

# A Dynamic Model of Predation\*

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## Abstract

We study an infinite-period dynamic predation model in which a dominant firm can either accommodate a weak rival or drive it out of the market. When the weak rival exits a new rival firm is born with some positive probability. We characterize the Markov perfect equilibria of the model and show that predation is an equilibrium strategy only when it is accompanied by a commitment by the dominant firm to also deter all future entry into the market. Predation is more profitable than accommodation for the dominant firm and may benefit consumers if they are relatively impatient and if the probability that a new rival will be born once the existing weak rival exits is high.

**JEL Classification:** D43, L41

**Keywords:** predation, preemption, accommodation, limit pricing, entry and exit, Markov strategies, Markov perfect equilibrium

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# 1 Introduction

Predatory pricing involves an allegation that a dominant firm has intentionally charged a low price in order to force a weaker rival firm to exit the market. This type of allegation is highly controversial. First, some scholars associated with the Chicago school, like Bork (1978, p. 154) claim that predatory pricing is “a phenomenon that probably does not exist.”<sup>1</sup> Indeed, the Supreme court of the U.S. adopted a similar view in its 1986 decision on *Matsushita v. Zenith* and wrote that “predatory pricing schemes are rarely tried, and even more rarely successful.”<sup>2</sup> On the other hand, other scholars like Broadly, Bolton, and Riordan (2000) and Edlin (2010) find evidence for the existence of predatory pricing in a variety of industries.

One reason for Bork’s claim that predatory pricing is not profitable (and hence rarely exists) is that if entry is relatively easy, then following the prey’s exit, the dominant firm will face a new entrant and will therefore be unable to raise its prices and recoup the losses it incurred during the predatory phase. This intuition, however, ignores the fact that whether new entry actually occurs during the recoupment period depends on the entrant’s expectation regarding the dominant firm’s future competitive behavior. Edlin (2010) claims that if a potential entrant expects the dominant firm to be aggressive and try to drive him out of the market once it enters, then it may prefer to stay out of the market altogether. Yet, if the dominant reacts to new entry with an aggressive behavior, then where is the recoupment of its losses during the original predatory phase? Clearly, if a dominant firm constantly needs to fight new entry with low prices, then it will be unable to ever recoup its losses from predatory behavior and hence it may not engage in predatory behavior in the first place. These arguments imply that in order to analyze the incentives of a dominant firm to engage in predatory behavior, one needs to consider a fully dynamic model since the dominant firm’s incentive to engage in predatory behavior in any given period and the potential entrant’s incentive to enter depend on how the game unfolds in future periods.

A second reason why predatory pricing is highly controversial is that some scholars argue that

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<sup>1</sup>Easterbrook (1981) raises similar doubts.

<sup>2</sup>*Matsushita Elec. Indus. Co. v. Zenith Radio Corp.*, 475 U.S. 574, 589 (1986).

predatory pricing is likely to harm consumers since it enables a dominant firm to drive a rival out of the market. Other scholars however point out that while the low price during the predatory phase are a sure thing (i.e., a “bird in hand,” the harm to consumers in the long run is only speculative since the prey may actually stay in the market after all, and even if it exits, the dominant firm may be unable to raise prices due to the threat of new entry. Indeed, in the U.S., Judge Breyer wrote in his decision on the *Barry Wright Corp. v. ITT Grinnell Corp.* case that “[T]he antitrust laws very rarely reject such beneficial ‘birds in hand’ for the sake of more speculative (future low-price) ‘birds in the bush’”.<sup>3</sup> The upshot then is that it is not clear if predatory pricing is feasible and if it is feasible whether it harms consumers.

To address these questions, we consider an infinite period model in which a dominant firm faces a weak rival and if it drives it out of the market, it faces new potential entry in every future period. We show that the model admits multiple equilibria. In one equilibrium the weak rival expects the dominant firm to accommodate entry and hence it enters. The dominant firm cannot profitably deter entry since it anticipates that future entrants will enter as they expect the dominant firm to accommodate them. As a result the dominant firm can never recoup its losses from predation so it never engages in predatory prices. As a result the entrant’s belief that the dominant firm will accommodate entry is vindicated. Predation can also be an equilibrium strategy provided that the entrant anticipates that the dominant firm will also fight new entry in all future periods. As a result, weak rivals will stay out of the market and hence once the dominant firm preys once, it becomes a monopoly forever and hence is able to recoup its losses from engaging in predatory behavior. Moreover, we also show that predation may actually benefit consumers provided that the discount factor is sufficiently large and the threat of new entry once the prey exits is sufficiently.

[Literature review to be added]

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<sup>3</sup>*Barry Wright Corp. v. ITT Grinnell Corp.*, 724 F.2d 227, 234 (1st Cir. 1983).

## 2 The model

Consider an infinite-period model with two price-setting firms that produce a homogeneous product and face a two-step demand function, illustrated in Figure 1: consumers wish to buy a fixed quantity, which we normalize to 1, and are willing to pay  $\alpha^h$  for the first  $\bar{Q}$  units and  $\alpha^\ell < \alpha^h$  for the remaining  $1 - \bar{Q}$  units.<sup>4</sup>

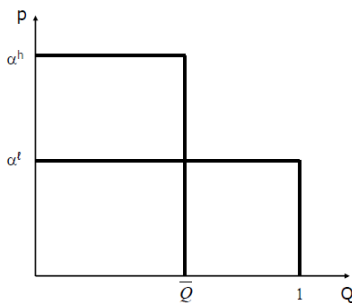


Figure 1: The demand function

Firm 1 is a dominant firm and has the capacity to serve the entire market. Firm 2 by contrast has a limited capacity, which enables it to serve only a fraction  $s < \bar{Q}$  of the market. To simplify the exposition, we normalize the marginal costs of both firms to 0 and also normalize the fixed cost of firm 1 to 0. These normalizations do not affect our qualitative results. Unlike firm 1, firm 2 does incur a fixed cost  $F$  in every period in which it operates in the market. Firm 2 can avoid the fixed cost  $F$  only by exiting the market, but then it dies, and a new firm 2 is born in the next period with probability  $\gamma$ . If a newborn firm 2 enters the market, it pays a one time entry cost  $E$  which is in addition to its fixed cost  $F$ .

The timing is as follows: at the beginning the first period, firm 1 sets a price,  $p_1$ . Given  $p_1$ , firm 2 either chooses to operate in the market and sets its own price  $p_2$ , or it exits the market. If firm 2 stays in the market, the strategic interaction between the two firms repeats itself in the next period:

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<sup>4</sup>For instance, imagine that there is a continuum of consumers of mass 1, each of whom is interested in buying a single unit. A mass  $\bar{Q}$  of consumers have a high willingness to pay  $\alpha^h$  and a mass  $1 - \bar{Q}$  of consumers have a low willingness to pay  $\alpha^\ell$ .

firm 1 sets  $p_1$ , and firm 2 either stays in the market and sets  $p_2$  or it exits. If firm 2 exits in any period, then it dies, and firm 1 serves the entire market at  $p_1$ . In the next period, a new firm 2 is born with probability  $\gamma$ . If a new firm 2 is not born, then firm 1 operates in the market as a monopoly, but in the next period, a new firm 2 is again born with probability  $\gamma$ . If a new firm 2 is born, then firm 1 sets a price  $p_1$ , and given  $p_1$ , the newborn firm 2 either enters the market, pays the entry cost  $E$ , and sets a price  $p_2$ , or it stays out of the market. In the latter case, firm 1 serves the entire market at  $p_1$ . The newborn firm dies if it did not enter, and in the next period a new firm 2 is born with probability  $\gamma$ . This sequence of events repeats itself in all periods.<sup>5</sup> We assume that all parameters (demand, cost, and the probability that a new firm 2 is born) are the same across all periods.

Notice that when firm 2 decides to operate in the market, it enjoys a second-mover advantage and hence sets  $p_2$  just below  $p_1$  and sells up to its capacity  $s$ . Firm 1 then serves the rest of the market, which is either  $1 - s$  if  $p_1 \leq \alpha^\ell$  or  $\bar{Q} - s$  if  $\alpha^\ell < p_1 \leq \alpha^h$ . On the other hand, if firm 2 stays out of the market or is not born at all, firm 1 serves the entire market at a price  $p_1$  and its sales are 1 if  $p_1 \leq \alpha^\ell$  and  $\bar{Q}$  if  $\alpha^\ell < p_1 < \alpha^h$ . Consequently, the per-period profits of the two firms are:

$$\pi_1 = \begin{cases} Qp_1 & \text{if firm 2 is out of the market and } \alpha^\ell < p_1 \leq \alpha^h, \\ p_1 & \text{if firm 2 is out of the market and } p_1 \leq \alpha^\ell, \\ (\bar{Q} - s)p_1 & \text{if firm 2 is in the market and } \alpha^\ell < p_1 \leq \alpha^h, \\ (1 - s)p_1 & \text{if firm 2 is in the market and } p_1 \leq \alpha^\ell, \end{cases} \quad (1)$$

and

$$\pi_2 = \begin{cases} 0 & \text{if firm 2 stays out of the market,} \\ s(p_1 - AC_O) & \text{if firm 2 just enters the market,} \\ s(p_1 - AC_I) & \text{if firm 2 stays in the market,} \end{cases} \quad (2)$$

where  $AC_O \equiv \frac{F+E}{s}$  is the average cost of firm 2 when it just enters the market, i.e., when it was initially “OUT” of the market, and  $AC_I \equiv \frac{F}{s}$  is its average cost when it is already “IN” the market.

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<sup>5</sup>We assume that firm 1 chooses its price before firm 2 in order to avoid a Bertrand outcome that would arise if both firms were to choose prices simultaneously. Our results can be generalized to a setting where the two firms produce differentiated products and compete by simultaneously setting prices.

The objective of each firm is to maximize the discounted sum of its profits. The intertemporal discount factor is  $\delta$ .

