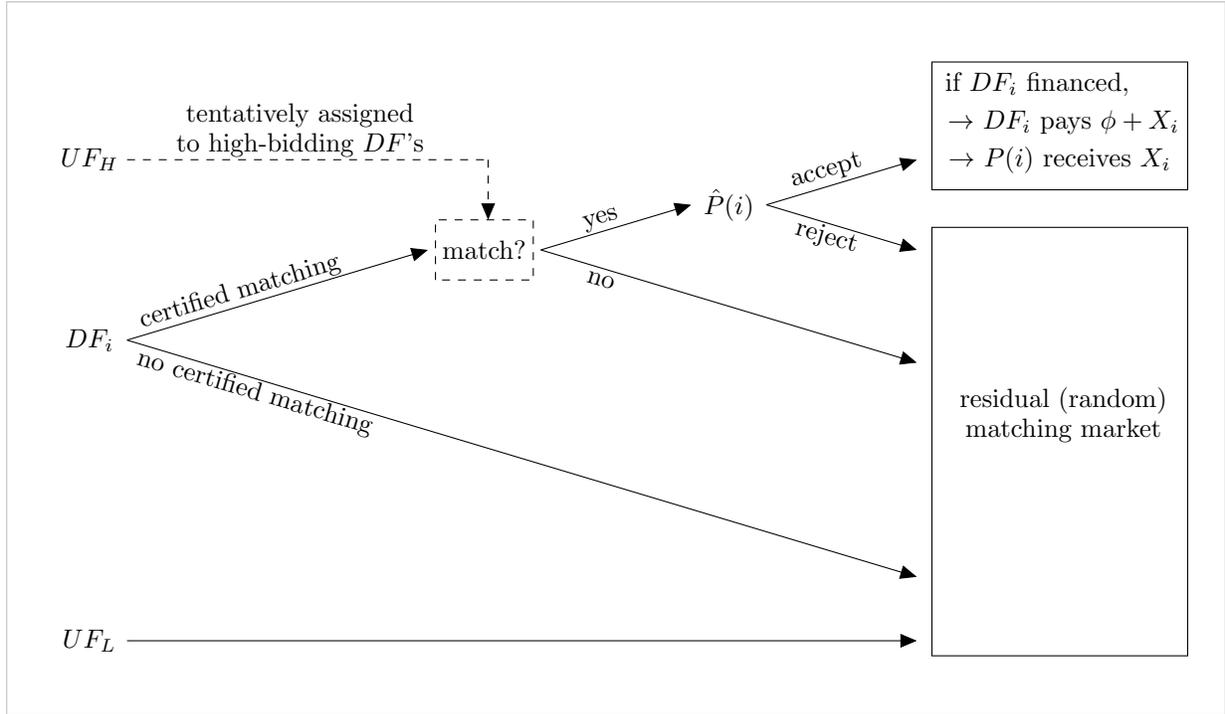
$t = 5$  (**Downstream financing.**) VC's choose whether or not to make a financing offer to their client's downstream partner, whose type they never observe directly. Surplus from informed financing is split according to Nash bargaining. If feasible, an uninformed debt market for downstream firms also opens. Downstream firms cannot credibly convey their business associations' quality to other agents in the economy.

$t = 6$  (**Downstream production and transfers.**) Downstream firms who secure financing transfer agreed funds to upstream partners and receive the input in exchange. Principal plus interest of upstream's firm debt are repaid up to the total funds generated by transfer payments. Downstream firms that obtain financing produce. Downstream firm's debt is repaid up to the amount generated by production.

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<sup>12</sup>Participation costs not being paid upfront is an assumption made for the sake of simplicity on two dimensions. First it rules out the case where firms do not choose certified matching only because they do not have internal funds; this paper is not about such a mechanism. Second it avoids having to consider two separate cases for downstream firms who participate in certified matching: those who paid  $\phi$  and were matched; and those who paid  $\phi$  but did not match.

<sup>13</sup>For simplicity, it is implicitly assumed that upstream firms cannot at this stage credibly convey whether they are VC-backed or not.



**Figure 1: Matching markets.** The figure details the sequence of events at  $t = 3$ .  $DF_i$  stands for downstream firm with label  $i$ ,  $UF_H$  ( $UF_L$ ) stands for high-type (low-type) upstream firms,  $P(i)$  ( $\hat{P}(i)$ ) is  $i$ 's (tentative) upstream partner.

The timing of the events depicts a situation where VC's have ongoing relationships with firms about which they learn gradually; and where the firms themselves are initially not too sure of their own quality. This does not seem unreasonable in the early stages of a high-tech sector. As the sector evolves, other interesting signaling behaviors may emerge; for example, a high-type upstream firm could be willing to accept a particularly stringent financing contract with a VC. This type of informational problem is however outside the scope of the paper. The key stage of the game is the upstream-downstream matching market ( $t = 3$ ). Figure 1 represents the sequence of events for this particular stage.

## 2.2 Equilibrium: definition

The game presented in the previous section has a potentially large set of equilibria, especially if mixed strategies and asymmetries are allowed. To keep the analysis tractable I will focus on symmetric pure-strategy perfect Bayesian equilibria, which are defined in more detail below.

**Definition 1** *A symmetric pure-strategy perfect Bayesian equilibrium is characterized by the following conditions a)-d).*

- a) A collection of pure strategies for firms and VC's, where these strategies are homogeneous across type at each information set.*
- b) A price for the certified matching market  $X_H$  (if it opens) and a price for the residual matching market  $X_L$ .*
- c) A rate for uninformed debt upstream  $r_u$  (if feasible) and a rate for uninformed debt downstream  $r_d$  (if feasible).*
- d) Every agent is behaving optimally given matching-market prices, uninformed debt rates, and other agents' actions; and holds correct beliefs at every information set.*

There are eight (candidate) types of the equilibria outlined in definition 1, which combine the status of the matching market (assortative matching or not) with the status of the upstream and downstream uninformed capital markets (feasible or not). The next section characterizes these equilibria, with a focus on the role played by the supply of informed capital in terms of obtaining generalized assortative matching in equilibrium. In this type of equilibrium all firms in the economy operate and high types match with one another.

## 3 Main results

### 3.1 Assortative matching with all firms operating

This section studies the main equilibrium of interest in the paper, where all firms operate and assortative matching obtains (generalized assortative matching). Section 3.1.1 establishes some initial assumptions regarding the parameter space that are useful to delineate the scope of the analysis. Sections 3.1.2 and 3.1.3 characterize the equilibrium.

#### 3.1.1 Preliminaries

It is first useful to derive expressions for the aggregate surplus of the economy in two cases.

**Lemma 1** *If all firms operate (downstream and upstream) and there is only residual matching, the (ex ante) aggregate surplus  $S$  (normalized by  $F$ ) is given by*

$$S = \frac{R}{4}(1 + 3\gamma) - (1 + \eta). \quad (3)$$

**Lemma 2** *If all firms operate and there is assortative matching, aggregate surplus is given by*

$$S = \frac{R}{2}(1 + \gamma) - \left(1 + \eta + \frac{\phi}{2}\right). \quad (4)$$

Next we make some assumptions regarding parameter relationships that are useful for the subsequent analysis. In particular, we take as a benchmark the state of the economy where all firms operate and there is no assortative matching, and require that this benchmark be economically unfeasible (assumption 1 below); this emphasizes the importance of assortative matching and simplifies the exposition. It is also necessary to require that the equilibrium of interest produces a positive aggregate surplus ex ante (assumption 2 below), otherwise this

equilibrium could not exist. Finally, assumption 3 is made more for technical convenience, as will become apparent in the subsequent analysis of section 3.1.2.

