The growth of emerging economies and global macroeconomic stability

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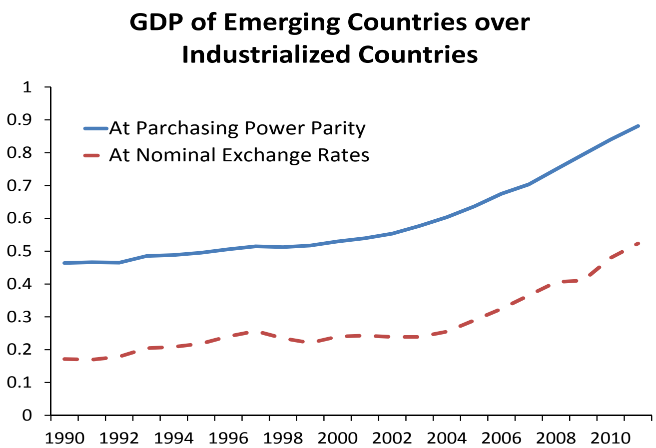
Abstract

This paper studies how the unprecedent growth within emerging countries during the last two decades has affected global macroeconomic stability in both emerging and industrialized countries. To address this question I develop a two-country model (representative of industrialized and emerging economies) where financial intermediaries play a central role in the domestic and international intermediation of funds. The main finding is that the growth of emerging economies has lead to a significant worldwide increase in the demand for safe financial assets. The greater demand for safe assets has increased the incentive of banks to leverage which in turn has contributed to greater financial and macroeconomic instability in both industrialized and emerging economies.

1 Introduction

During the last two decades we have witnessed unprecedent growth within emerging countries. As a result of the sustained growth, the size of these economies has increased dramatically compared to industrialized countries. The top panel of Figure 1 shows that, in PPP terms, the GDP of emerging countries at the beginning of the 1990s was 46 percent of the GDP of industrialized countries. This number has increased to 90 percent by 2011. When the GDP comparison is based on nominal exchange rates, the relative size of emerging economies has increased from 17 to 52 percent.

During the same period, emerging countries have increased the foreign holdings of safe assets. It is customary to divide foreign assets in four classes:



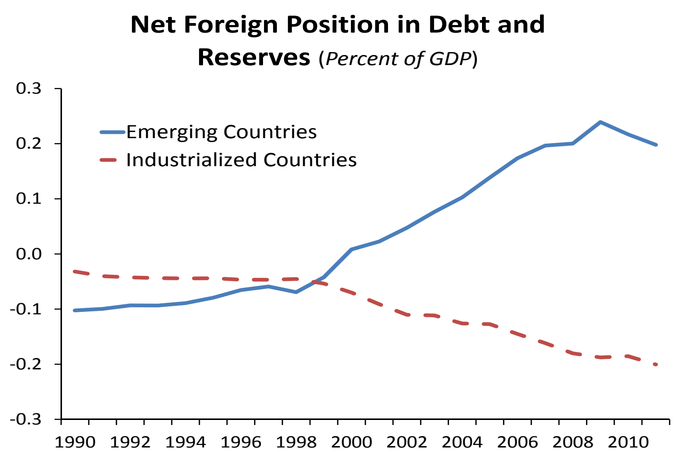


Figure 1: Gross domestic product and net foreign positions in debt instruments and international reserves of emerging and industrialized countries. **Emerging countries**: Argentina, Brazil, Bulgaria, Chile, China, Hong.Kong, Colombia, Estonia, Hungary, India, Indonesia, South Korea, Latvia, Lithuania, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Romania, Russia, South Africa, Thailand, Turkey, Ukraine, Venezuela. **Industrialized countries**: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United.Kingdom, United.States. **Sources**: World Development Indicators (World Bank) and External Wealth of Nations Mark II database (Lane and Milesi-Ferretti (2007)).

(i) debt instruments and international reserves; (ii) portfolio investments; (iii) foreign direct investments; (iv) other investments (see Gourinchas and Rey

(2007) and Lane and Milesi-Ferretti (2007)). The net foreign position in the first class of assets—debt and international reserves—is plotted in the bottom panel of Figure 1 separately for industrialized and emerging economies. Since the early 1990s, emerging countries have accumulated 'positive' net positions while industrialized countries have accumulated 'negative' net positions. Therefore, the increase in the relative size of emerging economies has been associated with a significant accumulation of safe financial assets by these countries.

There are several theories proposed in the literature to explain why emerging countries accumulate safe assets issued by industrialized countries. One explanation posits that emerging countries have been pursuing policies aimed at keeping their currencies undervalued and, to achieve this goal, they have been purchasing large volumes of foreign financial assets. Another explanation is based on differences in the characteristics of financial markets between emerging and industrialized countries. The idea is that lower financial development in emerging countries impairs the ability of these countries to create viable saving instruments for intertemporal smoothing (Caballero, Farhi, and Gourinchas (2008)) or for insurance purpose (Mendoza, Quadrini, and Ríos-Rull (2009)). Because of this, emerging economies turn to industrialized countries for the acquisition of these assets. A third explanation is based on greater idiosyncratic uncertainty faced by consumers and firms in emerging countries due, for example, to higher idiosyncratic risk or lower safety net provided by the public sector.

As briefly summarized above, the existing literature emphasizes that emerging economies tend to have an excess demand for safe financial assets. As the relative size of these countries increases, so does the global demand for these assets. The goal of this paper is to study how this affects financial and macroeconomic stability in both emerging and, especially, in industrialized countries.

To address this question I develop a two-country model where financial intermediaries play a central role in the intermediation of funds from agents in excess of funds (lenders) to agents in need of funds (borrowers). Financial intermediaries issue liabilities and make loans. Differently from recent macroeconomic models proposed in the literature,[1] I emphasize the central

[1]See, for example, Van den Heuvel (2008), Meh and Moran (2010), Brunnermeier and Sannikov (2010), Gertler and Kiyotaki (2010), Mendoza and Quadrini (2010), De Fiore and Uhlig (2011), Gertler and Karadi (2011), Boissay, Collard, and Smets (2010), Corbae and D'Erasmo (2012), Rampini and Viswanathan (2012), Adrian, Colla, and Shin (2013).

role of banks in issuing liabilities (or facilitating the issuance of liabilities) rather than its lending role for macroeconomic dynamics.

An important role played by bank liabilities is that they can be held by other sectors of the economy for insurance purposes. Then, when the stock of bank liabilities increases, agents that hold these liabilities are better insured and willing to engage in activities that are individually risky. In aggregate, this allows for sustained employment, production and consumption. However, when banks issue more liabilities, they also create the conditions for a liquidity crisis. A crisis generates a drop in the volume of intermediated funds and with it a fall in the stock of bank liabilities held by the nonfinancial sector. As a consequence of this, the nonfinancial sector will be less willing to engage in risky activities with a consequent contraction in real macroeconomic activity.

The probability and macroeconomic consequences of a liquidity crisis depend on the leverage chosen by banks, which in turn depends on the interest rate paid on their liabilities (funding cost). When the interest rate is low, banks have more incentives to leverage, which in turn increases the likelihood of a liquidity crisis. It is then easy to see how the growth of emerging countries could contribute to global economic instability. As the share of these countries in the world economy increases, the demand for assets issued by industrialized countries (bank liabilities in the model) rises. This drives down the interest rate paid by banks on their liabilities, increasing the incentives to take more leverage. But as the banking sector becomes more leveraged, the likelihood of a crisis starts to emerge and/or the consequences of a crisis become bigger. As long as a crisis does not materialize, the economy enjoys sustained levels of financial intermediation, asset prices and economic activity. But, eventually, the arrival of a crisis induces a reversal in financial intermediation with a fall in asset prices and overall economic activity.

The organization of the paper is as follows. Section 2 describes the model and characterizes the equilibrium. Section 3 applies the model to study how the growth of emerging economies affect the worldwide demand of financial assets and with it the financial and macroeconomic stability of both emerging and industrialized countries. Section 4 concludes.

2 Model

There are two countries in the model, indexed by $j \in \{1, 2\}$. The first country is representative of the industrialized countries and the second is representative of emerging economies. In each country there are two sectors: the entrepreneurial sector and the worker sector. Furthermore, there is an intermediation sector populated by many profit-maximizing banks that operate globally in a regime of international capital mobility. The role of banks is to facilitate the transfer of resources between entrepreneurs and workers. As we will see, the ownership of banks by country 1 or country 2 is not relevant. What is relevant is that banks operate globally, that is, they can sell liabilities and make loans in both countries.

The two countries differ in three dimensions: population, technology and financial markets. Country j is populated by a mass $N_j/2$ of atomistic entrepreneurs and a mass $N_j/2$ of atomistic workers. Therefore, N_j is the total population of country j.[2] The second difference between the two countries is in technology captured by the aggregate productivity parameter $\bar{z}_j$. The third difference is in financial markets development captured by two parameters: the volatility of the uninsurable idiosyncratic risk, σ_j, and the borrowing limit η_j. Therefore, country heterogeneity is fully captured by differences in four parameters: N_j (population), $\bar{z}_j$ (productivity), σ_j (uninsurable risk), and η_j (borrowing limit). The precise role played by each of the four parameters will be described shortly.

2.1 Entrepreneurial sector

In the entrepreneurial sector of country $j \in \{1, 2\}$ there is a mass $N_j/2$ of atomistic entrepreneurs, indexed by i, with lifetime utility,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln(c^i_{j,t}).$$

Entrepreneurs are individual owners of firms, each operating the production function $y^i_{j,t} = z^i_{j,t} h^i_{j,t}$, where $h^i_{j,t}$ is the input of labor supplied by workers in country j at the market wage $w_{j,t}$, and $z^i_{j,t}$ is an idiosyncratic productivity shock. In each country the idiosyncratic productivity is independently and identically distributed among firms and over time, with probability distribution $\Gamma_j(z)$. It would be convenient to assume that $\Gamma_j(z)$ is fully characterized by two country-specific parameters: the mean $\bar{z}_j$ and the standard deviation

[2]The relative population size of entrepreneurs and workers in each country is irrelevant for the properties of the model and the choice of 1/2 is only for convenience.

σ_j. Differences in the mean $\bar{z}_j$ captures cross-country differences in aggregate productivity while differences in the standard deviation σ_j captures cross-country differences in risk. Later I will interpret differences in σ_j as capturing differences in financial markets.[3]

As in Arellano, Bai, and Kehoe (2011), the input of labor $h^i_{j,t}$ is chosen before observing $z^i_{j,t}$, and therefore, labor is risky. To insure the risk, entrepreneurs have access to a market for non-contingent bonds at price q_t. As we will see later, the bonds held by entrepreneurs are the liabilities issued by banks. Notice that the market price of bonds does not have the subscript j because capital mobility implies that the price will be equalized across countries. Since the bonds cannot be contingent on the realization of the idiosyncratic shock $z^i_{j,t}$, they provide only partial insurance for entrepreneurs' consumption. Once we introduce the financial intermediation sector, the bonds held by entrepreneurs are the liabilities issued by intermediaries.

An entrepreneur i in country j enters period t with bonds $b^i_{j,t}$ and chooses the labor input $h^i_{j,t}$. After the realization of the idiosyncratic shock $z^i_{j,t}$, he/she chooses consumption $c^i_{j,t}$ and next period bonds $b^i_{j,t+1}$. The budget constraint is

$$c^i_{j,t} + q_t b^i_{j,t+1} = (z^i_{j,t} - w_{j,t})h^i_{j,t} + b^i_{j,t}. \quad (1)$$

Because labor $h^i_{j,t}$ is chosen before the realization of $z^i_{j,t}$, while the saving decision is made after the observation of $z^i_{j,t}$, it will be convenient to define $a^i_{j,t} = b^i_{j,t} + (z^i_{j,t} - w_{j,t})h^i_{j,t}$ the entrepreneur's wealth after production. Given the timing assumption, the input of labor $h^i_{j,t}$ depends on $b^i_{j,t}$ while the saving decision $b^i_{j,t+1}$ depends on $a^i_{j,t}$. The optimal entrepreneur's policies are characterized by the following lemma:

Lemma 2.1 *Let $\phi_{j,t}$ satisfy the condition* $\int_z \left\{ \frac{z - w_{j,t}}{1 + (z - w_{j,t})\phi_{j,t}} \right\} \Gamma_j(z) = 0.$ *The optimal entrepreneur's policies are*

$$\begin{aligned} h^i_{j,t} &= \phi_{j,t} b^i_{j,t}, \\ c^i_{j,t} &= (1-\beta) a^i_{j,t}, \\ q_t b^i_{j,t+1} &= \beta a^i_{j,t}. \end{aligned}$$

[3] I will interpret σ_j as the residual idiosyncratic risk that cannot be insured directly through financial markets (for example by selling a share of the business to external investors). Once interpreted this way it is easy to see that the residual risk faced by individuals is lower in countries with more developed financial markets.