In what follows we will maintain the following assumptions about the demand and cost parameters:

**Assumption 1:**  $\bar{Q}\alpha^h > \alpha^\ell$

**Assumption 2:**  $\alpha^\ell > \frac{(\bar{Q}-s)\alpha^h}{1-s}$

**Assumption 3:**  $\alpha^\ell \geq AC_O$

Assumption 1 implies that if firm 1 operates alone in the market, it will prefer to set a price of  $\alpha^h$  and earn a profit of  $\bar{Q}\alpha^h$  rather than set a price of  $\alpha^\ell$  and earn a profit of  $\alpha^\ell$ ; in other words,  $\alpha^h$  is a “monopoly” price. Absent Assumption 1, firm 1 will never set a price equal to  $\alpha^h$ . Assumption 2 ensures that if firm 1 chooses to accommodate firm 2, it would set a price of  $\alpha^\ell$  and make a profit of  $(1-s)\alpha^\ell$ , rather than set a price of  $\alpha^h$  and make a profit of  $(\bar{Q}-s)\alpha^h$ . Recalling that  $AC_O > AC_I$ , Assumption 3 ensures that firm 2 wishes to operate in the market when firm 1 sets the accommodation price  $\alpha^\ell$ .

Together, Assumptions 1 and 2 imply that the equilibrium price is lower when firm 2 competes with firm 1 than when firm 1 acts as a monopoly. Moreover, these assumptions imply that monopoly behavior by firm 1 leads to a deadweight loss since a fraction of the demand,  $1 - \bar{Q}$ , is not served. From a welfare perspective then, predatory behavior by firm 1 has a mixed effect on consumers: in the short run, it benefits consumers by leading to low predatory prices, but in the long run, it may hurt consumers since once firm 2 exits, a new firm 2 may not be born immediately and during this time, firm 1 will charge a high monopoly price.

### 3 The equilibria

We restrict attention to Markov strategies and solve for the Markov Perfect Equilibria (MPE) in our model. We begin in Subsection 3.1 with some definitions and preliminary observations. In Subsection

3.2, we solve for the limit prices that firm 1 needs to set when it wishes to keep firm 2 out of the market, and in Subsection 3.3, we fully characterize the set of MPE in our model.

### 3.1 Markov strategies and limit prices

A Markov strategy is a map from the set of payoff-relevant states into actions. In our model there are 3 payoff-relevant states: (i)  $I$  which corresponds to the case where firm 2 is already “IN” the market, (ii)  $O$  which means that firm 2 was just born and hence was “OUT” of the market in the last period, and (iii)  $M$  which corresponds to the monopoly case - firm 2 was “OUT” of the market in the last period and a new firm 2 was not born in the current period.

A Markov strategy for firm 1 is a map from the set of payoff relevant states,  $\{I, O, M\}$ , into a price,  $p_1$ . As mentioned earlier, firm 2 is a second mover and hence sets  $p_2$  just below  $p_1$  whenever it chooses to operate in the market. This implies in turn that a Markov strategy for firm 2 is a mapping from  $\{I, O, M\}$  and firm 1’s price into a decision to either operate in the market and undercut  $p_1$  slightly, or stay out of the market in which case it dies. A Markov Perfect Equilibrium (MPE), is a pair of Markov strategies which are mutual best-responses.

In state  $M$ , firm 1 is a monopoly and given Assumption 1, it will set  $p_1 = \alpha^h$  and sell  $\bar{Q}$  units. In states  $I$  and  $O$ , firm 2 is either already in the market or is just born and firm 1 must decide whether to accommodate it. There are therefore 4 possible equilibrium configurations, or regimes, that can arise:  $AA$ ,  $DA$ ,  $AD$ , and  $DD$ . Regime  $AA$  corresponds to the case where firm 1 “Accommodates” firm 2 both when it is already in the market (state  $I$ ) as well as when it is first born (state  $O$ ). In regime  $AD$ , firm 1 “Accommodates” an existing firm 2 (state  $O$ ), but “Deters” a newborn firm 2 (state  $O$ ). In regime  $DA$ , firm 1 “Deters” an existing firm 2 (state  $I$ ), but “Accommodates” a newborn firm 2 (state  $O$ ). Finally, in regime  $DD$  firm 1 “Deters” firm 2 irrespective of whether it is already in the market or is just born.<sup>6</sup>

To characterize the equilibrium prices in each regime, notice that Assumption 2 implies that

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<sup>6</sup>We will use the word “Deter” to refer to both the preemption of firm 2 when it is just born as well as to the predation of firm 2 when it is already in the market.

firm 1 should set  $p_1 = \alpha^\ell$  whenever it chooses to accommodate firm 2 (i.e., it plays “A”). Using equation (2), firm 2’s resulting discounted payoff in state  $\omega$  under regime  $\theta$  is given by

$$V_\omega^\theta = \begin{cases} s(\alpha^\ell - AC_O) + \delta V_I^\theta, & \omega = O, \theta = AA, DA, \\ s(\alpha^\ell - AC_I) + \delta V_I^\theta, & \omega = I, \theta = AA, AD, \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

The top line in (3) corresponds to the case where firm 2 is initially “OUT” of the market, but is accommodated when it just enters (regimes  $\cdot A$ ). The middle line corresponds to the case where firm 2 is already “IN” the market and is accommodated when it is in (regimes  $A\cdot$ ). The last line in (3) corresponds to the case where firm 2 stays out of the market, in which case it dies, and hence its payoff is 0. Using equation (3), we get

$$V_I^{AA} = V_I^{AD} = \frac{s(\alpha^\ell - AC_I)}{1 - \delta}, \quad (4)$$

$$V_O^{AA} = s(\alpha^\ell - AC_O) + \delta \frac{s(\alpha^\ell - AC_I)}{1 - \delta}, \quad (5)$$

$$V_O^{DA} = s(\alpha^\ell - AC_O), \quad (6)$$

and

$$V_I^{DA} = V_O^{AD} = V_I^{DD} = V_O^{DD} = 0. \quad (7)$$

By contrast, when firm 1 wishes to induce firm 2 to stay out of the market (i.e., it plays “D”), it needs to set an appropriate limit price, i.e., set the highest  $p_1$  that still induces firm 2 to stay out of the market. In the next subsection we characterize these prices.

### 3.2 Computing the limit prices

To compute the limit prices, we invoke the “one-stage-deviation principle” (see e.g., Fudenberg and Tirole, 1991). This principle states that in order to compute an MPE, it is sufficient to ensure that the equilibrium is immune to deviations only in the current period. Hence, the limit prices that firm 1 must choose when it plays  $D$  must leave firm 2 indifferent between (i) staying out of the market as



the equilibrium dictates, and (ii) deviating in the current period by operating in the market, but then reverting to its equilibrium strategy in all future periods.<sup>7</sup> For instance, the limit prices in regime  $DD$  must leave firm 2 indifferent between (i) staying out of the market and playing a one-stage deviation, and (ii) operating in the market for only one period (recall that on the equilibrium path of regime  $DD$ , firm 2 is supposed to stay out of the market).

Let  $p_\omega^\theta$  be the limit price that firm 1 charges in state  $\omega \in \{I, O\}$  under regime  $\theta \in \{AA, AD, DA, DD\}$  (in state  $M$  firm 2 is not born, so firm 1 does not need to set a limit price). On the equilibrium path, there are 4 limit prices that firm 1 charges:  $p_I^{DA}$ ,  $p_O^{AD}$ ,  $p_O^{DD}$ , and  $p_I^{DD}$ . For example,  $p_I^{DA}$  is the limit price that firm 1 charges when it accommodates firm 2's entry, but then induces it to exit once it is in the market (i.e., it plays  $D$  in state  $I$  and play  $A$  in state  $O$ ). Apart from these 4 limit prices, we also need to compute the limit prices that firm 1 would need to charge if it were to deviate from playing  $A$  to playing  $D$ . Although these limit prices are not actually used on the equilibrium path, they are nonetheless needed in order to compute the profit that firm 1 would make if it were to deviate from  $A$  to  $D$ . There are 4 such “shadow” limit prices:  $p_O^{AA}$ ,  $p_I^{AA}$ ,  $p_O^{DA}$ , and  $p_I^{AD}$ . For example,  $p_O^{AA}$  is the “shadow” limit price that firm 1 needs to charge in order to deter firm 2's entry into the market, given that once it enters, it stays in the market forever.

The following lemma characterizes the 8 different limit prices.

**Lemma 1:** *The limit prices that firm 1 sets when it plays  $D$  in state  $\omega$  are  $p_\omega^{DA} = p_\omega^{DD} = AC_\omega$  and  $p_\omega^{AD} = p_\omega^{AA} = p_\omega$ , where*

$$p_\omega \equiv AC_\omega - \delta \left( \frac{\alpha^\ell - AC_I}{1 - \delta} \right).$$

*The limit prices are feasible however only when they are nonnegative, since firm 2 can always stay in the market without producing. Hence, firm 1, however, can deter entry in regime  $AD$  by charging  $p_\omega$  only if*

$$\delta \leq \hat{\delta} \equiv \frac{AC_O}{AC_O + (\alpha^\ell - AC_I)}.$$

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<sup>7</sup>Firm 1 can always break this indifference by lowering the limit price slightly to ensure that firm 2 strictly prefers to stay out of the market. In what follows we will assume that when indifferent, firm 2 stays out of the market.

If this inequality fails, then there does not exist a sufficiently low limit price that can induce firm 2 to exit when it is just born under regime  $AD$ , so this regime cannot be an MPE. Likewise, firm 1 cannot induce firm 2 to stay out of the market if it is just born under regime  $AA$  when  $\delta \leq \widehat{\delta}$ . And if

$$\delta > \frac{AC_O}{\alpha^\ell},$$

then firm 1 cannot induce firm 2 to exit if it is already in the market under regimes  $AA$  or  $AD$ , where by Assumption A3,  $\frac{AC_O}{\alpha^\ell} < \widehat{\delta}$ . All limit prices are below  $\alpha^\ell$  - when firm 1 charges a limit price it serves the entire market.