**Assumption 1** *All firms operating with no assortative matching generates a strictly negative aggregate surplus. Using expression (3), this corresponds formally to*

$$R < \frac{4(1 + \eta)}{1 + 3\gamma}. \quad (5)$$

**Assumption 2** *Assortative matching with all firms operating generates a positive aggregate surplus. Using expression (4), this corresponds formally to*

$$R \geq \frac{2(1 + \eta) + \phi}{1 + \gamma}. \quad (6)$$

**Assumption 3** *It is never efficient to start a downstream operation if a downstream firm and/or its supplier are low-type, even if upstream costs are sunk. Formally,*

$$\gamma R < \eta. \quad (7)$$

### 3.1.2 Characterizing the equilibrium

Now we turn to the characterization of the equilibrium where all but one high-type downstream firms partner up with high-type upstream firms; and where all firms operate. For the latter condition to be verified, naturally it needs to be the case that uninformed capital markets open, both upstream and downstream. For now I will take this as given; later we will investigate when uninformed markets break down (section 3.1.3).

The first step in solving for the equilibrium is to derive the conditions that allow us to determine the main endogenous variables of interest,  $X_H$  and  $X_L$ , as a function of primitives.

This derivation is presented below in five steps.

1. Both types of downstream firms raise debt  $D$  in the following amount:

$$D = X_H + \phi + \eta(1 - \delta). \quad (8)$$

This is the amount of financing strictly required by high-type downstream firms with a high-type supplier; the low types naturally have to mimic this amount when facing the uninformed financial market, otherwise they reveal their type and get zero financing for sure. Firms could in principle raise an amount greater than  $D$ . If this were the case, then these additional funds could be used either to replace internal/entrepreneur financing, or to sit idly in the firm's balance sheet. The former case is equivalent to assuming that only the amount in (8) of outside financing is obtained, but where the firm has lower funds; the latter does not really matter for results.<sup>14</sup>

2. Under assumption 3, rational VC's only make financing offers to downstream firms that are partnered with high-type suppliers; i.e. only half of VC's will end up making offers. Let us denote the rate associated with these offers by  $r_{VC,d}$ .
3. Since VC's only make financing offers if their downstream potential client is a high type, the total measure of high-quality downstream firms that receives an offer is  $\mu/2$ . All other firms seek financing with the uninformed market, which I assume cannot observe whether a firm was "rejected" by a VC or not. The equilibrium rate for the

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<sup>14</sup>Moreover, if firms tend to misuse idle funds, it would then be optimal to provide only the strictly necessary funding.

downstream uninformed market,  $r_d$ , is set to solve

$$\underbrace{\left[ \frac{1/2 - \mu/2}{1 - \mu/2} \right] D(1 + r_d)}_{\text{high type}} + \underbrace{\left[ \frac{1/2}{1 - \mu/2} \right] [\gamma D(1 + r_d) + (1 - \gamma)0]}_{\text{low type}} = D, \quad (9)$$

which, using equation (8) and simplifying, yields

$$r_d = \frac{1 - \gamma}{1 + \gamma - \mu}. \quad (10)$$

4. The ex ante surplus for a downstream firm  $i$ , denoted by  $\pi_{d,i}$ , conditional on  $i$  being a high type and also matching with a high-type supplier, is the following:

$$\begin{aligned} E[\pi_{d,i} | \alpha_i \alpha_{P(i)} = 1] &= \underbrace{\mu \times [R - D(1 + r_d/2)]}_{\text{finds VC-financed high-type supplier}} + \\ &+ \underbrace{(1 - \mu) \times [R - D(1 + r_d)]}_{\text{finds non-VC-financed high-type supplier}} - \delta\eta, \end{aligned} \quad (11)$$

where we used the fact that with Nash bargaining, the informed rate is simply going to be one-half of the market rate (recall the real interest rate is 0), i.e.  $r_{VC,d} = r_d/2$ .<sup>15</sup> Since high-type downstream firms are competing amongst themselves (given the scarcity of high-type suppliers), in equilibrium their ex ante surplus needs to be zero. To see this, let us consider an  $X_H$  such that downstream firms do not wish to deviate to the residual matching market (to be checked later), and also where some surplus is generated. In this situation any one firm has the incentive to slightly increase their bid  $X_i$  and *make sure* they are matched with a high-type upstream partner,

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<sup>15</sup>The disagreement payoffs correspond to financing being conducted in the uninformed market, i.e.  $R - D(1 + r_d)$  for the downstream firm; and 0 for the VC.

instead of facing the probability  $1/(F/2)$  of being excluded. Equating expression (11) to zero and using expression (8) for debt  $D$ , one obtains the following expression for  $X_H$ :

$$X_H = \frac{R - \delta\eta}{1 + r_d \left(1 - \frac{\mu}{2}\right)} - \phi - \eta(1 - \delta) \quad (12)$$

This zero-surplus condition is naturally sufficient for low-type downstream firms not wanting to deviate to the certified matching market, since this would violate their participation constraint.

5. Under the assumption that firms' insiders are able to appropriate all ex ante excess funds—for example via higher wages (not modeled)—then the surplus of a downstream firm, conditional on it being partnered with a low-type supplier, is given by

$$\begin{aligned} \text{E}[\pi_{d,i} | \alpha_{P(i)} = 0] &= \gamma [R - D(1 + r_d)] - \overbrace{(X_L + \eta - D)}^{\text{necessary own funds for low-type downstream}} \\ &= \gamma [R - D(1 + r_d)] - \delta\eta + (X_H - X_L) + \phi. \end{aligned} \quad (13)$$

Note how the downstream firm in a low-type pair potentially saves own financial resources: certification costs are not incurred (and this is implicitly assumed to be unobservable to the uninformed market) and in principle it pays a lower amount to the supplier,  $X_L$  instead of  $X_H$  (also not observable to the uninformed market). The surplus in the above equation must be zero in equilibrium, in order to prevent high-type downstream firms from deviating to residual matching; since with this deviation a high-type upstream firm would be partnered with a low-type supplier for sure, expression

(13) is indeed the deviation payoff. This zero-surplus condition pins down  $X_L$ :

$$X_L = (X_H + \phi - \delta\eta) [1 - \gamma(1 + r_d)] + \gamma [R - \eta(1 + r_d)], \quad (14)$$

where in the simplification I made use of expression (8) for debt  $D$ . From the above expression, at least for small enough  $\gamma$ , one can see how the price of the certified matching market  $X_H$ , as well as participation costs  $\phi$ , push the price of the residual market  $X_L$  up. The intuition for this effect is that although high types downstream do not participate in the residual market in this equilibrium, they form a kind of “latent demand” for low-type upstream firms, given the competitive pressure in their own market. I note that the zero-surplus condition for low types would also naturally obtain in a slightly different setting where upstream firms are generally scarce (and not only high-type upstream firms); in that case  $X_L$  being high in equilibrium would be unrelated to  $X_H$  being high (I explore this further in section 4.3.1).

