Calling Circles: Network Competition with Non-Uniform Calling Patterns*

Steffen Hoernig† Roman Inderst‡ Tommaso Valletti§

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Abstract

We introduce a flexible model of telecommunications network competition with non-uniform calling patterns, accounting for the fact that customers tend to make most calls to a small set of contacts. Equilibrium call prices are distorted away from marginal cost, and competitive intensity is affected by the concentration of calling patterns. Contrary to previous predictions, jointly profit-maximizing access charges are set above termination cost in order to dampen competition if calling patterns are sufficiently concentrated. We discuss implications for regulating access charges as well as on- and off-net price discrimination.

Keywords: Network competition; non-uniform calling patterns; termination charges
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†Nova School of Business and Economics, Lisbon, and CEPR.

‡University of Frankfurt (IMFS), Imperial College London and CEPR.

§Imperial College London, University of Rome II and CEPR.

1 Introduction

Modern communication networks allow users to easily establish a large number of links, both on the same network and across networks. Still, most users' contacts are often limited to only a small fraction of all other users. Exchanges tend to be with friends and family or between people who have, for different reasons, close social links, such as members of user groups linked to business customers. While each user thus calls only a small fraction of all other users at all, the subset of links with whom he has also frequent communications may be even smaller. For instance, Shi et al. (2010) find, for a Chinese cellular network, that people make most of their calls ("80%") to a very small proportion ("20%") of their contacts.

These observations seem to contrast with a standard assumption that is frequently made in the literature on competition between telecommunications networks: the assumption of "uniform calling patterns", whereby each subscriber is assumed to be equally likely to call any other subscriber in the market (e.g., Armstrong, 1998; Laffont et al., 1998a, 1998b). As calls between networks (called "off-net calls") involve the payments of wholesale charges (also called access charges, or termination rates, in the literature), the assumption of uniform calling patterns has consequences on how access charges impact on consumers' retail expenses. When competing networks can set access charges strategically, Gans and King (2001) find that access charges are jointly set below cost. Under multi-part tariffs, this leads to higher prices for on-net than for off-net calls. Together with the assumption of uniform calling patterns, this implies that the fraction of call minutes that a subscriber makes on-net should be below the market share of the network to which he subscribes.

Instead, Birke and Swann (2006) document that calling patterns are heavily on-net biased, and that consumers tend to choose telecom providers following their calling clubs. In the UK, a recent report by Ofcom (2011) shows that there are no longer significant differences between on-net and off-net prices, albeit the share of on-net calls among all mobile calls is still significantly higher than what a uniform calling pattern would predict. In this paper we address this evidence by introducing non-uniform calling patterns in a tractable model of network competition.[1] Customers differ in their preferences for a par-

[1]In Italy, the subscriber market shares and the share of on-net calls compare as follows (on-net shares in brackets): TIM 36% (70%), Vodafone 34% (78%), Wind 20% (70%), and H3G 10% (33%). A similar picture emerges in Portugal: Over the years 2003-2009, the share of on-net calls in all national mobile-to-mobile calls has been fairly constant at 75%, while under a uniform calling pattern this share should

ticular network, and instead of stipulating that each subscriber calls any other subscriber with the same probability, we suppose that he is more likely to call other subscribers with similar preferences than those further away in preference space. For instance, the brand positioning of a network may be more appealing to a particular age group.[2] Likewise, differences in local network coverage could generate similar patterns of call preferences. In both cases, preferences for a particular provider are positively correlated with the probability of making calls to other people with similar preferences (e.g., youths tend to have more friends of the same age in the first case, and people interact more with neighbors in the second case). We analyze how non-uniform calling patterns, with the resulting high fraction of on-net calls, affect equilibrium outcomes.

When calling patterns are uniform, economic theory predicts that under multi-part tariffs variable prices for calls should be set equal to (perceived) marginal costs. Instead, with non-uniform calling patterns, we find that networks optimally deviate from such marginal-cost pricing in order to price discriminate. We derive a simple general pricing formula. The latter relates the deviation from marginal-cost pricing to the difference between the calling pattern of a network's "marginal subscriber", who is just indifferent between joining this and a competitor's network, and the average calling pattern of subscribers on this network.

An important implication of our results concerns networks' choice of profit-maximizing access charges. Under a uniform calling pattern, networks would choose access charges below cost to dampen competition (Gans and King, 2001). This induces off-net prices below on-net prices, leading to negative "tariff-mediated network effects" (Laffont et al., 1998b): The average call price on a smaller network will be lower due to the larger share of off-net calls. Consumers then prefer to join a smaller rather than a larger network, which dampens competition. However, when calling patterns are no longer uniform, the proportion of on- and off-net calls of the marginal subscriber is less closely tied to the respective market shares of the two networks. This diminishes the role of the aforemen-

have been at most 40%. In both instances, the on-net share is thus much larger than market shares alone would suggest, though the fact that there on-net calls are typically substantially cheaper does not allow to draw immediate inferences on whether these call imbalances are only tariff-induced or also due to what we term "calling circles".

[2]This seems to be the case, for instance, in the UK for customers of Virgin Mobile, who are typically young people attracted by the brand of the Virgin group. See http://www.ofcom.org.uk/research/cm/cmr09/. Also, when mobile operators sponsor different sport activities or clubs, subscribers may sort according to their respective preferences over sports or clubs.

tioned tariff-mediated network effects. We derive conditions for when, with non-uniform calling patterns, competition is dampened through higher and not lower access charges.

Our finding that access charges above cost can be used to dampen competition provides a rationale for policy invention aimed at *reducing* access charges. There is no scope for such policy intervention in the standard model with uniform calling patterns. That access charges above cost can dampen competition is related to, but conceptually different from, the standard logic of "raising rivals' cost". When, in a standard oligopoly model without network competition, firms cross-licence an essential input to each other, a higher marginal royalty unambiguously increases *both* a firm's own true cost *and* its opportunity cost of serving more customers. As acquiring the marginal customer thus becomes less attractive, this reduces the intensity of competition. To see the difference to the case of network competition, note that when off-net and on-net prices are the same, a change in access charges would *not* affect competition, as it would not change the profits that firms can make with the marginal subscriber: When calls between networks are balanced, for both networks the higher charges for off-net calls are exactly off-set by the higher revenues from received calls.[3] We show how the endogenous price difference between on-net and off-net prices, together with the degree of concentration of calling patterns, determine when lower or higher access charges dampen competition.

We contribute to the literature that analyzes how network effects affect the intensity of competition (e.g., Katz and Shapiro, 1985). Applied to communication networks, Gabrielsen and Vagstad (2008) and Calzada and Valletti (2008) introduce the notion of "calling clubs" to account for the fact that subscribers tend to call only a limited number of people, and analyze how these clubs deter subscribers from switching between networks. However, they assume that clubs only consist of subscribers with identical preferences, so that two "calling clubs" are always completely disjunct from each other. In our model clubs overlap, which seems more realistic.

Dessein (2003, 2004) and Armstrong (2004) allow for heterogeneity between high- and low-volume consumers and study the effects of net call inflows or outflows of specific consumer groups. Armstrong assumes inelastic call demand, so that call imbalances are

[3]This does no longer hold, however, when tariffs are restricted to be linear. Starting from a symmetric equilibrium, a deviating firm could only acquire market share through undercutting the marginal price, which would lead to an imbalance of calls: All of its customers would make more off-net calls than they receive. Higher access charges then dampen competition through this (out–of-equilibrium) imbalance (cf. Armstrong, 1998 and Laffont et al., 1998a).

translated into different "waterbed effects" in fixed payments. In Dessein's framework there is scope for marginal prices to differ from perceived marginal costs if either termination charges are far from cost or according to the heterogeneity between the two customer groups. In our model, we find that call prices are always distorted away from marginal cost if the calling pattern is not uniform, contrary to Dessein (2004). Since we consider a different type of heterogeneity, our work is complementary to Dessein's in this respect.