**Proof of Lemma 1:** By definition, a limit price (either an actual or a shadow limit price), leaves firm 2 indifferent between staying in the market and staying out of the market. In the latter case, firm 2 dies, so its profit is 0. If firm 2 stays in the market in state  $\omega$  under regime  $\theta$  and matches firm 1's price  $p_\omega^\theta$ , its current profit is  $s(p_\omega^\theta - AC_\omega)$  and its future profit is  $\delta V_I^\theta$  (by the “one-stage-deviation principle,” firm 2 reverts to its equilibrium strategy from the next period onward). Hence the limit prices are defined by

$$\underbrace{0}_{\text{Exiting}} = \underbrace{s(p_\omega^\theta - AC_\omega) + \delta V_I^\theta}_{\text{Staying in the market}} \quad \Rightarrow \quad p_\omega^\theta = AC_\omega - \frac{\delta V_I^\theta}{s}. \quad (8)$$

Note, however, that firm 1 cannot always induce firm 2 to exit the market, since firm 2 does not need to match  $p_\omega^\theta$  and can always stay in the market without producing. Hence, if  $p_\omega^\theta$  which solves equation (8) is negative, then  $-sAC_\omega + \delta V_I^\theta$ , which is firm 2's discounted payoff in state  $\omega$  under regime  $\theta$  if it does not produce in the current period, is strictly positive, so firm 2 will prefer to stay in the market. In that case, there does not exist a limit price that induces firm 2 to stay out of the market.

We are now ready to characterize the limit prices that firm 1 sets on the equilibrium path. In regime  $DD$ , firm 2 is supposed to stay out of the market in all periods, so  $V_I^{DD} = 0$ . The limit prices are  $p_O^{DD} = AC_O$  and  $p_I^{DD} = AC_I$ ; these prices ensure that firm 2 just breaks even if it operates in the market for a single period.

Likewise, in regime  $DA$ , firm 1 predates an existing firm 2 (i.e., plays  $D$  in state  $I$ ), so  $V_I^{DA} = 0$ . Therefore, once again the limit price is  $p_I^{DA} = AC_I$ .

In regime  $AD$ , firm 2 is accommodated if it is already IN the market, but is deterred when it is just born (firm 1 plays  $A$  in state  $I$  and  $D$  in state  $O$ ). Hence if firm 2 enters the market when it is just born, it stays in the market forever. Recalling from Assumption 2 that the accommodation price is  $\alpha^\ell$ , and noting that the average cost of firm 2 when it is already in the market is  $AC_I$ , the discounted infinite sum of future profits that firm 2 expects is  $V_I^{AD} = \frac{s(\alpha^\ell - AC_I)}{1 - \delta}$ , which is positive since by Assumption 3,  $\alpha^\ell > AC_O > AC_I$ . Using equation (8), the limit price is given by

$$p_O^{AD} = p_O \equiv AC_O - \delta \left( \frac{\alpha^\ell - AC_I}{1 - \delta} \right),$$

which is nonnegative provided that  $\delta \leq \widehat{\delta}$ . When  $\delta \leq \widehat{\delta}$  then, firm 1 can deter firm 2's entry by shading  $p_1$  below firm 2's average cost by an amount equal to the discounted infinite sum of firm 2's per-unit profits following entry. Assumption 3 ensures that  $p_O < AC_O < \alpha^\ell$ . If however  $\delta > \widehat{\delta}$ , firm 2 can only profitably enter the market when it is just born by staying in the market without producing.

We now turn to the shadow limit prices. We begin with  $p_O^{DA}$ , which is the limit price that deters the entry of a newborn firm 2, given that once it enters, firm 1's strategy is to induce it to exit. Hence  $V_I^{DA} = 0$ , so by equation (8),  $p_O^{DA} = AC_O$ .

Next, we consider  $p_I^{AD}$ , which is the limit price that induces firm 2 to exit if it is already IN the market. If firm 2 stays in the market despite the fact that firm 1 charges  $p_I^{AD}$ , it is then accommodated forever (under regime  $AD$  an existing firm 2 is accommodated), so as (4) shows,  $V_I^{AD} = \frac{s(\alpha^\ell - AC_I)}{1 - \delta}$ . By equation (8) then,

$$p_I^{AD} = p_I \equiv AC_I - \delta \left( \frac{\alpha^\ell - AC_I}{1 - \delta} \right).$$

Assumption 3 ensures that  $0 < p_I^{AD} < AC_I < \alpha^\ell$ . Clearly,  $p_I^{AD} \geq 0$  only if

$$\delta \leq \frac{AC_I}{\alpha^\ell}.$$

If  $\delta > \frac{AC_I}{\alpha^\ell}$ , then firm 1 cannot induce firm 2 to exit if it is already IN, so regime *AD* is surely an MPE.

Finally, we consider  $p_O^{AA}$  and  $p_I^{AA}$  which are the limit prices that induce firm 2 to stay out of the market in regime *AA*. In this regime, firm 2 is accommodated in all future periods if it stays in the market, and hence, as (4) shows,  $V_I^{AA} = \frac{s(\alpha^\ell - AC_I)}{1 - \delta}$ . Using equation (8),

$$p_\omega^{AA} = p_\omega \equiv AC_\omega - \delta \left( \frac{\alpha^\ell - AC_I}{1 - \delta} \right), \quad \omega = I, O.$$

Since all limit prices are below  $\alpha^\ell$ , firm 1 serves the entire market when it charges a limit price. Clearly,  $p_O^{AA} \geq 0$  if  $\delta \leq \widehat{\delta}$  and  $p_I^{AA} \geq 0$  if  $\delta \leq \frac{AC_I}{\alpha^\ell}$ . If these inequalities fail, firm 1 cannot deviate from its equilibrium strategy in regime *AA* and deter firm 2's entry into the market or drive an existing firm 2 to exit. ■

Lemma 1 shows that in regimes *DA* or *DD*, in which an existing firm 2 is deterred, the limit prices are equal to the average cost of firm 2 which is either  $AC_I$  or  $AC_O$ , depending on whether firm 2 is “IN” or “OUT” of the market. Intuitively, in regimes *DA* or *DD*, firm 2 can operate in the market for at most one period; hence limit prices equal to firm 2's average cost ensure that firm 2 cannot make a positive profit by operating in the market. On the other hand, in regimes *AD* or *AA*, firm 2 is accommodated when it is “IN” the market, so firm 1 needs to shade the limit price below the average cost of firm 2 by an amount equal to the discounted infinite sum of the per-unit profits that firm 2 can earn by staying in the market.

Lemma 1 also shows that in regimes *AD* and *AA* in which firm 2 expects to stay in the market forever if it managed to survive for at least one period, firm 1 can induce firm 2 to stay out of the market only if firm 2 is not too “patient.” Otherwise firm 2 will agree to sustain the highest current loss that firm 1 can impose on it ( $AC_I$  if it is already in the market and  $AC_O$  if it just enters) in order to reap off the future accommodation profits. In regimes *DA* and *DD* by contrast, firm 2 expects to stay in the market for at most one period and hence the limit prices are equal to its average costs. In other words, predation and preemption are always feasible in regimes *DA* and *DD* but not under regimes *AD* and *AA*.

To ensure that charging a limit price is not a dominant strategy for firm 1, we will impose the following assumption which is stronger than Assumption 3:

**Assumption 4:**  $(1 - s)\alpha^\ell > AC_O$

### 3.3 The conditions for an MPE

Having computed the limit prices, we are now ready to write the expected discounted payoff of firm 1. Using equation (1) and Lemma 1, the discounted infinite sum of firm 1's profits in state  $\omega$  under regime  $\theta$  is given by

$$Y_\omega^\theta = \begin{cases} \bar{Q}\alpha^h + \delta [\gamma Y_O^\theta + (1 - \gamma) Y_M^\theta], & \omega = M, \\ AC_I + \delta [\gamma Y_O^\theta + (1 - \gamma) Y_M^\theta], & \theta = DD, DA \text{ and } \omega = I, \\ AC_O + \delta [\gamma Y_O^\theta + (1 - \gamma) Y_M^\theta], & \theta = DD \text{ and } \omega = O, \\ p_O + \delta [\gamma Y_O^\theta + (1 - \gamma) Y_M^\theta], & \theta = AD \text{ and } \omega = O, \\ (1 - s)\alpha^\ell + \delta Y_I^\theta, & \text{otherwise.} \end{cases} \quad (9)$$

To understand this expression, note that in state  $M$ , firm 1 is a monopoly, and by Assumption 1 it sets  $p_1 = \alpha^h$  and sells  $\bar{Q}$  units. In the next period, firm 2 is born with probability  $\gamma$  and the discounted sum of firm 1's future profits is  $Y_O^\theta$ ; with probability  $1 - \gamma$ , firm 2 is not born and the discounted sum of firm 1's future profits is  $Y_M^\theta$ . The second, third, and fourth lines in equation (9) correspond to cases in which firm 1 sets a limit price,  $p_\omega^\theta$ , in the current period and keeps firm 2 out of the market. By Lemma 1, firm 1 serves the entire market in these cases. In the second line, the limit price is  $AC_I$ , in the third line it is  $AC_O$ , and in the fourth line it is  $p_O \equiv AC_O - \delta \left( \frac{\alpha^\ell - AC_I}{1 - \delta} \right)$ . Since firm 2 stays out of the market, the discounted sum of firm 1's profits from the next period onward is  $\gamma Y_O^\theta + (1 - \gamma) Y_M^\theta$ , exactly as in the first line. The last line in (9) corresponds to 3 cases where firm 2 is accommodated: (i)  $\theta = AA$ , (ii)  $DA$  and  $\omega = O$ , or (iii)  $\theta = AD$  and  $\omega = I$ . By Assumption 2, firm 1 prefers to set  $p_1 = \alpha^\ell$  in these cases, so its current profit is  $(1 - s)\alpha^\ell$ . Since firm 2 is accommodated, the discounted sum of firm 1's profits from the next period onwards is  $Y_I^\theta$ .