Proof 2.1 *See Appendix A.*

The demand for labor is linear in the initial wealth of the entrepreneur $b^i_{j,t}$. The term $\phi_{j,t}$ is defined by the condition $\int_z \left\{ \frac{z-w_{j,t}}{1+(z-w_{j,t})\phi_{j,t}} \right\} \Gamma_j(z) = 0$. Notice that the wage rate $w_{j,t}$ and the distribution of the shock $\Gamma_j(z)$ are country specific. Therefore, the value of $\phi_{j,t}$ differs across countries but it is the same for all entrepreneurs of each country. Since the distribution of the shock is fixed in the model, the only endogenous variable that affects $\phi_{j,t}$ is the wage rate $w_{j,t}$. Therefore, I denote this variable by the function $\phi_j(w_{j,t})$, which is strictly decreasing in its argument (the wage rate).

Because $\phi_j(w_{j,t})$ is the same for all entrepreneurs of country j, I can derive the aggregate demand for labor in country j as

$$H_{j,t} = \phi_j(w_{j,t}) \int_i b^i_{j,t} = \phi_j(w_{j,t}) B_{j,t},$$

where capital letters denote average (per-capita) variables.

The aggregate demand of labor depends negatively on the wage rate—which is a standard property—and positively on the financial wealth of entrepreneurs even if they are not financially constrained—which is a special property of this model. This property derives from the risk associated with hiring: entrepreneurs are willing to hire more labor when they hold more financial wealth as an insurance buffer.

Also linear is the consumption policy which follows from the logarithmic specification of the utility function. This property allows for linear aggregation. Another property worth emphasizing is that in a stationary equilibrium with constant $B_{j,t}$, the interest rate (the inverse of the price of bonds q_t) must be lower than the intertemporal discount rate, that is, $q_t > \beta$.[4]

[4] To see this, consider the first order condition of an individual entrepreneur for the choice of $b^i_{j,t+1}$. This is the typical euler equation that, with log preferences, takes the form $q_t/c^i_{j,t} = \beta \mathbb{E}_t(1/c^i_{j,t+1})$. Because individual consumption $c^i_{j,t+1}$ is stochastic, $\mathbb{E}_t(1/c^i_{j,t+1}) > 1/\mathbb{E}_t c^i_{j,t+1}$. Therefore, if $q_t = \beta$, we would have that $\mathbb{E}_t c^i_{j,t+1} > c^i_{j,t}$, implying that individual consumption would growth on average over time. But then aggregate consumption would not be bounded, which violates the hypothesis of a stationary equilibrium. I will come back to this property later.

2.2 Worker sector

In each country $j \in \{1, 2\}$ there is a mass $N_j/2$ of atomistic workers with lifetime utility,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(c_{j,t} - \alpha \bar{z}_j \frac{h_{j,t}^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \right),$$

where $c_{j,t}$ is consumption and $h_{j,t}$ is the supply of labor. Workers do not face idiosyncratic risks and the assumption of risk neutrality is not important for the key results of the paper as I will discuss below. Notice that the dis-utility from working also depends on productivity $\bar{z}_j$, which differs across countries. This guarantees that hours worked in the two countries are not too different and the model features balanced growth.

Worker can trade a non-reproducible asset available in per-capital fixed supply $\overline{K}_j$. The aggregate supply is $\overline{K}N_j$. Each unit of the asset produces $\bar{z}_j$ units of consumption goods. The variable $\bar{z}_j$ is also the average productivity of entrepreneurs. Therefore, the two countries are characterizes by the same productivity differentials in the entrepreneurial and household sectors. The asset $\overline{K}$ is divisible and can be traded at the market price $p_{j,t}$. We can interpret this asset as housing and Π_j as the services produced by one unit of housing. Workers can borrow at the gross interest rate R_t and face the individual budget constraint

$$c_{j,t} + l_{j,t} + (k_{j,t+1} - k_{j,t})p_{j,t} = \frac{l_{j,t+1}}{R_t} + w_{j,t}h_{j,t} + \bar{z}_j k_{j,t},$$

where $l_{j,t}$ is the loan contracted in period $t-1$ and due in the current period t, and $l_{j,t+1}$ is the new loan that will be repaid in the next period $t+1$. The interest rate on loans does not have the country subscript j because interest rates will be equalized across countries.

Debt is constrained by a borrowing limit. I will consider two specifications. In the first specification the borrowing limit takes the form

$$l_{j,t+1} \leq \eta_j, \tag{2}$$

where η_j is a constant parameter that could differ across countries. Later I will also consider a more complex borrowing constraint that depends on the collateral value of assets, that is,

$$l_{j,t+1} \leq \eta_j \mathbb{E}_t p_{j,t+1} k_{j,t+1}. \tag{3}$$

The borrowing constraint (2) allows me to characterize the equilibrium analytically. However, as we will see, with this specification of the borrowing constraint, the asset price $p_{j,t}$ is constant in equilibrium. Instead, when the borrowing constraint takes the form (3), the model also provides interesting predictions about the asset price $p_{j,t}$ but the full characterization of the equilibrium can be done only numerically.

Appendix C writes down the workers' problem and derives the first order conditions. When the borrowing constraint takes the form specified in (2), the optimality conditions are

$$\alpha \bar{z}_j h_{j,t}^{\frac{1}{\nu}} = w_{j,t}, \tag{4}$$

$$1 = \beta R_t (1 + \mu_{j,t}), \tag{5}$$

$$p_{j,t} = \beta \mathbb{E}_t (\bar{z}_j + p_{j,t+1}), \tag{6}$$

where $\beta \mu_{j,t}$ is the Lagrange multiplier associated with the borrowing constraint. When the borrowing constraint takes the form specified in (3), the first order conditions with respect to $h_{j,t}$ and $l_{j,t+1}$ are still (4) and (5) but the first order condition with respect to $k_{j,t+1}$ becomes

$$p_{j,t} = \beta \mathbb{E}_t \Big[\bar{z}_j + (1 + \eta_j \mu_{j,t}) p_{j,t+1} \Big]. \tag{7}$$

2.3 Equilibrium with direct borrowing and lending

Before introducing the financial intermediation sector it would be instructive to characterize the equilibrium with direct borrowing and lending. I will denote with capital letters the aggregate stock of bonds and loans, that is, $B_{j,t} = b_{j,t} N_j / 2$ and $L_{j,t} = l_{j,t} N_j / 2$.

With direct borrowing and lending, the worldwide bonds held by entrepreneurs are equal to the loans taken by workers, that is, $B_{1,t} + B_{2,t} = L_{1,t} + L_{2,t}$ and the interest rate on bonds is equal to the interest rate on loans, that is, $1/q_t = R_t$. Because of capital mobility and cross-country heterogeneity, the net foreign asset positions of the two countries are different from zero, that is, $B_{j,t} \neq L_{j,t}$.

Proposition 2.1 *In absence of aggregate shocks, the economy converges to a steady state in which workers borrow from entrepreneurs and* $q = 1/R > \beta$.

Proof 2.1 *See Appendix B*

The fact that the steady state interest rate is lower than the intertemporal discount rate is a consequence of the uninsurable risk faced by entrepreneurs. If $q = \beta$, entrepreneurs would continue to accumulate bonds without limit in order to insure the idiosyncratic risk. The supply of bonds from workers, however, is limited by the borrowing limit. To insure that entrepreneurs do not accumulate an infinite amount of bonds, the interest rate has to fall below the intertemporal discount rate.

The equilibrium in the labor market can be characterized as the simple intersection of aggregate demand and supply in each country as depicted in Figure 2. The aggregate demand in country j was derived in the previous subsection and takes the form $H^D_{j,t} = \phi_j(w_{j,t})B_{j,t}$. It depends negatively on the wage rate $w_{j,t}$ and positively on the aggregate wealth (bonds) of entrepreneurs, $B_{j,t}$. The supply of labor is derived from the households' first order condition (4) and takes the form $H^S_{j,t} = \left(\frac{w_{j,t}}{\alpha \bar{z}_j}\right)^\nu$.

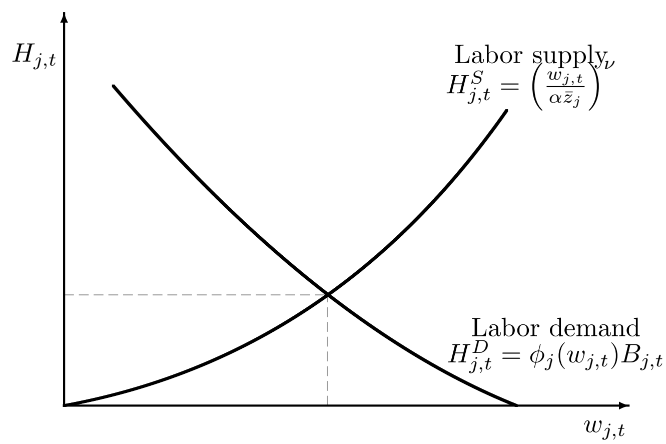


Figure 2: Labor market equilibrium.

The dependence of the demand of labor from the financial wealth of entrepreneurs is a key property of this model. When entrepreneurs hold a lower value of $B_{j,t}$, the demand for labor declines and in equilibrium there is lower employment and production. Importantly, the reason lower values of $B_{j,t}$ decreases the demand of labor is not because employers do not have funds to finance hiring or because they face a higher financing cost. In fact, employers do not need any financing to hire and produce. Instead, the transmission mechanism is based on the lower financial wealth of entrepreneurs which is held as a buffer to insure the idiosyncratic risk. This mechanism is clearly distinct from the traditional 'credit channel' where firms are in need of funds to finance employment (for example, because wages are paid in advance) or to finance investment.

The next step is to introduce financial intermediaries and show that a fall in $B_{j,t}$ could be the result of a crisis that originates in the financial sector.

Discussion The equilibrium is characterized by producers (entrepreneurs) that are net savers and workers that are net borrowers. Since it is customary to work with models in which firms are net borrowers, this property may seem counterfactual. However, when we consider the recent changes in the financial structure of US corporations, this property of the model is not inconsistent with the current structure of corporations. It is well known that during the last two and half decades, US corporations have increased their holdings of financial assets, suggesting that the proportion of financially dependent firms has declined significantly over time. This view is consistent with the study of Shourideh and Zetlin-Jones (2012) and Eisfeldt and Muir (2012). The large accumulation of financial assets by firms (often referred to as cash) is also observed in emerging countries (for example, in China). The model developed here is meant to capture the growing importance of firms that are no longer dependent on external financing.

The second remark is that the equilibrium outcome that the entrepreneurial sector is a net lender does not derive from the assumption that entrepreneurs are more risk-averse than workers. Instead, it follows from the assumption that only entrepreneurs are exposed to uninsurable risks. As long as producers face more risk than workers, the former would continue to lend to the latter even if workers were risk averse. Therefore, the assumption that workers are risk neutral is only made for analytical convenience and it is without loss of generality.

The final remark relates to the assumption that the idiosyncratic risk faced by entrepreneurs cannot be insured away (market incompleteness). Given that workers are risk neutral, it would be optimal for entrepreneurs to offer a wage that is contingent on the output of the firm. Although this is excluded by assumption, it is not difficult to extend the model so that the lack of insurance from workers is an endogenous outcome of information asymmetries. The idea is that, when the wage is state-contingent, firms could use their information advantage about the performance of the firm to gain opportunistically from workers. Since this is well known in the literature, to keep the model simple I have assumed that state contingent wages are not feasible.[5]

2.4 Financial intermediation sector

If direct borrowing is not feasible or is inefficient, financial intermediaries become important for transferring funds from lenders to borrowers and to create financial assets that could be held for insurance purposes.

To formalize this idea, suppose that direct borrowing implies a cost $\tilde{\tau}$. The analysis of the previous section can be trivially extended with the explicit consideration of this cost with the equilibrium characterized by $1/[(1-\tilde{\tau})q_t] = R_t$. In this economy, financial intermediaries play an important role because, by specializing in financial intermediation, they have the comparative advantage of raising funds at a cost τ lower than $\tilde{\tau}$.

Financial intermediaries are infinitely lived, profit-maximizing firms owned by workers. The assumption that they are owned by workers, as opposed to entrepreneurs, simplifies the analysis because workers are risk neutral while entrepreneurs are risk averse. Risk neutrality also implies that it is irrelevant whether banks are owned by domestic or foreign workers. Even if I use the term 'banks' as a reference to financial intermediaries, it should be clear that the financial sector is representative of all financial firms, not only commercial banks.

