Rivalry with Market, Efficiency (Technical) and Technological Uncertainty in the Adoption of New Technologies

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Abstract

We derive a duopoly pre-emption real options model for investments in new technologies where "market revenues", "efficiency after adoption" ("EAA") and "technological" uncertainty hold simultaneously. We obtain analytical or quasi-analytical solutions for the firms' value functions and investment thresholds for alternative scenarios. Somewhat surprisingly, we find that a relatively low "probability that a second and more efficient technology arrives", turns less relevant, or even negligible, the effect of market revenues and EAA uncertainty on firms' investment behaviour. A negative or moderately positive correlation between "market revenues" and EAA delays only slightly the investment of both firms, but a high positive correlation delays slightly the investment of the leader and significantly the investment of the follower.

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1. Introduction

Azevedo and Paxson (2012) study the effect of "market revenues" and "efficiency after adoption" ("EAA") uncertainty on the timing optimization of the adoption of a new technology for a duopoly market with a first-mover advantage. They argue that "in some circumstances the quality of a new technology becomes apparent only after adoption and so the assumption that a new technology, after adoption, will perform, technically, as the developer/adopter predicts is not appropriate for some investment decisions". Their results suggest that the EAA uncertainty (also called "technical uncertainty") has an asymmetric effect on the firms' investment behaviour, delaying significantly the investment of the follower and only slightly the investment of the leader; the correlation between "market revenues" and EAA has a negligible affect on the adoption time of both firms; and the size of the leader's "first-mover advantage", speeds up slightly the investment of the leader and delays significantly the investment of the follower.

We add to Azevedo and Paxson (2012) model the effect of "technological uncertainty", assuming that at the beginning of the investment game there is one technology available (tech 1) but the "probability that a second and more efficient technology (tech 2) arrives in the future" at a date not known in advance. We study the joint effect of rivalry (through a duopoly pre-emption game), and "market", "technical" and "technological" uncertainty, on firm's investment behaviour. Firms are allowed to invest only once, in tech 1 or in tech 2, and the arrival of tech 2 in the market is assumed to follow a Poisson distribution.

The importance of each of the uncertainties described above depends on the economic context underlying the investment decision. "Market revenues uncertainty" represents the uncertainty of changes in demand, price and competition (for example, income, tastes, and the pricing decisions of competitors can change unpredictably, or a substitute product might arrive making the firm's product suddenly obsolete). "EAA uncertainty" relates to the uncertainty regarding the performance of a technology that persists after being adopted.[3] "Technological uncertainty" is the uncertainty regarding the arrival of new, more efficient and possibly cheaper technologies in the future, whose importance in the timing optimization of investments relates to the evolution and the stage of the development of the industry where technologies are developed. For instance, in the early stages of the development of a new industry, there are a large number of innovations and sometimes the evolution of the industry is not clear. During those times, technological uncertainty reaches a maximum; as the industry matures and the technology standardizes, the rate of innovation tends to decrease and so does technological uncertainty.

[3] For an elaborated description of the economic rationale underlying the EAA variable and empirical evidence of its impact on firms' investment behaviour see Azevedo and Paxson, 2012, pp. 2-3.

For the sake of simplicity we assume that tech 2 is more efficient than tech 1 but costs the same. This means that as soon as tech 2 arrives, if both firms are idle, the adoption of tech 1 is never optimal for both firms. So the investment game resembles a standard leader/follower pre-emption real option game with no technological uncertainty, where firms have one option to invest only. We obtain analytical or quasi-analytical solutions for the value functions and investment thresholds of the leader and the follower for alternative investment scenarios.

Huisman (2001) considers “market revenues” and “technological” uncertainty, but neglects technical uncertainty. He shows that for a duopoly pre-emption game, “the optimal investment timing for both firms is governed, to a large extent, by the magnitude of the probability that a second technology becomes available within a given time period”. Technological uncertainty is also considered in Grenadier and Weiss (1997), but as a state variable that follows a gBm process (although technological progress is typically governed by random moves in one direction only, i.e., technological declines are not common). Our model has also some similarities with that of Paxson and Pinto (2005) in the sense that both use multi-stochastic variables, however, they focus on “market revenues” uncertainty only.

For monopolistic markets, Smith (2005) studies the effect of revenues and investment cost uncertainty on the adoption of two complementarity technologies and Murto (2007) the effect of revenues and technological uncertainty on the adoption of a new technology. Azevedo and Paxson (2011b) extend Smith (2005) to a duopoly pre-emption game. Hoppe (2002) provides a good survey about the literature on new technology adoption models. Coelli, et al. (1998) provides a good introduction about efficiency and productivity analysis. For an extensive and comprehensive literature review on real option game models see Azevedo and Paxson (2011a).

Our analytical derivations are organized in two main sections. In the first, we characterize the scenario where tech 2 is available (i.e., technological uncertainty is absent) and assume, in one case, that when tech 2 arrives the leader is active with tech 1 and the follower is idle, and in another, that when tech 2 arrives both firms are idle. In the second, we characterize the scenario where market, technical and technological uncertainty hold simultaneously, assuming that at the beginning of the investment game the leader is active with tech 1 and the follower is inactive, in one case, “committed” to the adoption of tech 1 and, in another, “committed” to the adoption of tech 2. We derive the firms’ value function and investment thresholds for all these scenarios.

To our knowledge, this is the first real option game model studying the combined effect of rivalry (through a duopoly pre-emption game) with market, technical and technological uncertainty. Due to the high number of market variables and investment scenarios underlying our model, to avoid unnecessary complexity, in section 3 we focus our comments on the most relevant results only. However, other alternative and also relevant analysis can also be provided. In Absence of technological uncertainty our results are in line with those of Azevedo and Paxson (2012). When we consider the join effect of market, technical and technological uncertainty we find that, somewhat surprisingly, a relatively low "probability that a second technology arrives" (technological uncertainty), can turn less relevant or even negligible the effect of market and technical uncertainty on firms' investment behaviour.

The paper is organized as follows. In section 2, we introduce the duopoly pre-emption game, describe the fundamental assumptions underlying the model and derive the firms' value functions and investment thresholds. In section 3, we do some sensitivity analysis and comment the most relevant results. In section 4, we conclude and give some suggestions for further research.

2. The Model

In games of timing the adoption of new technologies, the potential advantage of being the first to adopt may introduce an incentive for pre-empting the rival, speeding up the investment. Reinganum (1981) developed a deterministic game-theoretic approach, where the adoption of one firm is assumed to have a negative effect on the profits of the other firm, and the increase in profits due to the adoption is assumed to be greater for the leader than for the follower. Fudenberg and Tirole (1985) studied the adoption of a new technology using a deterministic framework and illustrate the effects of pre-emption in games of time. We use the Fudenberg and Tirole (1985) principle of "rent equalization" in our derivations (see Azevedo and Paxson 2011a, pp. 16-17, for an illustration about how this principle works in practice).

Suppose that there are two idle firms, *i* and *j*, considering the adoption of a new technology in a context where there is "market revenues" and "EAA" uncertainty. At the beginning of the investment game there is only one technology available (tech 1) but the probability that a second and more efficient technology (tech 2) arriving in the market at a date not known in advance. Tech 2 is more efficient than tech 1 and costs the same, i.e., $I_1 = I_2$, where I_1 is the cost of tech 1 and I_2 the cost of tech 2; firms can invest only once, in tech 1 or tech 2 ("one-shot" game); and the firm that adopts first (tech 1 or tech 2) gets a "first-mover market share advantage" ("FMA"). For the sake of simplicity, without losing any insight, firms are not allowed to invest at the same time.

Let $X(t)$ be the "market revenues" and $E_k(t)$ the EAA at a given (continuous) time *t*, with $k=\{0,1,2\}$, where "0", "1" and "2" means that the firm is "not active", operating with "tech 1", or operating with "tech 2", respectively, and $E_k(t) \in [0,1)$, where the lower limit represents a "catastrophic scenario", which occurs when after adoption the technology fails completely (operates with zero per cent efficiency), and the upper limit represents the (unlike) "perfect scenario", where after adoption the technology operates with 100 per cent efficiency. Between these two extreme scenarios there are, theoretically, an infinite number of other feasible scenarios.[4] $X(t)$ is expressed in monetary units and $E_k(t)$ is a dimensionless variable.

Both variables follow gBm processes given, respectively, by equations (1) and (2): [5]

$$dX = \mu_X Xdt + \sigma_X Xdz \tag{1}$$

$$dE_k = \mu_{E_k} Edt + \sigma_{E_k} Edz_k \tag{2}$$

where, μ_X and μ_{E_k} are the instantaneous conditional expected percentage changes in X and E_k per unit of time, respectively; σ_X and σ_{E_k} are the instantaneous conditional standard deviation of X_t and E_k per unit of time, respectively; and dz and dz_k are the increment of a standard Wiener process for X and E_k, respectively, with k defined previously. For convergence of the solution we assume that $r - \mu_X - \mu_{E_k} > 0$, where *r* is the riskless interest rate.

The arrival date of tech 2 is governed by a Poisson distribution with parameter λ and mean $1/\lambda\ (>0)$ defined by expression (3).

$$d\theta = \begin{cases} 1 & \text{with probability} \quad \lambda \\ 0 & \text{with probability} \quad 1\text{-}\lambda \end{cases} \tag{3}$$

where, $d\theta$ is the probability of arrival of tech 2.

The firm i's revenues flow is given by:

$$\varphi_k de_{k_i k_j} \tag{4}$$

where, φ_k is the technology *k* "efficiency weighted revenues" ("EWR"), given by $\varphi_k = (X)(E_k)$; $de_{k_i k_j}$ is a deterministic competition factor that represents the "proportion of the market revenues assigned to firm i for each investment scenario", which ensures a FMA through inequalities 5 and 6;

[4] In this section, $E_k(t)$ is defined as following a geometric Brownian (gBm) process. Consequently, the domain $E_k(t) \in [0,1)$ above seems inconsistent with a gBm. However, in reality, for most technologies/production processes, 100% (daily/monthly/annually) efficiency is rarely achieved.

[5] For simplicity of notation from hereafter we drop the subscript "t" and in some instance the sub-subscript "k".

$i, j = \{L, F\}$, where L means "leader" and F "follower".[6] The intuition underlying the leader's FMA is the same as that used in Dixit and Pindyck (1994) following Smets (1993).

Let $t = \tilde{\tau}$ be the arrival time of tech 2. For the leader, for $t < \tilde{\tau}$, inequality (5) holds:

$$de_{1_L 0_F} > de_{1_L 1_F} > de_{0_L 0_F} \tag{5}$$

The economic intuition for the inequality above is that, in terms of market share, for the leader, the best scenario, is when it operates with tech 1 alone ($de_{1_L 0_F}$); the second best scenario is when it adopts tech 1 and the follower does so later ($de_{1_L 1_F}$); the worst scenario is when it is idle with the follower ($de_{0_L 0_F}$).

For $t \geq \tilde{\tau}$ (after the arrival of tech 2) several other scenario are considered, depending on whether firms are active (with tech 1) or inactive when tech 2 arrives. There are three scenarios: (i) both firms are active with tech 1. In this case the arrival of tech 2 does not affect their investment behaviour due to the "one-shot" nature of the investment game; (ii) both firms are inactive. In this case, the adoption of tech 1 is never optimal since tech 2 is more efficient than tech 1 and cost the same (i.e., firms would behave as if they had only one technology available -tech 2); (iii) the leader is active with tech 1 and the follower is inactive. In this case the game is ended for the leader and the follower optimizes the adoption of tech 2 as if tech 2 was the unique technology available. Inequality (6) ensures the leader's FMA for these scenarios:

$$(de_{2_L 0_F} = de_{1_L 0_F}) > (de_{2_L 2_F} = de_{1_L 1_F}) > de_{1_L 2_F} \tag{6}$$

The economic intuition for the inequality above is that, in terms of market share, for the leader, the best scenario is when it operates alone with tech 1 or tech 2, ($de_{2_L 0_F} = de_{1_L 0_F}$); the second best scenario is when it adopts tech 1 or tech 2 and the follower does so latter, but firms operate with the same technologies ("technology-symmetry"), ($de_{2_L 2_F} = de_{1_L 1_F}$); the worst scenario is when it adopts tech 1 (before tech 2 being available), and tech 2 arrives immediately after being then adopted by the follower ($de_{1_L 2_F}$), i.e., firms operate with different technologies ("technology-asymmetry").

[6] To get the intuition about how the multiplicative form $\varphi_k = (X)(E_k)$ works in practice, suppose that firm i adopts tech 1 first, becoming the leader, and firm j adopts tech 1 later, becoming the follower; the market revenues are 100 million ($X = 100$), the EAA is 100 per cent ($E_1 = 1$), and after the follower adoption the market shares of the leader and the follower, as a proportion of the market revenues, are 60 and 40 per cent, respectively. In this case, the market EWR are $\varphi_1 = (X)(E_1) = 100(1.0) = 100$ million, and the competition factors are $de_{1_L 1_F} = 0.6$, for the leader, and $de_{1_F 1_L} = 0.4$, for the follower. In terms of firms' EWR (Eq. 4) this leads to 60 million ($\varphi_1 de_{1_L 1_F} = 100(0.60) = 60$), for the leader, and 40 million ($\varphi_1 de_{1_F 1_L} = 100(0.40) = 40$), for the follower, a FMA of $60 - 40 = 20$ million (see Azevedo and Paxson, 2012, p. 5 for more details).