We are now ready to specify the conditions under which each of the 4 possible regimes is an

MPE. As already mentioned, the “one-stage-deviation principle” implies that we only need to check that each regime is immune to a deviation by firm 1 in the current period, following which it reverts to its equilibrium strategy. Since charging  $\alpha^h$  is a dominant strategy in state  $M$ , we only need to consider firm 1’s strategy in states  $I$  and  $O$ . Note that a deviation by firm 1 from  $D$  to  $A$  in state  $\omega$  simply implies that it changes its price from  $p_\omega^\theta$  to  $\alpha^\ell$ . When firm 1 deviates from  $A$  to  $D$ , it should charge an appropriate “shadow” limit price (characterized by Lemma 1) instead of charging  $\alpha^\ell$ .

### 3.3.1 Regime AA

In regime  $AA$ , firm 1 always accommodates firm 2. On the equilibrium path then, firm 2 operates in the market forever. As a result, the discounted infinite sum of firm 1’s expected profits under regime  $AA$  is given by

$$Y_\omega^{AA} = \begin{cases} \bar{Q}\alpha^h + \delta [\gamma Y_O^{AA} + (1 - \gamma) Y_M^{AA}], & \omega = M, \\ (1 - s)\alpha^\ell + \delta Y_I^{AA}, & \omega = I, O. \end{cases}$$

Solving, we obtain

$$Y_I^{AA} = Y_O^{AA} = \frac{(1 - s)\alpha^\ell}{1 - \delta},$$

and

$$Y_M^{AA} = \frac{\bar{Q}\alpha^h + \delta\gamma Y_O^{AA}}{1 - \delta(1 - \gamma)}.$$

Regime  $AA$  is an MPE provided that firm 1 does not wish to make a one-shot deviation from  $A$  to  $D$  in either states  $I$  and  $O$  (but then revert to its equilibrium strategy and accommodate firm 2 in all future periods). By Lemma 2, however, such deviations are not feasible if  $\delta > \frac{AC_O}{\alpha^\ell + AC_O - AC_I}$ , so regime  $AA$  is surely an MPE. If  $\delta \leq \frac{AC_I}{\alpha^\ell}$ , there exist nonnegative limit prices that induce firm 2 to stay out of the market. By Lemma 1, the limit price in state  $\omega = I, O$  is  $p_\omega \equiv AC_\omega - \delta \left( \frac{\alpha^\ell - AC_I}{1 - \delta} \right)$  and at this price, firm 1 serves the entire market. Hence, the discounted infinite sum of firm 1’s expected

profits when it deviates from  $A$  to  $D$  in state  $\omega = I, O$  is

$$\begin{aligned}\tilde{Y}_\omega^{AA} &= \underbrace{p_\omega}_{\text{Current profit}} + \underbrace{\delta [\gamma Y_O^{AA} + (1 - \gamma) Y_M^{AA}]}_{\text{Future expected profit}} \\ &= \frac{\delta (1 - \gamma) \bar{Q} \alpha^h + (1 - \delta (1 - \gamma)) p_O}{1 - \delta}.\end{aligned}$$

The continuation profits in  $\tilde{Y}_\omega^{AA}$  reflect the idea that if firm 1 plays  $D$  in the current period, the state next period is  $O$  with probability  $\gamma$  and  $M$  with probability  $1 - \gamma$ . The second line in  $\tilde{Y}_\omega^{AA}$  is a weighted average of the discounted infinite sum of firm 1's monopoly profits  $\frac{\bar{Q} \alpha^h}{1 - \delta}$  and its deterrence profits  $\frac{p_O}{1 - \delta}$ , where the weight on the former,  $\delta (1 - \gamma)$ , is equal to the discounted probability that firm 1 will be a monopoly once it drives firm 2 out of the market. Notice that since  $\bar{Q} \alpha^h > (1 - s) \alpha^\ell > p_O$ , deviation from  $A$  to  $D$  involves a tradeoff: in the deviation period, firm 1's profit drops from  $(1 - s) \alpha^\ell$  to  $p_O$ , but in the following period, firm 1 may enjoy a monopoly profit. Hence, whether a deviation is profitable or not depends crucially on the likelihood that firm 1 becomes a monopoly once firm 2 exits,  $1 - \gamma$ , and the discount factor,  $\delta$ .

**Proposition 1:** *Regime AA is surely an MPE either if*

$$\delta > \frac{AC_O}{\alpha^\ell + AC_O - AC_I}, \quad (10)$$

or if

$$\bar{Q} \alpha^h - AC_O \leq \alpha^\ell - AC_I, \quad (11)$$

or when both (10) and (11) fail, if

$$\delta < \delta_1 \equiv \frac{(1 - s) \alpha^\ell - AC_O}{(\bar{Q} \alpha^h - AC_O) - (\alpha^\ell - AC_I)}, \quad (12)$$

where  $\delta_1 > 0$  when (11) fails. Otherwise, when (10), (11), and (12) fail, regime AA is an MPE

provided that

$$\gamma \geq \gamma_1 \equiv \frac{\delta \bar{Q} \alpha^h - (1-s) \alpha^\ell + (1-\delta) p_O}{\delta (\bar{Q} \alpha^h - p_O)}, \quad (13)$$

where  $p_O \equiv AC_O - \delta \left( \frac{\alpha^\ell - AC_I}{1-\delta} \right)$ . The critical value  $\gamma_1$  is below 1 for all  $\delta \in [0, 1]$  and is positive for  $\delta > \delta_1$ .

**Proof of Proposition 1:** Regime *AA* is an MPE provided that  $Y_I^{AA} \geq \tilde{Y}_I^{AA}$  and  $Y_O^{AA} \geq \tilde{Y}_O^{AA}$ . Noting that  $Y_O^{AA} = Y_I^{AA}$  while  $\tilde{Y}_O^{AA} > \tilde{Y}_I^{AA}$ , it follows that  $Y_O^{AA} \geq \tilde{Y}_O^{AA}$  is sufficient to ensure that regime *AA* is an MPE. By Lemma 2 though, deviation from *A* to *D* in state *O* is not feasible when  $\delta > \frac{AC_O}{\alpha^\ell + AC_O - AC_I}$ , so regime *AA* is surely an MPE in this case. Otherwise, if  $\delta \leq \frac{AC_O}{\alpha^\ell + AC_O - AC_I}$ , then using the definitions of  $Y_O^{AA}$  and  $\tilde{Y}_O^{AA}$ , the sufficient condition for MPE can be written as

$$(1-s) \alpha^\ell \geq \delta (1-\gamma) \bar{Q} \alpha^h + (1-\delta(1-\gamma)) p_O, \quad (14)$$

or  $\gamma \geq \gamma_1$ , where  $\gamma_1$  is defined by (13). If  $\gamma_1 < 0$ , then (14) holds for all  $\gamma \in [0, 1]$ . To determine the sign of  $\gamma_1$ , note that the denominator of  $\gamma_1$  is positive, since Assumptions 1 and 2 and Lemma 1 imply that  $\bar{Q} \alpha^h > \alpha^\ell > p_O$ . Hence the sign of  $\gamma_1$  depends on the sign of its numerator. Using the definition of  $p_O$ , the numerator of  $\gamma_1$  is

$$\begin{aligned} & \delta \bar{Q} \alpha^h - (1-s) \alpha^\ell + (1-\delta) \left( AC_O - \delta \left( \frac{\alpha^\ell - AC_I}{1-\delta} \right) \right) \\ = & \delta \left[ \left( \bar{Q} \alpha^h - AC_O \right) - \left( \alpha^\ell - AC_I \right) \right] - \left( (1-s) \alpha^\ell - AC_O \right). \end{aligned}$$

Since  $(1-s) \alpha^\ell - AC_O > 0$  by Assumption 4, this expression is negative, and hence  $\gamma_1 < 0$  if either (i)  $\bar{Q} \alpha^h - AC_O \leq \alpha^\ell - AC_I$ , or (ii)  $\bar{Q} \alpha^h - AC_O \geq \alpha^\ell - AC_I$  (in which case  $\delta_1 > 0$ ) and  $\delta < \delta_1$ . In both cases, (13) is satisfied for all  $\gamma \in [0, 1]$ . If  $\bar{Q} \alpha^h - AC_O \geq \alpha^\ell - AC_I$  and  $\delta > \delta_1$ , then  $\gamma_1 > 0$ . Assumption 4 ensures that  $(1-s) \alpha^\ell > AC_O > p_O$ , so  $\gamma_1 < 1$  for all  $\delta$ . ■

To interpret Proposition 1, note first that when  $\delta > \frac{AC_O}{\alpha^\ell + AC_O - AC_I}$ , firm 1 cannot induce firm 2 to exit the market since the discounted infinite sum of future profits that firm 2 earns by staying in



the market exceeds the maximal loss that firm 1 can impose on firm 2 in the current period (this loss is  $AC_\omega$ ). Given that predation is not feasible, regime  $AA$  is obviously an MPE.