Banks operate globally, that is, they sell liabilities and make loans to domestic and foreign agents. A bank starts the period with loans made to workers, l_t, and liabilities held by entrepreneurs, b_t. These loans and liabilities were made in the pervious period $t-1$. Since the interest rates on loans will be equalized across countries, banks are indifferent about the nationality of costumers (besides making sure that the borrowing constraints are not violated). Similarly, the interest rate paid by banks on their liabilities will be equalized across countries. Therefore, I will use the notation l_t and b_t without subscript j to denote the loans and liabilities of an individual bank.

[5]It could be claimed that in reality there are markets where some form of contingent claims are traded. For example, the sale corporate shares. The model accounts for this by interpreting σ_j as the residual risk that cannot be eliminated by trading in these markets. In the quantitative analysis I will use this idea to interpret cross-country differences in σ_j as reflecting differences in the characteristics of financial markets allowing for different degrees of insurance.

The difference between loans and liabilities is the bank equity $e_t = l_t - b_t$.

Renegotiation of bank liabilities Given the beginning of period balance sheet position, the bank could default on its liabilities. In case of default creditors have the right to liquidate the bank assets l_t but they may not be able to recover the full value of the liquidated assets. More specifically, in every period there is a probability λ_t that creditors can recover only a fraction $\underline{\xi} < 1$ of the liquidated bank assets (and with probability $1 - \lambda_t$ they will recover the full value). I will use the variable $\xi_t \in \{\underline{\xi}, 1\}$ to denote the fraction of the bank assets recoverable by creditors. The recovery value is the same for all banks (aggregate stochastic variable) and its value was unknown when a bank issued the liabilities b_t and made the loans l_t in the previous period $t - 1$.

The probability λ_t will be derived endogenously in the model. For the moment, however, it will be convenient to think of this probability as exogenously fixed at $\bar{\lambda}$.

Frictions arise from the possibility that the bank renegotiates its liabilities. More specifically, once the value of ξ_t becomes known at the beginning of period t, the bank could use the threat of default to renegotiate the outstanding liabilities. Under the assumption that the bank has the whole bargaining power, it could renegotiate its liabilities to $\xi_t l_t$. Of course, the bank will renegotiate only if the liabilities are bigger than the liquidation value, that is, $b_t > \xi_t l_t$. Therefore, after renegotiation, the residual liabilities of the bank are

$$\tilde{b}_t(b_t, l_t) = \begin{cases} b_t, & \text{if } b_t \le \xi_t l_t \\ \\ \xi_t l_t & \text{if } b_t > \xi_t l_t \end{cases} \tag{8}$$

Renegotiation also brings some cost for the bank. The renegotiation cost, denoted by $\tilde{\varphi}_t(b_{t+1}, l_t)$ takes the form,

$$\tilde{\varphi}_t(b_{t+1}, l_t) = \begin{cases} 0, & \text{if } b_t \le \xi_t l_t \\ \\ \varphi\left(\frac{b_t - \xi_t l_t}{l_t}\right) b_t & \text{if } b_t > \xi_t l_t \end{cases}, \tag{9}$$

where the function $\varphi(.)$ is strictly increasing and convex, differentiable and satisfies $\varphi(0) = \varphi'(0) = 0$.

The cost is incurred only if the bank renegotiates which arises only if the liabilities exceed the liquidation value of its assets, that is, $b_t > \xi_t l_t$.

However, with the cost to renegotiate, it is not certain that the bank would gain from renegotiating whenever $b_t > \xi_t l_t$. This is because the renegotiation cost could be bigger than the gain from debt relief, which is equal to $\xi_t l_t - b_t$. To eliminate this possibility, I make the following assumption:[6]

Assumption 1 *The renegotiation cost satisfies* $\varphi(1-\underline{\xi}) < 1-\underline{\xi}$.

The possibility that the bank renegotiates its liabilities implies potential losses for investors (entrepreneurs). This is fully internalized by the market when the bank issues the new liabilities b_{t+1} and makes the new loans l_{t+1}.

Denote by $\overline{R}_t^b$ the expected gross return from holding the market portfolio of bank liabilities issued in period t and repaid in period $t+1$ (that is, for the liabilities issued by the whole banking sector). Since banks are competitive, the expected return on the liabilities issued by an individual bank must be equal to the aggregate expected return $\overline{R}_t^b$. Therefore, the price of liabilities $q_t(b_{t+1}, l_{t+1})$ issued by an individual bank at time t must satisfy

$$q_t(b_{t+1}, l_{t+1})b_{t+1} = \frac{1}{\overline{R}_t^b}\mathbb{E}_t\tilde{b}_{t+1}(b_{t+1}, l_{t+1}). \tag{10}$$

The left-hand-side is the payment made by investors for the purchase of b_{t+1}. The term on the right-hand-side is the expected repayment in the next period, discounted by $\overline{R}_t^b$ (the expected market return). Since the bank could renegotiate in the next period in the event that $\xi_{t+1} = \underline{\xi}$, the actual repayment $\tilde{b}_{t+1}(b_{t+1}, l_{t+1})$ could differ from b_{t+1}. Arbitrage requires that the purchasing cost of b_{t+1} (the left-hand-side) is equal to the discounted value of the expected repayment (the right-hand-side).

Bank problem and optimality conditions The budget constraint of the bank can be written as

$$\tilde{b}_t(b_t, l_t) + \tilde{\varphi}_t(b_t, l_t) + \frac{l_{t+1}}{R_t^l} + d_t = l_t + (1-\tau)\left[\frac{\mathbb{E}_t\tilde{b}_{t+1}(b_{t+1}, l_{t+1})}{\overline{R}_t^b}\right], \tag{11}$$

[6] Banks do not borrow more than l_t because this will trigger renegotiation with probability 1. Therefore, renegotiation can only arise when $\xi_t = \underline{\xi}$. Provided that $b_t > \underline{\xi} l_t$, the debt reduction from renegotiating is $b_t - \underline{\xi} l_t$. This is a gain that is compared to the cost $\varphi(b_t/l_t - \underline{\xi})b_t$. Suppose that $b_t = l_t$ (maximum leverage). In this case the gain from renegotiation is $l_t - \underline{\xi} l_t$ while the cost is $\varphi(1-\underline{\xi})b_t$. Since $b_t < l_t$, we can verify that the gain is bigger than the cost if $\varphi(1-\underline{\xi}) < 1-\underline{\xi}$. The concavity of $\varphi(.)$ implies that this is also true when the bank chooses a leverage smaller than the maximum, that is, $b_t < l_t$.

where d_t are the dividends paid to shareholders (workers).

The budget constraint takes into account the renegotiation decision which determines the renegotiated liabilities $\tilde{b}_t(b_t, l_t)$ defined in (8) and the renegotiation cost $\tilde{\varphi}_t(b_t, l_t)$ defined in (9). The last term in the budget constraint denotes the funds raised by issuing new liabilities b_{t+1}. The arbitrage condition (10) implies that these funds are equal to $\mathbb{E}_t\tilde{b}_{t+1}(b_{t+1}, l_{t+1})/\overline{R}_t^b$. They are multiplied by $1-\tau$ because the bank incurs the operation cost τ to raise funds (cost that is significantly lower than the cost of direct borrowing $\tilde{\tau}$).

The optimization problem of the bank can be written recursively as

$$V_t(b_t, l_t) \quad = \quad \max_{d_t, b_{t+1}, l_{t+1}} \left\{ d_t + \beta \mathbb{E}_t V_{t+1}(b_{t+1}, l_{t+1}) \right\} \tag{12}$$

subject to (8), (9), (11).

The leverage chosen by the bank will never exceed 1 since the liabilities will be renegotiated with certainty. Once the probability of renegotiation is 1, a further increase in b_{t+1} does not increase the borrowed funds $[(1-\tau)/\overline{R}_t^b]\mathbb{E}_t\tilde{b}_{t+1}(b_{t+1}, l_{t+1})$ but raises the renegotiation cost. Therefore, Problem (12) is also subject to the constraint $b_{t+1} \leq l_{t+1}$.

Denote by $\omega_{t+1} = b_{t+1}/l_{t+1}$ the bank leverage. Appendix D shows that the first order conditions with respect to b_{t+1} and l_{t+1} can be expressed as

$$\frac{1-\tau}{\overline{R}_t^b} \quad \geq \quad \beta \left[1 + \frac{\theta(\omega_{t+1})\Big(\varphi'(\omega_{t+1} - \underline{\xi})\omega_{t+1} + \varphi(\omega_{t+1} - \underline{\xi})\Big)}{1 - \theta(\omega_{t+1})} \right], \tag{13}$$

$$\frac{1}{R_t} \quad \geq \quad \beta \left[1 + \theta(\omega_{t+1})\varphi'(\omega_{t+1} - \underline{\xi})\omega_{t+1}^2 + \theta(\omega_{t+1})\underline{\xi}\left(\frac{1-\tau}{\beta\overline{R}_t^b} - 1 \right) \right], \tag{14}$$

where $\theta(\omega_{t+1})$ is the probability that the bank renegotiates at $t+1$, which is equal to

$$\theta(\omega_{t+1}) = \begin{cases} 0, & \text{if} \quad \omega_{t+1} < \underline{\xi}, \\ \bar{\lambda}, & \text{if} \quad \underline{\xi} \leq \omega_{t+1} \leq 1, \\ 1, & \text{if} \quad \omega_{t+1} > 1. \end{cases}$$

The first order conditions are satisfied with equality if $\omega_{t+1} < 1$ and with inequality if $\omega_{t+1} = 1$. As observed above, for leverages bigger than 1 the bank would renegotiate with certainty, implying that $\omega_{t+1} \leq 1$.

Conditions (13) and (14) make clear that it is the leverage of the bank $\omega_{t+1} = b_{t+1}/l_{t+1}$ that matters, not the scale of operation b_{t+1} or l_{t+1}. This follows from the linearity of the intermediation technology and the risk neutrality of banks. The leverage matters because the renegotiation cost is convex in ω_{t+1}. These properties imply that in equilibrium all banks choose the same leverage (although they could chose different scales of operation).[7]

Further exploration of the first order conditions reveals that, if banks choose a low leverage $\omega_{t+1} < \underline{\xi}$, then the cost of liabilities (inclusive of the operation cost τ) and the lending rate must be equal to the discount rate, that is, $\overline{R}_t^b/(1-\tau) = R_t = 1/\beta$. However, if banks choose $\omega_{t+1} > \underline{\xi}$, the funding cost $\overline{R}_t^b/(1-\tau)$ must be smaller than the interest rate on loans, which is necessary to cover the renegotiation cost incurred with probability $\bar{\lambda}$. This property is stated formally in the next lemma.

Lemma 2.2 *If the leverage is $\omega_{t+1} \leq \underline{\xi}$, then $\frac{\overline{R}_t^b}{1-\tau} = R_t = \frac{1}{\beta}$. If the leverage is $\omega_{t+1} > \underline{\xi}$, then $\frac{\overline{R}_t^b}{1-\tau} < R_t < \frac{1}{\beta}$.*

Proof 2.2 *See Appendix E*

Therefore, once the leverage of banks exceeds $\underline{\xi}$, there is a spread between the funding rate (inclusive of the operation cost τ) and the lending rate. Intuitively, raising the leverage ω_{t+1} above $\underline{\xi}$ increases the expected renegotiation cost. The bank will choose to do so only if there is a spread between the cost of funds and the return on the investment. As the spread increases so does the leverage chosen by banks. When the leverage increases exceeds $\underline{\xi}$, banks could default with positive probability. This generates a loss of financial wealth for entrepreneurs, causing a macroeconomic contraction through the 'bank liabilities channel' as described earlier.

[7]Because the first order conditions (13) and (14) depend only on one individual variable, the leverage ω_{t+1}, there is no guarantee that these conditions are both satisfied for arbitrary values of $\overline{R}_t^b$ and R_t. In the general equilibrium, however, these rates adjust to clear the markets for bank liabilities and loans and both conditions will be satisfied.

2.5 Banking liquidity and endogenous ξ_t

To make ξ_t endogenous, I now interpret this variable as the liquidation price of bank assets. This price will be determined in equilibrium and the liquidity of the banking sector plays a central role in determining this price.

Assumption 2 *If a bank is liquidated, the assets l_t are divisible and can be sold either to other banks or to other sectors (workers and entrepreneurs). However, other sectors can recover only a fraction $\underline{\xi} < 1$.*

An implication of this assumption is that, in the event of liquidation, it is more efficient to sell the liquidated assets to other banks since they have the ability to recover the whole asset value l_t while other sectors can recover only $\underline{\xi} l_t$. This is a natural assumption since banks are likely to have a comparative advantage in the management of financial investments. However, in order for banks to acquired the assets, they need to be liquid.