Combining the equilibrium conditions derived in points 1-5 allows us to write the equilibrium prices of most interest to us,  $X_H$  and  $X_L$ , as a function of primitives only.

**Proposition 1** *In an equilibrium where all firms operate and assortative matching occurs,  $X_H$  and  $X_L$  have a unique solution:*

$$X_H = \frac{2R(1 + \gamma - \mu) + \delta\eta(1 - \gamma)(2 - \mu)}{4 - \mu(3 - \gamma)} - (\phi + \eta) \quad (15)$$

$$X_L = \left[ \frac{1}{4 - \mu(3 - \gamma)} \right] \left( R [2(1 + \gamma) - \mu(2 + \gamma - \gamma^2)] + \right. \\ \left. + \eta [\mu(3 - \gamma + 2\delta(1 - \gamma)) - 4 - 2\delta(1 - \gamma)] \right) \quad (16)$$

With these expressions for  $X_L$  and  $X_H$  we can now use a final equilibrium condition that

delivers the main result of the paper. In order for high-type upstream firms to accept the price in the certified matching market, it needs to be the case that  $X_H \geq X_L$ . This condition determines the minimum level for the supply of informed capital that is required to sustain the generalized assortative-matching equilibrium.

**Proposition 2** *The minimum level for the supply of informed capital that sustains an assortative-matching equilibrium where all firms operate, denoted by  $\mu_0$ , is given by*

$$\mu_0 = \begin{cases} 0 & , \text{ if } \phi < \phi_0 & (17) \\ \frac{4[\phi - \delta\eta(1 - \gamma)]}{\phi(3 - \gamma) + (1 - \gamma)(R\gamma - 3\delta\eta)} & , \text{ if } \phi \geq \phi_0, & (18) \end{cases}$$

where  $\phi_0$  is a minimum level for certified-matching-market participation costs, given by

$$\phi_0 := \delta\eta(1 - \gamma) \geq 0. \quad (19)$$

Naturally, for the above result to matter it needs to be the case that there exist regions of the parameter space where  $\mu_0 \in (0, 1)$  and uninformed capital markets do not break down. But let me postpone this analysis to section 3.1.3 and a numerical example that conforms to all required equilibrium conditions to section 3.3. Now I will turn to the intuition behind the result in proposition 2.

The market prices for upstream firms that support this equilibrium are both high, in the sense that  $X_H$  exhausts all of the surplus of high-type downstream firms; and  $X_L$  exhausts all surplus of low-type downstream firms. The former follows from high-type upstream firms being scarce; the latter is required in order to prevent high types downstream from deviating to the residual matching market. Given these characteristics of matching-market prices, it is the *wealth* of downstream firms, both high-type and low-type, that ultimately determines

these prices, and so a wedge between  $X_H$  and  $X_L$  is in essence a wedge between the wealth of high types downstream versus their low-type counterparts. One factor inducing a difference in wealth across types is precisely the cost at which they obtain funds. The difference in funding cost expands with the supply of informed capital  $\mu$ , via two channels. First, a higher  $\mu$  makes it more likely that high types find informed finance, which is cheaper. Second, the equilibrium rate of the uninformed market  $r_d$ —the cost of financing for low types—increases with  $\mu$ , i.e. these firms become poorer. Although the cost of informed financing for downstream high-type firms is also increasing in  $\mu$  (given Nash bargaining), it is increasing at a lower rate. For higher levels of  $\mu$  high-type downstream firms thus become more wealthy *relative* to their low-type counterparts, and the fact that they have to spend  $\phi$  in participation costs still allows them to afford a high  $X_H$ . Naturally this mechanism only matters if certified-matching costs are relatively high, as stated in the proposition.

### 3.1.3 Uninformed capital market feasibility

Let us start by analyzing the feasibility of upstream uninformed financial markets, under the assumption that downstream markets do not break down and that  $X_H \geq X_L$ . The upstream uninformed financial market sets the rate  $r_u$  competitively for a total debt amount of one unit, knowing that one-half of financed firms will turn out to be low-types. The formal condition for expected break even to be zero is given by

$$\frac{1}{2} \min((1 + r_u), X_H) + \frac{1}{2} \min((1 + r_u), X_L) = 1, \quad (20)$$

which solving for  $r_u$  yields

$$r_u = \begin{cases} 0 & , \text{ if } X_L \geq 1 \\ 1 - X_L & , \text{ if } X_L < 1. \end{cases} \quad (21)$$

The feasibility condition is defined implicitly by the event where low-type firms default; and high-type firms are at the default boundary:<sup>16</sup>

$$\frac{1}{2}X_H + \frac{1}{2}X_L = 1. \quad (23)$$

The above condition can be written in terms of a maximum level for the supply of informed capital  $\mu$ ; the result is contained in proposition 3. The intuition for the result is that as  $\mu$  increases, the cost of financing for all downstream firms also increases. This leads to a lower amount being transferred to upstream firms in equilibrium, which at some point makes the financing of these firms uneconomical. This breakdown may be welfare-destroying, a topic I discuss later in section 3.3.

**Proposition 3** *A sufficient condition for the failure of upstream uninformed capital markets is that the supply of informed capital be above the following threshold:*

$$\mu_1 := \frac{4[2(1 + \eta) + \phi - R(1 + \gamma)]}{\eta[6 - 2\gamma + \delta(1 - \gamma)] + (2 + \phi)(3 - \gamma) - R(4 + \gamma - \gamma^2)} \quad (24)$$

Next I study the conditions for the failure of downstream uninformed capital markets.

The boundary condition for default for downstream firms facing the uninformed market is

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<sup>16</sup>By allowing for a case where  $X_L < 1$ , I am actually forcing firms to not back out from production (at  $t = 4$ ), even after knowing they are low types (at  $t = 1$ ). One can think that it is socially unacceptable to “quit” at this stage, for reasons unspecified in the model. In any case, as illustrated by the numerical example in section 3.3,  $X_L > 1$  obtains for some regions of the parameter space in the main equilibrium of interest, so this does not seem a crucial issue for the main point of the paper.

that all funds available are used to pay for debt (principal plus interest). Formally, this is expressed as

$$\left[ \frac{1/2 - \mu/2}{1 - \mu/2} \right] R + \left[ \frac{1/2}{1 - \mu/2} \right] \gamma R = D, \quad (25)$$

which simplifies into the result in proposition 4.

**Proposition 4** *A sufficient condition for the failure of downstream uninformed capital markets is that the supply of informed capital be above the following threshold:*

$$\mu_2 := \frac{4\delta\eta}{2\delta\eta + R(1 - \gamma)} \quad (26)$$

Finally it is important to point out that the upstream uninformed capital market breaking down is a necessary and sufficient condition for the same to happen downstream. To see this, note that if the downstream uninformed capital market closes, it is never possible, even for high-type pairs, to operate downstream; and this destroys the incentive to produce/finance upstream if not financed by a VC. On the other hand, if the upstream uninformed capital market does not open, then there are no high-type pairs facing the downstream uninformed capital market; only the low types who did not get financing from the VC. This result is stated formally in corollary 1.