When  $\delta \leq \frac{AC_O}{\alpha^\ell + AC_O - AC_I}$ , predation is feasible, so  $AA$  is an MPE only if accommodation is more profitable for firm 1 than predation. Deviation from  $A$  to  $D$  in state  $O$  lowers the current profit of firm 1 from  $(1-s)\alpha^\ell$  to  $p_O$  (recall that  $p_O < AC_O < (1-s)\alpha^\ell$ ), but on the other hand, it raises the continuation profit of firm 1 from  $\delta Y_I^{AA}$  to  $\delta \left[ \gamma Y_O^{AA} + (1-\gamma) \frac{\bar{Q}\alpha^h + \delta\gamma Y_O^{AA}}{1-\delta(1-\gamma)} \right]$ . The latter exceeds the former since

$$\begin{aligned} \delta \left[ \gamma Y_O^{AA} + (1-\gamma) \frac{\bar{Q}\alpha^h + \delta\gamma Y_O^{AA}}{1-\delta(1-\gamma)} \right] &= \delta Y_O^{AA} + \delta(1-\gamma) \left[ \frac{\bar{Q}\alpha^h + \delta\gamma Y_O^{AA}}{1-\delta(1-\gamma)} - Y_O^{AA} \right] \\ &= \delta Y_O^{AA} + \delta(1-\gamma) \left[ \frac{\bar{Q}\alpha^h - (1-\delta)Y_O^{AA}}{1-\delta(1-\gamma)} \right] \\ &= \delta Y_O^{AA} + \delta(1-\gamma) \left[ \frac{\bar{Q}\alpha^h - (1-s)\alpha^\ell}{1-\delta(1-\gamma)} \right] > \delta Y_I^{AA}, \end{aligned}$$

where the inequality follows by noting that  $Y_I^{AA} = Y_O^{AA}$  and since the square bracketed expression is positive by Assumption 1. Hence, deviation from  $A$  to  $D$  entails a trade-off between a loss of current profit and a gain of future profits. The loss exceeds the gain whenever (i)  $\alpha^\ell - AC_I$  is large - the limit price that firm 1 needs to charge when deviating from  $A$  to  $D$  is low, (ii)  $\delta$  is low - firm 1 cares more about the current profit than future profits, and (iii)  $\gamma$  is large - firm 1's has only a small chance of being a monopoly following firm 2's exit).

### 3.3.2 Regime DD

In regime  $DD$  firm 1 induces firm 2 to stay out of the market both when it is already in the market as well as when it is just born. By Lemma 1, the required limit prices are  $AC_I$  and  $AC_O$  because under regime  $DD$ , firm 2 can operate in the market on the equilibrium path for at most one period, so a price equal to its average cost ensures that it cannot make a profit. From firm 1's point of view, the benefit of the  $DD$  regime is that with probability  $1-\gamma$ , a new firm 2 is not born in the next period, so firm 1 can earn a monopoly profit for at least one period.

The discounted infinite sum of firm 1's expected profits under regime  $DD$  is given by

$$Y_\omega^{DD} = \begin{cases} \bar{Q}\alpha^h + \delta [\gamma Y_O^{DD} + (1 - \gamma) Y_M^{DD}], & \omega = M, \\ AC_O + \delta [\gamma Y_O^{DD} + (1 - \gamma) Y_M^{DD}], & \omega = O, \\ AC_I + \delta [\gamma Y_O^{DD} + (1 - \gamma) Y_M^{DD}], & \omega = I. \end{cases}$$

The continuation profits in  $Y_\omega^{DD}$  reflect the idea that even if firm 2 is already in the market, on the equilibrium path, it should exit, so from the next period onwards, firm 1 is either a monopoly with probability  $1 - \gamma$ , or faces a newborn firm 2 with probability  $\gamma$ . Solving the system, we obtain

$$Y_\omega^{DD} = AC_\omega + \frac{\delta (\gamma AC_O + (1 - \gamma) \bar{Q}\alpha^h)}{1 - \delta}, \quad \omega = I, O,$$

and

$$Y_M^{DD} = \bar{Q}\alpha^h + \frac{\delta (\gamma AC_O + (1 - \gamma) \bar{Q}\alpha^h)}{1 - \delta}.$$

Regime  $DD$  is an MPE provided that firm 1 does not wish to make a one-shot deviation from  $D$  to  $A$  in either states  $I$  and  $O$ . By Assumption 2, if firm 1 deviates to  $A$ , it sets  $p_1 = \alpha^\ell$  and its profit in the current period increases from  $AC_\omega$  to  $(1 - s)\alpha^\ell$ . Since a one-shot deviation from  $D$  to  $A$  means that next period the state is  $I$ , the discounted sum of firm 1's profits in state  $\omega = I, O$  becomes

$$\begin{aligned} \tilde{Y}_\omega^{DD} &= \underbrace{(1 - s)\alpha^\ell}_{\text{Current profit}} + \underbrace{\delta Y_I^{DD}}_{\text{Future expected profit}}, \\ &= (1 - s)\alpha^\ell + \delta \left( AC_I + \frac{\delta (\gamma AC_O + (1 - \gamma) \bar{Q}\alpha^h)}{1 - \delta} \right). \end{aligned}$$

Notice that since  $Y_I^{DD} < \gamma Y_O^{DD} + (1 - \gamma) Y_M^{DD}$ , the continuation profit in  $\tilde{Y}_\omega^{DD}$  is lower than the continuation profit in  $Y_\omega^{DD}$ . This implies that a deviation from  $D$  to  $A$  involves a tradeoff between an increase in the current profit from  $AC_\omega$  to  $(1 - s)\alpha^\ell$  and a decrease in the future expected profits from  $\gamma Y_O^{DD} + (1 - \gamma) Y_M^{DD}$  to  $Y_I^{DD}$ .

**Proposition 2:** *Regime DD is an MPE provided that*

$$\gamma \leq \gamma_2 \equiv \frac{\delta \bar{Q} \alpha^h - (1-s) \alpha^\ell + (1-\delta) AC_I}{\delta (\bar{Q} \alpha^h - AC_O)}, \quad (15)$$

where  $\gamma_1 < \gamma_2 < 1$ . *The critical value  $\gamma_2$  is increasing and concave in  $\delta$  and is positive if and only if*

$$\delta > \delta_2 \equiv \frac{(1-s) \alpha^\ell - AC_I}{\bar{Q} \alpha^h - AC_I}. \quad (16)$$

*When  $\delta < \delta_2$  or when  $\gamma > \gamma_2$ , regime DD is not an MPE.*

**Proof of Proposition 2:** To ensure that DD is an MPE, we need to find conditions that ensure that  $Y_I^{DD} > \tilde{Y}_I^{DD}$  and  $Y_O^{DD} > \tilde{Y}_O^{DD}$ . Given that  $Y_I^{DD} < Y_O^{DD}$  and  $\tilde{Y}_I^{DD} = \tilde{Y}_O^{DD}$ , it is clear that if firm 1 does not wish to deviate from D to A in state I, then it also does not wish to deviate in state O. Hence, it is sufficient to find conditions that ensure that  $Y_I^{DD} \geq \tilde{Y}_I^{DD}$ . Using the definitions of  $Y_I^{DD}$  and  $\tilde{Y}_I^{DD}$ , this inequality is equivalent to

$$\delta(1-\gamma) \bar{Q} \alpha^h + \delta \gamma AC_O + (1-\delta) AC_I \geq (1-s) \alpha^\ell, \quad (17)$$

or  $\gamma \leq \gamma_2$ , where  $\gamma_2$  is defined by (15). It is easy to verify that since  $\bar{Q} \alpha^h > AC_O$  (Assumptions 1 and 3) and  $(1-s) \alpha^\ell > AC_I$  (Assumption 4), then  $\gamma_2$  is increasing and concave in  $\delta$ . Moreover, using the fact that  $\bar{Q} \alpha^h > AC_O > p_O$ ,

$$\begin{aligned} \gamma_2 - \gamma_1 &= \frac{z + (1-\delta) AC_I}{\delta (\bar{Q} \alpha^h - AC_O)} - \frac{z + (1-\delta) p_O}{\delta (\bar{Q} \alpha^h - p_O)} \\ &= \frac{z (AC_O - p_O) + (1-\delta) [\bar{Q} \alpha^h (AC_I - p_O) + p_O (AC_O - AC_I)]}{\delta (\bar{Q} \alpha^h - AC_O) (\bar{Q} \alpha^h - p_O)} \\ &> \frac{z (AC_O - p_O) + (1-\delta) [p_O (AC_I - p_O) + p_O (AC_O - AC_I)]}{\delta (\bar{Q} \alpha^h - AC_O) (\bar{Q} \alpha^h - p_O)} \\ &= \frac{((1-\delta) p_O + z) (AC_O - p_O)}{\delta (\bar{Q} \alpha^h - AC_O) (\bar{Q} \alpha^h - p_O)} > 0, \end{aligned}$$

where  $z \equiv \delta \bar{Q} \alpha^h - (1-s) \alpha^\ell$ . In addition,

$$\begin{aligned} \gamma_2 &\equiv \frac{\delta \bar{Q} \alpha^h - (1-s) \alpha^\ell + (1-\delta) AC_I}{\delta (\bar{Q} \alpha^h - AC_O)} \\ &< \frac{\delta \bar{Q} \alpha^h + (1-\delta) AC_O - (1-s) \alpha^\ell}{\delta (\bar{Q} \alpha^h - AC_O)} < 1, \end{aligned}$$

where the last inequality follows by Assumption 4. Noting that the denominator of  $\gamma_2$  is positive, while the numerator is increasing with  $\delta$ , negative at  $\delta = 0$  and positive at  $\delta = 1$ , it follows that  $\gamma_2 > 0$  if and only if  $\delta > \delta_2$ , where  $\delta_2$  is defined by (16). When  $\delta < \delta_2$ ,  $\gamma_2 < 0$  and hence (15) fails. ■

Intuitively, under regime *DD*, firm 1 induces firm 2 to stay out of the market by setting a limit price of  $AC_I$  when firm 2 is already in the market and  $AC_O$  when firm 2 is just born. By Assumption 4, the resulting current profit of firm 1 is below  $(1-s) \alpha^\ell$ , which is the profit that firm 1 can earn by accommodating firm 2. The sacrifice of current profit is profitable for firm 1 only if the discounted infinite sum of its future profits is sufficiently large. Not surprisingly, this is the case when firm 1 is sufficiently “patient” ( $\delta$  is large) and if there is a sufficiently high probability that it will enjoy a monopoly position next period ( $\gamma$  is sufficiently small). In other words, regime *DD* is an MPE only when  $\delta$  is relatively large and  $\gamma$  is relatively small.