Assumption 3 *Banks can purchase the assets of a liquidated bank only if they are liquid, that is, $b_t < \xi_t l_t$.*

A bank is liquid if it can issue new liabilities at the beginning of the period without renegotiating. Obviously, if the bank starts with $b_t > \xi_t l_t$—that is, the liabilities are bigger than the liquidation value of its assets—the bank will be unable to raise additional funds: potential investors know that the new liabilities (as well as the outstanding liabilities) are not collateralized and the bank will renegotiate immediately after receiving the funds.

To better understand these assumptions, consider the condition for not renegotiating, $b_t \leq \xi_t l_t$, where now $\xi_t \in \{\underline{\xi}, 1\}$ is the liquidation price of bank assets at the beginning of the period. If this condition is satisfied, banks have the option to raise additional funds at the beginning of the period to purchase the assets of a defaulting bank. This insures that the market price of the liquidated assets is $\xi_t = 1$. However, if $b_t > \xi_t l_t$ for all banks, there will not be any bank with unused credit. As a result, the liquidated assets can only be sold to non-banks and the price will be $\xi_t = \underline{\xi}$. Therefore, the value of liquidated assets depends on the financial decision of banks, which in turn depends on the expected liquidation value of their assets. This interdependence creates the conditions for multiple self-fulfilling equilibria.[8]

[8] Assumptions 2 and 3 are similar to the assumptions made in Perri and Quadrini (2011) but in a model without banks.

Proposition 2.2 *There exists multiple equilibria if and only if the leverage of the bank is within the two liquidation prices, that is,* $\underline{\xi} \leq \omega_t \leq 1$.

Proof 2.2 *See appendix F.*

Given the multiplicity, I assume that the equilibrium is selected stochastically through sunspot shocks. Denote by ε a sunspot variable that takes the value of 0 with probability λ and 1 with probability $1 - \lambda$. The probability of a low liquidation price, denoted by $\theta(\omega_t)$, is then equal to

$$\theta(\omega_t) = \begin{cases} 0, & \text{if} \quad \omega_t < \underline{\xi} \\ \lambda, & \text{if} \quad \underline{\xi} \leq \omega_t \leq 1 \\ 1, & \text{if} \quad \omega_t > 1 \end{cases}$$

If the leverage is sufficiently small ($\omega_t < \underline{\xi}$), banks do not renegotiate even if the liquidation price is low. But then the price cannot be low since banks remain liquid for any expectation of the liquidation price ξ_t, and therefore, for any draw of the sunspot variable ε. Instead, when the leverage is between the two liquidation prices ($\underline{\xi} \leq \omega_t \leq 1$), the liquidity of banks depends on the expectation of this price. Therefore, the equilibrium outcome depends on the realization of the sunspot variable ε. When $\varepsilon = 0$—which happens with probability λ—the market expects the low liquidation price $\xi_t = \underline{\xi}$, making the banking sector illiquid. On the other hand, when $\varepsilon = 1$—which happens with probability $1 - \lambda$—the market expects the high liquidation price $\xi_t = 1$ so that the banking sector remains liquid. The dependence of the probability $\theta(\omega_t)$ on the leverage of the banking sector plays an important role for the results of this paper.

2.6 General equilibrium

To characterize the general equilibrium I first derive the aggregate demand for bank liabilities from the optimal saving of entrepreneurs. I then derive the supply by consolidating the demand of loans from workers with the optimal policy of banks. In this section I assume that the borrowing limit for workers takes the simpler form specified in (2), which allows me to characterize the equilibrium analytically.

Demand for bank liabilities As shown in Lemma 2.1, the optimal saving of entrepreneurs takes the form $q_t b^i_{j,t+1} = \beta a^i_{j,t}$, where $a^i_{j,t}$ is the end-of-period wealth $a^i_{j,t} = \tilde{b}^i_t + (z^i_{j,t} - w_{j,t})h^i_{j,t}$. This lemma continues to hold even if the return from bank liabilities is stochastic since it depends on the realization of the sunspot shock.[9]

Since $h^i_{j,t} = \phi_j(w_{j,t})\tilde{b}^i_{j,t}$ (see Lemma 2.1), the end-of-period wealth can be rewritten as $a^i_{j,t} = [1 + (z^i_{j,t} - w_{j,t})\phi(w_{j,t})]\tilde{b}^i_{j,t}$. Substituting into the optimal saving and aggregating over all entrepreneurs (of total measure $N_j/2$) we obtain

$$q_t B_{j,t+1} = \beta \Big[1 + (\bar{z}_j - w_{j,t})\phi_j(w_{j,t})\Big]\tilde{B}_{j,t}. \tag{15}$$

This equation defines the aggregate demand for bank liabilities in country j as a function of the price of bank liabilities q_t, the wage rate $w_{j,t}$, and the beginning-of-period aggregate wealth of entrepreneurs $\tilde{B}_{j,t}$. Remember that the tilde sign denotes the financial wealth of entrepreneurs after banks have renegotiated their liabilities.

Using the equilibrium condition in the labor market, we can express the wage rate as a function of $\tilde{B}_t$. In particular, equalizing the demand for labor, $H^D_{j,t} = \phi_j(w_{j,t})\tilde{B}_{j,t}$, to the supply from workers, $H^S_{j,t} = (w_{j,t}/\alpha\bar{z}_j)^\nu$, the wage $w_{j,t}$ becomes a function of only $\tilde{B}_{j,t}$. We can then use this function to replace $w_{j,t}$ in (15) and express the demand for bank liabilities in country j as a function of only $\tilde{B}_{j,t}$ and q_t, that is,

$$B_{j,t+1} = \frac{s_j(\tilde{B}_{j,t})}{q_t},$$

where $s_j(\tilde{B}_{j,t})$ is strictly increasing in the wealth of entrepreneurs $\tilde{B}_{j,t}$. The total demand for bank liabilities is simply the sum of the demand from the two countries, that is,

$$B_{t+1} = \frac{s_1(\tilde{B}_{1,t}) + s_1(\tilde{B}_{2,t})}{q_t}. \tag{16}$$

[9] Lemma 2.1 was derived under the assumption that the bonds purchased by the entrepreneurs were not risky, that is, entrepreneurs receive $b_{j,t+1}$ units of consumption goods with certainty in the next period $t+1$. In the extension with financial intermediation, however, bank liabilities are risky since banks may renege on their liabilities. Because of the logarithmic utility, however, the lemma continues to hold. The proof requires only a trivial extension of the proof of Lemma 2.1 and is omitted.

Figure 3 plots this function for given values of $\tilde{B}_{1,t}$ and $\tilde{B}_{2,t}$. As we change $\tilde{B}_{1,t}$ and $\tilde{B}_{2,t}$, the slope of the demand function changes. More specifically, keeping the interest rate constant, higher initial values of wealth $\tilde{B}_{1,t}$ and $\tilde{B}_{2,t}$ imply higher demand for bank liabilities $B_{t+1} = B_{1,t+1} + B_{2,t+1}$.

Supply of bank liabilities The supply of bank liabilities is derived from consolidating the borrowing decisions of workers with the investment and funding decisions of banks.

According to Lemma 2.2, when banks are highly leveraged, that is, $\omega_{t+1} > \underline{\xi}$, the interest rate on loans must be smaller than the intertemporal discount rate ($R^l_t < 1/\beta$). From the workers' first order condition (5) we can see that $\mu_t > 0$ if $R^l_t < 1/\beta$. Therefore, the borrowing constraint for workers is binding. This implies that the aggregate loans received by workers in country j are equal to the borrowing limit multiplied by the mass of workers, that is, $L_{j,t+1} = \eta_j N_j/2$. The total loans made by banks is the sum of the loans made in the two countries $L_{t+1} = \eta_1 N_1/2 + \eta_2 N_2/2$. By definition, $B_{t+1} = \omega_{t+1} L_{t+1}$. We can then express the total supply of bank liabilities as $B_{t+1} = \omega_{t+1}[\eta_1 N_1 + \eta_2 N_2]/2$.

When the lending rate is equal to the intertemporal discount rate, instead, the demand for loans from workers is undetermined, which in turn implies indeterminacy in the supply of bank liabilities. In this case the liabilities of banks are demand determined. In summary, the supply of bank liabilities is

$$B^s(\omega_{t+1}) = \begin{cases} \text{Undetermined}, & \text{if} \quad \omega_{t+1} < \underline{\xi} \\ \left(\frac{\eta_1 N_1 + \eta_2 N_2}{2}\right) \omega_{t+1}, & \text{if} \quad \omega_{t+1} \geq \underline{\xi} \end{cases} \tag{17}$$

So far I have derived the supply of bank liabilities as a function of the bank leverage ω_{t+1}. However, the leverage of banks also depends on the cost of borrowing $\overline{R}^b_t/(1-\tau)$ through condition (13). The average expected return on bank liabilities for investors, $\overline{R}^b_t$, is in turn related to the price of bank liabilities q_t by the condition

$$q_t = \frac{1}{\overline{R}^b_t} \left[1 - \theta(\omega_{t+1}) + \theta(\omega_{t+1}) \left(\frac{\underline{\xi}}{\omega_{t+1}} \right) \right]. \tag{18}$$

The term in square brackets on the left-hand-side is the expected payment at time $t+1$ from holding one unit of bank liabilities. With probability

$1-\theta(\omega_{t+1})$ banks do not renegotiate and pay back one unit. With probability $\theta(\omega_{t+1})$, however, banks renegotiate and investors recover only a fraction $\underline{\xi}/\omega_{t+1}$ of the initial investment. The expected repayment is discounted by the market return $\overline{R}_t^b$ to obtain its current value. By arbitrage this must be equal to the price q_t for one unit of bank liabilities.

Using (18) to replace $\overline{R}_t^b$ in equation (13) we obtain a function that relates the price of bank liabilities q_t to their leverage ω_{t+1}. Finally, we can combine this function with $B_{t+1} = [\eta_1 N_1/2 + \eta_2 N_2/2]\omega_{t+1}$ from (17) to obtain the supply of bank liabilities as a function of q_t.

Figure 3 plots the supplies of bank liabilities which is undetermined when $1/q_t = (1-\tau)/\beta$ and strictly decreasing for lower values of $1/q_t$ until it reaches the maximum volume of loans that can be made to workers, that is, $L_{Max} = \eta_1 N_1/2 + \eta_2 N_2/2$.

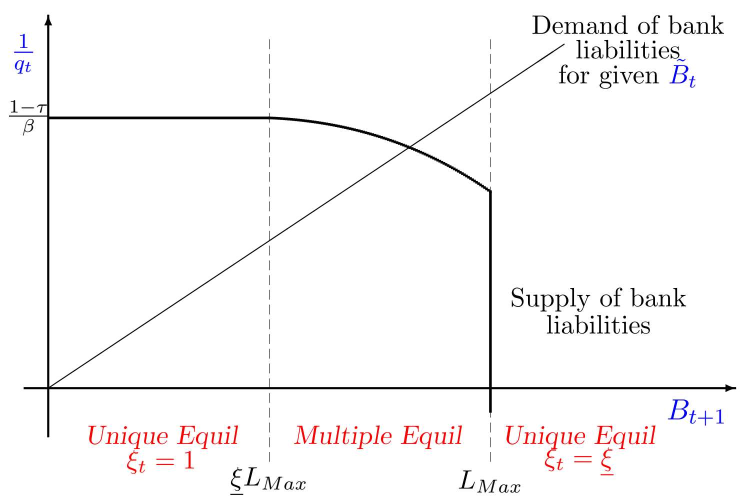


Figure 3: Demand and supply of bank liabilities.

Equilibrium The general equilibrium is characterized by the intersection of the demand and supply of bank liabilities as plotted in Figure 3. The supply (from banks) is increasing in the price of bank liability q_t while the demand (from entrepreneurs) is decreasing in the price q_t. The demand is plotted for a particular value of outstanding post-renegotiation liabilities

$\tilde{B}_t = \tilde{B}_{1,t} + \tilde{B}_{2,t}$. By changing the outstanding liabilities, the slope of the demand function changes.