Our model of calling patterns may be useful also for wider applications in the area of network economics (cf. Farrell and Klemperer, 2007). We combine exogenous call preferences for particular consumers with network externalities that are induced by endogenously set tariffs. The latter feature distinguishes our paper from other recent contributions on network externalities, such as Fjeldstad et al. (2010). Prices in our model depend on reciprocal access charges. Here, our analysis contributes to a large literature, starting with Armstrong (1998) and Laffont et al. (1998a, 1998b), on how networks gain from choosing (unregulated) reciprocal access charges. As we noted above, an important puzzle is the prediction of Gans and King (2001) that with two-part tariffs and discrimination between on- and off-net calls networks would jointly choose access charges below cost, which seems to be at odds with how networks set access charges in reality. Still within the constraints of uniform calling patterns, Lopez and Rey (2009) show that high access charges make foreclosure possible, Jullien et al. (2010) show how high access charges can arise from price discrimination between heavy and light users,[4] and in Armstrong and Wright (2009) they may arise under the simultaneous interconnection of mobile and fixed networks. As noted previously, in our model we also obtain for non-uniform calling patterns that off-net prices may, in equilibrium, be indeed higher than on-net prices.[5]

The rest of this paper is organized as follows. Section 2 introduces the model and the necessary notions related to calling patterns. In Section 3 we determine the on- and off-net pricing structure. Section 4 determines the market equilibrium. In Section 5 networks choose reciprocal access charges. Section 6 discusses the scope for various policy interventions, such as imposing restrictions on access charges or on price discrimination between on-net and off-net calls. Section 7 offers some concluding remarks.

[4]See, however, Hurkens and Jeon (2009), who find below-cost access charges in a model with elastic subscription demand.

[5]In the extant literature, when subscribers receive utility also from received calls, such a pricing structure is obtained by Jeon et al. (2004), but disappears once access charges are endogenized (Cambini and Valletti, 2008). See also Hermalin and Katz (2011).

2 A Model of Competition with Non-Uniform Calling Patterns

We consider competition between two interconnected telephone networks, 1 and 2, indexed by $i \neq j \in \{1, 2\}$. Both networks incur a fixed cost f to serve each subscriber. The marginal cost of providing a minute of a telephone call is $c \equiv c_O + c_T$, where c_O and c_T denote the costs borne by the originating and terminating network, respectively. As a result, the total marginal cost of an on-net call initiated and terminated on the same network is c. Networks pay each other a reciprocal access charge (or termination rate) a when a call initiated on network i is terminated on a different network j. The access mark-up is equal to $m \equiv a - c_T$. Thus, for an off-net call, the economic marginal cost is still c, but the "perceived" marginal cost for the network that initiates the call is $c + a - c_T = c + m$. This corresponds to the true marginal cost only when $m = 0$.

We follow much of the literature (cf. the Introduction) and let networks discriminate between on-net and off-net calls through the use of two-part tariffs:

$$T_i(q_{ii}, q_{ij}) = F_i + p_{ii}q_{ii} + p_{ij}q_{ij},$$

where F_i is the fixed monthly subscription fee that consumers pay to network i, p_{ii} and q_{ii} are the price and quantity of on-net call minutes, and p_{ij} and q_{ij} are the respective price and quantity for off-net call minutes from network i to network j. The advantage of such a simple multi-part tariff structure is that we will be able to go a long way in characterizing the equilibrium even with general demand and general calling patterns.

Consumer preferences over networks and call demand. The market consists of a continuum of size 1 of consumers. A consumer is indexed by his relative preference for the two networks, where we normalize the space of preferences to $x \in X \equiv [0, 1]$. To simplify expressions, we stipulate that consumers are uniformly distributed over X, though it is straightforward to extend results in this direction. The two networks' own "attributes" are represented by their locations at the two extremes $x_1 = 0$ and $x_2 = 1$. Preferences and networks' locations may relate to the brand image that networks have created through marketing and services directed at specific customer groups (cf. footnote 2).

If a consumer at location x subscribes to network i, he bears a disutility or "transport cost" $\tau |x - x_i|$ with $\tau > 0$. Consumers receive a fixed utility u_0 from being connected.

We assume that u_0 is large enough so that all consumers connect to some network. This convenient assumption may be understood to reflect the fact that most markets for fixed or mobile telephony are highly saturated.

Once a call is placed to somebody, its length depends on the call price. Given prices per minute of p_{ii} and p_{ij}, consumers demand calls of length $q_{ii} = q(p_{ii})$ and $q_{ij} = q(p_{ij})$, with demand elasticity $\eta(p) = -pq'(p)/q(p)$. The level of consumer surplus associated with this demand function is denoted by $v_{ii} = v(p_{ii})$ for on-net calls, and similarly $v_{ij} = v(p_{ij})$ for off-net calls. This indirect utility function $v(\cdot)$ has standard properties. In particular, it holds for the respective price p and quantity q that $dv/dp = -q$.

Calling patterns. The novel ingredient in our model is that consumers differ in their individual calling patterns. The latter are represented by a function $G(y|x)$ on X^2, the likelihood with which a consumer of preference ("location") x will call consumers at locations $y' \leq y$, with $G(0|x) = 0$ and $G(1|x) = 1$ for all $x \in X$.[6] Depending on whether the chosen recipient belongs to the same network or not, the respective call minutes will then equal q_{ii} or q_{ij}. $G(y|x)$ is assumed to be continuously differentiable in (x, y), and non-decreasing in y with density $g(y|x)$. This density may be zero for certain $(x, y) \in X^2$. A uniform calling pattern is obtained when $G(y|x) \equiv G^U(y|x) = y$.

We invoke two assumptions throughout our analysis. As much of the literature that we discussed in the Introduction, we stipulate that, across the whole market, networks have symmetric demand ("Network Symmetry"). Also, we make the assumption of "Communication Symmetry", implying that call preferences are reciprocal.

Network Symmetry $g(y|x) = g(1-y|1-x)$ *for all* $(x, y) \in X^2$.

Communication Symmetry $g(y|x) = g(x|y)$ *for all* $(x, y) \in X^2$.

Below we provide conditions for when, in equilibrium, there will be a unique cutoff customer type $\hat{x}$ such that all $x < \hat{x}$ subscribe to network 1 and all $x > \hat{x}$ subscribe to network 2. For given $\hat{x}$, we define the total expected number of on-net calls on network 1 by

$$L_{11}(\hat{x}) = \int_0^{\hat{x}} G(\hat{x}|x)dx,$$

[6] For our purposes, we could also suppose that each consumer makes the mass one of phone calls.

and the total expected number of off-net calls by

$$L_{12}(\hat{x}) = \int_0^{\hat{x}} [1 - G(\hat{x}|x)]\, dx.$$

Since network 1 has the mass $\hat{x}$ of subscribers who each makes a unit mass of calls, it holds that $L_{11}(\hat{x}) + L_{12}(\hat{x}) = \hat{x}$. In a similar manner, for network 2 we define on-net calls $L_{22}(\hat{x}) = \int_{\hat{x}}^1 [1 - G(\hat{x}|x)]\, dx$ and off-net calls $L_{21}(\hat{x}) = \int_{\hat{x}}^1 G(\hat{x}|x)dx$, with $L_{22}(\hat{x})+L_{21}(\hat{x}) = 1 - \hat{x}$. Note that Communication Symmetry implies that, on aggregate, traffic between networks is *balanced:* $L_{12}(\hat{x}) = L_{21}(\hat{x})$ for any given $\hat{x}$.

Our analysis rests on the following definition of "concentrated" calling patterns.