**Corollary 1** *A necessary condition for the equilibrium with assortative matching and all firms operating to obtain is that the supply of informed capital be below  $\min(\mu_1, \mu_2)$ .*

### 3.2 Other economic regimes

Now that we have characterized the main equilibrium of interest, it is natural to address the issue of what occurs if the conditions for this equilibrium are not met. Recall from section

2.2 that we had eight candidate types of equilibrium. With the result from corollary 1 this reduces to four, combining the pair (assortative-matching; no-assortative-matching) with the pair (uninformed-finance; no-uninformed-finance). Given our assumption regarding the surplus of the economy when all firms operate and there is no assortative matching (assumption 1), it follows immediately that it cannot be an equilibrium to have uninformed capital markets and no assortative matching. Basically this is an economy where collaborative innovation projects have negative NPV ex ante and so no initial financing is possible. Also note that in this economy VC's are equivalent to uninformed finance (there is nothing to learn about). There is thus only one other candidate equilibrium that survives, where only VC-backed firms operate. The surplus of the economy in this case is presented in lemma 3; assumption 2 implies that this surplus is always positive.

**Lemma 3** *If there is assortative matching but no uninformed financing, aggregate surplus is given by*

$$S = \mu \left[ \frac{R - (2 + \eta + \phi)}{2} \right]. \quad (27)$$

The mode of operation where only VC-backed firms operate is the only possible equilibrium (from the class in definition 1) whenever the supply of informed capital is too low ( $\mu < \mu_0$ ) or too high ( $\mu > \min(\mu_1, \mu_2)$ ). Finally, this mode of operation is *not* an equilibrium for the intermediate region of  $\mu$ . To see this note that given the sequence of decisions, upstream firms who did not obtain VC financing at  $t = 0$  still create value on average, as long as high-type downstream firms demand them later, i.e. choose to participate in the certified matching market. Since participating in the certified matching market later also creates value for these downstream firms, the only possible subgame-perfect equilibrium necessarily implies that all firms operate. This would potentially be different if the firms were moving simultaneously, and where a coordination failure could obtain.

### 3.3 Numerical example

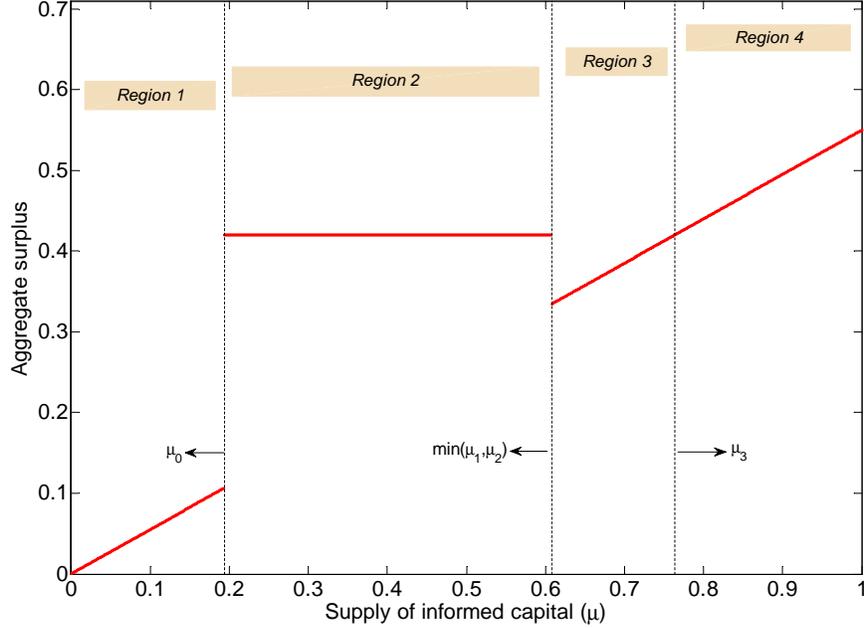
This section presents a numerical example of an economy where there exists an intermediate region for the supply of informed capital. Table 1 shows the main endogenous variables for three different levels of  $\mu$ , all consistent with the equilibrium where all firms operate; the examples in the table illustrate how the increase in the supply of informed capital drives a wedge between the certified market price of upstream firms  $X_H$  and that of their low-quality counterparts,  $X_L$ .

**Table 1: Supply of informed capital and endogenous outcomes.** The table shows the equilibrium level of the endogenous variables of interest: upstream and downstream cost of uninformed capital,  $r_u$  and  $r_d$ ; certified-matching and residual-matching market prices,  $X_H$  and  $X_L$ . All scenarios refer to the equilibrium where all firms operate and assortative matching obtains.

Supply of informed capital	$r_d$	$r_u$	$X_H$	$X_L$
$\mu = 0.2$	0.33	0.00	1.32	1.32
$\mu = 0.3$	0.36	0.00	1.30	1.23
$\mu = 0.4$	0.39	0.00	1.26	1.12
$\mu = 0.5$	0.43	0.01	1.23	0.99

Figure 2 shows the variation in equilibrium aggregate surplus—normalized by  $F$ , the number of firms in each sector—for varying levels of  $\mu$ , the supply of informed capital. Region 1 in the figure represents a state (or stage) of the economy where very few firms operate and are all backed by informed capital. Aggregate surplus is low because there are few firms operating; the surplus per operating firm is actually high, since there are no low-type downstream firms operating. If we look at the picture as representing a life cycle of a high-tech/VC cluster, then this would correspond to an initial phase of slow and steady growth, similar to the initial stage of an S-shaped technology curve.<sup>17</sup>

<sup>17</sup>An early reference on the patterns of technological expansion (and the famous S-curve) is Rogers (1962).



**Figure 2: Supply of informed capital and aggregate surplus.** The figure plots the aggregate equilibrium surplus, normalized by  $F$ , for varying supply of informed capital  $\mu$ .  $\mu_0$  ( $\min(\mu_1, \mu_2)$ ) is the minimal (maximal) threshold necessary to sustain the assortative-matching equilibrium where all firms operate. Parameter choice:  $R = 10.8$ ,  $\eta = 6.2$ ,  $\phi = 1.5$ ,  $\gamma = 0.55$ ,  $\delta = 0.5$ .

Figure 2 shows that as the critical threshold  $\mu_0$  is hit, the size of the economy experiences a strong jump; in a dynamic interpretation this jump compares to the accelerating stage of an S-curve. Interestingly, the jump in economic surplus is driven entirely by an increase in the size of the financial sector, since the number of firms  $F$  and their characteristics are kept unchanged. The equilibrium in region 2 has all firms in the economy operating, but under an efficient generalized assortative-matching regime. This is possible because now uninformed financial markets can operate and the simultaneous existence of informed and uninformed capital allows for the differences in cost of financing that make efficient matching incentive-compatible. It is perhaps interesting to note that in this model, uninformed financial markets are initially enabled by the presence of enough informed intermediaries. It is only after a second threshold is hit (region 3) that the adverse selection induced by the presence of

informed intermediaries leads to a breakdown of uninformed markets (and this is just a standard mechanism). In regions 3 and 4 aggregate surplus again increases linearly in  $\mu$ .

At least for the choice of parameters in figure 2, the transition from region 2 to region 3 is welfare-destroying. This stems from a bargaining externality: VC's extract a high surplus ex post (downstream) via Nash bargaining and this limits the amount of funds that downstream firms can pass to upstream partners.<sup>18</sup> At some point it is economically efficient to start an uninformed-financed upstream operation, but the firm will not have high enough sales to make the funding opportunity viable for uninformed (upstream) investors (note how  $X_H$  and  $X_L$  decrease in  $\mu$  in table 1). I do not focus on this result because it depends critically on the assumption about Nash bargaining. The main result holds even for competitive supply of informed capital (see section 4.3.2); and in this case the bargaining externality that leads to a drop in surplus from region 3 to region 4 is absent.