The figure also indicates the regions with unique or multiple equilibria. When the price q_t is equal to $\beta/(1-\tau)$, banks are indifferent in the choice of leverage $\omega_{t+1} \leq \underline{\xi}$. When the funding price falls below this value, however, the optimal leverage starts to increase above $\underline{\xi}$ and the economy enters in the region with multiple equilibria. Once the leverage reaches $\omega_{t+1} = 1$, a further increases in the price for bank liabilities does not lead to higher leverages since the choice of $\omega_{t+1} > 1$ would cause renegotiation with probability 1.[10]

Given the initial entrepreneurial wealth $\tilde{B}_t$, the intersection of demand and supply of bank liabilities determines the price q_t, which in turn determines the next period wealth of entrepreneurs $\tilde{B}_{t+1}$. In absence of renegotiation we have $\tilde{B}_{t+1} = B_{t+1}$, where B_{t+1} is determined by equation (16). In the event of renegotiation (assuming that we are in a region with multiple equilibria) we have $\tilde{B}_{t+1} = (\underline{\xi}/\omega_{t+1})B_{t+1}$. The new $\tilde{B}_{t+1}$ will determine a new slope for the demand of bank liabilities, and therefore, new equilibrium values of q_t and B_{t+1}. Depending on the parameters, the economy may or may not reach a steady state. A key parameter determining the convergence to a steady state is the intermediation cost τ.

Proposition 2.3 *There exists $\hat{\tau} > 0$ such that: If $\tau \geq \hat{\tau}$, the economy converges to a steady state without renegotiation. If $\tau < \hat{\tau}$, the economy never converges to a steady state but switches stochastically between equilibria with and without renegotiation depending to the realization of the sunspot ε.*

Proof 2.3 *See Appendix G*

In order to converge to a steady state, the economy has to reach an equilibrium in which renegotiation never arises. This can happen only if the price of bank liabilities is equal to $q_t = \beta/(1-\tau)$. With this price banks do not have incentive to leverage because the funding cost, $1/q_t$, is equal to the return on loans. For this to be an equilibrium, however, the demand for bank liabilities must be sufficiently low which cannot be the case when $\tau = 0$. With $\tau = 0$, in fact, the steady state price q_t must be equal to β. But then entrepreneurs continue to accumulate bank liabilities without bound for precautionary motive. The demand for bank liabilities

[10] The dependence of the existence of multiple equilibria from the leverage of the economy is also a feature of the sovereign default model of Cole and Kehoe (2000).

will eventually become bigger than the supply (which is bounded by the borrowing constraint of workers), driving the price above β. As the price for bank liabilities increases, multiple equilibria become possible.

Bank leverage and crises Figure 3 illustrates how the type of equilibria depends on the bank leverage. When banks increase their leverage, the economy switches from a state in which the equilibrium is unique (no crises) to a state with multiple equilibria (and the possibility of financial crises). But even if the economy was already in a state with multiple equilibria, the increase in leverage implies that the consequences of a crisis are more severe. In fact, when the economy switches from a good equilibria to a bad equilibria, the bank liabilities are renegotiated to $\underline{\xi}L_{Max}$. Therefore, bigger are the liabilities B_t issued by banks and larger are the losses incurred by entrepreneurs holding these liabilities. Larger financial losses incurred by entrepreneurs then imply larger declines in the demand for labor in both countries, which in turn cause larger macroeconomic contractions.

3 Quantitative analysis

As shown in the introduction, emerging countries have experienced unprecedent economic growth during the last two decades. The goal of this section is to study how the growth has affected financial and macroeconomic stability in both industrialized and emerging countries.

To address this question, I calibrate the model using data for industrialized and emerging countries at the beginning of the 1990s. Starting in 1991 I then simulate the model from 1991 to 2011 assuming that during this period the relative productivity $\bar{z}_{2,t}/\bar{z}_{1,t}$ has changed. The change in productivity is equal to the observed ratio of GDP between emerging and industrialized countries. For the analysis of this section I use the borrowing limit specified in 3 which allows for the price of the fixed asset to change over time.

Parametrization The period in the model is a quarter and the discount factor is set to $\beta = 0.9825$, implying an annual intertemporal discount rate of about 7%. The parameter ν in the utility function of workers is the elasticity of the labor supply. I set this elasticity to the high value of 50. The reason to use this high value is to capture, in a simple fashion, possible wage rigidities. In fact, with a very high elasticity, wages are almost constant.

The alternative would be to model explicitly downward wage rigidities but this requires an additional state variable and would make the computation of the model more demanding. The utility parameter α is chosen to have an average working time of about 0.3.

The average productivity in country 1 (industrialized countries) is assumed to be fixed and normalized to $\bar{z}_{1,t} = 1$. The average productivity of country 2 (emerging countries) from 1991 to 2011 is set to the ratio of GDP in emerging and industrialized countries at market prices. As shown in Figure 1, the GDP ratio increases from 17% in 1990 to 52% in 2011.

I interpret the production from the fixed asset $\bar{z}\bar{k}$ as the value of housing services. I will then set $\bar{k}$ so that the contribution of these services to total output (entrepreneurial production plus housing services) is about 15%. Total production is $\bar{z}h + \bar{z}\bar{k}$. Since $h = 0.3$, this requires $\bar{k} = 0.055$.

The parameter η_j determines the fraction of fixed asset that can be used as a collateral in country j. This parameter limits the volume of assets that can be created by a country. Cross-country differences in this parameter captures differences in the ability of countries to create financial assets in the spirit of (Caballero et al. (2008)). I set $\eta_1 = 0.6$ (for industrialized countries) and $\eta_2 = 0.2$ (for emerging economies).

The idiosyncratic productivity shock z follows a truncated normal distribution with mean $\bar{z}_j$ and standard deviation of $\bar{z}_j\sigma_j$. The parameter σ_j is the residual risk that cannot be insured through state-contingent financial contracts. More developed financial markets allow for better insurance, and therefore, lower residual risk σ_j. Thus, cross-country differences in σ_j captures differences in financial markets as in Mendoza et al. (2009). I set $\sigma_1 = 0.3$ for industrialized countries and $\sigma_2 = 0.6$ for emerging economies.

The last set of parameters to calibrate pertain to the banking sector. The liquidation fraction $\underline{\xi}$ is set to 0.75 and the probability that the sunspot takes the value $\varepsilon = 0$ (which could lead to a bank crisis) is set to 2 percent ($\lambda = 0.02$). Therefore, provided that the economy is in a region that admits multiple equilibria, a crisis arises on average every fifty quarters. The renegotiation cost is assumed to be quadratic, that is, $\varphi(.) = (.)^2$ and the operation cost for banks is set to $\tau = 0.006$.

Numerical exercise Given the parameter values described above, I simulate the model for 700 quarters (175 years) using a random sequence of draws of the sunspot shock. In the first 500 quarters the relative productivity of country 2 (emerging economies) is constant at the 1990 level. Starting at quarter 501 (which corresponds to the first quarter of 1991), agents learn that the relative productivity of emerging economies will change during the next 80 quarters (from 1991 to 2011) after which it stabilizes at the level observed in 2011. The change in relative productivity during these 80 quarters is equal to the change in relative GDP measured at market prices as shown in Figure 1.

Since there are sunspot shocks that could shift the economy from one type of equilibrium to the other, the dynamics of the economy depend on the actual realizations of the shock. To better illustrate the stochastic nature of the model, I repeat the simulation 1,000 times (with each simulation performed over 700 periods as described above).

Simulation results Figure 4 plots the average of the 1,000 repeated simulations as well as the 5th and 95th percentiles. The range of variation between the 5th and 95th percentiles provides information about the potential volatility of the economy at any point in time.

The first panel plots the productivity of emerging countries. Since the productivity of industrialized countries has been normalized to 1, this represents the relative productivity of emerging countries. Productivity is exogenous in the model and the 1991 change represents a structural break. The next three panels plot banks leverage and the interest rates paid by banks on liabilities and earned on loans. The remaining panels show the dynamics of asset prices and labor in each of the two countries.

The first point to notice is that, following the increase in relative productivity of emerging countries, the interval delimited by the 5th and 95th percentiles for the repeated simulations widens dramatically. Therefore, financial and macroeconomic volatility increases substantially as we move to the 2000s. In this particular simulation, the probability of a bank crisis is always positive, even before the structural break in 1991. However, after the structural change, the consequence of a bank crisis could be much bigger since the distance between the 5th and 95th percentiles is significantly wider.

Besides the increase in financial and macroeconomic volatility, the figure reveals other interesting patterns. First, as the relative size of emerging economies increases, banks raise their leverage while the interest rate on their liabilities declines. The economy also experiences a decline in the interest rate on loans which in turn allows for a boom in asset prices. Labor, however,

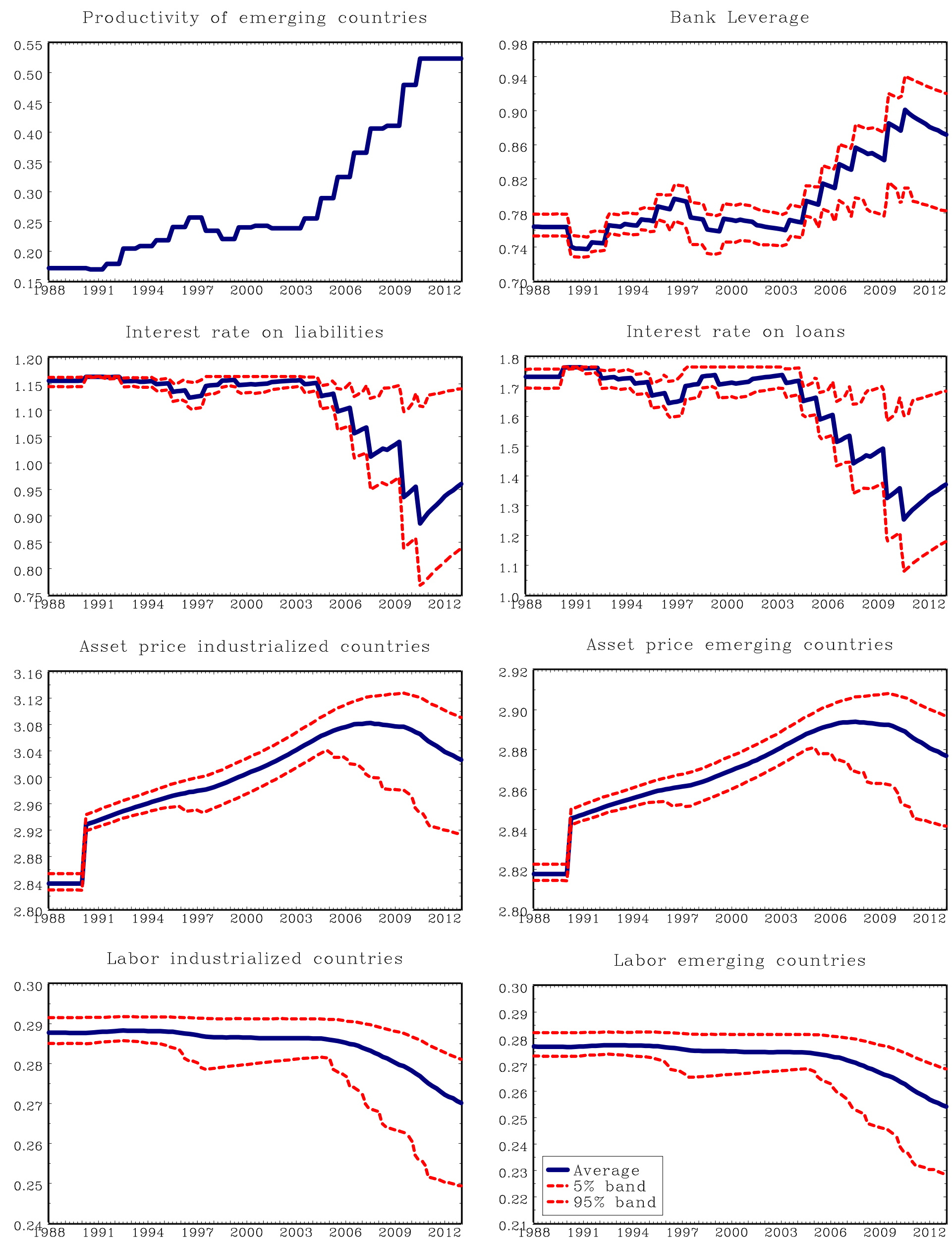


Figure 4: Change in productivity of emerging countries 1991-2011. Responses of 1,000 simulations.

declines on average.

The asset price boom is a direct consequence of the interest rate decline on loans. Since part of the holding of real assets can be financed with loans issued by banks, the decline in the interest rate makes the financing of these assets cheaper for workers, raising their price.

The average decline in labor can be explained as follows. As emerging countries become bigger, they demand more financial assets that in the model are issued by banks. Banks increase the supply but not enough to compensate for the overall increase in demand. As a result, in equilibrium entrepreneurs will hold less financial assets relatively to they scale of production. This implies lower insurance and, therefore, less demand of labor.