### 3.3.3 Regime DA

In regime *DA*, firm 1 induces firm 2 to exit when firm 2 is already in the market, but then it accommodates the entry of a newborn firm 2. On the equilibrium path then, a newborn firm 2 enters the market for only one period. By Lemma 1, the limit price that firm 1 charges when firm 2 is already in the market is  $AC_I$  (firm 2 expects to exit after one period, so a limit price of  $AC_I$  ensures that it cannot make a profit by staying). Hence, the discounted infinite sum of firm 1’s expected

profits under regime  $DA$  is given by

$$Y_{\omega}^{DA} = \begin{cases} \bar{Q}\alpha^h + \delta [\gamma Y_O^{DA} + (1 - \gamma) Y_M^{DA}], & \omega = M, \\ (1 - s)\alpha^{\ell} + \delta Y_I^{DA}, & \omega = O, \\ AC_I + \delta [\gamma Y_O^{DA} + (1 - \gamma) Y_M^{DA}], & \omega = I. \end{cases}$$

Solving, we obtain

$$Y_I^{DA} = \frac{\delta\gamma(1 - s)\alpha^{\ell} + \delta(1 - \gamma)\bar{Q}\alpha^h + (1 - \delta(1 - \gamma))AC_I}{(1 - \delta)(1 + \delta\gamma)},$$

$$Y_O^{DA} = (1 - s)\alpha^{\ell} + \delta Y_I^{DA},$$

and

$$Y_M^{DA} = \bar{Q}\alpha^h - AC_I + Y_I^{DA}.$$

Regime  $DA$  is an MPE provided that firm 1 does not wish to make a one-shot deviation from  $A$  to  $D$  in state  $O$  and from  $D$  to  $A$  in state  $I$ . By Lemma 1, the limit price necessary to induce firm 2 to stay out in state  $O$  is equal to  $AC_O$  (recall that in regime  $DA$  firm 2 is deterred once it is in the market). Given this limit price, and noting that once firm 1 plays  $D$  in the current period, the state next period is either  $O$  with probability  $\gamma$  or  $M$  with probability  $1 - \gamma$ , the discounted infinite sum of firm 1's profits if it deviates from  $A$  to  $D$  in state  $O$  is

$$\tilde{Y}_O^{DA} = \underbrace{AC_O}_{\text{Current profit}} + \underbrace{\delta [\gamma Y_O^{DA} + (1 - \gamma) Y_M^{DA}]}_{\text{Future expected profit}}.$$

If firm 1 deviates in state  $I$  from  $D$  to  $A$ , then by Assumption 2, it sets  $p_1 = \alpha^{\ell}$  in the current period and makes a current profit of  $(1 - s)\alpha^{\ell}$ . Following the deviation, the state next period is  $I$ . Hence, the discounted infinite sum of firm 1's expected profits if it deviates from  $D$  to  $A$  in state  $I$  is

$$\tilde{Y}_I^{DA} = \underbrace{(1 - s)\alpha^{\ell}}_{\text{Current profit}} + \underbrace{\delta Y_I^{DA}}_{\text{Future expected profit}}.$$

Given the payoffs of firm 1 on the equilibrium path and following deviations, regime  $DA$  is an MPE provided that  $Y_O^{DA} \geq \tilde{Y}_O^{DA}$  and  $Y_I^{DA} \geq \tilde{Y}_I^{DA}$ . In the next proposition, we prove however that the two inequalities cannot hold simultaneously.

**Proposition 3:** *Regime  $DA$  cannot be an MPE.*

**Proof of Proposition 3:** Using the definitions of  $Y_O^{DA}$ ,  $Y_I^{DA}$ , and  $Y_M^{DA}$ , condition  $Y_O^{DA} \geq \tilde{Y}_O^{DA}$  is equivalent to

$$(1-s)\alpha^\ell \geq \delta(1-\gamma)\bar{Q}\alpha^h + (1+\delta\gamma)AC_O - \delta AC_I. \quad (18)$$

Using the definition of  $Y_I^{DA}$ , condition  $Y_I^{DA} \geq \tilde{Y}_I^{DA}$  is equivalent to

$$\delta(1-\gamma)\bar{Q}\alpha^h + (1+\delta\gamma)AC_I - \delta AC_I \geq (1-s)\alpha^\ell. \quad (19)$$

Since  $AC_O > AC_I$ , conditions (18) and (19) cannot hold simultaneously. ■

Proposition 3 shows that it is never optimal for firm 1 to accommodate a newborn firm 2 for a single period and then induce it to exit. To understand the intuition, note that in order to induce firm 2 to exit, firm 1 needs to charge a limit price of  $AC_I$ . This action lowers the current profit of firm 1 from  $(1-s)\alpha^\ell$  to  $AC_I$ , but has the advantage of allowing firm 1 to enjoy a monopoly position next period with probability  $1-\gamma$ . But if the expected gain from monopoly position exceeds the loss of current profit, then it should also be profitable for firm 1 to deter entry when firm 2 is just born: the required limit price in that case exceeds the limit price when firm 2 is already in the market ( $AC_O$  rather than  $AC_I$ ), while the expected gain from being a monopoly in the next period is equal to  $\frac{\alpha^\ell - AC_I}{1-\delta}$  in both cases. Hence, if it pays firm 1 to induce firm 2 to exit, then it should also pay it to deter firm 2's entry into the market in the first place. As a result,  $DA$  cannot be an MPE.

### 3.3.4 Regime AD

In regime  $AD$ , firm 1 accommodates an existing firm 2 but deters the entry of a newborn firm 2. Since the game begins with firm 2 being in the market, on the equilibrium path, firm 2 will stay in the market

forever. By Lemma 1, the limit price that firm 1 needs to set in order to deter the entry of a newborn firm 2 is  $p_O \equiv AC_O - \delta \left( \frac{\alpha^\ell - AC_I}{1 - \delta} \right)$ . Lemma 2 shows however that this limit price is feasible only when  $\delta \leq \frac{AC_O}{\alpha^\ell + AC_O - AC_I}$ ; when  $\delta > \frac{AC_O}{\alpha^\ell + AC_O - AC_I}$ , firm 1 cannot deter the entry of a newborn firm 2 since the discounted infinite sum of future profits that firm 2 expects when it enters the market exceed the loss it incurs in the period it enters; hence, regime  $AD$  is not feasible.

The discounted infinite sum of firm 1's profits under regime  $AD$  when it is feasible is given by

$$Y_\omega^{AD} = \begin{cases} \bar{Q}\alpha^h + \delta [\gamma Y_O^{AD} + (1 - \gamma) Y_M^{AD}], & \omega = M, \\ p_O + \delta [\gamma Y_O^{AD} + (1 - \gamma) Y_M^{AD}], & \omega = O, \\ (1 - s)\alpha^\ell + \delta Y_I^{AD}, & \omega = I. \end{cases}$$

Solving, we obtain

$$Y_I^{AD} = \frac{(1 - s)\alpha^\ell}{1 - \delta},$$

$$Y_O^{AD} = \frac{\delta(1 - \gamma)\bar{Q}\alpha^h + (1 - \delta(1 - \gamma))p_O}{1 - \delta},$$

and

$$Y_M^{AD} = \frac{(1 - \delta\gamma)\bar{Q}\alpha^h + \delta\gamma p_O}{1 - \delta},$$

Note that  $Y_I^{AD} = \frac{(1-s)\alpha^\ell}{1-\delta}$  because under regime  $AD$ , firm 1 accommodates firm 2 if it is already in the market, so firm 1's payoff in state  $I$  is simply equal to the discounted infinite sum of its accommodation profits. On the other hand,  $Y_O^{AD}$  is a weighted average of the discounted infinite sum of monopoly profits  $\frac{\bar{Q}\alpha^h}{1-\delta}$  and the deterrence profits  $\frac{p_O}{1-\delta}$ . The weight on the former,  $\delta(1 - \gamma)$ , is equal to the discounted probability that firm 1 will be a monopoly next period.

If firm 1 makes a one-shot deviation from  $D$  to  $A$  in state  $O$ , it charges  $\alpha^\ell$  and earns  $(1 - s)\alpha^\ell$  in the current period. Once it is in the market, firm 2 stays in the market forever, so the discounted infinite sum of firm 1's profits under such deviation is

$$\tilde{Y}_O^{AD} = \underbrace{(1 - s)\alpha^\ell}_{\text{Current profit}} + \underbrace{\delta Y_I^{AD}}_{\text{Future expected profit}} = \frac{(1 - s)\alpha^\ell}{1 - \delta}.$$

This expression is equal to  $Y_I^{AD}$  because in both cases firm 2 is accommodated forever.

Next, consider a one-shot deviation by firm 1 from  $A$  to  $D$  in state  $I$ . By Lemma 1, firm 1 charges in this case a limit price of  $p_I \equiv AC_I - \delta \left( \frac{\alpha^\ell - AC_I}{1-\delta} \right)$ , which is equal to the average cost of firm 2 given that it is already in the market, minus the discounted infinite sum of its future per-unit profits from operating in the market. The latter is needed because if firm 2 does not exit, it is accommodated forever and earns  $\alpha^\ell - AC_I$  per unit in every period. The limit price  $p_I$  is feasible however only when  $\delta \leq \frac{AC_I}{\alpha^\ell}$ ; otherwise there is no nonnegative price that can induce firm 2 to exit the market and hence deviation from  $A$  to  $D$  is not feasible. The discounted sum of firm 1's profits when it deviates from  $A$  to  $D$  in state  $I$  is therefore

$$\begin{aligned} \tilde{Y}_I^{AD} &= \underbrace{p_I}_{\text{Current profit}} + \underbrace{\delta [\gamma Y_O^{AD} + (1-\gamma) Y_M^{AD}]}_{\text{Future expected profit}} \\ &= \frac{\delta(1-\gamma)\bar{Q}\alpha^h + \delta\gamma p_O + (1-\delta)p_I}{1-\delta}. \end{aligned}$$

The last line in  $\tilde{Y}_I^{AD}$  is a weighted average of the discounted infinite sum of firm 1's monopoly profit  $\frac{\bar{Q}\alpha^h}{1-\delta}$ , its deterrence profit  $\frac{p_O}{1-\delta}$ , and its predation profit  $\frac{p_I}{1-\delta}$ .