The follower's market share is a complement of the leader's, i.e., $de_{k_F k_L} = (1 - de_{k_L k_F})$, so the following inequality holds for the follower.

$$de_{2_F 1_L} > (de_{2_F 2_L} = de_{1_F 1_L}) > (de_{2_L 0_F} = de_{1_L 0_F}) \quad (7)$$

In section 3, we use the following base inputs for our illustrative sensitivity analysis:

		Investment Scenario (Competition Factors: Proportion of the market EWR)					
		Leader, tech 1 Follower, inactive	Leader, tech 2 Follower, inactive	Leader, tech 1 Follower, tech 2	Leader, tech 1 Follower, tech 1	Leader, tech 2 Follower, tech 2	Both firms inactive
Leader's Market Share		$de_{1_L 0_F}$	$de_{2_L 0_F}$	$de_{1_L 2_F}$	$de_{1_L 1_F}$	$de_{2_L 2_F}$	$de_{0_L 0_F}$
		1.0	**1.0**	**0.55**	**0.70**	**0.70**	**0.0**
Follower's Market Share		$de_{0_F 1_L}$	$de_{0_F 2_L}$	$de_{2_F 1_L}$	$de_{1_F 1_L}$	$de_{2_F 2_L}$	$de_{0_F 0_L}$
		0.0	**0.0**	**0.45**	**0.30**	**0.30**	**0.0**
Total Market Share		**1.0**	**1.0**	**1.0**	**1.0**	**1.0**	**0.0**

Table 1: Deterministic Competition Factors

Inequalities (5), (6) and (7), and the information in Table 1, ensure the following conditions: (i) when the leader is active alone it gets 100% of the market share; (ii) at the instant the follower becomes active with tech 1, being the leader active with tech 1, it gets 30% of the market share (i.e., the leader's market share drops from 100% to 70%; (iii) at the instant the follower becomes active with tech 2, being the leader active with tech 1, it gets 45% of the market share (i.e., the leader's market share drops from 100% to 55%; iv) when both firms are active, the leader's market share is higher than the follower's market share, due to the FMA, but the leader's FMA is lower when it operates with the follower in a "technology-asymmetry" (the leader with tech 1 and the follower with tech 2) than when it operates in a "technology-symmetry" (both firms with the same technology, tech 1 or tech 2). [7] Our assumption is that the follower benefits in terms of market share from operating with a more efficient technology but not to the point of offsetting completely the leader's FMA; (v) when firms are active, the sum of their market shares is 100 per cent and when they are inactive their market shares are null.

Notice that the relation between the competition factors above, inequalities (5), (6) and (7), and the information in Table 1, should not be seen as static applying to all duopoly markets, technologies and industries, but as leader/follower (ex-post) market share relations that need to be studied for each investment decision.

[7] Notice that the scenario where the leader adopts tech 2 and the follower adopts tech 1 is impossible, given that as soon as tech 2 is available the adoption of tech 1 is never optimal for both firms.

2.1 Technology 2 is Available, $t \geq \tilde{\tau}$

As soon as tech 2 is released technological uncertainty disappears. We assume that tech 1 and tech 2 cost the same and tech 2 is more efficient than tech 1. Therefore, for $t \geq \tilde{\tau}$ it is never optimal to adopt tech 1 for both firms. In this section we study two scenarios for when tech 2 arrives: (i) both firms are idle, and (ii) the leader is active with tech 1 and the follower is idle. Figures 2 and 3 below illustrate the investment thresholds for both cases.

(i) **Scenario 1**: tech 2 arrives with both firms idle

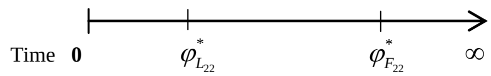

Figure 2 –Firms' investment thresholds

Where, $\varphi^*_{L_{22}}$ and $\varphi^*_{F_{22}}$ are the leader's and the follower's investment thresholds to adopt tech 2, respectively.

(ii) **Scenario 2**: tech 2 arrives with the leader active with tech 1 and the follower idle

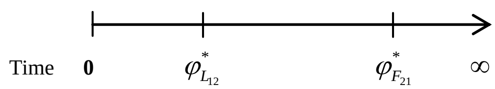

Figure 3 –Firms' investment thresholds

Where, $\varphi^*_{L_{12}}$ is the leader's investment threshold to adopt tech 1 assuming that the follower adopts, later, tech 2 when it arrives and $\varphi^*_{F_{21}}$ is reached. $\varphi^*_{F_{21}}$ is the follower's investment threshold to adopt tech 2 when the leader is active with tech 1.

This is a "one-shot" investment game so the game ends for both firms as soon as they adopt one of the technologies. Consequently, for scenario (2) above, the arrival of tech 2 affects the value of the investment of the active (operating with tech 1) leader but not its investment behaviour since its game is ended.

In the next subsections, the methodology used to derive the firms' value functions and investment thresholds is based on a "real option-backward induction" framework. We derive first the value function and investment threshold of the follower (end of the game) and work then backward to derive the value function and investment threshold of the leader, assuming that the follower invests as our (previous) derivations suggests. For each firm and investment scenario we use the adequate "value matching" and "smooth-past" conditions and, in some instances, the knowledge that the firms' value functions are continuous and differentiable at the point where firms invest. For the investment threshold of the leader we use the Fudenberg and Tirole (1985) "principle of rent equalization", which provides the information that, in a leader/follower pre-emption game, the leader invest at the point where the rents of both firms equalise (i.e., the firms' value functions cross).

2.1.1 Follower: Scenario 1 *(tech 2 arrives with both firms idle)*

Let $F_{F_{22}}(X,E_2)$ be the follower's option value to adopt tech 2 for a context where technological uncertainty is absent (tech 2 is available) and the leader is active with tech 2. Setting the returns on the option equal to the expected capital gain on the option and using Ito's lemma, we obtain the partial differential equation (8) for the value function of the follower for the region where it waits to adopt tech 2.

$$\frac{1}{2}\frac{\partial^2 F_{F_{22}}}{\partial X^2}\sigma_x{}^2X^2+\frac{1}{2}\frac{\partial^2 F_{F_{22}}}{\partial E^2}\sigma_E{}^2E^2+\frac{\partial^2 F_{F_{22}}}{\partial X\partial E}XE\sigma_x\sigma_E\rho_{XE}+\frac{\partial F_{F_{22}}}{\partial X}\mu_x X+\frac{\partial F_{F_{22}}}{\partial E}\mu_E E-rF_{F_{22}}=0 \tag{8}$$

Using similarity methods[8] we can find an explicit closed-form solution for X and E, using the following change in the variables: $\varphi_2=(X)(E_2)$. Doing the respective substitutions in Equation (8) we get the Ordinary Differential Equation (ODE) (9), which describes the follower's option value as a function of φ_2 (for the sake of simplicity and when confusion is not possible, we drop the subscript "2").

$$\frac{1}{2}\varphi^2\sigma_m{}^2\frac{\partial^2 F_{F_{22}}(\varphi)}{\partial\varphi^2}+\varphi\left(\sigma_X\sigma_E\rho_{XE}+\mu_X+\mu_E\right)\frac{\partial F_{F_{22}}(\varphi)}{\partial\varphi}-rF_{F_{22}}(\varphi)=0 \tag{9}$$

where, $\sigma_m^2=\sigma_X{}^2+\sigma_E{}^2+2\rho_{XE}\sigma_X\sigma_E$.

See derivation in Appendix A, section 1.

The ODE (9) has an analytical solution, whose general form is given by,

$$F_{F_{22}}(\varphi)=A\varphi^{\beta_1}+B\varphi^{\beta_2} \tag{10}$$

where, A and B are constants to be determined, using the boundary conditions ("value-matching" and "smooth-pasting"), and β_1 and β_2 are the roots of a characteristic quadratic function of an Euler's type ordinary differential equation given by,

$$0.5\sigma_m^2\beta(\beta-1)+(\rho\sigma_X\sigma_E+\mu_X+\mu_E)\beta-r=0 \tag{11}$$

Solving the equation above for β we get two roots, one positive, β_1, and one negative, β_2, given by:

$$\beta_{1(2)}=\frac{0.5\sigma_m^2-(\rho_{XE}\sigma_X\sigma_E+\mu_X+\mu_E)+(-)\sqrt{\left(-0.5\sigma_m^2+\rho_{XE}\sigma_X\sigma_E+\mu_X+\mu_E\right)^2+2r\sigma_m^2}}{\sigma_m^2} \tag{12}$$

[8] For a detailed discussion about similarity methods see Bluman and Cole (1974).

To find the follower's value ($F_{F_{22}}$) and investment threshold ($\varphi^*_{F_{22}}$) the following boundary conditions apply to Equation (10):

$$F_{F_{22}}(\varphi^*_{F_{22}}) = \frac{\varphi^*_{F_{22}} de_{2_F 2_L}}{r - \mu_X - \mu_E} - I_2 \tag{13}$$

$$\frac{\partial}{\partial \varphi} F_{F_{22}}(\varphi^*_{F_{22}}) = \frac{de_{2_F 2_L}}{r - \mu_X - \mu_E} \tag{14}$$

$$F_{F_{22}}(0) = 0 \tag{15}$$

Conditions (13) and (14) are the "value-matching" and "smooth-pasting" conditions, respectively, and ensure continuity and differentiability of the value function at the investment threshold. Condition (15) ensures that the option value is worthless at the absorbing barrier $\varphi = 0$. Consequently, in equation (10) $B = 0$. Solving together Equations (10) and (13)-(15) after some algebraic manipulation yields the follower's investment threshold and value function, given by Equation (16) and Expression (17), respectively.

$$\varphi^*_{F_{22}} = \frac{\beta_1}{\beta_1 - 1} \frac{(r - \mu_X - \mu_E)}{de_{2_F 2_L}} I_2 \tag{16}$$

With β_1 given by expression (12). The follower's value function is given by,

$$F_{F_{22}}(\varphi) = \begin{cases} A\varphi^{\beta_1} & \varphi < \varphi^*_{F_{22}} \\ \dfrac{\varphi de_{2_F 2_L}}{r - \mu_X - \mu_E} - I_2 & \varphi \geq \varphi^*_{F_{22}} \end{cases} \tag{17}$$

With

$$A = \frac{de_{2_F 2_L}}{r - \mu_X - \mu_E} \frac{{\varphi^*_{F_{22}}}^{(1-\beta_1)}}{\beta_1} \tag{18}$$

2.1.2 Leader: Scenario 1 *(tech 2 arrives with both firms idle)*

Assuming that the follower adopts tech 2 as soon as $\varphi^*_{F_{22}}$ is reached, the leader's payoff is given by,

$$E\left[\int_{\tau}^{T_{2_F}} \varphi de_{2_L 0_F} e^{-r\tau} d\tau - I_2 + \int_{T_{2_F}}^{\infty} \varphi^*_{F_{22}} de_{2_L 2_F} e^{-r\tau} d\tau\right] \tag{19}$$

The first integral represents the leader's payoff for the period where it is alone in the market, where τ is the leader's adoption time of tech 2 and T_{2_F} is the follower's adoption time of tech 2; the second integral is the leader's payoff for the period where both firms are active with tech 2; I_2 is the investment cost in tech 2. Applying the methodology used in Dixit and Pindyck (1994), pp. 309-315, we get the expression (20) for the leader's value function.

$$F_{L_{22}}(\varphi)=\begin{cases}\dfrac{\varphi de_{2_L0_F}}{r-\mu_X-\mu_E}-I_2+\dfrac{\varphi(de_{2_L2_F}-de_{2_L0_F})}{r-\mu_X-\mu_E}\left(\dfrac{\varphi}{\varphi^*_{F_{22}}}\right)^{\beta_1} & \varphi<\varphi^*_{F_{22}}\\ \dfrac{\varphi de_{2_L2_F}}{r-\mu_X-\mu_E} & \varphi\geq\varphi^*_{F_{22}}\end{cases} \tag{20}$$

where, $\frac{\varphi de_{2_L0_F}}{r-\mu_X-\mu_E}-I_2$ is the leader's payoff at the instant it adopts tech 2 if it operates alone forever; $\frac{\varphi(de_{2_L2_F}-de_{2_L0_F})}{r-\mu_X-\mu_E}\left(\frac{\varphi}{\varphi^*_{F_{22}}}\right)^{\beta_1}$ is derived using the continuity condition of $F_{L_{22}}(\varphi)$ at $\varphi^*_{F_{22}}$, it is negative given that $(de_{2_L2_F}-ds_{2_L0_F})<0$ (see inequality 6) and corresponds to the correction factor that incorporates the fact that in the future if $\varphi^*_{F_{22}}$ is reached the follower will adopt tech 2 and the leader's profits will be reduced; [9] $\frac{\varphi de_{2_L2_F}}{r-\mu_X-\mu_E}$ is the leader's payoff when active with the follower, both with tech 2, from $\varphi^*_{F_{22}}$ until infinity.