After threshold  $\mu_3$  in the figure, the economy is operating at a higher level of surplus than in region 2; it is however not surprising that the maximum level of informed capital ( $\mu = 1$ ) generates the maximum level of surplus in this economy. Threshold  $\mu_3$  is defined formally in proposition 5.

**Proposition 5** *As long as  $\min(\mu_1, \mu_2) < 1$ , then there exists a threshold for the supply of informed capital, given by*

$$\mu_3 := \frac{R(1 + \gamma) - 2(1 + \eta) - \phi}{R - (2 + \eta + \phi)}, \quad (28)$$

*such that for all  $\mu \geq \mu_3$  the surplus of the economy is larger in the VC-backed-only-firms equilibrium than in the equilibrium where all firms operate.*

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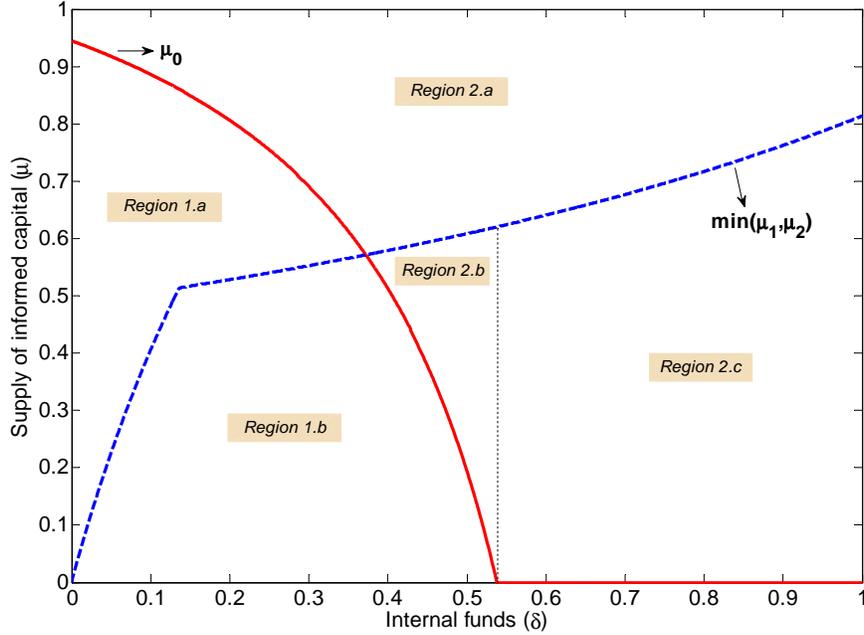
<sup>18</sup>Consistent with this explanation, in figure 2 it is the case that  $\min(\mu_1, \mu_2) = \mu_1$ .

Finally I note that empirically regions 3 and 4 in figure 2 may never be observed, if it is costly to have so much informed capital confined to a single cluster (not modeled), either due to required public investments in the appropriate institutional framework or to some initial costs VC's have to incur to be able to undertake their economic activity. If it is inexpensive to have an arbitrarily large supply of informed capital, then the main point of the paper becomes immaterial in situations such as the one depicted in figure 2; since it is preferable to function in a regime where most high types downstream operate in a partnership with high types upstream, and no low types downstream operate. The argument of the paper is not that a significant amount of informed capital is beneficial, but rather that increasing the supply of informed capital beyond a *critical threshold* creates a significant discontinuous jump in economic surplus.

## 4 Additional analyses

### 4.1 The role of internal funds

So far I have focused on the role played by a minimum supply of (outside) informed capital ( $\mu$ ) in obtaining an efficient assortative-matching equilibrium where all firms operate. However, another important factor for this equilibrium to obtain is the level of internal funds  $\delta$ . Figure 3 presents five possible regions where the economy may be operating, as a function of the level of internal funds (horizontal axis) and the supply of informed capital (vertical axis). The figure can also be interpreted as capturing two other dimensions of the economy: the total funding availability in the economy (which expands along the NE direction); and the aggregate capital structure (where going NW implies a higher ratio of outside-to-inside funds). The five regions are defined mainly by the conditions that sustain the assortative-



**Figure 3: Internal funds, supply of informed capital, and economic regimes.** The red solid line represents the minimum level of informed capital required to support an assortative-matching equilibrium where all firms operate ( $\mu_0$ ), as a function of internal funds  $\delta$ . The blue dashed line maps the maximum level of informed capital ( $\min(\mu_1, \mu_2)$ ) such that uninformed markets do not break down, also as a function of internal funds  $\delta$ . Remaining parameter choice is the same as in figure 2:  $R = 10.8$ ,  $\eta = 6.2$ ,  $\phi = 1.5$ ,  $\gamma = 0.55$ .

matching equilibrium of interest:  $\mu \geq \mu_0$  represents the minimum-VC-supply condition;  $\mu \leq \min(\mu_1, \mu_2)$  guarantees that uninformed markets do not break down.

Inspection of figure 3 immediately reveals that there is a substitution effect between internal funds and the supply of informed capital, since the  $\mu_0$ -threshold is (weakly) downward sloping. Regions 1.a and 1.b in the figure depict the case where the  $\mu_0$ -threshold is not satisfied; which implies that the economy is necessarily in a VC-only equilibrium, as explained in section 3.2. Region 2.a. represents the case where the minimum-VC-supply criterion is satisfied, but where informed markets fail. Region 2.b is the one relevant for the paper's main argument: a minimum positive supply of VC's is required to sustain the equilibrium where all firms operate. Region 2.c shows that this argument is no longer binding if internal

funds are large enough.

Perhaps the most interesting feature of figure 3 is that there exist both a minimum and maximum level for internal funds that need to be verified in order for the mechanism of interest to be relevant. This is more general than just the numerical example in figure 3, as stated in propositions 6 and 7.

**Proposition 6** *A strictly positive level of internal funds  $\delta$  is necessary in order for downstream uninformed markets not to break down.*

**Proposition 7** *If internal funds  $\delta$  are smaller than the following threshold,*

$$\bar{\delta} := \frac{\phi}{\eta(1 - \gamma)}, \tag{29}$$

*then a positive supply of informed capital is not a necessary condition to obtain the assortative-matching equilibrium with all firms operating.*

The intuition for proposition 7 is that only because firms are financially constrained (small  $\delta$ ) does the minimum-VC-supply mechanism matter; otherwise the wealth differential comes simply from the fact that good projects are valuable enough. As for proposition 6, the reason why downstream uninformed markets break down is that under the intense competition for upstream targets in the certified matching market, high-type downstream firms with little internal funds are willing to increase their bid ( $X_H$ ) to value-destroying levels, as long as this is achieved via the expropriation of outside financiers. Since the latter anticipate this problem, no uninformed capital is available in these conditions.