Regime  $AD$  is an MPE provided that  $Y_O^{AD} \geq \tilde{Y}_O^{AD}$  and  $Y_I^{AD} \geq \tilde{Y}_I^{AD}$ . The next proposition characterizes the conditions under which the two inequalities hold.

**Proposition 4:** *Regime  $AD$  is not an MPE if either (10) or (11) hold, or (iii) both (10) and (11) fail but  $\delta < \delta_1$ , where  $\delta_1$  is defined by (12). Otherwise, when*

$$\frac{AC_O}{\alpha^\ell} \leq \delta < \frac{AC_O}{\alpha^\ell + AC_O - AC_I},$$

*regime  $AD$  is an MPE provided that  $\gamma \leq \gamma_1$ , where  $\gamma_1$  is defined by (13) and is positive since (11) fails and  $\delta > \delta_1$ . Finally, when  $\delta < \frac{AC_O}{\alpha^\ell}$ , regime  $AD$  is an MPE if  $\gamma_0 \leq \gamma \leq \gamma_1$ , where*

$$\gamma_0 \equiv \frac{\delta\bar{Q}\alpha^h - (1-s)\alpha^\ell + (1-\delta)p_I}{\delta(\bar{Q}\alpha^h - p_O)}, \quad (20)$$



with  $p_I \equiv AC_I - \delta \left( \frac{\alpha^\ell - AC_I}{1 - \delta} \right)$ . The critical value  $\gamma_0$  is below  $\gamma_1$  and is positive if and only if

$$\delta \geq \delta_0 \equiv \frac{(1 - s)\alpha^\ell - AC_I}{\bar{Q}\alpha^h - \alpha^\ell}. \quad (21)$$

**Proof of Proposition 4:** By Lemma 2, regime  $AD$  cannot be an MPE if  $\delta \leq \frac{AC_O}{\alpha^\ell + AC_O - AC_I}$  because then, there does not exist  $p_O \geq 0$  that enables firm 1 to deter the entry of a newborn firm 2. If  $\frac{AC_O}{\alpha^\ell} \leq \delta < \frac{AC_O}{\alpha^\ell + AC_O - AC_I}$ , there exists  $p_O \geq 0$  that deters the entry of a newborn firm 2, but there does not exist  $p_I \geq 0$  that induces an existing firm 2 to exit. Hence, deviation from  $A$  to  $D$  is not feasible in state  $I$ , so to ensure that  $AD$  is an MPE, we only need to ensure that firm 1 does not wish to deviate from  $D$  to  $A$  in state  $O$ , i.e.,  $Y_O^{AD} \geq \tilde{Y}_O^{AD}$ . Using the definitions of  $Y_O^{AD}$  and  $\tilde{Y}_O^{AD}$ , this inequality can be written as

$$\delta(1 - \gamma)\bar{Q}\alpha^h + (1 - \delta(1 - \gamma))p_O \geq (1 - s)\alpha^\ell, \quad (22)$$

or  $\gamma \leq \gamma_1$ , where  $\gamma_1$  is defined by (13). From Proposition 1, however, we know that  $\gamma_1 < 0$  if either (i)  $\bar{Q}\alpha^h - AC_O \leq \alpha^\ell - AC_I$ , or (ii)  $\bar{Q}\alpha^h - AC_O \geq \alpha^\ell - AC_I$  and  $\delta < \delta_1$ . In both cases, (22) cannot hold, so  $AD$  cannot be an MPE. If  $\bar{Q}\alpha^h - AC_O \geq \alpha^\ell - AC_I$  and  $\delta > \delta_1$ , then  $\gamma_1 > 0$  and hence (22) is satisfied for small enough values of  $\gamma$ .

Finally, when  $\delta < \frac{AC_O}{\alpha^\ell}$ , both  $p_O$  and  $p_I$  are feasible, so we also need to ensure that firm 1 does not wish to deviate from  $A$  to  $D$  in state  $I$ , i.e.,  $Y_I^{AD} \geq \tilde{Y}_I^{AD}$ . Using the definitions of  $Y_I^{AD}$  and  $\tilde{Y}_I^{AD}$ , this inequality can be written as

$$(1 - s)\alpha^\ell \geq \delta(1 - \gamma)\bar{Q}\alpha^h + (1 - \delta(1 - \gamma))p_O - (1 - \delta)(p_O - p_I), \quad (23)$$

or  $\gamma \geq \gamma_0$ , where  $\gamma_0$  is defined by (20). Notice that since  $p_O > p_I$ , then  $\gamma_0 < \gamma_1$ . Moreover, since Assumptions 1 and 2 and Lemma 1 imply that  $\bar{Q}\alpha^h > \alpha^\ell > p_O$ , the sign of  $\gamma_0$  depends on the sign of

its numerator, which using the definition of  $p_I$ , is given by

$$\begin{aligned}
& \delta \bar{Q} \alpha^h - (1-s) \alpha^\ell + (1-\delta) \left( AC_I - \delta \frac{\alpha^\ell - AC_I}{1-\delta} \right) \\
= & \delta \left( \bar{Q} \alpha^h - \alpha^\ell \right) - \left( (1-s) \alpha^\ell - AC_I \right) \\
= & \left( \bar{Q} \alpha^h - \alpha^\ell \right) (\delta - \delta_0),
\end{aligned}$$

where  $\delta_0$  is defined by (12). It is now easy to see that  $\gamma_0 \geq 0$  as  $\delta \geq \delta_0$ .  $\blacksquare$

Proposition 4 shows that regime  $AD$  can be an MPE only when  $\delta < \frac{AC_O}{\alpha^\ell + AC_O - AC_I}$ , otherwise firm 1 cannot deter the entry of a newborn firm 2 into the market because the discounted infinite sum of future profits that firm 2 expects when it enters exceeds any loss that firm 2 bears in the period in which it enters.

When  $\delta$  is intermediate, firm 1 can deter entry in state  $O$  but cannot deviate from  $A$  to  $D$  in state  $I$  since the maximal loss that firm 2 can bear is smaller than its discounted infinite sum of future profits when it stays in the market (in regime  $AD$ , firm 2 is accommodated if it stays in the market since the deviation from  $A$  to  $D$  is for only one period). Hence regime  $AD$  is an MPE if firm 1 does not wish to deviate from  $D$  to  $A$  in state  $O$ . The condition that ensures that this is the case is  $\gamma \leq \gamma_1$ ; this condition is the opposite of the condition that ensures that regime  $AA$  is an MPE, since there we had to ensure that firm 1 does not wish to deviate from  $A$  to  $D$  in state  $O$ , taking into account the fact that, as in regime  $AD$ , firm 2 stays in the market forever if it is accommodated. Intuitively, regime  $AD$  is an MPE only when  $\gamma$  is relatively small since then firm 1 is very likely to remain a monopoly once firm 2 exits and hence  $D$  is particularly attractive. However when  $\bar{Q} \alpha^h - AC_O \leq \alpha^\ell - AC_I$  or when  $\bar{Q} \alpha^h - AC_O \geq \alpha^\ell - AC_I$  and  $\delta < \delta_1$ ,  $D$  is not profitable for firm 1 no matter how small  $\gamma$  is. The reason is that when  $\bar{Q} \alpha^h - AC_O \leq \alpha^\ell - AC_I$  the limit price required to deter entry is very low and when  $\delta < \delta_1$  the discounted infinite sum of future monopoly profits that firm 1 earns once it deters entry is not sufficiently large to justify the sacrifice of current profit associated with charging a limit price. Hence, regime  $AD$  can be an MPE when  $\delta$  is intermediate only if (i)  $\bar{Q} \alpha^h - AC_O \geq \alpha^\ell - AC_I$  and (ii)  $\delta > \delta_1$  and (iii)  $\gamma \leq \gamma_1$ .

Finally, when  $\delta$  is small, regime  $AD$  can be an MPE provided that (i)  $\bar{Q}\alpha^h - AC_O \geq \alpha^\ell - AC_I$  and (ii)  $\delta > \delta_1$  and (iii)  $\gamma_0 \leq \gamma \leq \gamma_1$ . The requirement that  $\gamma \geq \gamma_0$  is needed to ensure that firm 1 does not wish to deviate from  $A$  to  $D$  in state  $I$  ( $D$  is not very attractive when  $\gamma$  is high since then firm 1 is high likely to face a newborn firm 2 once the current firm 2 exits the market).

It should be noted that when  $\delta \geq \frac{AC_O}{\alpha^\ell}$  or  $\delta < \delta_1$ ,  $AA$  and  $AD$  cannot be both MPE: the conditions that ensure that  $AA$  is an MPE ensure that  $AD$  is not an MPE and vice versa. Only when  $\frac{AC_O}{\alpha^\ell} \leq \delta < \frac{AC_O}{\alpha^\ell + AC_O - AC_I}$  can  $AA$  and  $AD$  be both MPE.

### 3.4 Comparing the different equilibrium regimes

Having characterized the conditions for the various equilibrium regimes, we now establish the following results. First, combined, Propositions 2 and 3 imply the following:

**Corollary 1:** *Predation is an equilibrium behavior in our model only if it is accompanied by the predator's commitment to also deter all future entry. Short-term predation accompanied by accommodation of future entry is never an MPE.*

Second, we now examine how the conditions for predation to be arise in equilibrium are affected by the various exogenous parameters of the model. We will say that predation is facilitated when  $\gamma_2$  increases (the set of values of  $\delta$  and  $\gamma$  for which regime  $DD$  is an MPE becomes wider) and hindered when  $\gamma_2$  decreases.