Definition. *Calling pattern G_2 is more concentrated than calling pattern G_1 if customers with location $0 < x < 1/2$, i.e. those who prefer network 1, are less likely under G_2 to call customers with a stronger preference for network 2 than under G_1: For all $y \in [x, 1)$ it holds that $1 - G_2(y|x) < 1 - G_1(y|x)$.*

This definition takes into account that always $G(0|x) = 0$, $G(1|x) = 1$, and $G(1/2|1/2) = 1/2$.[7] By Network Symmetry, this definition also extends to locations $1/2 < x < 1$: It follows that for all $y \in (0, x]$ we have $G_2(y|x) < G_1(y|x)$, i.e., customers with a preference for network 2 are less likely under G_2 to call customers with a preference for network 1 than under G_1. In the following, we simply will call a calling pattern "concentrated" when, according to the above definition, it is more concentrated than the uniform calling pattern G^U.

Calling pattern example. In what follows, we will obtain explicit expressions for our equilibrium characterization, as well as additional implications, by using the following family of calling patterns. Each customer calls with probability $1 - \lambda$ randomly someone else, while with probability λ he makes a call to someone in his personal "calling circle". For customers who are not too close to the "corners" $x = 0, 1$, calling circles are defined symmetrically around the customer's own location, namely by a distribution function $H(z)$ with support $z \in [-\varepsilon, \varepsilon]$ for some $\varepsilon \in (0, 1/2)$ and symmetric density $h(z)$. We then obtain the following parameterized family of distribution functions:

$$G^{\lambda}(y|x) = (1 - \lambda)\, y + \lambda H(y - x). \tag{1}$$

[7]Hence, for the sake of brevity we have not included into the definition the calling patterns at the "corners" $x = 0, 1$. There, for $y = x$, the requirements can only hold weakly, as $G(0|0) = 0$ and $G(1|1) = 1$.

Realistically, ε should be relatively small, so that the specification of $G^{\lambda}(y|x)$ in (1), which is valid for $\varepsilon \leq x \leq 1-\varepsilon$, applies to most consumers. In fact, for our subsequent analysis it is then inconsequential how we specify calling patterns for those consumers who have the most extreme brand preferences: $x < \varepsilon$ and $x > 1-\varepsilon$. For completeness, we specify there

$$G^{\lambda}(y|x) = (1-\lambda)\, y + \lambda\left[H(y-x) + H(y+x) - H(2-y-x)\right].$$

Literally speaking, the bits of the calling clubs that "stick out" are thereby folded back into the preference space. The chosen specification satisfies everywhere Network Symmetry and Communication Symmetry. It is further convenient to denote the dispersion of calls within the calling circle by[8]

$$\delta = \int_{-\varepsilon}^{\varepsilon} |x|\, h(x)dx = 2\int_{0}^{\varepsilon} xh(x)dx \leq \varepsilon.$$

With this notation at hand, we obtain, when the marginal consumer satisfies $\hat{x} \in [\varepsilon, 1-\varepsilon]$, for on-net calls of network 1, through partial integration that

$$L_{11}(z) = (1-\lambda)\, z^2 + \lambda \int_0^z H(z-x)dx = (1-\lambda)\, z^2 + \lambda\left(z - \frac{\delta}{2}\right). \qquad (2)$$

Utility. Given $\hat{x}$, for any consumer x the net utility from subscribing to network 1 is given by

$$V_1(x,\hat{x}) = v_1(x,\hat{x}) + u_0 - F_1 - \tau x,$$

where

$$v_1(x,\hat{x}) = G(\hat{x}|x)v(p_{11}) + \left[1 - G(\hat{x}|x)\right] v(p_{12}).$$

Recall that on network 1 a consumer of type x makes on-net calls with probability $G(\hat{x}|x)$ and off-net calls with the complementary probability $1 - G(\hat{x}|x)$.

If the consumer subscribes, instead, to network 2, his utility is

$$V_2(x,\hat{x}) = v_2(x,\hat{x}) + u_0 - F_2 - \tau(1-x),$$

with

$$v_2(x,\hat{x}) = \left[1 - G(\hat{x}|x)\right] v(p_{22}) + G(\hat{x}|x)v(p_{21}).$$

[8]For example, if H is a uniform distribution with density $h(z) = 1/2\varepsilon$ then $\delta = \varepsilon/2$, or if H is a tent distribution with density $h(z) = (\varepsilon - |z|)/\varepsilon^2$ we have $\delta = \varepsilon/3$.

Market game. At $t = 1$, for any given reciprocal access charge, networks compete for consumers by simultaneously making contract offers T_i. At $t = 2$, consumers subscribe and place calls. At this stage, all payoffs are realized. Below in Section 5, we will also consider an initial stage $t = 0$ where networks jointly choose a profit-maximizing reciprocal access charge.

Section 3 solves for networks' optimal call prices for given market shares. Here, the focus is on networks' optimal price discrimination strategy for on-net and off-net calls. Section 4 determines the symmetric Nash equilibrium in multi-part tariffs. Section 5 considers the choice of profit-maximizing access charges.

3 Using On- and Off-Net Tariffs for Price Discrimination

Given the contract T_1, each subscriber at location $x \leq \hat{x}$ of network 1 yields expected profits equal to the sum of the fixed part F_1, the expected call profits

$$\pi_1(x, \hat{x}) = G(\hat{x}|x)(p_{11} - c)q(p_{11}) + [1 - G(\hat{x}|x)]\,(p_{12} - c - m)q(p_{12}),$$

and the expected termination profits

$$R_{12}(x, \hat{x}) = mq(p_{21}) \int_{\hat{x}}^{1} g(x|x')dx'.$$

We can thus write the total expected profits that network 1 obtains from a given subscriber at location x as

$$\Pi_1(x, \hat{x}) = \pi_1(x, \hat{x}) + F_1 + R_{12}(x, \hat{x}) - f.$$

Total expected profits of network 1 can now be written as

$$\begin{aligned} \overline{\Pi}_1\left(\hat{x}\right) &= \int_0^{\hat{x}} \Pi_1(x, \hat{x})dx \\ &= \hat{x}\left(F_1 - f\right) + L_{11}(\hat{x})(p_{11} - c)q(p_{11}) \\ &\quad + L_{12}(\hat{x})(p_{12} - c - m)q(p_{12}) + L_{21}(\hat{x})mq(p_{21}). \end{aligned} \tag{3}$$

Similar expressions can be obtained for network 2.

Optimal prices. Take the marginal subscriber $\hat{x}$, and thus the networks' market shares $\hat{x}$ and $1-\hat{x}$, as given. We consider how networks optimally choose on- and off-net prices so as to maximize profits, holding these market shares constant.

More specifically, we consider the following program. We take as given the gross utility level that the marginal consumer must obtain:

$$V_1(\hat{x}, \hat{x}) \geq \overline{V}.$$

Note that in equilibrium the utility level $\overline{V}$ will be determined by the offer of the competing network, i.e., $\overline{V} = V_2(\hat{x}, \hat{x})$. For given $\hat{x}$ and $\overline{V}$, we then solve for the choices p_{11} and p_{12} that maximize $\bar{\Pi}_1$. We first relax this program by only considering the participation constraint of the marginal consumer $x = \hat{x}$ but not those of consumers $x < \hat{x}$, and then state a sufficient condition for when (both on- and off-equilibrium) the solution to the relaxed program is indeed a solution to the original one.