## 4.2 Equilibrium rate of informed upstream capital

This section briefly discusses how the equilibrium VC rate for upstream firms is obtained, and I will focus on the assortative-matching equilibrium where all firms operate. For simplicity I will first assume that  $r_u = 0$  (i.e.  $X_L \geq 1$ ). The surplus of VC financing is assumed to be split according to Nash bargaining. If agreement is achieved, the ex ante payoffs for the firm and for the VC are, respectively,

$$\begin{cases} \frac{1}{2}(X_L + X_H) - (1 + r_{VC,u}) & \text{(Upstream firm)} & (30) \\ r_{VC,u} + \frac{1}{2}\left(D\frac{r_d}{2}\right) & \text{(VC).} & (31) \end{cases}$$

In turn, the disagreement payoffs correspond to a situation where the firm declines the VC opportunity and goes to the uninformed market; these payoffs are given below:

$$\begin{cases} \frac{1}{2}(X_L + X_H) - 1 & \text{(Upstream firm)} & (32) \\ 0 & \text{(VC)} & (33) \end{cases}$$

Combining the agreement and disagreement payoffs, one arrives at the result stated in proposition 8.

**Proposition 8** *If  $r_u = 0$ , the equilibrium rate charged by venture capitalists to the upstream firm they are initially matched with is negative, and given by the following expression:*

$$r_{VC,u} = -\frac{(1 - \gamma)(R - \delta\eta)}{4[4 - \mu(3 - \gamma)]} \quad (34)$$

Average upstream firms obtain a discounted rate because VC's obtain a positive ex post surplus from this associative activity (they may find a high-quality downstream firm later

on). This result mimics the one in Montgomery (1991), where the initial employee obtains a higher-than-market wage because of the option value associated with a future potential hiring of her high-type “friend”. According with this theory one would thus expect to empirically find that venture capitalists’ initial ventures exhibit low average returns. The argument that VC choices are influenced by learning considerations has been previously made by Sørensen (2008), although learning is not occurring via the assortative-matching mechanism.

It is perhaps interesting to note that  $r_{VC,u}$  declines with  $\mu$ , the supply of informed capital, although this is totally unrelated to competitive forces in the VC market. In fact, in my setting this obtains because a higher  $\mu$  implies that VC’s extract a higher absolute surplus ex post (higher  $r_d$ ).

If we analyze the case where  $r_u \neq 0$ , i.e. initial financing is risky, then the model delivers persistence in VC returns, which is an empirically documented regularity of VC investments (Kaplan and Schoar, 2005). This result is stated in proposition 9 and follows directly from the assortative-matching mechanism: either the VC finds a high type initially, which delivers a high initial return upstream and a good investment opportunity downstream; or the VC learns that the initial venture is a low type, in which case it earns a low return initially and the competitive return subsequently.

**Proposition 9** *If  $X_L$  is smaller than a threshold  $\underline{X}_L$  (defined in the appendix), then two cases occur with equal probability:*

1. *The VC experiences a high realized return upstream (greater than the average return for VC’s); which is followed by a high realized return downstream (greater than the competitive average return of zero).*
2. *The VC experiences a low realized return upstream (lower than the average return*

for VC's); which is followed by non-participation (or earning the competitive average return of zero).

## 4.3 Extensions

### 4.3.1 A closer look at the role of scarcity

The scarcity of upstream high types plays an important role in the analysis above. It is this scarcity that drives up the price of the certified matching market; and, indirectly, the price of the residual matching market as well. This section shows that this is not the only setting where the main result obtains. In fact, what is required is that there is scarcity of upstream firms, either just for high types, as previously assumed, or in general. I will still focus on the sustainability of an assortative-matching equilibrium where all firms operate.

Assume now that upstream firms are generally scarce, for example there are  $F - 2$  upstream firms (but let us keep considering a large even  $F$ ). Also, let us assume that it is still the case that  $1/2$  of these firms are high types; but now high types downstream are scarcer, totaling  $\beta \times F$ , with  $\beta < 0.5$ . In this setting, the participation constraint of low types downstream is binding, given the general scarcity of upstream firms; this pins down  $X_L$ . In this equilibrium it needs to be the case that  $X_H = X_L = X$ , since this is the minimum amount that high types downstream need to bid in order to secure a partner via the certified matching market. Using equations (8) and (13) we obtain an expression for  $X$ :

$$X = \frac{1}{1 + r_d} \left[ R - \left( \frac{\delta\eta - \phi}{\gamma} \right) \right] - \phi - \eta(1 - \delta) \quad (35)$$

To see that the main mechanism is still at play, assume otherwise; in particular let us set the supply of informed capital  $\mu = 0$  and check whether the incentive-compatibility

constraint for high types downstream is verified. According to equation (11) (or equation (12)), high types downstream have a positive surplus as long as

$$X \leq \frac{R - \delta\eta}{1 + r_d(1 - \mu/2)} - \phi - \eta(1 - \delta) = \Big|_{\mu=0} \frac{R - \delta\eta}{1 + r_d} - \phi - \eta(1 - \delta). \quad (36)$$

Combining expressions (35) and (36) we have

$$\frac{1}{1 + r_d} \left[ R - \left( \frac{\delta\eta - \phi}{\gamma} \right) \right] \leq \frac{R - \delta\eta}{1 + r_d} \Leftrightarrow \phi \leq \delta\eta(1 - \gamma),$$

which is the threshold for certified-matching costs which we previously derived in proposition 2. This establishes that a minimum level for the supply of informed capital is still necessary in this case. In fact, this level is the same as the one derived before in proposition 2, since  $\mu_0$  is implicitly determined by  $X_H = X_L$ ; an equality which we assumed at the outset for this new setting. The difference relative to the first case is that now  $X_H = X_L$  even for  $\mu > \mu_0$ ; whereas before this slack in the incentive-compatibility constraint of high types downstream was actually being transferred to high types upstream, via  $X_H > X_L$ .

Another way to make the argument that scarcity of the party incurring the certified-matching costs is what drives the results is to show that they break down otherwise. More specifically, consider now that upstream firms are abundant—in general and in terms of high types—and their participation constraint is binding. This implies, in equilibrium,  $X_L = X_H = 1$ . Using equations (11) and (13), the incentive-compatibility constraint of high types downstream is satisfied as long as

$$\begin{aligned} \mu[R - D(1 + r_d/2)] + (1 - \mu)[R - D(1 + r_d)] - \delta\eta &\geq \gamma[R - D(1 + r_d)] - \delta\eta - \phi \Leftrightarrow \\ R(1 - \gamma) &\geq D[\mu(1 + r_d/2) + (1 - \mu)(1 + r_d) - \gamma(1 + r_d)] - \phi. \end{aligned}$$

Assuming  $\mu = 0$ , the above simplifies to

$$R(1 - \gamma) \geq D(1 + r_d)(1 - \gamma) - \phi \Leftrightarrow R \geq D(1 + r_d) - \frac{\phi}{1 - \gamma}. \quad (37)$$

Since in the conjectured equilibrium uninformed financial markets do not break down,<sup>19</sup> a necessary condition for this to be the case is naturally that  $R > D(1 + r_d)$ ; since otherwise not even high types would repay principal plus interest. This immediately implies that condition (37) is verified. In short, even with  $\mu = 0$  the assortative-matching equilibrium with all firms operating would obtain.

### 4.3.2 Competitive informed capital

It is intuitive that for the main mechanism to operate, it cannot be the case that informed financiers extract too high of a surplus. If they do, this compromises the wedge in cost of financing across types. But what happens if venture capitalists provide capital at better terms than those associated with Nash bargaining? Presumably this would facilitate, at the margin, the beneficial impact of informed capital in terms of matching. To analyze this setting I consider the extreme case of competitive rates offered by informed capitalists. Proposition 10 contains the main results, which confirm the intuition.