**Corollary 2:** *Predation is facilitated when firm 2 has a larger capacity ( $s$  increases) and has higher average costs ( $AC_I$  and  $AC_O$  increase) and when the monopoly profit of firm 1 are higher ( $\bar{Q}\alpha^h$  increases), but is hindered when the accommodation price is higher ( $\alpha^\ell$  increase).*

**Proof of Corollary 2:** It is easy to see from (15) that  $\gamma_2$  is increasing with  $s$ ,  $AC_I$ , and  $AC_O$ , but decreasing with  $\alpha^\ell$ . Differentiating  $\gamma_2$  with respect to  $\bar{Q}\alpha^h$  reveals that

$$\frac{\partial \gamma_2}{\partial \bar{Q}\alpha^h} \equiv \frac{(1-s)\alpha^\ell - (1-\delta)AC_I - AC_O}{\delta(\bar{Q}\alpha^h - AC_O)^2} > 0,$$

where the inequality follows by Assumption 4. ■

Intuitively, increases in  $AC_I$  and  $AC_O$  allow firm 1 to induce firm 2 to stay out of the market at higher limit prices and hence lower the cost of regime  $DD$  from firm 1's point of view. An increase in firm 1's monopoly profit,  $\bar{Q}\alpha^h$ , makes predation more profitable, since the benefit of predation in our model comes from the fact that firm 1 can become a monopoly if a new firm 2 is not born. Finally, either an increase in firm 2's capacity or in the accommodation price  $\alpha^\ell$  raise the accommodation profit of firm 1 and hence make predation relatively less attractive.

Next we examine the profitability of the various equilibrium configurations:

**Proposition 5:** *Regime  $DD$  is more profitable for firm 1 than regimes  $AA$  and  $AD$ .*

**Proof of Proposition 5:** Under regimes  $AA$  and  $AD$ , firm 2 is accommodated forever, so its equilibrium payoff is

$$Y_I^{AA} = Y_I^{AD} = \frac{(1-s)\alpha^\ell}{1-\delta}.$$

Under regime  $DD$ , firm 2 is deterred forever, so its equilibrium payoff is

$$Y_O^{DD} = AC_I + \frac{\delta(\gamma AC_O + (1-\gamma)\bar{Q}\alpha^h)}{1-\delta}.$$

Noting that  $Y_O^{DD}$  is decreasing with  $\gamma$  and recalling that  $DD$  is an equilibrium if  $\gamma \leq \gamma_2$ , it follows that

$$Y_O^{DD} \geq AC_I + \frac{\delta(\gamma_2 AC_O + (1-\gamma_2)\bar{Q}\alpha^h)}{1-\delta} = \frac{(1-s)\alpha^\ell}{1-\delta},$$

where the last equality follows from the definition of  $\gamma_2$  (see condition (17) above). ■

[It should be noted that for  $DD$  to be an MPE, deterrence must be more profitable than accommodation in both states  $I$  (firm 2 is already in the market) and  $O$  (firm 2 is just born). As a result, the conditions that ensure that regime  $DD$  is an MPE also ensure that  $DD$  is more profitable for firm 1 than regime  $AA$ . Formally, the left-hand side of (17) is equal to  $Y_I^{DD}$  (firm 1's equilibrium payoff under regime  $DD$ ), while the right-hand side is lower than  $Y_O^{AA}$  (firm 1's equilibrium payoff

under regime  $AA$ ). Hence, (17) implies  $Y_I^{DD} > Y_O^{AA}$ . In sum, when  $DD$  is an MPE, it yields firm 1 higher profits than  $AA$ .]

## 4 Welfare implications

In this section we explore the welfare consequences of predation. As is well known, predatory behavior involves a tradeoff from consumers' point of view: in the short-run, predatory behavior allow consumes to enjoy low predatory prices. In the long-run however, the prey exits the market and the dominant firm raises prices, potentially to the monopoly level. in our model though, the tradeoff is more involved since firm 1 potentially faces new entrants after firm 2 exits and hence it may have to charge low limit prices even in the future. To explore the welfare implications of predatory behavior in the context of our dynamic model, note that when firm 1 acts as a monopoly, it sets a price  $\alpha^h$  and hence there is no consumer surplus. On the other hand, under accommodation, the price is  $\alpha^\ell$  and consumer surplus is

$$S_A \equiv \bar{Q} (\alpha^h - \alpha^\ell).$$

And, when firm 1 charges a limit price  $p_\omega$ , consumer surplus is

$$S_\omega \equiv S_A + \alpha^\ell - p_\omega.$$

Given that regime  $DA$  is never an MPE in our model we will focus only on regimes  $AA$ ,  $DD$ , and  $AD$ . Using the above expressions, the discounted infinite sum of consumer surplus in state  $\omega$  under regime  $\theta = AA, DD, AD$  is given by:

$$CS_\omega^\theta = \begin{cases} 0 + \delta [\gamma CS_O^\theta + (1 - \gamma) CS_M^\theta], & \omega = M, \\ S_\omega + \delta [\gamma CS_O^\theta + (1 - \gamma) CS_M^\theta], & \theta = DD, \\ S_O + \delta [\gamma CS_O^\theta + (1 - \gamma) CS_M^\theta], & \theta = AD \text{ and } \omega = O, \\ S_A + \delta CS_I^\theta, & \theta = AA \text{ or } \theta = AD \text{ and } \omega = I. \end{cases} \quad (24)$$

To understand this expression, note that in state  $M$ , firm 1 sets  $p_1 = \alpha^h$  and there is no consumer surplus in the current period. In the next period, the discounted sum of consumer surplus is  $CS_O^\theta$  with probability  $\gamma$  (a new firm 2 is born) and  $CS_M^\theta$  with probability  $1 - \gamma$  (a new firm 2 is not born). In regime  $DD$  and in state  $O$  in regime  $AD$ , firm 2 is induced to stay out of the market and hence the continuation value of consumer surplus are as in state  $M$ . The current value of consumer surplus reflects the fact that firm 1 charges a limit price in the current period. The last line in (24) corresponds to cases where firm 2 is accommodated; the current value of consumer surplus is  $S_A$  while the continuation value is  $CS_I^\theta$ .

Solving, we obtain the following values of consumer surplus:

State Regime	$M$	$O$	$I$
$AA$	$\frac{\delta\gamma S_A}{(1-\delta)(1-\delta(1-\gamma))}$	$\frac{S_A}{1-\delta}$	$\frac{S_A}{1-\delta}$
$DD$	$\frac{\delta\gamma S_O}{1-\delta}$	$\frac{(1-\delta(1-\gamma))S_O}{1-\delta}$	$S_I + \frac{\delta\gamma S_O}{1-\delta}$
$AD$	$\frac{\delta\gamma S_O}{1-\delta}$	$\frac{(1-\delta(1-\gamma))S_O}{1-\delta}$	$\frac{S_A}{1-\delta}$

Using the expressions in the table, we can now report the following result:

**Proposition 6:** *Predation in regime  $DD$  improves the welfare of consumers relative to regimes  $AA$  and  $AD$  in which, on the equilibrium path, firm 2 is accommodated forever, when  $\delta \leq \frac{\alpha^\ell - AC_I}{S_A}$ . When  $\delta > \frac{\alpha^\ell - AC_I}{S_A}$ , predation can still improve the welfare of consumers relative to regime  $AA$  if*

$$\gamma > \bar{\gamma} \equiv \frac{(1-\delta)(\delta S_A - (\alpha^\ell - AC_I))}{\delta[(1-\delta)S_A + \alpha^\ell - \delta AC_I + (1-\delta)AC_O]}. \quad (25)$$

**Proof of Proposition 6:** Under regimes  $DD$ , the limit price in state  $I$  is  $p_\omega = AC_\omega$ . Using the

expressions in the table,

$$\begin{aligned}
\Delta(\gamma) &\equiv CS_I^{DD} - CS_I^{AA} \\
&= S_I + \frac{\delta\gamma S_O}{1-\delta} - \frac{S_A}{1-\delta} \\
&= \frac{(1-\delta)(\alpha^\ell - AC_I) + \delta\gamma(\alpha^\ell - AC_O) - \delta(1-\gamma)S_A}{1-\delta},
\end{aligned}$$

where  $\Delta(\gamma)$  is increasing with  $\gamma$ . Note that

$$\Delta(1) = \frac{(1-\delta)(\alpha^\ell - AC_I) + \delta(\alpha^\ell - AC_O)}{(1-\delta)^2} > 0.$$

Hence,  $\Delta(1) > 0$  for sufficiently large values of  $\gamma$ . Also note that

$$\Delta(0) = \frac{(1-\delta)(\alpha^\ell - AC_I) - \delta S_A}{1-\delta}.$$

If  $\delta \leq \frac{\alpha^\ell - AC_I}{\alpha^\ell - AC_I + S_A}$ , then  $\Delta(0) \geq 0$  so  $\Delta(\gamma) \geq 0$  for all  $\gamma$  in which case, predation actually benefits consumers. Otherwise, if  $\delta > \frac{\alpha^\ell - AC_I}{\alpha^\ell - AC_I + S_A}$ , then  $\Delta(\gamma) > 0$  only if  $\gamma > \bar{\gamma}$ , where  $\bar{\gamma} \in (0, 1)$  is defined by (25). ■

Intuitively, predation involves a tradeoff from the consumers' point of view: in the current period it is beneficial as it involves low limit prices, but in the long-run it may be harmful as it may lead to high monopoly prices. Proposition 6 shows that when  $\delta$  is small (consumers are "impatient" and care more about their short-term surplus), then predation benefits consumers. The proposition also shows that predation benefits consumers when the likelihood that a new firm 2 will be born,  $\gamma$ , is high. The reason of course is that whenever a new firm 2 is born, firm 1 charges a low limit price to deter its entry. This low limit price benefits consumers.

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