Let now

$$\hat{L}_{11}(\hat{x}) = \hat{x}G(\hat{x}|\hat{x})$$

be the total number of on-net calls on network 1 that *would* arise if all subscribers of network 1 had the same calling pattern as the marginal subscriber $\hat{x}$. With symmetric market shares we have $\hat{L}_{11}(1/2) = 1/4$. Define likewise the number of off-net calls that *would* obtain if all subscribers had the same calling pattern as the marginal subscriber

$$\hat{L}_{12}(\hat{x}) = \hat{x}(1 - G(\hat{x}|\hat{x})).$$

For network 2 we define $\hat{L}_{22}(\hat{x}) = (1-\hat{x})(1 - G(\hat{x}|\hat{x}))$ and $\hat{L}_{21}(\hat{x}) = (1-\hat{x})G(\hat{x}|\hat{x})$.

Proposition 1 *Take the relaxed program of the two networks, where for each firm only the participation constraint of a given marginal customer $\hat{x}$ binds. Then, network i's prices for on-net calls satisfy*

$$\frac{p_{ii} - c}{p_{ii}} = \frac{1}{\eta(p_{ii})}\left(1 - \frac{\hat{L}_{ii}(\hat{x})}{L_{ii}(\hat{x})}\right), \tag{4}$$

while those for off-net calls satisfy

$$\frac{p_{ij} - c - m}{p_{ij}} = \frac{1}{\eta(p_{ij})}\left(1 - \frac{\hat{L}_{ij}(\hat{x})}{L_{ij}(\hat{x})}\right). \tag{5}$$

When τ is sufficiently large, then for any market share $\hat{x}$ these expressions characterize the optimal prices for the two networks in the unrelaxed program.

Proof. See Appendix.

When the calling pattern of the average infra-marginal subscriber is the same as that of the marginal subscriber $\hat{x}$, as is the case with the uniform calling pattern G^U, then $\hat{L}_{ii}(\hat{x}) = L_{ii}(\hat{x})$ and $\hat{L}_{ij}(\hat{x}) = L_{ij}(\hat{x})$. Proposition 1 then yields the standard (perceived) marginal-cost pricing result $p_{ii} = c$ and $p_{ij} = c + m$. Yet, when the marginal subscriber makes more off-net calls and fewer on-net calls than the average subscriber, then it holds that $p_{ii} > c$ and $p_{ij} < c + m$. Intuitively, raising the on-net price above marginal cost and lowering the off-net price below marginal cost allows the network to extract more of the "information rent" of infra-marginal subscribers, who will not switch networks when they have to cede slightly more of their surplus. The pricing formula in Proposition 1 trades off the increase in profits that is made from infra-marginal subscribers with the compensation that must be given to the marginal subscriber in terms of an adjusted fixed fee.

To further foster the intuition, take the case with symmetric market shares, $\hat{x} = 1/2$, and also suppose for the moment that access is at cost, $m = 0$. In this case, suppose now that contrary to the derived optimal tariffs, a network would charge uniform prices. Note that the marginal subscriber now cares equally about on-net and off-net call charges, and so does not mind if the on-net charge rises so long as there is a corresponding reduction in the off-net charge. However, the infra-marginal users of the respective network make less off-net calls, so that the network's profits are increased if the on-net call charge is raised and the off-net charge reduced.

Symmetric market shares. Given our assumption of Network Symmetry, for the subsequent characterization of a market equilibrium the case with symmetric market shares, $\hat{x} = 1/2$, will be most prominent. Observe first that from the definition of more concentrated calling patterns, the respective numbers of on-net calls, $L_{11}(1/2)$ and $L_{22}(1/2)$, are strictly higher when calling patterns are more concentrated. Likewise, the respective numbers of off-net calls, $L_{12}(1/2)$ and $L_{21}(1/2)$, are strictly lower. As, given symmetry, the marginal customer at $\hat{x} = 1/2$ always makes half of his calls on-net and the other half off-net, we have, regardless of how concentrated calling patterns are, $\hat{L}_{11}(1/2) = \hat{L}_{12}(1/2) = 1/4$. From these observations we have immediately that as calling patterns become more concentrated relative to G^U prices in Proposition 1 become more distorted: The multiplier $1 - \frac{\hat{L}_{ii}(1/2)}{L_{ii}(1/2)} > 0$ in expression (4) increases, which pushes up $p_{ii} > c$, and the multiplier

$1-\frac{\hat{L}_{ij}(1/2)}{L_{ij}(1/2)}<0$ in expression (5) decreases, which pushes down $p_{ij}<c+m$.

For our following analysis it is convenient to restate this result by introducing some additional notation. We define μ as a measure of how the average number of on-net calls per subscriber changes as the marginal subscriber is shifted away from the symmetric market share $\hat{x}=1/2$:

$$\mu=\frac{d}{d\hat{x}}\left(\frac{L_{11}(\hat{x})}{\hat{x}}\right)\Big|_{\hat{x}=1/2}. \tag{6}$$

Recall now that from Communication Symmetry it follows that traffic between networks is always balanced: $L_{12}(\hat{x})=L_{21}(\hat{x})$. From Network Symmetry, in turn, we have, in addition, that $L_{21}(\hat{x})=L_{12}(1-\hat{x})$. At $\hat{x}=1/2$ these two identities can only hold when

$$L_{12}'(1/2)=0. \tag{7}$$

In words, a marginal change of $\hat{x}$ at $\hat{x}=1/2$ does not affect the number of off-net calls that are made from network 1.[9] Using expression (7), we obtain the following useful transformation of expression (6):

$$\mu=-\frac{d}{d\hat{x}}\left(\frac{L_{12}(\hat{x})}{\hat{x}}\right)\Big|_{\hat{x}=1/2}=\frac{L_{12}(\hat{x})}{\hat{x}^2}-\frac{L_{12}'(\hat{x})}{\hat{x}}\Big|_{\hat{x}=1/2}=4L_{12}(1/2).$$

Note now that $L_{12}(1/2)=\hat{x}^2|_{\hat{x}=1/2}=1/4$ holds for a uniform calling pattern, so that in this case we have $\mu=1$. Instead, for all more concentrated calling patterns the number of off-net calls at $\hat{x}=1/2$ is strictly smaller, so that $\mu<1$. In fact, from our definition of a more concentrated calling pattern we have that μ strictly decreases as the calling pattern becomes more concentrated. Note in addition that for the marginal consumer we have, given Network Symmetry, $\hat{L}_{11}(1/2)=1/4$. Taken together, these simplified expressions for $L_{12}(1/2)$ and $\hat{L}_{11}(1/2)$ can now be substituted into the expression for the optimal on-net prices of network 1 in expression (4). Given Network Symmetry, the same can be done for network 2, while with regards to off-net prices we can simply use that $\hat{L}_{ij}(1/2)=1/4$ and that $L_{ij}(1/2)=1/2-L_{ii}(1/2)$. We thereby obtain the following results for symmetric market shares.

[9] More explicitly, Network and Communication Symmetry together imply that the following two effects are exactly offsetting when we consider a change of the marginal consumer at $\hat{x}=1/2$: On the one hand, an increase in $\hat{x}$ expands the market share of network 1 and thus the total number of calls that are made from this network; on the other hand, as more consumers switch to network 1, calls that were previously off-net are turned into on-net calls.

Corollary 1 *With symmetric market shares, on-net and off-net prices are*

$$\begin{aligned} \frac{p_{ii}-c}{p_{ii}} &= \frac{1}{\eta(p_{ii})}\left(\frac{1-\mu}{2-\mu}\right), \\ \frac{p_{ij}-c-m}{p_{ij}} &= -\frac{1}{\eta(p_{ij})}\left(\frac{1-\mu}{\mu}\right), \end{aligned} \tag{8}$$

where $\mu < 1$ holds for concentrated calling patterns and decreases strictly as calling patterns become more concentrated.