**Proposition 10** *If informed capital to downstream firms is provided at a competitive rate, then the minimum supply of informed capital that sustains an assortative-matching equilibrium where all firms operate is given by*

$$\tilde{\mu}_0 := \frac{2[\phi - \delta\eta(1 - \gamma)]}{\phi(2 - \gamma) + (1 - \gamma)(R\gamma - 2\delta\eta)}. \quad (38)$$

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<sup>19</sup>Notice that adverse selection is at its minimum, since  $\mu = 0$ .

*This new threshold is always lower than the threshold derived in proposition 2.*

### **4.3.3 Informed capital and the boundaries of the firm**

The main result also has implications about the boundaries of the firm in this type of setting. More specifically, if firms were to pair up via a merger before types are revealed, this would in essence lead to a breakdown of the economy (important information is now not revealed early enough), at least under our assumptions regarding surplus without assortative matching. This mechanism, at the margin, would make firms prefer to operate as stand-alones. Furthermore, even if firms incur certified-matching costs and pair up assortatively, perhaps this is hard to convey to outside financiers when operations are integrated, since now low-type pairs have an incentive to coordinate on falsifying information about their true quality, just in order to obtain funds. Note that this problem is not present if upstream firms and downstream firms operate as stand-alones: the upstream firm has no incentive to hide its type after matching has been conducted and after it receives funds from the VC.<sup>20</sup>

## **5 Conclusion**

This paper proposes a model where it is efficient for high-quality firms to match up, but costly certification may impede such matching from obtaining in equilibrium. I show that, under certain conditions, a minimum supply of informed capital alleviates this problem, by inducing a wedge in the cost of financing across firm types that in turn makes costly matching incentive-compatible for high types. The mechanism presented in the paper suggests a novel role for financial intermediaries in enhancing innovation and economic growth.

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<sup>20</sup>This argument is related to the discussion on stage financing in venture-capital settings; see for example Admati and Pfleiderer (1994) and Gompers (1995).

## Appendix – Proofs

**Proof of lemma 1.** If all firms are matched at random, then  $1/4$  of all pairs will comprise a high-type upstream firm and a high-type downstream firm. For these pairs, and noting that certified-matching costs are not incurred in this case, total surplus is  $R - (1 + \eta)$ ; for other pairs in the economy surplus corresponds to  $\gamma R - (1 + \eta)$ . We thus obtain an expected surplus (normalized by  $F$ ) of

$$S = \frac{1}{4}[R - (1 + \eta)] + \frac{3}{4}[\gamma R - (1 + \eta)].$$

Simplifying the above expression yields equation (3) in the lemma. ■

**Proof of lemma 2.** With assortative matching, and for large  $F$ , approximately all high-type upstream firms match up with all high-type downstream firms; but the latter incur certified-matching costs. Total surplus for these pairs is  $R - (1 + \eta + \phi)$ . For other pairs in the economy surplus corresponds to  $\gamma R - (1 + \eta)$ . Combining the two types of surplus, in the aggregate we thus obtain

$$S = \frac{1}{2}[R - (1 + \eta + \phi)] + \frac{1}{2}[\gamma R - (1 + \eta)],$$

which simplifies into expression (4) in the lemma. ■

**Proof of lemma 3.** In this equilibrium,  $1/2$  of upstream firms that partnered with a VC will be a high type; and so its downstream partner will obtain a financing offer. For these pairs, who comprise a measure of  $\mu/2$  in the economy, total surplus is then  $R - (1 + \eta + \phi)$ . The other  $1/2$  upstream firms turn out to be low types and accordingly their downstream partners will not obtain the funding necessary to engage in production (nor will they pay certified-

matching costs). Since upstream firms' type is only revealed to the VC via production, low types upstream still need to incur the initial cost of 1. Combining the two cases we thus obtain the following for the aggregate surplus:

$$S = \frac{\mu}{2}[R - (1 + \eta + \phi)] + \frac{\mu}{2}(-1).$$

This simplifies into expression (27).■

**Proof of proposition 1.** Expressions (15) and (16) in the proposition are obtained by inserting the expression for the uninformed rate in equation (10) into equations (12) and (14).■

**Proof of proposition 2.** The denominator in expressions (15) and (16) is positive for all  $\mu \in [0, 1]$  (our domain of interest). Therefore,  $X_H \geq X_L$  is equivalent to

$$\begin{aligned} &2R(1 + \gamma - \mu) + \delta\eta(1 - \gamma)(2 - \mu) - (\phi + \eta)[4 - \mu(3 - \gamma)] \geq \\ &2R(1 + \gamma) - \mu R(2 + \gamma - \gamma^2) + \eta[\mu(3 - \gamma + 2\delta(1 - \gamma)) - 4 - 2\delta(1 - \gamma)]. \end{aligned}$$

Simplifying the above expression with respect to  $\mu$  yields the threshold in equation (18).

The next step is to establish that expression (18) increases with certified-matching costs  $\phi$ :

$$\begin{aligned} \frac{\partial \mu_0}{\partial \phi} \geq 0 &\Leftrightarrow \frac{4}{\phi(3 - \gamma) + (1 - \gamma)[R\gamma - 3\delta\eta]} \geq \frac{4(3 - \gamma)[\phi - \delta\eta(1 - \gamma)]}{[\phi(3 - \gamma) + (1 - \gamma)[R\gamma - 3\delta\eta]]^2} \Leftrightarrow \\ &(1 - \gamma)[R\gamma - 3\delta\eta] \geq -(3 - \gamma)\delta\eta(1 - \gamma) \Leftrightarrow R \geq \delta\eta, \end{aligned}$$

which is always true under assumption 2 and the fact that  $\delta < 1$ . The threshold  $\phi_0$  is obtained by setting the numerator in (18) to zero.■

**Proof of proposition 3.** Combining the expressions for  $X_H$  and  $X_L$  (equations (15) and (16)) with the expression that determines the default boundary (equation (23)), the breakdown condition  $X_H + X_L < 2$  is equivalent to

$$\frac{R[4(1 + \gamma - \mu) - \mu(\gamma - \gamma^2)] - (\phi + 2\eta)[4 - \mu(3 - \gamma)] + \delta\eta\mu(1 - \gamma)}{2[4 - \mu(3 - \gamma)]} < 2.$$

Simplifying the above expression with respect to  $\mu$  yields threshold (24) in the proposition. ■

**Proof of proposition 4.** Using equations (8) and (25), the breakdown condition for downstream uninformed financial markets can be written as

$$\begin{aligned} R \left( \frac{1/2 - \mu/2 + 1/2\gamma}{1 - \mu/2} \right) < X_H + \phi + \eta(1 - \delta) &\Leftrightarrow \\ R \left( \frac{1 - \mu + \gamma}{2 - \mu} \right) < \frac{2R(1 + \gamma - \mu) + \delta\eta(1 - \gamma)(2 - \mu)}{4\mu(3 - \gamma)} - \eta\delta, &\quad (\text{A.1}) \end{aligned}$$

where in the second step I used equation (15) to substitute for  $X_H$ . The inequality appears to be quadratic in  $\mu$  but it simplifies to a linear relationship; equation (A.1) is equivalent to

$$\begin{aligned} R(1 + \gamma - \mu)[4 - \mu(3 - \gamma) - 2(2 - \mu)] < \delta\eta(2 - \mu)[2 - \mu - \gamma(2 - \mu) - 4 + \mu(3 - \gamma)] &\Leftrightarrow \\ R(1 + \gamma - \mu)\mu(\gamma - 1) < \delta\eta(2 - \mu)2(\mu - 1 - \gamma) &\Leftrightarrow R\mu(1 - \gamma) > \delta\eta(2 - \mu)2. \end{aligned}$$