It is important to point out that, although the underlying financial and macroeconomic volatility has increased, this does not mean that we can observe it in the actual data. It is conceivable that the recent crisis is the only negative realization of the sunspot shock during the last 20 years. If this is the case, then the dynamics of the economy observed during the last two decades would appear quite stable until the 2008 crisis. Since the probability of a negative sunspot shock is very low (only 2% per quarter), the probability of positive realizations from 1991 to 2008 is about 25 percent. Therefore, the scenario is very plausible. It also fits with anecdotal evidence for which 2008 is the only truly worldwide financial crisis observed during the last 20 years.

A second remark is that, although labor falls in the average of all repeated simulations, the actual dynamics of labor during the 20 years that followed the 1991 break could be increasing or decreasing depending on the actual realizations of the sunspot shocks.

To show this point, I repeat the experiment shown in Figure 4 but for a particular sequence of sunspot shocks. More specifically, I simulate the model under the assumption that, starting in the first quarter of 1991, the economy experiences a sequence of draws of the sunspot variable $\varepsilon = 1$ until the second quarter of 2008. Then in the third quarter of 2008 the draw of the sunspot becomes $\varepsilon = 0$ but returns to $\varepsilon = 1$ from the fourth quarter of 2008 and in all subsequent quarters.

This particular sequence of sunspots captures the idea that expectations may have turned pessimistic in the fourth quarter of 2008 leading to a sudden financial and macroeconomic crisis. The simulated variables are plotted in Figure 5. The continuous line is still the average at time t of the 1,000 simulations. However, differently from the previous graph, starting from the first quarter of 1991 the sequences of draws for the sunspot shock is always the same for all 1,000 repeated simulations (which explain why the 5th and 95th percentiles have been omitted).

As we can seen from Figure 5, as long as the draws of the sunspot variable is $\varepsilon = 1$, asset prices continue to increase and the input of labor does not drop. However, a single realization $\varepsilon = 1$ of the sunspot shock can trigger a large decline in labor. Furthermore, even if the negative shock is only for one period and there are no crises afterwards, the recovery in the labor market is extremely slow. This is because the crisis generates a large decline in the financial wealth of employers and it will take a long time for them to rebuilt the lost wealth through savings.

Another way of showing the importance of the growth of emerging countries for macroeconomic stability is by conducting the following counterfactual exercise. I repeat the simulation under the assumption that the productivity of emerging countries does not growth but remains at the pre-1991 level for the whole simulation period. This counterfactual exercise tells us how the financial and macroeconomic dynamics in response to the same shocks would have changed in absence of the growth of emerging economies. The resulting simulation is shown by the dashed line in Figure 5.

Without the growth of emerging economies, the same sequence of sunspot shocks would have generated a much smaller financial and macroeconomic expansion before 2008 as well as a much smaller contraction in the third quarter of 2008. Therefore, the increase in foreign demand for financial assets issued by industrialized countries could have contributed to the observed expansion of the financial sector in industrialized countries but it also created the conditions for greater financial and macroeconomic fragility that became evident once the crisis materialized.

4 Discussion and conclusion

The sustained high growth experienced by emerging economies has increased their share of the world economy. An implication of this is that the economic performance of these countries is becoming more and more important for the performance of industrialized countries. The view that emerging countries are a collection of small open economies that are highly dependent on industrialized countries but they are of negligible importance for industrialized economies is no longer valid. The recent growth of emerging countries has made this view obsolete.

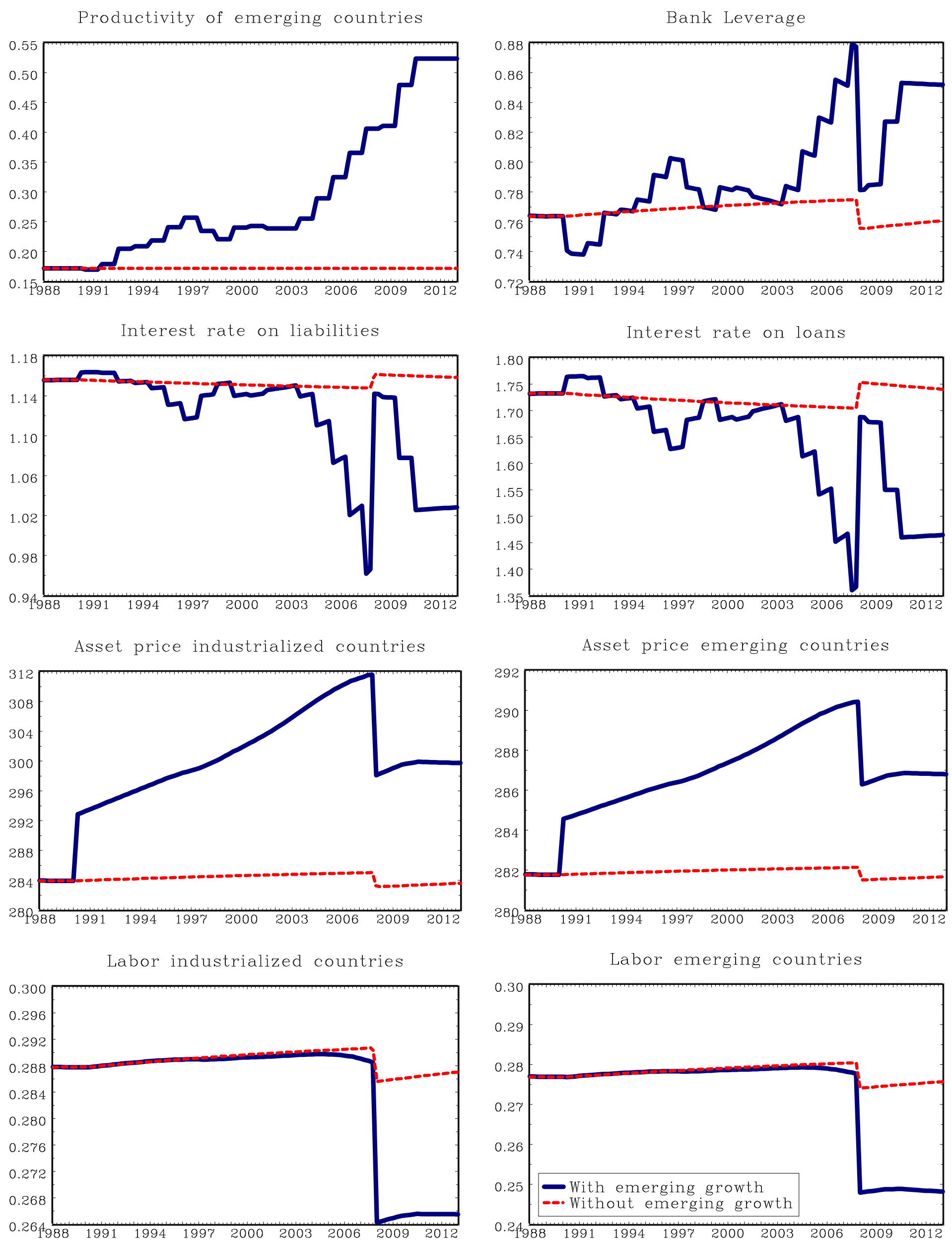


Figure 5: Change in productivity of emerging countries 1991-2011. Responses of 1,000 simulations with same draws of the sunspot variable starting in 1991, with the exception of third quarter of 2008.

There are many channels through which emerging economies could affect industrialized countries and in this paper I emphasized one of these channels: the increased external demand for financial assets issued by industrialized countries. In particular, I have shown that the increased demand for financial assets raises the incentives of financial intermediaries in industrialized countries to leverage. On the one had, this allows for the expansion of the financial sector with positive effects on real macroeconomic variables. On the other, it increases the fragility of the financial system, raising the probability and/or the consequences of a crisis.

These results are illustrated with a model in which the banking sector plays a central role in the intermediation of funds and, therefore, in the creation of financial assets. The paper emphasizes a special channel through which banks can affect the real sector of the economy: the issuance of liabilities held by the nonfinancial sector for insurance purposes. When the supply of bank liabilities or their value are low, agents are less willing to engage in risky activities and this causes a macroeconomic contraction.

The analysis of the paper also shows that booms and busts in financial intermediation can be driven by self-fulfilling expectations about the liquidity of the banking sector. When the economy expects the banking sector to be liquid, banks have an incentive to leverage and this allows for an economic boom. But as the leverage increases, the banking sector becomes vulnerable to pessimistic expectations about its liquidity, creating the conditions for a financial crisis. The increase in external demand for financial assets from emerging economies amplifies this mechanism because, by reducing the funding cost, it increases the incentive of banks to leverage.

In reality, financial assets that can be held for precautionary reasons are also created directly in nonfinancial sectors. For example, firms and governments issue liabilities that are directly held by nonfinancial sectors. Still, financial intermediaries play an important role in the direct issuance of instruments such as corporate and government bonds. Financial intermediaries also play an important role in the secondary market for these assets. Therefore, difficulties in financial intermediation is likely to affect the functioning and valuation of all financial markets. It is for this reason that in this paper I focused on the operation of financial intermediaries.

An important feature of the model economy studied here is that the expansion of the financial sector improves the allocation efficiency. This is because the issuance of bank liabilities provides insurance instruments for entrepreneurs, encouraging them to hire more labor. However, the expansion of the financial sector is associated with higher leverage, making the financial system more vulnerable to crises. Therefore, from a policy perspective, there is a trade-off: allowing for the expansion of the macro-economy at the cost of deeper crises. The study of optimal policies will be a subject for future research.

Appendix

A Proof of Lemma 2.1

The optimization problem of an entrepreneur can be written recursively as

$$\begin{aligned} V_t(b_t) &= \max_{h_t} \mathbb{E}_t \tilde{V}_t(a_t) \\ & \text{subject to} \\ & a_t = b_t + (z_t - w_t)h_t \end{aligned} \tag{19}$$

$$\begin{aligned} \tilde{V}_t(a_t) &= \max_{b_{t+1}} \Big\{ \ln(c_t) + \beta \mathbb{E}_t V_{t+1}(b_{t+1}) \Big\} \\ & \text{subject to} \\ & c_t = a_t - \frac{b_{t+1}}{R_t} \end{aligned} \tag{20}$$

Since the information set changes from the beginning of the period to the end of the period, the optimization problem has been separated according to the available information. In sub-problem (19) the entrepreneur chooses the input of labor without knowing the productivity z_t. In sub-problem (20) the entrepreneur allocates the end of period wealth in consumption and savings after observing z_t.

The first order condition for sub-problem (19) is

$$\mathbb{E}_t \frac{\partial \tilde{V}_t}{\partial a_t}(z_t - w_t) = 0.$$

The envelope condition from sub-problem (20) gives

$$\frac{\partial \tilde{V}_t}{\partial a_t} = \frac{1}{c_t}.$$

Substituting in the first order condition we obtain

$$\mathbb{E}_t \left(\frac{z_t - w_t}{c_t} \right) = 0. \tag{21}$$

At this point we proceed by guessing and verifying the optimal policies for employment and savings. The guessed policies take the form:

$$h_t = \phi_t b_t \tag{22}$$

$$c_t = (1-\beta) a_t \tag{23}$$

Since $a_t = b_t + (z_t - w_t)h_t$ and the employment policy is $h_t = \phi_t b_t$, the end of period wealth can be written as $a_t = [1 + (z_t - w_t)\phi_t]b_t$. Substituting in the guessed consumption policy we obtain

$$c_t = (1 - \beta)\Big[1 + (z_t - w_t)\phi_t\Big]b_t. \tag{24}$$

This expression is used to replace c_t in the first order condition (21) to obtain

$$\mathbb{E}_t\left[\frac{z_t - w_t}{1 + (z_t - w_t)\phi_t}\right] = 0, \tag{25}$$

which is the condition stated in Lemma 2.1.

To complete the proof, we need to show that the guessed policies (22) and (23) satisfy the optimality condition for the choice of consumption and saving. This is characterized by the first order condition of sub-problem (20), which is equal to

$$-\frac{1}{c_t R_t} + \beta\mathbb{E}_t\frac{\partial V_{t+1}}{\partial b_{t+1}} = 0.$$

From sub-problem (19) we derive the envelope condition $\partial V_t/\partial b_t = 1/c_t$ which can be used in the first order condition to obtain

$$\frac{1}{c_t} = \beta R_t\mathbb{E}_t\frac{1}{c_{t+1}}.$$

We have to verify that the guessed policies satisfy this condition. Using the guessed policy (23) and equation (24) updated one period, the first order condition can be rewritten as

$$\frac{1}{a_t} = \beta R_t\mathbb{E}_t\frac{1}{[1 + (z_{t+1} - w_{t+1})\phi_{t+1}]b_{t+1}}.$$

Using the guessed policy (23) we have that $b_{t+1} = \beta R_t a_t$. Substituting and rearranging we obtain

$$1 = \mathbb{E}_t\left[\frac{1}{1 + (z_{t+1} - w_{t+1})\phi_{t+1}}\right]. \tag{26}$$

The final step is to show that, if condition (25) is satisfied, then condition (26) is also satisfied. Let's start with condition (25), updated by one period. Multiplying both sides by ϕ_{t+1} and then subtracting 1 in both sides we obtain

$$\mathbb{E}_{t+1}\left[\frac{(z_{t+1} - w_{t+1})\phi_{t+1}}{1 + (z_{t+1} - w_{t+1})\phi_{t+1}} - 1\right] = -1.$$

Multiplying both sides by -1 and taking expectations at time t we obtain (26).