4 Market Equilibrium

To characterize the market equilibrium, it is convenient to introduce one more piece of notation. We denote by

$$\widehat{G}(\hat{x}) = G(\hat{x}|\hat{x})$$

the number of on-net calls of the marginal customer $\hat{x}$. Clearly, $\widehat{G}(0) = 0$, $\widehat{G}(1/2) = 1/2$, and $\widehat{G}(1) = 1$. We assume that $\widehat{G}$ is differentiable at $\hat{x} = 1/2$, and further define

$$\hat{\mu} = \frac{d}{d\hat{x}} G(\hat{x}|\hat{x})\bigg|_{\hat{x}=1/2} = \widehat{G}'(1/2). \tag{9}$$

Since $\widehat{G}(\hat{x})$ is the marginal subscriber's number of on-net calls on network 1, its derivative, $\hat{\mu}$, measures how this number varies as the marginal subscriber changes. With a uniform calling pattern, we have $\hat{\mu} = 1$, since the number of calls that the marginal customer, just as any other customer, makes to a given network is just equal to the respective market share. When the calling pattern is concentrated, then $\hat{\mu} < 1$, and a more concentrated calling pattern implies that $\hat{\mu}$ decreases. Intuitively, when the calling pattern is more concentrated, then as the marginal customer shifts, the fraction of calls that he makes to either network is less closely tied to networks' market shares but more closely to the customer's own location. Before we further illustrate this with the help of our example, we first provide a formal argument for why $\hat{\mu}$ decreases as calling patterns become more concentrated. Recall that, by network symmetry, for any two calling patterns G_1 and G_2 it holds that $\widehat{G}_1(1/2) = 1/2 = \widehat{G}_2(1/2)$. If G_2 is more concentrated than G_1 according to our definition, then for all $0 < \hat{x} < 1/2$ $(1/2 < \hat{x} < 1)$ it holds that $\widehat{G}_2(\hat{x}) > (<)\ \widehat{G}_1(\hat{x})$, so that the respective derivatives at $\hat{x} = 1/2$ must indeed satisfy $\hat{\mu}_2 < \hat{\mu}_1$.

In terms of our example, where we use the family of calling patterns $G^{\lambda}(\cdot)$, we apply the previously introduced definition (6) to (2), obtaining that $\mu = 1 - \lambda(1 - 2\delta)$, with

$\delta \le \varepsilon < 1/2$. Recall that μ measures the number of off-net calls for a network relative to that of on-net calls, where $\mu = 1$ for the uniform calling pattern. In the example, μ is intuitively strictly lower when it is more likely that any customer makes a call to his calling circle (higher λ) or when the circle is more concentrated (low δ). Furthermore, we have from (1) and (9) that $\hat{\mu} = 1 - \lambda$. This expression is particularly revealing. With a uniform calling pattern, we already observed that as the marginal customer $\hat{x}$ increases, his fraction of calls made to network 1 increases one-by-one with the market share of this network, so that $\hat{\mu} = 1$. This clearly corresponds to the case where $\lambda = 0$ in our example, i.e., when there are no calling circles. At the other extreme, when a customer *only* makes calls to his calling circle and no random calls, then the fraction of calls that the marginal customer makes to either network is completely independent of his location, so that $\hat{\mu} = 0$ when $\lambda = 1$.

For our following analysis both the parameter μ, as defined in expression (6), and the newly defined parameter $\hat{\mu}$ will play a key role. Both strictly decrease as the calling pattern becomes more concentrated but capture different aspects of calling patterns. As the calling pattern becomes more concentrated, a shift of the marginal customer around $\hat{x} = 1/2$ has a smaller impact on the average number of on-net calls per subscriber, i.e., these are less closely tied to the identity of the marginal costumer; and also the number of on-net calls of the marginal costumer is less closely tied to market share. Somewhat loosely speaking, the decrease in both μ and $\hat{\mu}$ results from the fact that as calling patterns become more concentrated, market shares are less relevant for how much customers make use of on-net instead of off-net calls.

The marginal cost of expanding market share. Throughout the subsequent analysis we assume existence of a unique symmetric equilibrium in pure strategies.[10] As the marginal consumer must be indifferent between the offers of the two networks, i.e., $V_1(\hat{x}, \hat{x}) = V_2(\hat{x}, \hat{x})$, we have

$$F_1 = F_2 + v_1(\hat{x}, \hat{x}) - v_2(\hat{x}, \hat{x}) + \tau(1 - 2\hat{x}). \tag{10}$$

[10] A proof of existence and uniqueness of equilibrium follows the same steps as in Laffont et al. (1998b) and is therefore omitted here. It again involves the condition that transport cost τ must be high enough, in order to avoid multiple solutions for equilibrium market shares arising due to network effects.

From this we obtain

$$\frac{dF_1}{d\hat{x}} = \widehat{G}'(\hat{x})\left(v_{11} + v_{22} - v_{12} - v_{21}\right) - 2\tau,$$

which at a symmetric equilibrium candidate and after substituting from (9) becomes

$$\left.\frac{dF_i}{d\hat{x}}\right|_{\hat{x}=1/2} = -2\left[\tau - \hat{\mu}\left(v_{ii} - v_{ij}\right)\right]. \tag{11}$$

Expression (11) captures how expensive it is for a network to shift the marginal subscriber and, thereby, capture market share. In the original Hotelling model the respective marginal cost would be just 2τ. This remains true when on- and off-net call prices are identical, as then $v_{ii} = v_{ij}$. On the other hand, when off-net calls are more expensive than on-net calls, $v_{ii} > v_{ij}$ holds and tariff-mediated network externalities are created. It then becomes *less* expensive for a network to expand its market share. For $p_{ii} > p_{ij}$ and thus $v_{ii} < v_{ij}$ the opposite holds. Importantly, the marginal cost of expanding a network's market share (11) is affected by the value of $\hat{\mu}$ if there are tariff-mediated network externalities. Recall that $\hat{\mu} = 1$ holds with uniform calling patterns, while it is strictly lower with concentrated calling patterns. Thus tariff-intermediated network externalities, as captured in expression (11), become gradually less important as calling patterns become more concentrated. This observation will be important below when we analyze how access charges are optimally chosen so as to dampen competition.

Equilibrium profits. We will now derive networks' fixed fees and, from (3), profits in a symmetric equilibrium. Given the tariff of network 2 and the optimal structure of call prices discussed above, network 1 maximizes its profits by adjusting its fixed fee, or equivalently, its market share.

In a symmetric equilibrium, we have $p_{ii} = p_{jj}$ and $p_{ij} = p_{ji}$. It is then convenient to denote the per-call profits from on-net calls by

$$r_{ii} \equiv (p_{ii} - c)q(p_{ii}).$$

Also consider

$$(p_{ij} - c - m)q(p_{ij}) + mq(p_{ji}), \tag{12}$$

which represents network i's profits from an exchange of one pair of off-net calls with network j.

Recall now that for off-net calls the perceived marginal cost is given by $c+m$. Note next that in a symmetric equilibrium we have that $p_{ij} = p_{ji}$, so that also the respective quantities are equal: $q(p_{ij}) = q(p_{ji})$ when $\hat{x} = 1/2$. Then, expression (12) simplifies to

$$r_{ij} \equiv (p_{ij} - c)q(p_{ij}).$$

In the proof of Proposition 2 below we show how, after substituting for $dF_1/d\hat{x}$ from (11) and using symmetry, we can solve the first-order condition for profit-maximization to obtain

$$F^* = f + \tau - r_{ii} - \hat{\mu}\left(v_{ii} - v_{ij}\right). \tag{13}$$

The equilibrium fixed fee increases in the per-customer fixed cost and in the transport cost, as usual. In fact, in a standard Hotelling model without network externalities, we would have $F^* = f + \tau$. There are now two differences: On-net and off-net prices may be different, so that $v_{ii} \neq v_{ij}$, and on-net revenues are not zero, given that prices are not set equal to (perceived) costs when calling patterns are non-uniform. Substituting F^* back into expression (3) for profits leads to the following outcome.