This is a pre-emption game where the Fudenberg and Tirole (1985) principle of rent equalization applies. Hence, the leader adopts tech 2 at the point where the value functions of both firms cross. Consequently, equalizing Equations (17) and (20), for $\varphi<\varphi^*_{F_{22}}$, we get Equation (21).

$$\frac{\varphi de_{2_L0_F}}{r-\mu_X-\mu_E}-I_2+\frac{\varphi(de_{2_L2_F}-de_{2_L0_F})}{r-\mu_X-\mu_E}\left(\frac{\varphi}{\varphi^*_{F_{22}}}\right)^{\beta_1}-A\varphi^{\beta_1}=0 \tag{21}$$

Replacing in Equation (21) φ by $\varphi^*_{L_{22}}$ and using standard numerical methods to solve for $\varphi^*_{L_{22}}$ we get the leader's investment threshold.

2.1.3 Follower: Scenario 2 *(tech 2 arrives with the leader active with tech 1 and the follower idle)*

Let $F_{F_{21}}$ be the value function of an idle follower for the context where tech 2 is available and the leader is active with tech 1. Replacing in ODE (9) $F_{F_{22}}$ by $F_{F_{21}}$ we get ODE (22),

$$\frac{1}{2}\varphi^2\sigma_m{}^2\frac{\partial^2F_{F_{21}}(\varphi)}{\partial\varphi^2}+\varphi\left(\sigma_X\sigma_E\rho_{XE}+\mu_X+\mu_E\right)\frac{\partial F_{F_{21}}(\varphi)}{\partial\varphi}-rF_{F_{21}}(\varphi)=0 \tag{22}$$

where, $\sigma_m^2=\sigma_X{}^2+\sigma_E{}^2+2\rho_{XE}\sigma_X\sigma_E$.

The differential Equation (22) has an analytical solution, whose general form is given by,

$$F_{F_{21}}(\varphi)=C\varphi^{\psi_1}+D\varphi^{\psi_2} \tag{23}$$

[9] This term equals the leader's loss discounted back from the (random) time at which the follower adopt tech 2. The term $(\varphi/\varphi^*_{2_F})^{\beta_1}$ is interpreted as a stochastic discount factor equal to the present value of $1 received when the variable φ hits $\varphi^*_{F_{22}}$ (see Pawlina and Kort, 2006, p. 10).

where, C and D are constants to be determined, using the "value-matching" and "smooth-pasting" conditions, and ψ_1 and ψ_2 are the roots of a characteristic quadratic function of an Euler's type ordinary differential equation given by,

$$0.5\sigma_m^2\psi(\psi-1)+(\rho\sigma_X\sigma_E+\mu_X+\mu_E)\psi-r=0 \tag{24}$$

Solving the equation above for ψ we get two roots, one positive, ψ_1, and one negative, ψ_2, given by:

$$\psi_{1(2)}=\frac{0.5\sigma_m^2-(\rho_{XE}\sigma_X\sigma_E+\mu_X+\mu_E)+(-)\sqrt{\left(-0.5\sigma_m^2+\rho_{XE}\sigma_X\sigma_E+\mu_X+\mu_E\right)^2+2r\sigma_m^2}}{\sigma_m^2} \tag{25}$$

In this case the following boundary conditions apply to Equation (23):

$$F_{F_{21}}(\varphi_{F_{21}}^*)=\frac{\varphi_{F_{21}}^* de_{2_F 1_L}}{r-\mu_X-\mu_X}-I_2 \tag{26}$$

$$\frac{\partial}{\partial\varphi}F_{F_{21}}(\varphi_{F_{21}}^*)=\frac{de_{2_F 1_L}}{r-\mu_X-\mu_E} \tag{27}$$

$$F_{F_{21}}(0)=0 \tag{28}$$

Condition (28) ensures that the option value is worthless at the absorbing barrier $\varphi=0$. Consequently, in equation (23), $D=0$. Solving together Equations (23) and (26)-(28) after some algebraic manipulation yields the follower's investment threshold and value function, given by Equation (29) and Expression (30), respectively.

$$\varphi_{F_{21}}^*=\frac{\psi_1}{\psi_1-1}\frac{(r-\mu_X-\mu_E)}{de_{2_F 1_L}}I_2 \tag{29}$$

With ψ_1 given by expression (25). The follower's value function is given by,

$$F_{F_{21}}(\varphi)=\begin{cases} C\varphi^{\psi_1} & \varphi<\varphi_{F_{21}}^* \\ \dfrac{\varphi de_{2_F 1_L}}{r-\mu_X-\mu_E}-I_2 & \varphi\geq\varphi_{F_{21}}^* \end{cases} \tag{30}$$

With

$$C=\frac{de_{2_F 1_L}}{r-\mu_X-\mu_E}\frac{\varphi_{F_{21}}^{*\ (1-\psi_1)}}{\psi_1} \tag{31}$$

2.1.4 Leader: Scenario 2 *(tech 2 arrives with the leader active with tech 1 and the follower idle)*

Assuming that the follower adopts tech 2 as soon as $\varphi_{F_{21}}^*$ is reached, the leader's payoff is given by,

$$E\left[\int_{\tau}^{T_{2_F}}\varphi de_{1_L 0_F}e^{-r\tau}d\tau-I_1+\int_{T_{2_F}}^{\infty}\varphi_{F_{21}}^* de_{1_L 2_F}e^{-r\tau}d\tau\right] \tag{32}$$

The first integral represents the leader's payoff for period where it is alone in the market, where τ is the leader's adoption time of tech 1 and T_{2_F} is the follower's adoption time of tech 2; the second integral is the leader's payoff for the period where both firms are active (the leader with tech 1 and the follower with tech 2); I_1 is the investment cost in tech 1.

Proceeding as for (20), we get the expression (33) for the leader's value function.

$$F_{L_{12}}(\varphi)=\begin{cases}\dfrac{\varphi de_{1_L0_F}}{r-\mu_X-\mu_E}-I_1+\dfrac{\varphi(de_{1_L2_F}-de_{1_L0_F})}{r-\mu_X-\mu_E}\left(\dfrac{\varphi}{\varphi^*_{F_{21}}}\right)^{\psi_1} & \varphi<\varphi^*_{F_{21}}\\[2ex] \dfrac{\varphi de_{1_L2_F}}{r-\mu_X-\mu_E} & \varphi\geq\varphi^*_{F_{21}}\end{cases} \tag{33}$$

where, $\frac{\varphi de_{1_L0_F}}{r-\mu_X-\mu_E}-I_1$ corresponds to the leader's payoff at the instant it adopts tech 1, if it operates alone forever; $\frac{\varphi(de_{1_L2_F}-de_{1_L0_F})}{r-\mu_X-\mu_E}\left(\frac{\varphi}{\varphi^*_{F_{21}}}\right)^{\psi_1}$ is derived using the continuity condition of $F_{L_{12}}(\varphi)$ at $\varphi^*_{F_{21}}$, is negative given that $(de_{1_L2_F}-ds_{1_L0_F})<0$ (see inequality 6), and corresponds to the correction factor that incorporates the fact that in the future if $\varphi^*_{F_{21}}$ is reached the follower will adopt tech 2 and the leader's profits will be reduced; $\frac{\varphi de_{1_L2_F}}{r-\mu_X-\mu_E}$ is the leader's payoff if both firms are active (the leader with tech 1 and the follower with tech 2), from $\varphi^*_{F_{21}}$ until infinity.

The leader's investment threshold is determined by equalizing Equations (30) and (33), for $\varphi<\varphi^*_{F_{21}}$, leading to Equation (34),

$$\frac{\varphi de_{1_L0_F}}{r-\mu_X-\mu_E}-I_1+\frac{\varphi(de_{1_L2_F}-de_{1_L0_F})}{r-\mu_X-\mu_E}\left(\frac{\varphi}{\varphi^*_{F_{21}}}\right)^{\psi_1}-C\varphi^{\psi_1}=0 \tag{34}$$

Replacing in Equation (34) φ by $\varphi^*_{L_{12}}$ and solving in order to $\varphi^*_{L_{12}}$ we get the leader's investment threshold.

2.2 Technology 2 is not available, $t<\tilde{\tau}$

We consider the following scenarios for the beginning of the investment game: (i) the leader is active with tech 1 and the follower is idle "committed" to the adoption of tech 2 (i.e., the follower adopts tech 2 when it arrives and the investment threshold is reached); (ii) the leader is active with tech 1 and the follower is idle "committed" to the adoption of tech 1 (i.e., the follower's threshold to adopts tech 1 is reached before tech 2 arrives). [10]

(i) Scenario 1: Leader active with tech 1 and follower "committed" to the adoption of tech 2

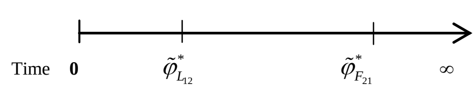

Figure 4 – Firms' Investment Threshold

[10] Notice that the adoption of tech 2 by the follower is conditioned on its arrival (not known in advance) and the investment threshold reached. Figures 4 and 5 illustrate the investment thresholds for both cases.

Where, $\tilde{\varphi}^*_{L_{12}}$ is the leader's threshold to adopt tech 1 assuming that the follower adopts, later, tech 2 when $\tilde{\varphi}^*_{F_{21}}$ is reached; $\tilde{\varphi}^*_{F_{21}}$ is the follower's thresholds to adopt tech 2 when the leader is active with tech 1.

(ii) Scenario 2: Leader active with tech 1 and follower "committed" to the adoption of tech 1

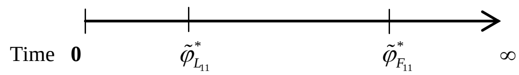

Figure 5 - Firms' Investment Thresholds

Where, $\tilde{\varphi}^*_{L_{11}}$ is the leader's threshold to adopt tech 1 assuming that the follower adopts, later, tech 1 when $\tilde{\varphi}^*_{F_{11}}$ is reached; $\tilde{\varphi}^*_{F_{11}}$ is the follower's threshold to adopt tech 1 when the leader is active with tech 1.

2.2.1 Follower: Scenario 1 *(leader active with tech 1 and follower "committed" to the adoption of tech 2)*

Let assume that the leader is active with tech 1 and the follower is inactive considering the adoption of (the not yet available) tech 2. In this context condition (35) is satisfied for the follower:

$$r\tilde{F}_{F_{21}}(\varphi) = \lim_{dt\to 0}\frac{1}{dt}E\left[d\tilde{F}_{F_{21}}(\varphi)\right] \tag{35}$$

Where $\tilde{F}_{F_{21}}(\varphi)$ is the follower's value function for the scenario (1) above.

Considering technological uncertainty through expression (3) and applying Ito's Lemma to equation (35) yields,

$$E\left[d\tilde{F}_{F_{21}}(\varphi)\right] = (1-\lambda dt)\left(\frac{1}{2}\sigma_m^2\varphi^2\frac{\partial^2\tilde{F}_{F_{21}}(\varphi)}{\partial\varphi^2}dt + \left(\sigma_X\sigma_X\rho+\mu_X+\mu_E\right)\varphi\frac{\partial\tilde{F}_{F_{21}}(\varphi)}{\partial\varphi}dt\right) + \lambda dt\left(F_{F_{21}}(\varphi)-\tilde{F}_{F_{21}}(\varphi)\right) \tag{36}$$

Notice that in (36) the first term represents the follower's value for the region where tech 2 is not available and the second term represents the follower's value for the region where tech 2 is available. Substituting expression (36) into equation (35) we obtain,

$$\frac{1}{2}\sigma_m^2\varphi^2\frac{\partial^2\tilde{F}_{F_{21}}(\varphi)}{\partial\varphi^2} + \left(\sigma_X\sigma_E\rho+\mu_X+\mu_E\right)\varphi\frac{\partial\tilde{F}_{F_{21}}(\varphi)}{\partial\varphi} - (r+\lambda)\tilde{F}_{F_{21}}(\varphi) + \lambda F_{F_{21}}(\varphi) = 0 \tag{37}$$

Using the two possible expressions for $F_{F_{21}}(\varphi)$ (see Equation 30), we get the following solution:

$$\tilde{F}_{F_{21}}(\varphi) = \begin{cases} Y\varphi^{\beta_3} + A\varphi^{\beta_1} & \varphi < \tilde{\varphi}^*_{F_{21}} \\ W\varphi^{\beta_4} + \dfrac{\varphi de_{2_F1_L}}{(r-\mu_X-\mu_E)}\dfrac{\lambda}{(r-\mu_X-\mu_E+\lambda)} - \dfrac{\lambda I_2}{r+\lambda} & \varphi \geq \tilde{\varphi}^*_{F_{21}} \end{cases} \tag{38}$$

Where, β_1 and the constant A are given by Equations (12) and (18), respectively, and the constants Y and W are given by Equations (39) and (40), respectively -derived by solving the continuity and differentiability conditions for $\tilde{F}_{F_{21}}(\varphi)$ at $\varphi = \tilde{\varphi}^*_{F_{21}}$ (see derivation in Appendix A, section 2.1):

$$Y = \frac{\left(\tilde{\varphi}^*_{F_{21}}\right)^{-\beta_3}\left[r\left(r-\mu_X-\mu_E\right)\beta_4 + \left(r-(\mu_X+\mu_E)\beta_1\right)\lambda\beta_4 - \left(r-\mu_X-\mu_E\right)(r+\lambda)\beta_1\right]I_2}{(r+\lambda)\left(r-\mu_X-\mu_E+\lambda\right)(\beta_1-1)(\beta_3-\beta_4)} \tag{39}$$

$$W=\frac{\left(\tilde{\varphi}^{*}_{F_{21}}\right)^{-\beta_4}\left[r\left(r-\mu_X-\mu_E\right)\beta_3+\left(r-\left(\mu_X+\mu_E\right)\beta_1\right)\lambda\beta_3-\left(r-\mu_X-\mu_E\right)\left(r+\lambda\right)\beta_1\right]I_2}{\left(r+\lambda\right)\left(r-\mu_X-\mu_E+\lambda\right)\left(\beta_1-1\right)\left(\beta_3-\beta_4\right)} \tag{40}$$

where, $Y<0$ and $W>0$ (see proof in Appendix A, sections 2.2 and 2.3).