Further simplification with respect to  $\mu$  yields expression (26). ■

**Proof of proposition 5.** The expression in the proposition is obtained by equalizing the surplus from lemma 2 (equation (4)) to the surplus from lemma 3 (equation (27)):

$$\frac{2(1 + \eta) + \phi}{1 + \gamma} = \mu \left[ \frac{R - (2 + \eta + \phi)}{2} \right]$$

Solving the above for  $\mu$  gives the result. ■

**Proof of proposition 6.** If  $\delta = 0$ , then the condition for breakdown of downstream uninformed capital markets, as per proposition 4 (equation (26)), is immediately verified for any  $\mu > 0$ . ■

**Proof of proposition 7.** The result follows directly from the definition of  $\phi_0$  in proposition 2. ■

**Proof of proposition 8.** Using equations (30)-(33), the normalized agreement payoffs for the upstream firm and the VC are

$$\begin{cases} -r_{VC,u} & \text{(Upstream firm)} \\ \frac{Dr_d}{4} + r_{VC,u} & \text{(VC)}. \end{cases}$$

According to Nash bargaining, total (normalized) surplus is now shared equitatively, i.e.

$$r_{VC,u} = -\frac{Dr_d}{8}.$$

Using expressions (8), (10), and (15), one can write the above equation as a function of primitives only:

$$\begin{aligned} -\frac{Dr_d}{8} &= -\frac{r_d[X_H + \phi + \eta(1 - \delta)]}{8} = -\frac{r_d}{8} \left[ \frac{R - \delta\eta}{1 + r_d(1 - \mu/2)} \right] = \\ &\left[ \frac{\gamma - 1}{8(1 + \gamma - \mu)} \right] \left[ \frac{R - \delta\eta}{1 + \left( \frac{1-\gamma}{1+\gamma-\mu} \right) (1 - \mu/2)} \right] \end{aligned}$$

Further simplification yields equation (34) in the proposition. ■

**Proof of proposition 9.** The disagreement payoffs are the same as with  $r_u = 0$ ; under the assumption that upstream debt is risky for the VC (i.e. low types default), the agreement payoffs are given by

$$\begin{cases} \frac{1}{2} [X_H - (1 + r_{VC,u})] & \text{(Upstream firm)} \\ \frac{1}{2} \left( r_{VC,u} + D \frac{r_d}{2} \right) + \frac{1}{2} (X_L - 1) & \text{(VC)}. \end{cases}$$

Equating the normalized payoffs we can write

$$\begin{aligned} X_H - 1 - r_{VC,u} - X_H - X_L + 2 &= r_{VC,u} + D \frac{r_d}{2} + X_L - 1 \Leftrightarrow \\ 2 &= 2r_{VC,u} + D \frac{r_d}{2} + 2X_L \Leftrightarrow r_{VC,u} = 1 - X_L - D \frac{r_d}{4}. \end{aligned}$$

Using the expression for debt downstream (8), we can simplify the above equation:

$$r_{VC,u} = 1 - X_L - [X_H + \phi + \eta(1 - \delta)] \frac{r_d}{4}.$$

Replacing  $X_H$  using equation (12), we obtain

$$r_{VC,u} = 1 - X_L - \left[ \frac{X_L - \gamma[R - \eta(1 + r_d)]}{1 - \gamma(1 + r_d)} + \eta \right] \frac{r_d}{4}. \quad (\text{A.2})$$

The condition that low types default is equivalent to  $r_{VC,u} < X_L - 1$ ; combining this with (A.2) we can write

$$\begin{aligned} X_L - 1 &< 1 - X_L - \frac{r_d}{4} \left[ \frac{X_L - \gamma R + \eta}{1 - \gamma(1 + r_d)} \right] \Leftrightarrow \\ X_L &< \frac{8[1 - \gamma(1 + r_d)] - r_d(\eta - \gamma R)}{8[1 - \gamma(1 + r_d)] + r_d} =: \underline{X}_L, \end{aligned}$$

the threshold in the proposition. If the parameters are such that this threshold is met, then in the conjectured equilibrium a high realized return of  $r_{VC,u}$  (no default) happens if and only if the upstream firm is a high type; and this return is naturally higher than the average return for VC's upstream (simple average of  $r_{VC_u}$  and  $X_L - 1$ ). The fact that the return downstream is higher than the competitive return follows directly from the fact that  $r_{VC,d}$  is positive and certain for high types. ■

**Proof of proposition 10.** The surplus for high-type pairs downstream in equation (11) now becomes

$$E[\pi_d | \alpha_i \alpha_{P(i)}] = \mu(R - D) + (1 - \mu)[R - D(1 + r_d)] - \delta\eta.$$

Setting the above expression to zero, and using equation (8) for debt  $D$ , the price of the certified-matching market is given by

$$X_H = \frac{R - \delta\eta}{1 + r_d(1 - \mu)} - \phi - \eta(1 - \delta), \quad (\text{A.3})$$

instead of expression (12). The surplus of low types is still given by equation (13) and accordingly  $X_L$  by expression (14). Using equations (14) and (A.3), the condition for sustainability of the equilibrium,  $X_H \geq X_L$ , can now be written as

$$\begin{aligned} X_H &\geq (X_H + \phi - \delta\eta) [1 - \gamma(1 + r_d)] + \gamma [R - \eta(1 + r_d)] \Leftrightarrow \\ (1 + r_d)\gamma \left[ \frac{R - \delta\eta}{1 + r_d(1 - \mu)} \right] &\geq \phi - \delta\eta + \gamma R \Leftrightarrow \\ r_d [\phi - \delta\eta - \mu(\phi - \delta\eta + \gamma R)] &\leq \delta\eta(1 - \gamma) - \phi. \end{aligned} \quad (\text{A.4})$$

The equilibrium uninformed rate is still given by equation (10). Combining this with (A.4)

the condition becomes

$$(1 - \gamma) [\phi - \delta\eta - \mu(\phi - \delta\eta + \gamma R)] \leq [\delta\eta(1 - \gamma) - \phi] (1 + \gamma - \mu).$$

Solving for  $\mu$  yields the threshold in the proposition. To prove the second statement in the proposition we need to show

$$\frac{2[\phi - \delta\eta(1 - \gamma)]}{\phi(2 - \gamma) + (1 - \gamma)(R\gamma - 2\delta\eta)} \leq \frac{4[\phi - \delta\eta(1 - \gamma)]}{\phi(3 - \gamma) + (1 - \gamma)(R\gamma - 3\delta\eta)} \Leftrightarrow$$

$$\phi(\gamma - 1) - R\gamma(1 - \gamma) - \delta\eta(1 - \gamma) \leq 0,$$

which is always true given that every individual term in the above expression is negative. ■

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