Proposition 2 *In a symmetric equilibrium, profits for each network are equal to*

$$\bar{\Pi}^* = \frac{1}{2}\left[\tau - \hat{\mu}(v_{ii} - v_{ij}) + \frac{\mu}{2}\left(r_{ij} - r_{ii}\right)\right]. \tag{14}$$

Proof. See Appendix.

In the original Hotelling model profits would be equal to $\tau/2$. Thus, the first term in expression (14) captures, in the traditional manner, how profits depend on the substitutability of networks' services. The second term in expression (14) captures the effect of the tariff-mediated network externalities on the marginal subscriber. If the term $\hat{\mu}(v_{ii} - v_{ij})$ is positive, as a result of off-net prices above on-net prices, then these externalities are positive and it is easier to capture market share. When on-net prices are above off-net prices, on the other hand, then these externalities are negative, and it is more costly to capture market share. Importantly, when it is easier to capture market share, competition is more intense, and equilibrium profits are lower. The relevance of this term depends on the relevance of the calling circle of the *marginal* consumer, as described by $\hat{\mu}$. In fact, we already commented on this in detail after deriving expression (11) above.

We come now to the third term in expression (14), which describes how profits due to infra-marginal subscribers change with the difference between on- and off-net prices. Starting from symmetric market shares, when a network deviates and captures more market share, it increases the number of on-net calls at the expense of decreasing the number of both outgoing and incoming off-net calls. Observe also that $r_{ij} - r_{ii}$ captures the true difference in profits between off-net and on-net calls, i.e., evaluated at the true marginal cost. The third term in (14) thus captures how, through turning off-net calls into on-net calls, a marginal increase in market share, starting from $\hat{x} = 1/2$, impacts on profits. As this effect works through all subscribers on a given network, its importance depends on the *average* behavior of subscribers, as described by the term μ.

Taken together, the second and third terms in expression (14) capture the tariff-induced *costs* and *benefits* from acquiring customers.[11] Both costs and benefits decrease for more concentrated calling patterns, i.e., when both μ and $\hat{\mu}$ decrease. Generally, there is thus not a monotonic impact of more or less concentrated calling patterns on network profits, at least not while access charges are chosen exogenously and thus remain unchanged.

Waterbed effect. Before we ask in the following Section how networks would optimally set the access price so as to maximize equilibrium profits, as derived in Proposition 2, we shed more light on the derived expressions by considering the so-called "waterbed effect". As we discuss in more detail below, in many jurisdictions around the world, wholesale access charges are subject to some form of regulation. While changes in access charges should obviously directly affect the price for off-net calls and networks' termination revenues, policy-makers also have a practical interest in understanding how their intervention may influence the structure of *other* prices and, ultimately, the customer's bill, which is frequently referred to as a "waterbed" or "seesaw" effect. This possible rebalancing in the price structure is an information readily obtainable from price data.[12] In particular, with multi-part tariffs a change in the access price translates into a change in off-net prices and

[11] Note that the derivation of the form of fixed fees (13) and profits (14) does not depend on call prices being at their equilibrium levels or even being non-uniform. If firms were constrained to offer *uniform* two-part tariffs, i.e., if they were to charge identical prices for on- and off-net calls, then the second and the third term in expression (14) would be zero. Profits would be constant at $\tau/2$ and thus independent of the level of the access charge and the shape of the calling pattern. Under uniform pricing, therefore, the access charge is profit-neutral even for non-uniform calling patterns. See Section 6 for a more detailed discussion of the impact of price regulation.

[12] Cf. Genakos and Valletti (2011).

the fixed fees.

Inspection of (13) shows that the access mark-up m has an indirect effect on the equilibrium fixed fee through the off-net indirect utility, v_{ij}, since the latter depends on the off-net price p_{ij} and thus on m. Suppose now that the elasticity of demand for call minutes is constant and in the elastic range: $\eta(p) = \eta > 1$. In this case, from Corollary 1 we can solve for

$$p_{ij} = (c+m)\frac{\mu\eta}{\mu(\eta-1)+1},$$

which does not depend on $\hat{\mu}$ but increases in m and μ. For the fixed fee we obtain

$$\frac{dF^*}{dm} = -\hat{\mu}q_{ij}\frac{dp_{ij}}{dm} = -\hat{\mu}\frac{p_{ij}q_{ij}}{c+m}.$$

Recall now that a more concentrated calling pattern leads to a lower $\hat{\mu}$, which dampens the waterbed effect, i.e., it mitigates the decrease in the fixed fee. This happens because with concentrated calling patterns additional marginal subscribers will make relatively more off-net calls. These customers become even less attractive with a higher access charge, so that the compensation in the fixed fee offered is lower. Apart from the direct effect through $\hat{\mu}$ there is also an indirect effect of a change in concentration of calling patterns, namely through μ: A lower value of μ, as implied by a more concentrated calling pattern, reduces p_{ij} and revenues $p_{ij}q_{ij}$ (since demand is elastic), thus the indirect effect of μ also dampens the waterbed effect. Thus we can conclude unambiguously that under a more concentrated calling pattern the waterbed effect on fixed fees will be smaller.

For the family of calling patterns G^λ, we can proceed somewhat further. In particular, in the limit, when consumers make *only* calls to their circle ($\lambda = 1$), the fixed fee is *independent* of the access charge, such that there is *no* waterbed effect on the fixed fee.[13] Generally, we have from

$$\frac{dF^*}{dm} = -(1-\lambda)(c+m)^{-\eta}\left(1-\frac{1}{\eta}+\frac{1}{\eta}\frac{1}{1-\lambda(1-2\delta)}\right)^{\eta-1}$$

that when calling patterns are very concentrated ($\lambda \to 1$), there should only be a marginal waterbed effect with respect to the fixed fee.

[13] The access charge has, however, an effect on off-net prices also in this case.

5 Dampening Competition through Access Charges

In this section we will determine the jointly profit-maximizing reciprocal access charge a. This is the access charge that networks would want to negotiate if they were free to choose between themselves.

When firms set the reciprocal access charge, they maximize joint profits, which in a symmetric equilibrium are equal to $2\bar{\Pi}^*$. There are now two opposing effects that need to be considered, corresponding to the second and third terms in expression (14) for profits. We referred to these terms as the costs and benefits from capturing additional market share. Take first the costs, i.e., the second term in expression (14). Decreasing the access charge pushes off-net prices down (cf. Proposition 1), leading to a decrease in the utility difference $v_{ii} - v_{ij}$. This makes joining a smaller network more attractive for customers and, thereby, dampens competition through increasing the costs of capturing market share. From this perspective alone firms should thus lower a.

However, lower off-net prices decrease the profits $r_{ij} = (p_{ij} - c)q_{ij}$ from making and receiving off-net calls, at least as long as the off-net price induced by a lies below the "monopoly" call price[14]

$$p_M = \arg\max_p [(p-c)q(p)].$$

Recall that a does not affect the price of on-net calls, p_{ii}. Hence, when a decreases and thus also the off-net price p_{ij} decreases, then the difference between off-net and on-net profits, $r_{ij} - r_{ii}$, decreases as well. From expression (14) this pushes equilibrium profits $\bar{\Pi}^*$ down. Put differently, when a increases, the effect that works through an increase in the benefit of acquiring customers leads to less intense competition and thus increases profits.