In expression (38), in the first row, $A\varphi^{\beta_1}$ is the option value to adopt tech 2, with the constant A given by Equation (18); $Y\varphi^{\beta_3}$ is a negative correction factor to reflect the fact that $\tilde{\varphi}^{*}_{F_{21}}$ can be reached with tech 2 not available, with the constant Y given by Equation (39); in the second row, $W\varphi^{\beta_4}$ is the option value to adopt tech 2, with the constant W given by Equation (40). Notice that if the threshold to adopt tech 2 is reached and tech 2 is not available, the follower still holds the option to adopt tech 2; $\frac{\varphi de_{2_F1_L}}{(r-\mu_X-\mu_E)}\frac{\lambda}{(r-\mu_X-\mu_E+\lambda)}$ is the expected present value of the follower's payoff from the adoption of tech 2 until infinity and reflects the fact that, for $\varphi\geq\tilde{\varphi}^{*}_{F_{21}}$, the value of the adoption of tech 2 is conditioned on its arrival; $\frac{\lambda I_2}{r+\lambda}$ is the expected present value of the follower's investment cost incurred at the time tech 2 is adopted (see derivation Appendix A, section 3).

The variables β_3 and β_4 are, respectively, the positive and the negative roots of the quadratic equation (41), given by equation (42):

$$0.5\sigma_m^2\beta(\beta-1)+(\rho\sigma_X\sigma_E+\mu_X+\mu_E)\beta-(r+\lambda)=0 \tag{41}$$

$$\beta_{3(4)}=\frac{0.5\sigma_m^2-(\rho_{XE}\sigma_X\sigma_E+\mu_X+\mu_E)+(-)\sqrt{\left(-0.5\sigma_m^2+\rho_{XE}\sigma_X\sigma_E+\mu_X+\mu_E\right)^2+2(r-\lambda)\sigma_m^2}}{\sigma_m^2} \tag{42}$$

Notice that $Y<0$ implies that before the arrival of tech 2 and for $\varphi<\tilde{\varphi}^{*}_{F_{21}}$, $\tilde{F}_{F_{21}}(\varphi)<F_{F_{21}}(\varphi)$.[11]

In this case the following "value-matching" condition applies:

$$\tilde{F}_{F_{21}}(\tilde{\varphi}^{*}_{F_{21}})=\frac{\tilde{\varphi}^{*}_{F_{21}}de_{2_F1_L}}{r-\mu_X-\mu_X}-I_2 \tag{43}$$

Replacing $\tilde{F}_{F_{21}}(\varphi)$ (expression 38) in (43), for $\varphi<\tilde{\varphi}^{*}_{F_{21}}$, we obtain (44).

$$Y\varphi^{\beta_3}+A\varphi^{\beta_1}-\frac{r-\mu_X-\mu_E}{de_{2_F1_L}}+I_2=0 \tag{44}$$

Replacing in (44) φ by $\tilde{\varphi}^{*}_{F_{21}}$ we determine the follower's threshold.

[11] Compare expressions (38) and (30). $\tilde{F}_{F_{21}}(\varphi)$ is the follower's value function for the scenario where at the beginning of the investment game tech 2 is not available, the leader is active with tech 1 and the follower is idle and "committed" to the adoption of tech 2 (Expression 38); $F_{F_{21}}(\varphi)$ is the follower's value function for the scenario where at the beginning of the investment game tech 2 is available, the leader is active with tech 1 and the follower is idle (Expression 30).

2.2.2 Leader: Scenario 1 *(leader active with tech1 and follower "committed" to the adoption of tech 2)*

Assuming that the leader adopts tech 1 with the follower inactive and the follower waits for tech 2 and adopts it if available and the respective investment threshold is reached, the leader's expected payoff at the instant (τ) it adopts tech 1 is given by:

$$\tilde{F}_{L_{12}}(\varphi) = E\left[\int_{t=\tau}^{T_{2_F}} \varphi de_{1_L 0_F} e^{-r\tau} dt + \left(F_{L_{12}}\right) e^{-r\tau} - I_1\right] \tag{45}$$

where the integral represents the leader's payoff for the period where it is alone in the market; the second term is the leader's value function for the period where both firms are active (the leader with tech 1 and the follower with tech 2 – see expression 33); I_1 is the leader's investment cost in tech 1. Taking expectations and using the appropriate boundary conditions, we get the following expression:

$$\tilde{F}_{L_{12}}(\varphi) = \begin{cases} E\varphi^{\beta_3} + \dfrac{\varphi de_{1_L 0_F}}{r-\mu_X-\mu_E} - I_1 + \dfrac{\varphi(de_{1_L 2_F} - de_{1_L 0_F})}{r-\mu_X-\mu_E}\left(\dfrac{\varphi}{\tilde{\varphi}^*_{F_{21}}}\right)^{\beta_3} & \varphi < \tilde{\varphi}^*_{F_{21}} \\ G\varphi^{\beta_4} + \dfrac{\varphi de_{1_L 0_F}}{(r-\mu_X-\mu_E+\lambda)} + \dfrac{\varphi de_{1_L 2_F}}{(r-\mu_X-\mu_E)}\dfrac{\lambda}{(r-\mu_X-\mu_E+\lambda)} & \varphi \geq \tilde{\varphi}^*_{F_{21}} \end{cases} \tag{46}$$

where $E\varphi^{\beta_3}$ and $G\varphi^{\beta_4}$ are both positive (see proof in Appendix, sections 4.2 and 4.3) and correct for the fact that the follower's threshold to adopt tech 2 can be reached before tech 2 is available, which favours the payoff of an active leader. The constants E and G are given by Equations (47) and (48), respectively, derived using the continuity and differentiability conditions for $\tilde{F}_{L_{12}}(\varphi)$ at $\varphi = \tilde{\varphi}^*_{F_{21}}$ (see Appendix A, section 4.1); $\frac{\varphi de_{1_L 0_F}}{r-\mu_X-\mu_E} - I_1$ is the leader's payoff at the time of the adoption of tech 1 if it operates alone forever; $\frac{\varphi(de_{1_L 2_F} - de_{1_L 0_F})}{r-\mu_X-\mu_E}\left(\frac{\varphi}{\varphi^*_{F_{21}}}\right)^{\psi_1}$ is derived using the continuity condition of $F_{L_{12}}(\varphi)$ at $\tilde{\varphi}^*_{F_{21}}$, it is negative given that $(de_{1_L 2_F} - ds_{1_L 0_F}) < 0$ (see inequality 6), and corresponds to the correction factor that incorporates the fact that in the future if $\tilde{\varphi}^*_{F_{21}}$ is reached the follower adopts tech 2 and the leader's payoff is reduced. The rest of the terms are defined as for expression (38).

$$E = \frac{\left(\tilde{\varphi}^*_{F_{21}}\right)^{1-\beta_3}\left[(r-\mu_X-\mu_E)(\beta_1-\beta_4) + \lambda(\beta_1-1)\right]\left(de_{1_L 0_F} - de_{1_L 2_F}\right)}{(r-\mu_X-\mu_E+\lambda)(r-\mu_X-\mu_E)(\beta_3-\beta_4)} \tag{47}$$

$$G = \frac{\left(\tilde{\varphi}^*_{F_{21}}\right)^{1-\beta_4}\left[(r-\mu_X-\mu_E)(\beta_1-\beta_3) + \lambda(\beta_1-1)\right]\left(de_{1_L 0_F} - de_{1_L 2_F}\right)}{(r-\mu_X-\mu_E+\lambda)(r-\mu_X-\mu_E)(\beta_3-\beta_4)} \tag{48}$$

The leader' investment threshold expression to adopt tech 1, $\tilde{\varphi}^*_{L_{12}}$, is determined by equalizing equations (38) to (46), for $\varphi < \tilde{\varphi}^*_{F_{21}}$, from where we obtain Equation (49),

$$Y\varphi^{\beta_3}+A\varphi^{\beta_1}-E\varphi^{\beta_3}-\frac{\varphi de_{1_L0_F}}{(r-\mu_X-\mu_E)}+I_1-\frac{\varphi(de_{1_L2_F}-de_{1_L0_F})}{r-\mu_X-\mu_E}\left(\frac{\varphi}{\tilde{\varphi}^*_{F_{21}}}\right)^{\beta_3}=0 \quad (49)$$

Replacing in Equation (49) φ by $\tilde{\varphi}^*_{L_{12}}$ and solving in order to $\tilde{\varphi}^*_{L_{12}}$ we get the leader's investment threshold.

2.2.3 Follower: Scenario 2 *(leader active with tech 1 and follower "committed" to the adoption of tech 1)*

Using the same procedure as above for scenario 1 we obtain the follower's value function for the scenario where the leader is active with tech 1 and the follower adopts tech 1: [12]

$$\tilde{F}_{F_{11}}(\varphi)=\begin{cases} J\varphi^{\psi_1}+C\varphi^{\beta_3}+H\varphi^{\beta_3} & \varphi\in\left[0,\tilde{\varphi}^*_{F_{21}}\right) \\ J\varphi^{\psi_1}+P\varphi^{\beta_4}+\frac{\varphi de_{2_F1_L}}{(r-\mu_X-\mu_E)}\frac{\lambda}{(r-\mu_X-\mu_E+\lambda)}-\frac{\lambda I_2}{r+\lambda} & \varphi\in\left[\tilde{\varphi}^*_{F_{21}},\tilde{\varphi}^*_{F_{11}}\right) \\ \frac{\varphi de_{1_F1_L}}{r-\mu_X-\mu_E}-I_1 & \varphi\in\left[\tilde{\varphi}^*_{F_{11}},\infty\right) \end{cases} \quad (50)$$

where, $J\varphi^{\psi_1}$ is the option value to adopt tech 1, with the constant J given by Equation (51); $C\varphi^{\psi_1}$ is the option value to adopt tech 2, with constant C given by Equation (31); $H\varphi^{\beta_3}$ is a negative correction factor which takes into account the fact that tech 2 may not be available when $\tilde{\varphi}^*_{F_{21}}$ is reached, with constant H given by Equation (52); $P\varphi^{\beta_4}$ is equal to $W\varphi^{\beta_4}$, as for W (see expression 38), $P>0$ and reflects the fact that $\tilde{\varphi}^*_{F_{21}}$ can be reached before tech 2 is available (if so the follower will keep the option to adopt tech 2); $\frac{\varphi de_{2_F1_L}}{(r-\mu_X-\mu_E)}\frac{\lambda}{(r-\mu_X-\mu_E+\lambda)}$ and $\frac{\lambda I_2}{r+\lambda}$ have the same meanings as those described for expression (38); $\frac{\varphi de_{1_F1_L}}{r-\mu_X-\mu_E}-I_1$ is the follower's payoff when both firms operate with tech 1 from $\tilde{\varphi}^*_{F_{11}}$ until infinity, where I_1 is the investment cost in tech 1.

Solving simultaneously the continuity and differentiability conditions for $\tilde{F}_{F_{11}}(\varphi)$ at $\varphi=\tilde{\varphi}^*_{F_{21}}$ (expression 38) and the "value matching" and the "smooth pasting" conditions for $\tilde{F}_{F_{11}}(\varphi)$ at $\varphi=\tilde{\varphi}^*_{F_{11}}$ (expression 50, second row), we determine the constants H, J and P and the follower's investment threshold $\tilde{\varphi}^*_{F_{11}}$:

$$J=\frac{(1-\beta_4)}{\beta_3-1}W(\tilde{\varphi}^*_{F_{11}})^{(\beta_4-\beta_3)}+\frac{r}{(\beta_3-1)(r+\lambda)}I_1(\tilde{\varphi}^*_{F_{11}})^{-\beta_3} \quad (51)$$

$$H=J+\frac{\beta_4}{\beta_3}W(\tilde{\varphi}^*_{F_{21}})^{(\beta_4-\beta_3)}-\frac{de_{2_F1_L}}{\beta_3(r-\mu_X-\mu_E+\lambda)}(\tilde{\varphi}^*_{F_{21}})^{(1-\beta_3)} \quad (52)$$

$$P=W \quad (53)$$

See derivations in Appendix A, section 5. We find that $P=W$, where W is given by Equation (40).