B Proof of Proposition 2.1

As shown in Lemma 2.1, the optimal saving of entrepreneurs takes the form $b^i_{t+1}/R^b_t = \beta a^i_t$, where a^i_t is the end-of-period wealth $a^i_t = b^i_t + (z^i_t - w_t)h^i_t$. Since $h^i_t = \phi(w_t)b^i_t$ (see Lemma 2.1), the end-of-period wealth can be rewritten as $a^i_t = [1 + (z^i_t - w_t)\phi(w_t)]b^i_t$. Substituting into the optimal saving and aggregating over all entrepreneurs we obtain

$$B_{t+1} = \beta R^b_t \Big[1 + (\bar{z} - w_t)\phi(w_t) \Big] B_t. \tag{27}$$

This equation defines the aggregate demand for bonds as a function of the interest rate R^b_t, the wage rate w_t, and the beginning-of-period aggregate wealth of entrepreneurs B_t. Notice that the term in square brackets is bigger than 1. Therefore, in a steady state equilibrium where $B_{t+1} = B_t$, the condition $\beta R < 1$ must be satisfied.

Using the equilibrium condition in the labor market, I can express the wage rate as a function of B_t. In particular, equalizing the demand for labor, $H^D_t = \phi(w_t)\tilde{B}_t$, to the supply from workers, $H^S_t = (w_t/\alpha)^\nu$, the wage w_t can be expressed as a function of only $\tilde{B}_t$. We can then use this function to replace w_t in (27) and express the demand for bank liabilities as a function of only B_t and R^b_t as follows

$$B_{t+1} = s(B_t)\, R^b_t. \tag{28}$$

The function $s(B_t)$ is strictly increasing in the wealth of entrepreneurs, B_t.

Consider now the supply of bonds from workers. For simplicity I assume that the borrowing constraint takes the form specified in equation (2), that is, $l_{t+1} \leq \eta$. Using this limit together with the first order condition (5), we have that, either the interest rate satisfies $1 = \beta R^b_t$ or workers are financially constrained, that is, $B_{t+1} = \eta$. When the interest rate is equal to the inter-temporal discount rate (first case), we can see from (27) that $B_{t+1} > B_t$. So eventually, the borrowing constraint of workers becomes binding, that is, $B_{t+1} = \eta$ (second case). When the borrowing constraint is binding, the multiplier μ_t is positive and condition (5) implies that the interest rate is smaller than the inter-temporal discount rate. So the economy has reached a steady state. The steady state interest rate is determined by condition (28) after setting $B_t = B_{t+1} = \eta$. This is the only steady state equilibrium.

When the borrowing constraint takes the form (3), the proof is more involved but the economy also reaches a steady state with $\beta R < 1$.

C First order conditions for workers

The optimization problem of a worker can be written recursively as

$$\begin{aligned} V_t(l_t, k_t) &= \max_{h_t, l_{t+1}, k_{t+1}} \left\{ c_t - \alpha \frac{h_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} + \beta V_{t+1}(l_{t+1}, k_{t+1}) \right\} \\ & \text{subject to} \\ & c_t = w_t h_t + \chi k_t + \frac{l_{t+1}}{R_t^l} - l_t - (k_{t+1} - k_t) p_t \\ & \eta \geq l_{t+1}. \end{aligned}$$

Given $\beta\mu_t$ the lagrange multiplier associated with the borrowing constraint, the first order conditions with respect to h_t, l_{t+1}, k_{t+1} are, respectively,

$$\begin{aligned} & -\alpha h_t^{\frac{1}{\nu}} + w_t = 0, \\ & \frac{1}{R_t^l} + \beta \frac{\partial V_{t+1}(l_{t+1}, k_{t+1}}{\partial l_{t+1}} - \beta\mu_t = 0, \\ & -p_t + \beta \frac{\partial V_{t+1}(l_{t+1}, k_{t+1}}{\partial k_{t+1}} = 0. \end{aligned}$$

The envelope conditions are

$$\begin{aligned} \frac{\partial V_t(l_{t+1}, k_{t+1}}{\partial l_{t+1}} &= -1, \\ \frac{\partial V_t(l_{t+1}, k_{t+1}}{\partial k_{t+1}} &= \chi + p_t. \end{aligned}$$

Updating by one period and substituting in the first order conditions we obtain (4), (5), (6).

When the borrowing constraint takes the form $\eta \mathbb{E}_t p_{t+1} k_{t+1} \geq l_{t+1}$, the first order condition with respect to k_{t+1} becomes

$$-p_t + \beta \frac{\partial V_{t+1}(l_{t+1}, k_{t+1}}{\partial k_{t+1}} + \eta\beta\mu_t \mathbb{E}_t p_{t+1} = 0,$$

Substituting the envelope condition we obtain (7).

D First order conditions for problem (12)

The first order conditions for problem (12) with respect to b_{t+1} and l_{t+1} are

$$\frac{1-\tau}{\overline{R}_t^b}\mathbb{E}_t\frac{\partial\tilde{b}_{t+1}}{\partial b_{t+1}} = \beta\mathbb{E}_t\left[\frac{\partial\tilde{b}_{t+1}}{\partial b_{t+1}}+\frac{\partial\tilde{\varphi}_{t+1}}{\partial b_{t+1}}+\gamma_t\right] \tag{29}$$

$$\frac{1}{R_t^l} = \frac{1-\tau}{\overline{R}_t^b}\mathbb{E}_t\frac{\partial\tilde{b}_{t+1}}{\partial l_{t+1}}+\beta\mathbb{E}_t\left[1-\frac{\partial\tilde{b}_{t+1}}{\partial l_{t+1}}-\frac{\partial\tilde{\varphi}_{t+1}}{\partial l_{t+1}}+\gamma_t\right], \tag{30}$$

where γ_t is the Lagrange multiplier associated with constraint $b_{t+1}\leq l_{t+1}$.

I now use the definition of $\tilde{b}_{t+1}$ and $\tilde{\varphi}_{t+1}$ provided in equations (8) and (9) to derive the following terms

$$\begin{aligned}
\mathbb{E}_t\frac{\partial\tilde{b}_{t+1}}{\partial b_{t+1}} &= 1-\theta(\omega_{t+1}),\\
\mathbb{E}_t\frac{\partial\tilde{b}_{t+1}}{\partial l_{t+1}} &= \theta(\omega_{t+1})\underline{\xi},\\
\mathbb{E}_t\frac{\partial\tilde{\varphi}_{t+1}}{\partial b_{t+1}} &= \theta(\omega_{t+1})\Big[\varphi'(\omega_{t+1}-\underline{\xi})\omega_{t+1}+\varphi(\omega_{t+1}-\underline{\xi})\Big],\\
\mathbb{E}_t\frac{\partial\tilde{\varphi}_{t+1}}{\partial l_{t+1}} &= -\theta(\omega_{t+1})\varphi'(\omega_{t+1}-\underline{\xi})\omega_{t+1}^2,
\end{aligned}$$

where $\theta(\omega_{t+1})$ is the probability of renegotiation defined as

$$\theta(\omega_{t+1})=\begin{cases}0, & \text{if}\quad \omega_{t+1}<\underline{\xi}\\ \lambda, & \text{if}\quad \underline{\xi}\leq\omega_{t+1}\leq 1\\ 1, & \text{if}\quad \omega_{t+1}>1\end{cases}$$

Substituting in (29) and (30) and re-arranging we obtain

$$\frac{1-\tau}{\overline{R}_t^b} = \beta\left[1+\frac{\theta(\omega_{t+1})\Big(\varphi'(\omega_{t+1}-\underline{\xi})\omega_{t+1}+\varphi(\omega_{t+1}-\underline{\xi})\Big)+\gamma_t}{1-\theta(\omega_{t+1})}\right], \tag{31}$$

$$\frac{1}{R_t^l} = \beta\left[1+\theta(\omega_{t+1})\varphi'(\omega_{t+1}-\underline{\xi})\omega_{t+1}^2+\left(\frac{1-\tau}{\beta\overline{R}_t^b}-1\right)\theta(\omega_{t+1})\underline{\xi}+\gamma_t\right] \tag{32}$$

where the multiplier γ_t is zero if $\omega_{t+1}<1$ and positive if $\omega_{t+1}=1$. Eliminating γ_t we obtain (13) and (14) with can be satisfied with the inequality sign if $\gamma_t>0$.

E Proof of Lemma 2.2

Let's consider the first order conditions (31) and (32). When $\omega_{t+1} < \underline{\xi}$, the default probability is $\theta(\omega_{t+1}) = 0$ and the first order conditions are satisfied with equality. Therefore, they simplify to

$$\begin{aligned}\frac{1-\tau}{\overline{R}_t^b} &= \beta, \\ \frac{1}{R_t^l} &= \beta,\end{aligned}$$

which proves the first part of the lemma.

Now suppose that $\omega_{t+1} > \underline{\xi}$. In this case the probability of default is $\theta(\omega_{t+1}) = \lambda$ and the first order conditions can be written as

$$\frac{1-\tau}{\overline{R}_t^b} = \beta\left[1 + \frac{\lambda\varphi'(\omega_{t+1} - \underline{\xi})\omega_{t+1} + \lambda\varphi(\omega_{t+1} - \underline{\xi}) + \gamma_t}{1-\lambda}\right], \tag{33}$$

$$\frac{1}{R_t^l} = \beta\left[1 + \lambda\varphi'(\omega_{t+1} - \underline{\xi})\omega_{t+1}^2 + \lambda\underline{\xi}\left(\frac{1-\tau}{\beta\overline{R}_t^b} - 1\right) + \gamma_t\right], \tag{34}$$

where the multiplier γ_t is bigger than zero if $\omega_{t+1} = 1$.

That $\overline{R}_t^b/(1-\tau)$ and R_t^l are both smaller than $1/\beta$ is obvious from the above two conditions. What is not immediate to see is that $\overline{R}_t^b/(1-\tau) < R_t^l$. To show this, let's first use equation (33) to eliminate $\overline{R}_t^b$ in equation (34). After some re-arrangement I can rewrite equation (34) as

$$\frac{1}{R_t^l} = \beta\left[1 + \frac{\lambda\varphi'(\omega_{t+1} - \underline{\xi})\omega_{t+1}\mathbf{K_1} + \lambda\varphi(\omega_{t+1} - \underline{\xi})\mathbf{K_2} + \gamma_t\mathbf{K}_3}{1-\lambda}\right], \tag{35}$$

where

$$\begin{aligned}\mathbf{K}_1 &= (1-\lambda)\omega_{t+1} + \lambda\underline{\xi}, \\ \mathbf{K}_2 &= \lambda\underline{\xi}, \\ \mathbf{K}_3 &= 1 - \lambda + \lambda\underline{\xi}.\end{aligned}$$

Because ω_{t+1}, $\underline{\xi}$ and λ are all smaller than 1, the terms $\mathbf{K}_1$, $\mathbf{K}_2$ and $\mathbf{K}_3$ are also smaller than 1.

To prove that $\overline{R}_t^b/(1-\tau) < R_t^l$ I need to show that the right-hand-side of (33) is bigger than the right-hand-side of (35). This requires to show that

$$\begin{aligned}&\lambda\varphi'(\omega_{t+1} - \underline{\xi})\omega_{t+1} + \lambda\varphi(\omega_{t+1} - \underline{\xi}) + \gamma_t > \\ &\qquad \lambda\varphi'(\omega_{t+1} - \underline{\xi})\omega_{t+1}\mathbf{K_1} + \lambda\varphi(\omega_{t+1} - \underline{\xi})\mathbf{K_2} + \gamma_t\mathbf{K}_3.\end{aligned}$$

This follows directly from $\mathbf{K}_1$, $\mathbf{K}_2$ and $\mathbf{K}_3$ being smaller than 1.