The first effect is stronger when $\hat{\mu}$ is large, which is the case for a less concentrated calling pattern. Then, a shift of the marginal customer has a larger effect on his share of on-net calls. The other previously described effect of a change in a, which works through a change in the benefit of acquiring customers when a increases, also increases with a less concentrated calling pattern, i.e., with a larger μ, since then more off-net calls will be transformed into on-net calls as the marginal customer changes. As shown in the proof of the Proposition 3 below, from these two opposing effects we obtain that the optimal off-net price that the networks wish to jointly implement through their choice of the access

[14] For simplicity, we stipulate here that this problem has a unique finite solution p_M. This corresponds to $\eta > 1$ in the constant elasticity case, with $p_M = c\eta/(\eta - 1)$.

charge is indeed strictly decreasing in $\hat{\mu}$ and strictly increasing in μ:

$$\frac{p_{ij}-c}{p_{ij}}=\frac{1}{\eta\left(p_{ij}\right)}\left(1-\frac{2\hat{\mu}}{\mu}\right). \tag{15}$$

When calling patterns are uniform, as in Laffont et al. (1998b), such that $\hat{\mu}=1$ and $\mu=1$, from (15) we obtain immediately $p_{ij}<c$, which together with Proposition 1 results in $m<0$, or $a<c_T$, as in Gans and King (2001). In order to obtain the optimal access charge we substitute the equilibrium prices from Proposition 1 into (15).

Proposition 3 *The jointly profit-maximizing access mark-up is*

$$m^{*}=c\frac{1-2\hat{\mu}}{\mu\left[\eta\left(p_{ij}\right)-1\right]+2\hat{\mu}}. \tag{16}$$

In particular, the corresponding access charge $a^{*}=c_T+m^{*}$ is strictly above the cost of termination ($a^{*}>c_T$) if calling patterns are sufficiently concentrated so that

$$\hat{\mu}<\frac{1}{2}, \tag{17}$$

while $a^{*}<c_T$ applies when $\hat{\mu}>\frac{1}{2}$.

Proof. See Appendix.

Whether the profit-maximizing access charge is above or below the cost of termination is thus closely tied to the calling pattern of the marginal customer, as summarized by the parameter $\hat{\mu}$. When calling patterns are sufficiently concentrated, the result that Gans and King (2001) obtained with a uniform calling pattern is overturned, as then the access charge is chosen *above* cost in order to dampen competition. In other words, the balance between the costs and benefits of expanding market share changes for more concentrated calling patterns, with the benefits of higher off-net prices outweighing their cost.

Note that expression (16) for the access charge a^{*} (or the respective margin m^{*}) is only implicit, given that $\eta(p_{ij})$ on the right-hand side depends on the access charge (cf. Proposition 1). This is, however, no longer the case with isoelastic call demand, $\eta(p_{ij})=\eta$ - a specification that we have already made use of when discussing the waterbed effect. Then, we can substitute η into (16) to immediately obtain the optimal access charge margin.

Proposition 3 provides a clear cut-off result on when the profit-maximizing access charge is above or below the cost of termination c_T, namely whether $\hat{\mu}<\frac{1}{2}$ or $\hat{\mu}>\frac{1}{2}$. This supports our claim that we predict above-cost access charges when calling patterns are sufficiently concentrated. As we noted, however, the optimal access charge need not shift monotonically as we make calling patterns more concentrated. Still, this is the case in our example, for which we obtain from substitution

$$m^* = c\frac{2\lambda - 1}{(\eta + 1)(1 - \lambda) + 2(\eta - 1)\lambda\delta}. \tag{18}$$

Note first that here the access charge margin is strictly negative if and only if consumers make a "random" call less than half of the time (as $1 - \lambda < 1/2$). Further, using that $\lambda \leq 1$ and that $\delta < 1$, next to $\eta > 1$, confirms that $dm^*/d\lambda > 0$, so that the access charge (margin) strictly increases as it becomes more likely that consumers call their respective calling circles.[15] When the calling circle becomes itself less dispersed, as δ decreases, this leads likewise to an increase of m^* if $\lambda > 1/2$.

On-net vs. off-net prices. Having determined the jointly profit-maximizing access charge, we can finally return to the question of whether on-net prices will be higher or lower than off-net prices if networks adopt this access charge. While price discrimination makes off-net prices lower than the respective cost, the access charge effect pushes in the opposite direction, to the extent that $m^* > 0$, as characterized by Proposition 3. The following result shows that the outcome is determined by the relative strength of these two countervailing effects.

Proposition 4 *The on-net price is lower than the off-net price, at the profit-maximizing access charge, iff*

$$\hat{\mu} < \frac{1}{2 - \mu}\frac{\mu}{2}. \tag{19}$$

Proof. See Appendix.

As we already noted, the effects of more concentrated calling patterns involve contradictory forces in this case, and in contrast to Proposition 3 we thus do not obtain clear-cut results in general. Still, for our example we obtain a simple cut-off rule. To see this, it is first instructive to reproduce the price discrimination result for the example, where we obtain

$$\frac{p_{ii} - c}{p_{ii}} = \frac{1}{\eta}\frac{\lambda(1 - 2\delta)}{1 + \lambda(1 - 2\delta)}, \quad \text{and} \quad \frac{p_{ij} - c - m}{p_{ij}} = -\frac{1}{\eta}\frac{\lambda(1 - 2\delta)}{1 - \lambda(1 - 2\delta)}. \tag{20}$$

[15] Formally, the sign of the derivative is given by $(\eta + 1) + 2\delta(\eta - 1)$.

From this we see clearly the direct effect of concentration on the (relative) profit margin, but also how the off-net price is additionally affected by the margin m on the access charge. Substituting for the optimal choice m^* from (18), we obtain that the on-net price is below the off-net price when

$$\lambda \geq \lambda^* = \frac{1 - 6\delta + \sqrt{9 - 28\delta + 36\delta^2}}{4(1 - 2\delta)}. \tag{21}$$

That is, in our example calling circles must be sufficiently relevant for off-net calls to be more expensive than on-net calls. Note that the condition is stricter when δ is low, i.e., when calls made to the calling circles are less dispersed. This effect arises because, as we observed, less dispersion of calls made to calling circles implies lower off-net prices, which counters the effect of the rising access charge.

Recall from the Introduction, that there is evidence that calls are more concentrated on-net than what would be predicted by market shares alone ("call imbalances"). In our model, where in equilibrium market shares are symmetric, there are two forces at work, namely consumers' exogenous calling partners and the relation of on-net to off-net prices. As we noted above, one possible interpretation of the quantity q is in terms of minutes per call. Then, in our example, for all $\lambda \geq \lambda^*$ both the non-uniform calling patterns and the difference in on-net vs. off-net prices work towards increasing the imbalance in terms of call minutes. Recall again that condition (21), which implies that on-net prices are (weakly) below off-net prices, is stricter than the condition $\hat{\mu} = 1 - \lambda \leq 1/2$, which implies that the access charge is (weakly) above cost. For instance, suppose that the "calling circle" $H(\cdot)$ is uniform and comprises 60% of all consumers, which are then called uniformly, so that $\delta = 0.15$.[16] Then, we obtain $\lambda^* = 88\%$: To ensure that on-net prices are not above off-net prices, a consumer must make at least 88% of all calls to his circle.[17] The fraction of calls that are made on-net (cf. expression (2)) is then equal to 81%. In our example, where

[16]The "width" of the calling circle is 2ε, and from footnote 8, with a uniform distribution it is $\delta = \varepsilon/2$.