[12] In (50), notice that tech 2 is more efficient than tech 1 and costs the same. Hence, ceteris paribus, the investment threshold to adopt tech 2 is lower than the investment threshold to adopt tech 1.

There is no closed-form solution for the follower's investment threshold, but using the value matching condition at $\tilde{\varphi}^*_{F_{11}}$ and the information from Expression (50) and Equations (51)-(53) we get Equations (54) from where we determine $\tilde{\varphi}^*_{F_{11}}$.

$$(\beta_3-\beta_4)P(\tilde{\varphi}^*_{F_{11}})^{\beta_4}+\frac{(\beta_3-1)\lambda\tilde{\varphi}^*_{F_{11}}de_{2_F1_L}}{(r-\mu_X-\mu_E+\lambda)(r-\mu_X-\mu_E)}-\frac{(\beta_3-1)\tilde{\varphi}^*_{F_{11}}de_{1_F1_L}}{r-\mu_X-\mu_E}+\frac{r\beta_3\lambda I_2}{r+\lambda}=0 \tag{54}$$

2.2.4 Leader: Scenario 2 *(leader active with tech 1 and follower "committed" to the adoption of tech 1)*

Assuming that the leader adopts tech 1 before tech 2 arrives and the follower is "committed" to adopt tech 1, for $\varphi \geq \tilde{\varphi}^*_{F_{11}}$ (both firms are active with tech 1), the leader's payoff is given by:

$$\tilde{F}_{L_{11}}(\varphi)=\frac{\varphi de_{1_L1_F}}{r-\mu_X-\mu_E} \tag{55}$$

At the instant the leader adopts tech 1 (T_{1_L}) its expected payoff is given by:

$$F_{L_{11}}(\varphi)=E\left[\int_{t=T_{1_L}}^{T_{1_F}}\varphi de_{1_L0_F}e^{-rt}dt-I_1+\left(e^{-rT}\Big|_{t\leq T_{1_F}}\right)F_{L_{12}}(\varphi)+\int_{T_{1_F}}^{\infty}\varphi de_{1_L1_F}e^{-rt}dt\right] \tag{56}$$

The first integral represents the leader's payoff while alone in the market (operating with tech 1); the second integral is the leader's payoff for the period where both firms are active with tech 1; $\left(e^{-rT}\Big|_{t\leq T_{1_F}}\right)F_{L_{12}}(\varphi)$ is the present value of the leader's payoff given by expression (33), which takes into account the fact that, before the follower adopts tech 1, there is the possibility that tech 2 arrives and the respective threshold reached, reducing the value of the leader. The leader's value function is given by:

$$\tilde{F}_{L_{11}}(\varphi)=\begin{cases} L\varphi^{\beta_3}+\dfrac{\varphi(de_{1_L2_F}-de_{1_L0_F})}{r-\mu_X-\mu_E}\left(\dfrac{\varphi}{\varphi^*_{F_{21}}}\right)^{\beta_1}+\dfrac{\varphi de_{1_L0_F}}{r-\mu_X-\mu_E}-I_1 & \varphi\in\left[\tilde{\varphi}^*_{L_{12}},\tilde{\varphi}^*_{F_{21}}\right) \\ M\varphi^{\beta_3}+G\varphi^{\beta_4}+\dfrac{\varphi de_{1_L0_F}}{(r-\mu_X-\mu_E+\lambda)}+\dfrac{\varphi de_{1_L2_F}}{(r-\mu_X-\mu_E)}\dfrac{\lambda}{(r-\mu_X-\mu_E+\lambda)} & \varphi\in\left[\tilde{\varphi}^*_{F_{21}},\tilde{\varphi}^*_{F_{11}}\right) \\ \dfrac{\varphi de_{1_L1_F}}{r-\mu_X-\mu_E} & \varphi\in\left[\tilde{\varphi}^*_{F_{11}},\infty\right) \end{cases} \tag{57}$$

where, in the first row represents the leader's value at the instant it adopts tech 1; $L\varphi^{\beta_3}$ corrects the fact that tech 2 has to arrive for the follower to adopt it, which favours the leader; $\dfrac{\varphi(de_{1_L2_F}-de_{1_L0_F})}{r-\mu_X-\mu_E}\left(\dfrac{\varphi}{\varphi^*_{F_{21}}}\right)^{\beta_1}$ is negative and represents the fact that if tech 2 arrives and $\tilde{\varphi}^*_{F_{21}}$ is reached the follower adopts tech 2, reducing the leader's value; $\dfrac{\varphi de_{1_L0_F}}{r-\mu_X-\mu_E}-I_1$ is the present value of the leader's payoff for the scenario where it operates alone with tech 1 forever; in the second row, $M\varphi^{\beta_3}$ values the possibility that φ rises above $\tilde{\varphi}^*_{F_{11}}$ before tech 2 arrives. This has a positive and a negative effect on the leader's value. A negative effect, because if the follower adopts tech 1 the leader loses its monopoly, a positive effect, because if the follower adopts tech 1 it loses the option to adopt tech 2. Hence, the signal for the constant M depends on the market conditions; $G\varphi^{\beta_4}$ and $\frac{\varphi de_{1_L 2_F}}{(r-\mu_X-\mu_E)} \frac{\lambda}{(r-\mu_X-\mu_E+\lambda)}$ have the same meanings as those described for expression (46); $\frac{\varphi de_{1_L 1_F}}{r-\mu_X-\mu_E}$ is the leader's payoff for the scenario where both firms operate with tech 1 from $\tilde{\varphi}^*_{F_{11}}$ until infinity. For details of the derivation of (57) and the constants L and M see Appendix A, section 6.

The leader's threshold to adopt tech 1 assuming that the follower waits for tech 2 and adopts it as soon as it arrives and the threshold is reached is determined using expression (49). The leader's threshold to adopt tech 1 assuming that the follower is "committed" to adopt tech 1 is derived by equalizing the leader's and the follower's value functions, expressions (57) for $\varphi \in \left[\tilde{\varphi}^*_{L_{12}}, \tilde{\varphi}^*_{F_{21}}\right)$, and (50) for $\varphi \in \left[0, \tilde{\varphi}^*_{F_{21}}\right)$, leading to equations (58).

$$L\varphi^{\beta_3} + \frac{\varphi(de_{1_L 2_F} - de_{1_L 0_F})}{r-\mu_X-\mu_E}\left(\frac{\varphi}{\varphi^*_{F_{21}}}\right)^{\beta_1} + \frac{\varphi de_{1_L 0_F}}{r-\mu_X-\mu_E} - I_1 - J\varphi^{\beta_3} - C\varphi^{\psi_1} - H\varphi^{\beta_3} = 0 \tag{58}$$

Replacing in (58) φ by $\tilde{\varphi}^*_{L_{11}}$ we determine a numerical solution for the leader's investment threshold.

3. Sensitivity Analysis

In this section we do some sensitivity analysis to study the effect of the most important parameters of our model on the investment thresholds of the leader and the follower. In our illustrative results we use the following base parameters:

$X(t)$	$E_1(t)$	$E_2(t)$	$\varphi_1(t)$	$\varphi_2(t)$	I_1	I_2	σ_X	σ_E	μ_X	μ_E	r	λ	ρ_{XE_k}
100	0.70	0.85	70.0	85.0	100	100	0.20	0.20	0.02	0.02	0.07	0.20	0.0

Table 3 – Market Variables

$de_{1_L 0_F}$	$de_{0_F 1_L}$	$de_{1_L 2_F}$	$de_{2_F 1_L}$	$de_{1_L 1_F}$	$de_{1_F 1_L}$	$de_{2_L 2_F}$	$de_{2_F 2_L}$
1.0	0.0	0.55	0.45	0.70	0.30	0.70	0.30

Table 4 – Competition Factors

In Table 2 we clarify the notation.

Technological Uncertainty?	**Notation**	**Description**
	$\tilde{\tau}$	Arrival time of tech 2
(**NO**) $t \geq \tilde{\tau}$	$\varphi^*_{L_{22}}$	Leader's threshold to adopt tech 2 assuming that tech 2 arrives with both firms idle
	$\varphi^*_{F_{22}}$	Follower's threshold to adopt tech 2 assuming that tech 2 arrives with both firms idle
	$\varphi^*_{L_{12}}$	Leader's threshold to adopt tech 1 assuming that when tech 2 arrives the leader is active with tech 1 and the follower is idle
	$\varphi^*_{F_{21}}$	Follower's threshold to adopt tech 2 assuming that when tech 2 arrives the leader is active with tech 1 and the follower is idle
(**YES**) $t < \tilde{\tau}$	$\tilde{\varphi}^*_{L_{12}}$	Leader's threshold to adopt tech 1 assuming that the follower is committed to the adoption of tech 2 and tech 2 is not available
	$\tilde{\varphi}^*_{F_{21}}$	Follower's threshold to adopt tech 2 assuming that the leader adopts tech 1 and tech 2 is not available
	$\tilde{\varphi}^*_{L_{11}}$	Leader's threshold to adopt tech 1 assuming that the follower adopts tech 1 and tech 2 is not available
	$\tilde{\varphi}^*_{F_{21}}$	Follower's threshold to adopt tech 2 assuming that the leader adopts tech 1 and tech 2 is not available

Table 2 –Notation: Firms' Investment Threshold

3.1 Tech 2 is Available, $t \geq \tilde{\tau}$

3.1.1 Both Firms idle

Tech 2 is more efficient than tech 1 and costs the same, so when both firms are idle and tech 2 arrives the adoption of tech 1 is never optimal for both firms, i.e., they behave as if there was one technology (tech 2) available only. The investment game turns therefore in a standard leader/follower pre-emption game where firms have one option to invest.

3.1.2 Leader is active with tech 1 and the Follower is idle

This is a "one-shot" game and so as soon as firms exercise one of the options available (option to adopt tech 1 or option to tech 2), the game ends. If when tech 2 arrives the leader is active with tech 1 and the follower is idle, the follower behaves as if it was in a monopoly-like regarding the option to adopt tech 2.

The worst scenario for the leader is when it adopts tech 1 and tech 2 arrives soon after and its threshold is reached with the follower idle. The leader would be in a better position if when tech 2 arrives both firms were idle. This happens because the (assumed) leader's FMA is lower when the leader is active with tech 1 and the follower with tech 2 than when the leader and the follower are active both with tech 1 (see Table 1). More specifically, when the leader is active with tech 1 and the follower with tech 2 (technology-asymmetry), the leader gets 55% and the follower 45% of the market share, and when both firms are active with tech 2 (technology-symmetry), the leader gets 70% and the follower 30% of the market share. Firms can also operate with "technology-symmetry" when both adopt tech 1. We assumed, however, that they get the same market share as when both adopt tech 2. Our model allows simulations of both of these scenarios, adjusting accordingly its parameters.

3.1.3 Results

In Figures 6 and 7 we show the sensitivity of the investment thresholds of the leader and the follower, respectively, to changes in the correlation between the market revenues and the EAA, for the scenarios 3.1.1 and 3.12.

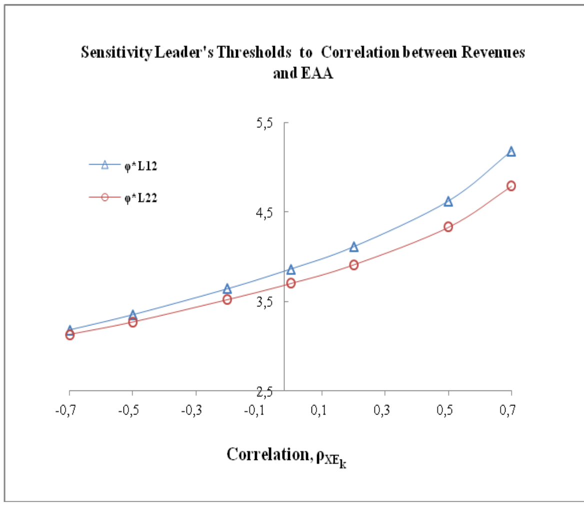

Figure 6

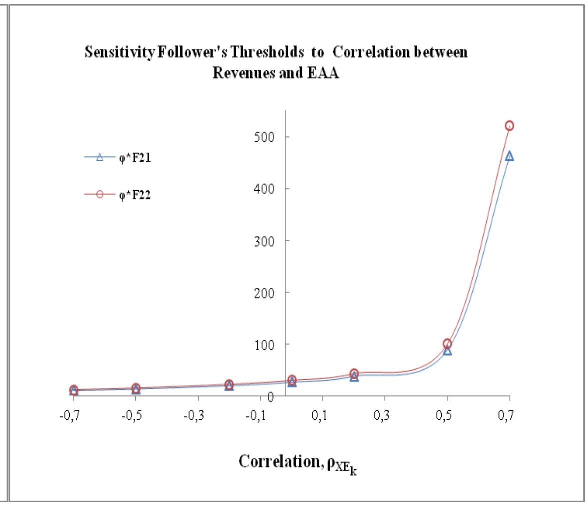

Figure 7

For both figures the line marked with a circle identifies the case where when tech 2 arrives both firms are idle and the line marked with a triangle identifies the case where when tech 2 arrives the leader is active with tech 1 and the follower is idle. The results show that the higher is the correlation the higher the investment threshold for both firms (i.e., the later the adoption). The sensitivity of the leader is however more or less constant for the all range of correlation values used while the sensitivity of the follower increases abruptly, ceteris paribus, for a correlation around $\rho_{XE_k} = 0.50$.