F Proof of Proposition 2.2

Banks make decisions at two different stages. At the beginning of the period they choose whether to renegotiate the debt and at the end of the period they choose the funding and lending policies. Given the initial states, b_t and l_t, the renegotiation decision boils down to a take-it or leave-it offer made by each bank to its creditors for the repayment of the debt. Denote by $\tilde{b}_t = f(b_t, l_t, \xi_t^e)$ the offered repayment. This depends on the individual liabilities b_t, individual assets l_t, and the expected liquidation price of assets ξ_t^e. The superscript e is to make clear that the bank decision depends on the expected price in the eventuality of liquidation. Obviously, the best repayment offer made by the bank is

$$f(b_t, l_t, \xi_t^e) = \begin{cases} b_t, & \text{if} \quad b_t \leq \xi_t^e l_t \\ \\ \xi_t^e l_t, & \text{if} \quad b_t > \xi_t^e l_t \end{cases}, \qquad (36)$$

which is accepted by creditors whenever the actual liquidation price is bigger than the expected price ξ_t^e.

After the renegotiation stage, banks choose the funding and lending policies, b_{t+1} and l_{t+1}. These policies depend on the two interest rates, $\overline{R}_t^b$ and R_l, and on the probability distribution of the next period liquidation price ξ_{t+1}. Since we could have multiple equilibria, the next period price could be stochastic. Suppose that the price could take two values, $\underline{\xi}$ and 1, with the probability of the low value defined as

$$\theta(\omega_{t+1}) = \begin{cases} 0, & \text{if} \quad \omega_{t+1} < \underline{\xi} \\ \\ \lambda, & \text{if} \quad \underline{\xi} \leq \omega_{t+1} \leq 1 \\ \\ 1, & \text{if} \quad \omega_{t+1} > 1. \end{cases}$$

The variable $\omega_{t+1} = b_{t+1}/l_{t+1}$ represents the leverage of all banks in a symmetric equilibrium, that is, they all choose the same leverage. For the moment the symmetry of the equilibrium is an assumption. I will then show below that in fact banks do not have incentives to deviate from the leverage chosen by other banks.

Given the above assumption about the probability distribution of the liquidation price, the funding and lending policies of the bank are characterized in Lemma 2.2 and depend on $\overline{R}_t^b$ and R_t^l. In short, if $\overline{R}_t^b/(1-\tau) = R_t^l$, then the optimal policy of the bank is to choose a leverage $\omega_{t+1} \leq \underline{\xi}$. If $\overline{R}_t^b/(1-\tau) < R_t^l$, the optimal leverage is $\omega_{t+1} > \underline{\xi}$.

Given the assumption that the equilibrium is symmetric (all banks choose the same leverage ω_{t+1}), multiple equilibria arise if the chosen leverage is $\omega_{t+1} \in \{\underline{\xi}, 1\}$.

In fact, once we move to the next period, if the market expects $\xi^e_{t+1} = \underline{\xi}$, all banks are illiquid and they choose to renege on their liabilities (given the renegotiation policy (36)). As a result, there will not be any bank that can buy the liquidated assets of other banks. Then the only possible price that is consistent with the expected price is $\xi_{t+1} = \underline{\xi}$. On the other hand, if the market expects $\xi^e_{t+1} = 1$, banks are liquid and, if one bank reneges, creditors can sell the liquidated assets to other banks at the price $\xi_{t+1} = 1$. Therefore, it is optimal for banks not to renegotiate consistently with the renegotiation policy (36).

The above proof, however, assumes that the equilibrium is symmetric, that is, all banks choose the same leverage. To complete the proof, we have to show that there is no incentive for an individual bank to deviate from the leverage chosen by other banks. In particular, I need to show that, in the anticipation that the next period liquidation price could be $\xi_{t+1} = \underline{\xi}$, a bank do not find convenient to chose a lower leverage so that, in the eventuality that the next period price is $\xi_{t+1} = \underline{\xi}$, the bank could purchase the liquidated asset at a price lower than 1 and make a profit (since the unit value for the bank of the liquidated assets is 1.

If the price at $t+1$ is $\xi_{t+1} = \underline{\xi}$, a liquid bank could offer a price $\underline{\xi} + \epsilon$, where ϵ is a small but positive number. Since the repayment offered by a defaulting bank is $\underline{\xi} l_{t+1}$, creditors prefer to sell the assets rather than accepting the repayment offered by the defaulting bank. However, if this happens, the expectation of the liquidation price $\xi^e = \underline{\xi}$ turns out to be incorrect ex-post. Therefore, the presence of a single bank with liquidity will raise the expected liquidation price to $\underline{\xi} + \epsilon$. But even with this new expectation, a bank with liquidity can make a profit by offering $\underline{\xi} + 2\epsilon$. Again, this implies that the expectation turns out to be incorrect ex-post. This mechanism will continue to raise the expected price to $\xi^e_{t+1} = 1$. At this point the liquid bank will not offer a price bigger than 1 and the ex-post liquidation price is correctly predicted to be 1. Therefore, as long as there is a single bank with liquidity, the expected liquidation price must be 1. But then a bank cannot make a profit in period $t+1$ by choosing a lower leverage in period t with the goal of remaining liquid in the next period. This proves that there is no incentive to deviate from the policy chosen by other banks.

Finally, the fact that multiple equilibria cannot arise when $\omega_t < \underline{\xi}$ is obvious. Even if the price is $\underline{\xi}$, banks remain liquid.

G Proof of Proposition 2.3

Given a fixed interest rate R^b, the aggregate demand for bank liabilities, equation (16), has a converging fix point $B^*(R^b)$. The fixed point is increasing in R^b and converges to infinity as R^b converges to $1/\beta$. This implies that, if $\tau = 0$, then the leverage of banks is always bigger than $\underline{\xi}$. To show this, suppose that banks choose a leverage of $\omega < \underline{\xi}$. According to conditions (13) and (14), we have that $R^b = R^l = 1/\beta$. But when $R^b = 1/\beta$ the demand of bank liabilities is unbounded in the limit. This implies that to reach a stable equilibrium without renegotiation (that is, $\omega < \underline{\xi}$), R^b must be smaller than $1/\beta$. This requires τ to be sufficiently big. In fact, when $\tau > 0$ and $\omega < \underline{\xi}$, we have $R^b/(1-\tau) = R^l = 1/\beta$. Since the demand for bank liabilities is increasing in R^b, there must be some $\hat{\tau} > 0$ such that, for $\tau > \hat{\tau}$, the equilibrium is characterized by $\omega < \underline{\xi}$. This implies that the economy is not subject to crises and converges to a steady state. For $\tau < \hat{\tau}$, instead, the equilibrium is characterized by $\omega > \underline{\xi}$. In this case the economy is subject to self-fulfilling crises and, therefore, it does not converge to a steady state.

H Numerical solution

I describe the numerical procedure to solve the model with the endogenous borrowing constraint specified in (3). I first describe the numerical procedure when the external demand for bank liabilities is fixed (without loss of generality set to zero). In this case I can solve for the stochastic stationary equilibrium. I will then describe the numerical procedure when the external demand for bank liabilities changes over time, inducing a transition dynamics.

H.1 Stationary equilibrium without structural break

The states of the economy are given by the bank liabilities B_t, the bank loans L_t and the realization of the sunspot shock ε_t. These three variables are important in determining the renegotiation liabilities $\tilde{B}_t$. However, once we know the renegotiated liabilities $\tilde{B}_t$, this becomes the sufficient state for solving the model. Therefore, in the computation I will solve for the recursive equilibrium using $\tilde{B}_t$ as a state variable.

I will use the following equilibrium conditions:

$$H_t = \phi(w_t)\tilde{B}_t, \tag{37}$$

$$\frac{B_{t+1}}{R^b_t} = \beta A_t, \tag{38}$$

$$A_t = \tilde{B}_t + (1-w_t)H_t \tag{39}$$

$$\alpha H_t^{\frac{1}{\nu}} = w_t, \tag{40}$$

$$1 = \beta R^l_t(1+\mu_t), \tag{41}$$

$$p_t = \beta \mathbb{E}_t\Big[\chi + (1+\eta\mu_t)p_{t+1}\Big], \tag{42}$$

$$L_{t+1} = \eta \mathbb{E}_t p_{t+1}, \tag{43}$$

$$\frac{1-\tau}{\overline{R}_t^b} \geq \beta \left[1 + \frac{\theta(\omega_{t+1}) \Big(\varphi'(\omega_{t+1} - \underline{\xi})\omega_{t+1} + \varphi(\omega_{t+1} - \underline{\xi}) \Big)}{1 - \theta(\omega_{t+1})} \right], \tag{44}$$

$$\frac{1}{R_t^l} \geq \beta \left[1 + \theta(\omega_{t+1})\varphi'(\omega_{t+1} - \underline{\xi})\omega_{t+1}^2 + \theta(\omega_{t+1})\underline{\xi} \left(\frac{1-\tau}{\beta \overline{R}_t^b} - 1 \right) \right], \tag{45}$$

$$\overline{R}_t^b = \left[1 - \theta(\omega_{t+1}) + \theta(\omega_{t+1}) \left(\frac{\underline{\xi}}{\omega_{t+1}} \right) \right] R_t^b, \tag{46}$$

$$\omega_{t+1} = \frac{B_{t+1}}{L_{t+1}} \tag{47}$$

Equations (37)-(39) come from the aggregation of the optimal policies of entrepreneurs (labor demand, savings and end of periods wealth). Equations (40)-(43) come from the optimization problem of workers (labor supply, optimal borrowing, optimal holding of the fixed asset, and borrowing constraint). Notice that the borrowing constraint of workers (equation (43) is not always binding. However, when it is not binding and the multiplier is $\mu_t = 0$, workers' borrowing is not determined. Therefore, without loss of generality I assume that in this case workers borrow up to the limit. This explains why the borrowing constraint is always satisfied with equality. Equations (44)-(45) are the first order conditions of banks. These conditions are satisfied with equality if $\omega_{t+1} < 1$ and with inequality if $\omega_{t+1} = 1$. Equation (46) defines the expected return on bank liabilities given the price of these liabilities, that is, the inverse of R_t^b. The final equation (47) simply defines leverage.

One complication in solving this system of equations is that the expectation of the next period price of the fixed asset, $\mathbb{E}_t p_{t+1}$, is unknown. All we know is that the next period price is a function of $\tilde{B}_{t+1}$, that is, $p_{t+1} = P(\tilde{B}_{t+1})$. If I knew the function $P(\tilde{B}_{t+1})$, for any given state $\tilde{B}_t$, the above conditions would be a system of 11 equations in 11 variables: H_t, A_t, μ_t, w_t, p_t, R_t^b, R_t^l, $\overline{R}_t^b$, B_{t+1}, L_{t+1}, ω_{t+1}. Notice that $\tilde{B}_{t+1}$ is a known function of B_{t+1}, L_{t+1} and the realization of the sunspot shock ε. Therefore, I can compute the expectation of the next period price p_{t+1} if I know the function $P(\tilde{B}_{t+1})$. We can then solve the 11 equations for the 11 variables and this would provide a solution for any given state $\tilde{B}_t$.

The problem is that I do not know the function $P(\tilde{B}_{t+1})$. Therefore, the procedure will be based on a parametrization of an approximation of this function. In particular, I approximate $P(\tilde{B}_{t+1})$ with a piece-wise linear function over a grid for the state variable $\tilde{B}_t$. I then solve the above system of equations at each grid point for $\tilde{B}_t$. As part of the solution I obtain the current price p_t. I then use the solution for the current price to update the approximated function $P(\tilde{B}_{t+1})$ at the grid point. I repeat the iteration until convergence, that is, the values guessed for $P(\tilde{B}_{t+1})$ at each grid point must be equal (up to a small rounding number) to the values of p_t obtained by solving the model (given the guess for $P(\tilde{B}_{t+1})$).

H.2 Equilibrium with structural break

When the external demand for bank liabilities changes over time or there are innovations that change the operation cost of banks, the economy transits from a stochastic equilibrium to a new stochastic equilibrium. This requires to solve for the transition and the solution method is based on the following steps.

1. I first compute the stochastic equilibrium under the regime that proceeds the structural break (the foreign demand for bank liabilities is constant at the initial level and/or the operation cost of banks is constant at the initial level).

2. I then compute the stochastic equilibrium under the terminal regime (the foreign demand of bank liabilities remains constant at the new level after the transition and/or the operation cost of banks remains constant at the new level).

3. At this point I solve the model backward at any time t starting at the terminal period when the external demand and the operation cost remain constant at the new levels. At each t I solve the system (37)-(47) using the approximated function $P_{t+1}(\tilde{B}_{t+1})$ found at time $t+1$. In the first backward step (last period of the transition), $P_{t+1}(\tilde{B}_{t+1})$ is the approximated price function found in the stochastic stationary equilibrium after the break (see previous computational step).

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