In Figures 8 and 9 are our illustrative results for the effect of the volatility of the EAA of technology *k* on the investment thresholds of the leader and the follower, respectively. Again, the line marked with a circle identifies the case where when tech 2 arrives both firms are idle (so they will adopt tech 2) and the line marked with a triangle identifies the case where when tech 2 arrives the leader is active with tech 1 and the follower is idle (so the leader adopts etch 1 and the follower tech 2).

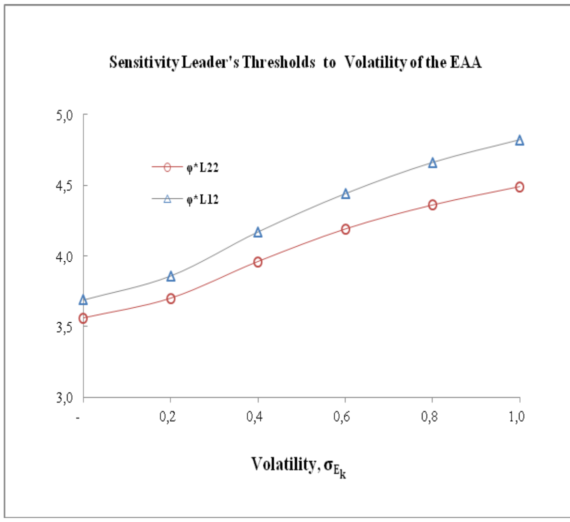

Figure 8

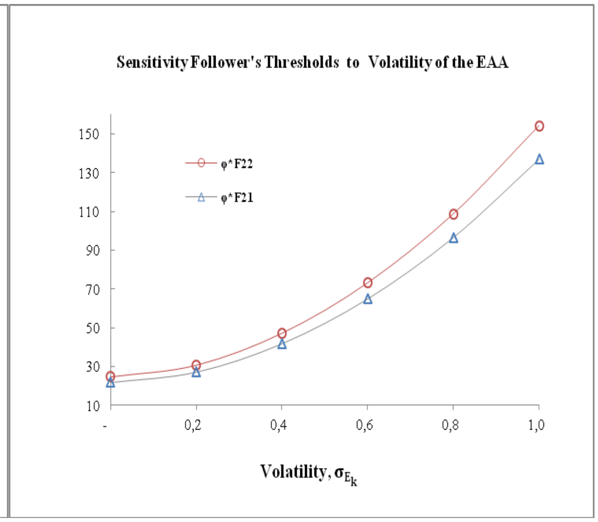

Figure 9

For both firms and the two scenarios the higher is the volatility of the EAA the higher the firm's investment threshold (i.e., the later the adoption).

3.2 Tech 2 is not available, $t < \tilde{\tau}$

Due to the high number of market variables and investment scenarios underlying our model, to avoid unnecessary complexity, in this section we provide and comment on the most relevant results only. However, other potentially relevant results and analysis are neglected.

3.2.1 Leader is active with tech 1 and the follower is committed to adopt tech 2

In Figure 10 are our illustrative results for the sensitivity of the firms' investment thresholds to changes in the "probability that tech 2 arrives" (λ). We show the sensitivity of the leader's threshold to adopt tech 1 assuming that when tech 2 arrives the leader is active with tech 1 and the follower is idle, and the sensitivity of the follower's threshold to adopt tech 2 assuming that when tech 2 arrives the leader is active with tech 1.

The leader's threshold to adopt tech 1 assuming that the follower will adopt tech 2, is not sensitive to changes in λ; the follower's threshold to adopt tech 2 assuming that the leader is active with tech 1 is very sensitive to changes in λ for low values of λ, i.e., as λ increases from zero up to around λ=0.20, the follower's threshold decreases significantly reaching a minimum; for λ>0.20 it becomes insensitive to changes in λ.

The difference between the behaviour of the leader and the follower is due to the FMA, which affects the leader (turning the other model variables less relevant) and does not affect the follower.

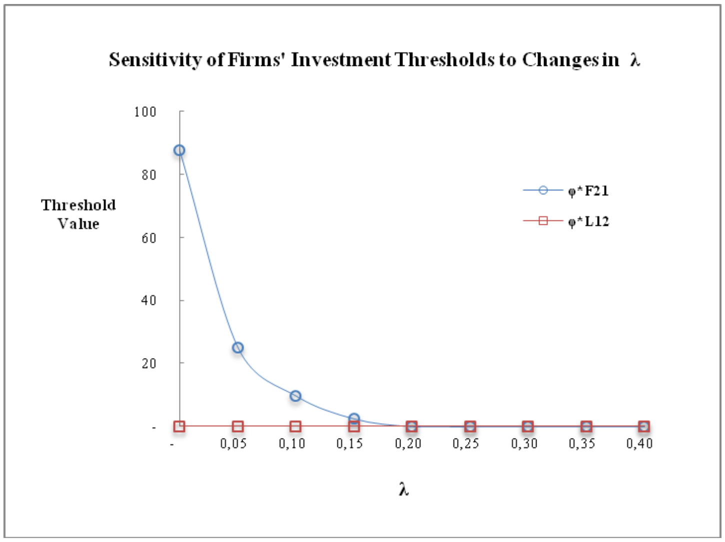

Figure 10

In Figure 11 are our illustrative results for the sensitivity of the follower's investment threshold to the combined effect of changes in the "probability that tech 2 arrives" (λ) and the volatility of the EAA of tech 2 (σ_{E_2}). This result is consistent with that of Figures 10 (sensitivity of the follower's threshold to changes in λ) where we used as base case $\sigma_{E_k} = 0.20$.

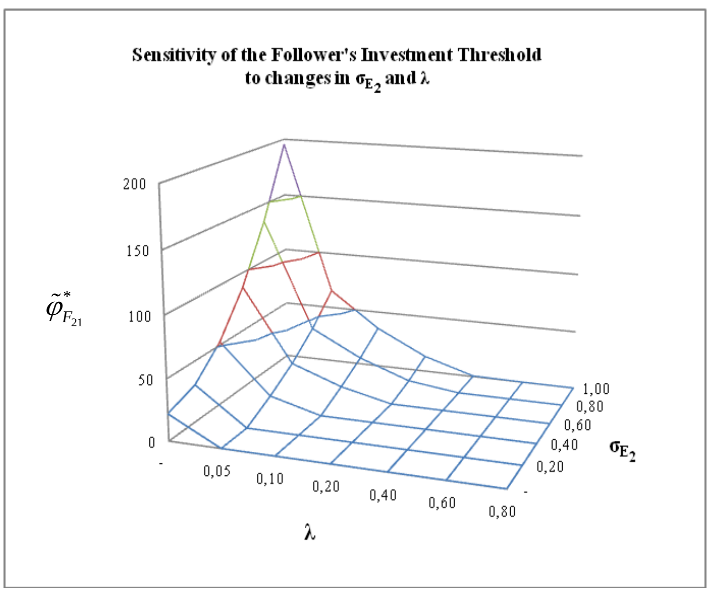

Figure 11

For very low values of λ, small changes in σ_{E_2} affects significantly the follower's threshold. As λ increases from zero up to 0.4, the effect of changes in the volatility of tech 2 on the follower's threshold becomes less relevant, and irrelevant for λ>0.4. The conclusion is that the follower's threshold to adopt tech 2 is sensitive to both λ and σ_{E_2} , but as λ increases the effect of s_E becomes less relevant or even negligible in the follower's investment behaviour. This is a somewhat surprising result.

In Figure 12 are our illustrative results for the sensitivity of the follower's investment threshold to adopt tech 2 to changes in λ and the leader's FMA.

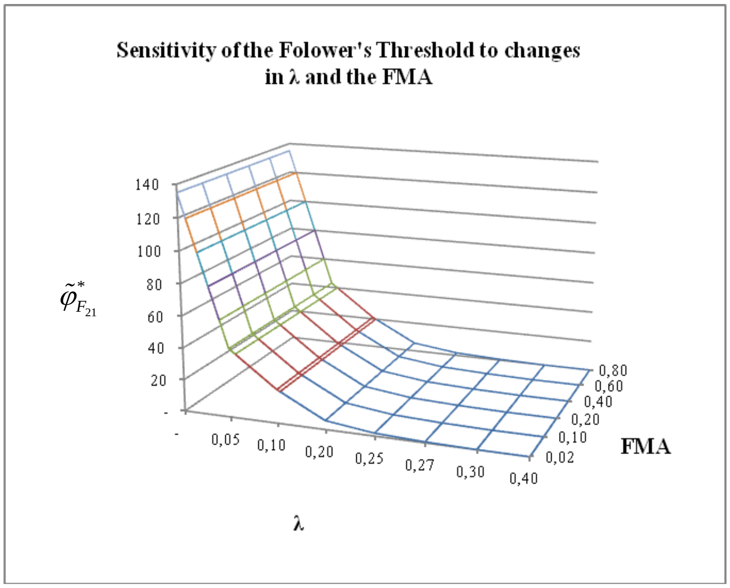

Figure 12

The leader's FMA has a negligible effect on the follower's investment threshold to adopt tech 2. For very low values of "probability that tech 2 arrives" (λ) the higher is λ the lower the follower's threshold. For λ>0.25 its effect on the follower's threshold becomes irrelevant. This is an interesting result since it contradicts the result of Azevedo and Paxson (2012), where (p. 12) they show that, in absence of technological uncertainty, the follower's investment threshold is very sensitive to the leader's FMA. Apparently, a moderate "probability that tech 2 arrives" turns the leader's FMA irrelevant to the follower's investment behaviour.

In Figure 13 are our illustrative results for the sensitivity of the leader's investment threshold to adopt tech 1 (assuming that the follower adopts tech 2), to changes in λ and the FMA.

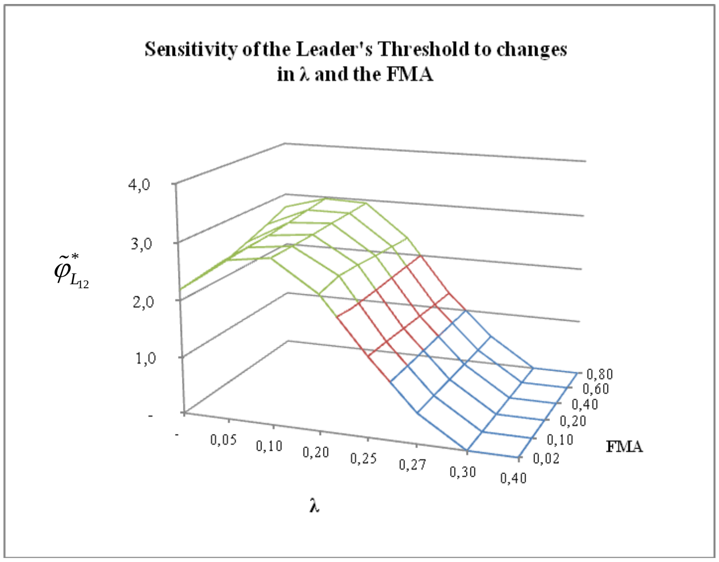

Figure 13

For very low "probability that tech 2 arrives" (λ), the higher the FMA the slightly higher is the leader's investment threshold (the later the adoption). As λ increases from zero, the FMA becomes less relevant or even negligible. There is a sort of leader's "probability that tech 2 arrives-related "inflection strategy" (λ^*), around $\lambda^* = 0.10$, where, for $\lambda < \lambda^*$ the higher is λ the higher the leader's threshold, for $\lambda > \lambda^*$ the higher is λ the lower the leader's threshold. This result reflects perhaps the fact that the leader's threshold for this scenario is derived assuming that it is committed to adopt tech 1 and the follower will adopt tech 2 but conditioned on its arrival and the respective investment threshold being reached.

4. Conclusions

To our knowledge this is the first real options game model studying the simultaneous effect of rivalry with market, technical and technological uncertainty. Our results show that the "probability that a second (and more efficient) technology arrives" has a significant effect on the investment threshold of the leader and the follower. Somewhat surprisingly, when technological uncertainty is considered, for relatively moderate high probability that a second technology arrives, other (usually) important real options model factors, such as the volatility of the underlying variables and leader's FMA, become significant less relevant or even negligible. Negative or relatively low positive correlation between "market revenues" and "EAA", ρ_{XE_k} , affects slightly the investment threshold of both firms (the higher the correlation the later is the adoption). A high positive correlation between "market revenues" and "EAA" delays slightly the investment threshold of the leader and significantly the investment threshold of the follower.

Our real option game model setting is based on the assumption that there is a FMA (“pre-emption game”). It would be interesting to relax this assumption and extend it to the case where there is a second-mover advantage (“attrition game”). This would apply to cases where due to spill over information the follower benefits from costless information about the technical quality (efficiency) of the technology. We use a competition framework where the FMA is based on deterministic competition factors defined as proportions of the market revenues. Although mathematically challenging, it would be interesting to refine this assumption using dynamic market share for both firms.

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Appendix A

1. Derivation - Ordinary Differential Equation (9)

Let rewrite Equation (8) as,

$$\frac{1}{2}\frac{\partial^2 F_F}{\partial X^2}\sigma_x{}^2 X^2 + \frac{1}{2}\frac{\partial^2 F_F}{\partial E^2}\sigma_E{}^2 E^2 + \frac{\partial^2 F_F}{\partial E \partial X} XE\sigma_x\sigma_E\rho + \frac{\partial F_F}{\partial X}\mu_x X + \frac{\partial F_F}{\partial E}\mu_E E - rF_F = 0 \tag{A1}$$

In order to reduce the homogeneity of degree two in the underlying variables to homogeneity of degree one similarity methods can be used. Let $\varphi = (X)(E)$, so:

$$F_F(X,E) = F_F(\varphi)$$
$$\frac{\partial F_F(X,E)}{\partial E} = \frac{\partial F_F(\varphi)}{\partial \varphi} X$$
$$\frac{\partial F_F(X,E)}{\partial X} = \frac{\partial F_F(\varphi)}{\partial \varphi} E$$
$$\frac{\partial^2 F_F(X,E)}{\partial E^2} = \frac{\partial^2 F_F(\phi_2)}{(\partial \varphi)^2} X^2$$
$$\frac{\partial^2 F_F(X,E)}{\partial X^2} = \frac{\partial^2 F_F(\varphi)}{(\partial \varphi)^2} E^2$$
$$\frac{\partial^2 F_F(X,E)}{\partial X\ \partial E} = \frac{\partial^2 F_F(\varphi)}{(\partial \varphi)^2} XE + \frac{\partial F_F(\varphi)}{\partial \varphi}$$

Substituting back to Equation (A1) we obtain Equation (9), rewritten here as:

$$\frac{1}{2}\varphi^2\sigma_m{}^2\frac{\partial^2 F_F(\varphi)}{\partial \varphi^2} + \varphi\left(\sigma_X\sigma_E\rho_{XE} + \mu_X + \mu_E\right)\frac{\partial F_F(\varphi)}{\partial \varphi} - rF_F(\varphi) = 0 \tag{A2}$$

where, $\sigma_m^2 = \sigma_X{}^2 + \sigma_E{}^2 + 2\rho_{XE}\sigma_X\sigma_E$.

2. Derivation and Proofs

2.1 Expressions (39) and (40)

In Expression (38) the two regions, $\varphi < \tilde{\varphi}^*_{F_{21}}$ and $\varphi \geq \tilde{\varphi}^*_{F_{21}}$, must meet at $\varphi = \tilde{\varphi}^*_{F_{21}}$ and $\tilde{F}_{F_{21}}(\varphi)$ must be continuous and differentiable along φ. Therefore,

$$Y(\tilde{\varphi}^*_{F_{21}})^{\beta_3} + A(\tilde{\varphi}^*_{2_F})^{\beta_1} = W(\tilde{\varphi}^*_{F_{21}})^{\beta_4} + \frac{\tilde{\varphi}^*_{F_{21}} de_{2_F 1_L}}{(r-\mu_X-\mu_E)}\frac{\lambda}{(r-\mu_X-\mu_E+\lambda)} - \frac{\lambda I_2}{r+\lambda} \tag{A3}$$

$$\beta_3 Y(\tilde{\varphi}^*_{F_{21}})^{(\beta_3-1)} + \beta_1 A(\tilde{\varphi}^*_{2_F})^{(\beta_1-1)} = \beta_4 W(\tilde{\varphi}^*_{F_{21}})^{(\beta_4-1)} + \frac{de_{2_F 1_L}}{(r-\mu_X-\mu_E)}\frac{\lambda}{(r-\mu_X-\mu_E+\lambda)} \tag{A4}$$

Substituting A from Equation (18) and solving for Y and W, after some algebraic manipulation, we get equations (39) and (40).

2.2 Proof: $Y < 0$

Rewriting equation (39) as (A5),

$$Y = \frac{\left(\tilde{\varphi}^*_{F_{21}}\right)^{-\beta_3}\left[r\left(r-\mu_X-\mu_E\right)\beta_4 + \left(r-(\mu_X+\mu_E)\beta_1\right)\lambda\beta_4 - \left(r-\mu_X-\mu_E\right)\left(r+\lambda\right)\beta_1\right]I_2}{\left(r+\lambda\right)\left(r-\mu_X-\mu_E+\lambda\right)\left(\beta_1-1\right)\left(\beta_3-\beta_4\right)} \tag{A5}$$

We know that $\tilde{\varphi}^*_{F_{21}}$, $(r-\mu_X-\mu_E) > 0$, r, λ, β_1, β_3 and I_2 are all positive and $\beta_4 < 0$.

Simplifying the numerator: let $v_1 = \left(\tilde{\varphi}^*_{F_{21}}\right)^{-\beta_3}$, $a = r\left(r-\mu_X-\mu_E\right)$, $b = \left(r-(\mu_X+\mu_E)\beta_1\right)\lambda$, $c = \left(r-\mu_X-\mu_E\right)\left(r+\lambda\right)\beta_1$. Simplifying the denominator: let $d = \left(r+\lambda\right)\left(r-\mu_X-\mu_E+\lambda\right)\left(\beta_1-1\right)$, and $e = \left(\beta_3-\beta_4\right)$. Substituting these terms in (A5) and rewriting yields:

$$Y = \frac{v_1(a\beta_4 + b\beta_4 - c)I_2}{d(e)} \tag{A6}$$

From the information above we conclude that a, b, c and d (given that $\beta_1 > 1$) are all positive. From Equation (42) we can see that $\beta_3 > 0$ and $\beta_4 < 0$. So $e > 0$ and the denominator is positive. The nominator is negative since v_1, a, b, c and I_2 are positive and $\beta_4 < 0$. Hence, $Y < 0$.

2.3 Proof: $W > 0$

Rewriting equation (40) as (A7),

$$W = \frac{\left(\tilde{\varphi}^*_{F_{21}}\right)^{-\beta_4}\left[r\left(r-\mu_X-\mu_E\right)\beta_3 + \left(r-\left(\mu_X+\mu_E\right)\beta_1\right)\lambda\beta_3 - \left(r-\mu_X-\mu_E\right)\left(r+\lambda\right)\beta_1\right]I_2}{\left(r+\lambda\right)\left(r-\mu_X-\mu_E+\lambda\right)\left(\beta_1-1\right)\left(\beta_3-\beta_4\right)} \tag{A7}$$

We know that $\tilde{\varphi}^*_{F_{21}}$, $(r-\mu_X-\mu_E)>0$, r, λ, β_1, β_3 and I_2 are all positive and $\beta_4 < 0$. Simplifying the numerator: let $v_2 = \left(\tilde{\varphi}^*_{F_{21}}\right)^{-\beta_4}$, $a = r\left(r-\mu_X-\mu_E\right)$, $b = \left(r-\left(\mu_X+\mu_E\right)\beta_1\right)\lambda$, $c = \left(r-\mu_X-\mu_E\right)\left(r+\lambda\right)\beta_1$. Simplifying the denominator: let $d = \left(r+\lambda\right)\left(r-\mu_X-\mu_E+\lambda\right)\left(\beta_1-1\right)$ and $e = \left(\beta_3-\beta_4\right)$. Substituting in (A7) and rewriting yields:

$$W = \frac{v_2(a\beta_3 + b\beta_3 - c)I_2}{d(e)} \tag{A8}$$

From the information above we conclude that a, b, c and d (given that $\beta_1 > 1$) are all positive. From Equation (42) we can see that $\beta_3 > 0$ and $\beta_4 < 0$, so $e > 0$. Therefore, both the numerator and the denominator are positive. Hence $W > 0$.

3. Derivation – second and third terms of Expression (38), for $\varphi \geq \tilde{\varphi}^*_{F_{21}}$

In expression (38), for $\varphi \geq \tilde{\varphi}^*_{F_{21}}$, the second term is the expected present value of the follower's profit flows generated from the moment it adopts tech 2 onwards, derived as follow:

$$\begin{aligned}
&E\left[e^{-rT_{2_F}}\frac{\varphi_t de_{2_F1_L}}{r-\mu_X-\mu_E}\right] = \\
&= \frac{de_{2_F1_L}}{r-\mu_X-\mu_E}E\left[e^{-rT_{2_F}}\varphi_t\right] \\
&= \frac{de_{2_F1_L}}{r-\mu_X-\mu_E}\int_{t=0}^{\infty}\lambda e^{(-\lambda t)}e^{-rt}E\left[\varphi_t\right]dt \\
&= \frac{de_{2_F1_L}}{r-\mu_X-\mu_E}\int_{t=0}^{\infty}\lambda e^{(-\lambda t)}e^{(-rt)}\varphi_t e^{(\mu_X+\mu_E)t}dt \\
&= \frac{\lambda}{(r-\mu_X-\mu_E+\lambda)}\frac{\varphi_t de_{2_F1_L}}{(r-\mu_X-\mu_E)}
\end{aligned} \tag{A9}$$

The third term is the expected present value of the investment cost, I_2, which has to be paid at the time of the adoption, T_{2_F}, derived as follow:

$$\begin{aligned}
&E\left[I_2 e^{(-rT_{2_F})}\right] = \\
&= I_2\int_{t=0}^{\infty}\lambda e^{(-\lambda t)}e^{(-rt)}dt \\
&= I_2\frac{\lambda}{r+\lambda}
\end{aligned} \tag{A10}$$

4. Derivation and Proofs

4.1 Equations (47) and (48)

In Expression (46) the two regions, $\varphi < \tilde{\varphi}^*_{F_{21}}$ and $\varphi \geq \tilde{\varphi}^*_{F_{21}}$, must met at $\varphi = \tilde{\varphi}^*_{F_{21}}$ and $\tilde{F}_{L_{12}}(\varphi)$ must be continuous and differentiable along φ. Therefore,

$$E(\tilde{\varphi}^*_{F_{21}})^{\beta_3} + \frac{\tilde{\varphi}^*_{F_{21}}(de_{1_L2_F} - de_{1_L0_F})}{r - \mu_X - \mu_E}\left(\frac{\tilde{\varphi}^*_{F_{21}}}{\tilde{\varphi}^*_{F_{21}}}\right)^{\beta_3} + \frac{\tilde{\varphi}^*_{F_{21}} de_{1_L0_F}}{(r - \mu_X - \mu_E)} = G(\tilde{\varphi}^*_{F_{21}})^{\beta_4} + \frac{\tilde{\varphi}^*_{F_{21}} de_{1_L0_F}}{(r - \mu_X - \mu_E + \lambda)} + \frac{\tilde{\varphi}^*_{F_{21}} de_{1_L2_F}}{(r - \mu_X - \mu_E)} \frac{\lambda}{(r - \mu_X - \mu_E + \lambda)} \tag{A11}$$

$$\beta_3 E(\tilde{\varphi}^*_{F_{21}})^{(\beta_3 - 1)} + \frac{(de_{1_L2_F} - de_{1_L0_F})}{r - \mu_X - \mu_E} + \frac{de_{1_L0_F}}{(r - \mu_X - \mu_E)} = \beta_4 G(\tilde{\varphi}^*_{F_{21}})^{(\beta_4 - 1)} + \frac{de_{1_L0_F}}{(r - \mu_X - \mu_E + \lambda)} + \frac{de_{1_L2_F}}{(r - \mu_X - \mu_E)} \frac{\lambda}{(r - \mu_X - \mu_E + \lambda)} \tag{A12}$$

Solving in order to E and G, after some algebraic manipulation we get equations (47) and (48).

4.2 Proof: $E > 0$

Rewriting equation (47) as (A15),

$$E = \frac{\left(\tilde{\varphi}^*_{F_{21}}\right)^{1-\beta_3}\left[(r - \mu_X - \mu_E)(\beta_1 - \beta_4) + \lambda(\beta_1 - 1)\right]\left(de_{1_L0_F} - de_{1_L2_F}\right)}{(r - \mu_X - \mu_E + \lambda)(r - \mu_X - \mu_E)(\beta_3 - \beta_4)} \tag{A15}$$

We know that $\tilde{\varphi}^*_{F_{21}}$, $(r - \mu_X - \mu_E) > 0$, r, λ, β_1 and β_3 are all positive and $\beta_4 < 0$. Simplifying the numerator: let $v_3 = \left(\tilde{\varphi}^*_{2_F}\right)^{(1-\beta_3)}$, $a = (r - \mu_X - \mu_E)$, $b = \beta_1 - \beta_4$, $c = \lambda(\beta_1 - 1)$, $d = \left(de_{1_L0_F} - de_{1_L2_F}\right)$. Simplifying the denominator: let $u = (r - \mu_X - \mu_E + \lambda)$ and $e = (\beta_3 - \beta_4)$. Substituting in (A15) and rewriting yields:

$$E = \frac{v_3\left[a(b)c)\right]d}{u(a)e} \tag{A16}$$

All terms v_3, a, b, c and d are positive (for c note that $\beta_1 > 1$). From equation (42) we can see that $\beta_3 > 0$ and $\beta_4 < 0$, so $e > 0$). Hence, $E > 0$.

4.3 Proof: $G > 0$

Rewriting equation (48) as (A17),

$$G = \frac{\left(\tilde{\varphi}^*_{F_{21}}\right)^{1-\beta_4}\left[(r - \mu_X - \mu_E)(\beta_1 - \beta_3) + \lambda(\beta_1 - 1)\right]\left(de_{1_L0_F} - de_{1_L2_F}\right)}{(r - \mu_X - \mu_E + \lambda)(r - \mu_X - \mu_E)(\beta_3 - \beta_4)} \tag{A17}$$

We know that $\tilde{\varphi}^*_{F_{21}}$, $(r - \mu_X - \mu_E) > 0$, r, λ, β_1 and β_3 are all positive and $\beta_4 < 0$. Simplifying the numerator: let $v_4 = \left(\tilde{\varphi}^*_{F_{21}}\right)^{1-\beta_4}$, $a = (r - \mu_X - \mu_E)$, $b = \beta_1 - \beta_3$, $c = \lambda(\beta_1 - 1)$, $d = \left(de_{1_L0_F} - de_{1_L2_F}\right)$. Simplifying the denominator: let $u = (r - \mu_X - \mu_E + \lambda)$ and $e = (\beta_3 - \beta_4)$. Substituting in (A17) and rewriting yields:

$$G = \frac{v_4\left[a(b)c)\right]d}{u(a)e} \tag{A18}$$

For $\lambda = 0$, $b = \beta_1 - \beta_3 = 0$ (compare equations 12 and 40). Defining the numerator of (A18) with β_3 as a function of λ and taking its second derivative we can see that it is positive. In addition, we know that v_4, a, c and d are all positive (for c note that $\beta_1 > 1$). From equation (42) we can see that $\beta_3 > \beta_4$, so $e > 0$. Hence $G > 0$.

5 Derivation – Expressions (51) – (53)

Solving the continuity and differentiability conditions for $\tilde{F}_{F_{11}}(\varphi)$ at $\varphi=\tilde{\varphi}^*_{F_{21}}$ and the value-matching and smooth-pasting conditions for $\tilde{F}_{F_{11}}(\varphi)$ at $\varphi=\tilde{\varphi}^*_{F_{11}}$ we get the expressions for J, H, P and $\tilde{\varphi}^*_{F_{11}}$ (notice that Y is given by Equation 39).

$$J\tilde{\varphi}^{*\ \psi_1}_{F_{21}}+C\tilde{\varphi}^{*\ \beta_1}_{F_{21}}+H\tilde{\varphi}^{*\ \beta_3}_{F_{21}}=J\tilde{\varphi}^{*\ \psi_1}_{F_{21}}+P\tilde{\varphi}^{*\ \beta_4}_{F_{21}}+\frac{\tilde{\varphi}^*_{F_{21}}de_{2_F1_L}}{r-\mu_X-\mu_E}\frac{\lambda}{r-\mu_X-\mu_E+\lambda}-\frac{\lambda I_2}{r+\lambda} \tag{A19}$$

$$\beta_3J\tilde{\varphi}^{*\ (\psi_1-1)}_{F_{21}}+\beta_1Y\tilde{\varphi}^{*\ (\beta_1-1)}_{F_{21}}+\beta_3H\tilde{\varphi}^{*\ (\beta_3-1)}_{F_{21}}=\beta_3J\tilde{\varphi}^{*\ (\psi_1-1)}_{F_{21}}+\beta_4P\tilde{\varphi}^{*\ (\beta_4-1)}_{F_{21}}+\frac{de_{2_F1_L}}{r-\mu_X-\mu_E}\frac{\lambda}{r-\mu_X-\mu_E+\lambda} \tag{A20}$$

$$J\tilde{\varphi}^{*\ \beta_3}_{F_{11}}+P\tilde{\varphi}^{*\ \beta_4}_{F_{11}}+\frac{\lambda}{r-\mu_X-\mu_E+\lambda}\frac{\tilde{\varphi}^*_{F_{11}}de_{2_F1_L}}{r-\mu_X-\mu_E}-\frac{\lambda I_2}{r+\lambda}=\frac{\tilde{\varphi}^*_{F_{11}}de_{1_F1_L}}{r-\mu_X-\mu_E}-I_1 \tag{A21}$$

$$\beta_3J\tilde{\varphi}^{*\ (\beta_3-1)}_{F_{11}}+\beta_4P\tilde{\varphi}^{*\ (\beta_4-1)}_{F_{11}}+\frac{\lambda}{r-\mu_X-\mu_E+\lambda}\frac{de_{2_F1_L}}{r-\mu_X-\mu_E}=\frac{de_{1_F1_L}}{r-\mu_X-\mu_E} \tag{A22}$$

The equation system (A19)-(A22) has four equations and four unknowns. Solving for J, H, P and $\tilde{\varphi}^*_{F_{11}}$, after some algebraic manipulation, yields Equations (51), (52), (53) and (54). We find that the constant $P=W$ (see Equation 40).

6 Derivation - Expression (57)

Let the first integral of equation (56) be:

$$Z(\varphi)=E\left[\int_{t=T_{1_L}}^{T_{1_F}}\varphi(t)\left(de_{1_L0_F}\right)e^{-rt}dt\right] \tag{A21}$$

$$rZ(\varphi)=\varphi\left(de_{1_L0_F}\right)+\lim_{dt\to0}\frac{1}{dt}E\left[dZ(\varphi)\right] \tag{A22}$$

Ito's lemma gives:

$$E\left[dZ(\varphi)\right]=(1-\lambda dt)\left(\frac{1}{2}\sigma_m^2\varphi^2\frac{\partial^2Z(\varphi)}{\partial\varphi^2}dt+(\sigma_X\sigma_E\rho+\mu_X+\mu_E)\varphi\frac{\partial Z(\varphi)}{\partial\varphi}dt\right)+\lambda dt(0-Z(\varphi)) \tag{A23}$$

Leading to:

$$\frac{1}{2}\sigma_m^2\varphi^2\frac{\partial^2Z(\varphi)}{\partial\varphi^2}+\left(\sigma_X\sigma_E\rho_{XE}+\mu_X+\mu_E\right)\varphi\frac{\partial Z(\varphi)}{\partial\varphi}-\left(r+\lambda\right)Z(\varphi)+\varphi\left(de_{1_L0_F}\right)=0 \tag{A24}$$

With solution:

$$Z(\varphi)=C_1\varphi^{\beta_3}+C_2\varphi^{\beta_4}+\frac{\varphi\left(de_{1_L0_F}\right)}{r-\mu_X-\mu_E+\lambda} \tag{A25}$$

$C_2=0$ since $\beta_4<0$ and as φ increases the value of the leader should increase. Using the absorbing barrier condition $Z(0)=0$ and the condition that ensures that at the follower's investment threshold the leader's option value is null, i.e., $Z(\tilde{\varphi}^*_{F_{11}})=0$ we conclude that and C_1 is given by,

$$C_1=(\tilde{\varphi}^*_{F_{11}})^{1-\beta_3}\frac{-de_{1_L0_F}}{r-\mu_X-\mu_E+\lambda} \tag{A26}$$

Let the second integral of equation (56) be:

$$W(\varphi)=\left[e^{-rT}\Big|_{T\le T_{1_F}}\right]F_{L_{12}}(\varphi)+\int_{t=\min(t,T_{1_F})}^{\infty}\left[\varphi\left(de_{1_L1_F}\right)e^{-rt}\Big|_{T>T_{1_F}}\right]dt \tag{A27}$$

The function $W(\varphi)$ must satisfy the Bellman equation for $\varphi<\tilde{\varphi}^*_{F_{11}}$:

$$rW(\varphi)=(1-\lambda dt)\left(\frac{1}{2}\sigma_m^2\varphi^2\frac{\partial^2 W(\varphi)}{\partial\varphi^2}dt+\left(\sigma_X\sigma_E\rho_{XE}+\mu_X+\mu_E\right)\varphi\frac{\partial W(\varphi)}{\partial\varphi}dt\right)+\lim_{dt\to 0}\frac{1}{dt}\left(\lambda dt(F_{F_{21}}(\varphi)-W(\varphi)\right) \tag{A28}$$

Leading to:

$$\frac{1}{2}\sigma_m^2\varphi^2\frac{\partial^2 Z(\varphi)}{\partial\varphi^2}+\left(\sigma_X\sigma_E\rho+\mu_X+\mu_E\right)\varphi\frac{\partial Z(\varphi)}{\partial\varphi}-\left(r-\lambda\right)W(\varphi)+\lambda F_{F_{21}}(\varphi)=0 \tag{A29}$$

With solution:

$$W(\varphi)=\begin{cases} B_1\varphi^{\beta_3}+B_2\varphi^{\beta_4}+A_{12}\varphi^{\beta_1}+\dfrac{\varphi de_{1_L0_F}}{r-\mu_X-\mu_E}\dfrac{\lambda}{(r-\mu_X-\mu_E+\lambda)} & \varphi<\tilde{\varphi}^*_{F_{21}} \\ B_3\varphi^{\beta_3}+B_4\varphi^{\beta_4}+\dfrac{\varphi de_{1_L2_F}}{r-\mu_X-\mu_E}\dfrac{\lambda}{(r-\mu_X-\mu_E+\lambda)} & \varphi\geq\tilde{\varphi}^*_{F_{21}} \end{cases} \tag{A30}$$

Using the boundary conditions: $W(0)=0$ we get the constant $B_2=0$. The rest of the constants are determined by solving the continuity and differentiability condition at $\varphi=\tilde{\varphi}^*_{F_{21}}$ and using the boundary condition $W(\tilde{\varphi}^*_{F_{11}})=\frac{\tilde{\varphi}^*_{F_{11}}\left(de_{1_L1_F}\right)}{r-\mu_X-\mu_E}$, leading to:

$$B_1=B_3+E \tag{A31}$$

$$B_4=G \tag{A31}$$

where E and G are given by equations (47) and (48), respectively, and B_3 is given by:

$$B_3=(\tilde{\varphi}^*_{F_{11}})^{-\beta_3}\left(\frac{\tilde{\varphi}^*_{F_{11}}de_{1_L1_F}}{r-\mu_X-\mu_E}-\frac{\lambda\tilde{\varphi}^*_{F_{11}}de_{1_L2_F}}{(r-\mu_X-\mu_E+\lambda)(r-\mu_X-\mu_E)}\right)-G(\varphi^*_{1_F})^{(\beta_4-\beta_3)} \tag{A32}$$

Combining equations (55), (A25) and (A30) we get equation (57), rewritten here as (A33)

$$\tilde{F}_{L_{11}}(\varphi)=\begin{cases} L\varphi^{\beta_3}+\dfrac{\varphi(de_{1_L2_F}-de_{1_L0_F})}{r-\mu_X-\mu_E}\left(\dfrac{\varphi}{\varphi^*_{F_{21}}}\right)^{\beta_1}+\dfrac{\varphi de_{1_L0_F}}{r-\mu_X-\mu_E}-I_1 & \varphi\in\left[\tilde{\varphi}^*_{L_{12}},\tilde{\varphi}^*_{F_{21}}\right) \\ M\varphi^{\beta_3}+G\varphi^{\beta_4}+\dfrac{\varphi de_{1_L0_F}}{(r-\mu_X-\mu_E+\lambda)}+\dfrac{\varphi de_{1_L2_F}}{(r-\mu_X-\mu_E)}\dfrac{\lambda}{(r-\mu_X-\mu_E+\lambda)} & \varphi\in\left[\tilde{\varphi}^*_{F_{21}},\tilde{\varphi}^*_{F_{11}}\right) \\ \dfrac{\varphi de_{1_L1_F}}{r-\mu_X-\mu_E} & \varphi\in\left[\tilde{\varphi}^*_{F_{11}},\infty\right) \end{cases} \tag{A33}$$

where,

$$L=C_1+B_3+E \tag{A34}$$

$$M=C_1+B_3 \tag{A35}$$

With C_1, B_3 and E given by (A26), (A32) and (48), respectively.