[17]Note that the number of calls that are ultimately made to the calling circle equals the sum of λ and the "random" component $2\varepsilon(1 - \lambda)$.

market shares are symmetric, the imbalance ratio of calls is also easily calculated as[18]

$$\frac{L_{ii}(1/2)/\hat{x}}{L_{ij}(1/2)/(1-\hat{x})} = \frac{2-\mu}{\mu} = \frac{1+\lambda(1-2\delta)}{1-\lambda(1-2\delta)},$$

which for the chosen numerical specification is equal to 4.2 when $\lambda = \lambda^*$. The higher we choose λ, implying that on-net calls become cheaper than off-net calls, the larger will be the imbalance ratio.[19]

6 Regulation

The consumer surplus, in a symmetric equilibrium, is

$$CS = u_0 + 2L_{ii}(1/2)v_{ii} + 2L_{ij}(1/2)v_{ij} - F^* - 2\int_0^{1/2} \tau x dx.$$

Substituting for F^* from (13), we have

$$CS = u_0 - f - \frac{5}{4}\tau + (r_{ii} + v_{ii}) + \left(\frac{\mu}{2} - \hat{\mu}\right)(v_{ij} - v_{ii}). \tag{22}$$

In a standard Hotelling model consumer surplus would be equal to total welfare minus τ, as the latter is then equal to firms' total profits. The introduction of differential on-net and off-net prices in our model with network externalities leads now, in particular, to the following differences: First, the sum $r_{ii} + v_{ii}$ is not equal to first-best surplus (cf. the price discrimination result in Proposition 1); and, second, there is the presence of the additional term $\left(\frac{\mu}{2} - \hat{\mu}\right)(v_{ij} - v_{ii})$. We discuss these terms in more detail below. Further, social welfare is given by the sum of consumer surplus and firm's profits:

$$W = 2\bar{\Pi}^* + CS.$$

In what follows, we consider various policy interventions aimed at maximizing either total welfare W or consumer surplus CS.

[18]For completeness, in terms of minutes we obtain the imbalance ratio

$$\frac{2-\mu}{\mu}\frac{q(p_{ii})}{q(p_{ij})} = \frac{1+\lambda(1-2\delta)}{1-\lambda(1-2\delta)}\left(\frac{\eta + \frac{1-\lambda-2\delta\lambda}{1-\lambda+2\delta\lambda}}{\eta - \frac{(1-2\delta)\lambda}{1+\lambda-2\delta\lambda}}\right)^{-\eta}.$$

[19]For a comparison, the Italian data in footnote 1 imply imbalance ratios of, respectively, 4.2 (TIM), 6.9 (Vodafone), 9.2 (Wind), and 4.4 (H3G).

6.1 Regulating Access Charges

We ask next how the networks' profit-maximizing access charge, as determined in Proposition 3, compares with the access charge that a social planner would optimally set.

Setting access charges to maximize welfare. Suppose that a policy maker's objective is to maximize total welfare W. Presently, and in line with most regulatory practice, the only instrument that we consider is the regulation of the symmetric access charge a. From the perspective of maximizing welfare, it only matters to the extent that it affects the efficiency of off-net prices. (Recall that on-net prices are not affected by a, while the market outcome along the Hotelling line is always symmetric.) As is intuitive and shown more formally in the proof of the subsequent proposition, a is thus efficiently set so that the resulting off-net price is equal to cost: $p_{ij} = c$. After substituting into the prices obtained from Proposition 1, this yields the welfare-maximizing access charge

$$a^W = c_T + c\frac{1-\mu}{\mu}\frac{1}{\eta(c)}. \tag{23}$$

Proposition 5 *A social planner would set $a = a^W$, as given by (23), when he wants to maximize total welfare. This results in an access price strictly above cost for all concentrated calling patterns. The profit-maximizing access charge a^*, as implied by (16), is socially optimal if and only if*

$$\hat{\mu} = \mu/2.$$

When $\hat{\mu} < \mu/2$ (respectively, $\hat{\mu} > \mu/2$) then $a^W < a^*$ (respectively, $a^W > a^*$).

Proof. See Appendix.

From condition (19), which is stricter than $\hat{\mu} < \mu/2$ if $\mu < 1$, we can see that whenever networks would opt for a termination charge below the socially optimal value then the resulting on-net price would be above the off-net price. Thus for only slightly concentrated calling patterns the qualitative findings of Gans and King (2001) continue to hold.

On the other hand, recall that $\mu = 1$ holds for a uniform calling pattern, so that then $a^W = c_T$. Instead, we have $a^W > c_T$ for all concentrated calling patterns. Intuitively, for non-uniform calling patterns, it is efficient to set the access charge above cost as firms tend to set off-net prices below perceived costs, given its use as a discrimination device.

Proposition 5 also derives conditions when the market outcome would result in the efficient access charge, a^W, which is only the case when $\hat{\mu} = \mu/2$.

As for Proposition 4, we again obtain a more clear-cut result for our particular example. We obtain from (23) that

$$a^W = c_T + c\frac{\lambda(1-2\delta)}{1-\lambda(1-2\delta)}\frac{1}{\eta},$$

so that the welfare-maximizing access charge a^W is strictly higher when calling circles become more relevant (higher λ) or when they become less dispersed (lower δ). Still, we have $a^W < a^*$ whenever the calling circle is sufficiently relevant, i.e., if λ is sufficiently high. The precise condition, obtained from $\hat{\mu} < \mu/2$, is that

$$\lambda > \frac{1}{1+2\delta}. \tag{24}$$

Hence, for our example we would predict that a welfare-maximizing regulator would want to set access charges strictly *below* the unconstrained market equilibrium level whenever localized calling patterns are sufficiently important. This case does not arise for uniform calling patterns ($\lambda = 0$).

Setting access charges to maximize consumer surplus. Suppose now, instead, that the social planner would want to maximize consumer surplus. The social planner's optimal solution is then immediately obtained by looking at the last term in expression (22): When $\hat{\mu} < \mu/2$, to maximize consumer surplus the social planer would want to push down the access charge as far as possible (e.g., adopt a bill-and-keep system), thereby decreasing p_{ij} and increasing v_{ij}. Instead, when $\hat{\mu} > \mu/2$, the social planner would want to push up the access charge as far as possible, thereby essentially fully choking off the demand for off-net calls. Hence, the social planner's program to maximize consumer surplus gives rise to a "bang-bang" solution.

This extreme outcome is clearly due to the stylized nature of our model, as this abstracts, for instance, from elastic participation in the market (we assumed full coverage). Still, it illustrates the previously discussed interaction between calling pattern concentration and how the intensity of competition is affected by price discrimination between off-net and on-net calls and thus by the associated tariff-induced network externalities. In particular, despite shutting down off-net communications and the associated surplus when

$\hat{\mu} > \mu/2$, in this range the externality effect intensifies competition for the market via lower fixed fees, to the benefit of consumers.

Proposition 6 *When $\hat{\mu} < \mu/2$ holds, consumer surplus CS is strictly decreasing in the prevailing access charge a. When $\hat{\mu} > \mu/2$ holds, it is strictly increasing in a.*

Incidentally, note thus that when $\hat{\mu} < \mu/2$ holds, then the access charge that prevails in the unregulated equilibrium will be too high *both* from the perspective of maximizing welfare and from the perspective of maximizing consumer surplus. This is another instance where we see the role played by concentrated calling patterns, as this possibility would never arise under a uniform distribution. Note finally that as the threshold in Proposition 6 is the same as that in Proposition 5, also the respective cutoff result for the example applies (cf. condition (24)).

6.2 Imposing Uniform Pricing

In our model networks can price discriminate between on-net and off-net calls. A common theme in the IO literature is to analyze how firms' ability to price discriminate affects welfare and consumer surplus.[20] In what follows we explore such a policy given its practical relevance.

The degree of on-net/off-net price discrimination has in fact been a subject of concern for a number of competition authorities and/or regulators in Europe and elsewhere. For example, in April 2008, Germany's Federal Competition Authority ("Bundeskartellamt") initiated proceedings against the two largest incumbent mobile telephone network operators, T-Mobile and Vodafone D2, "on the suspicion of the abuse of a joint dominant position on grounds of price differentiation between calls within their own networks (on-net) and calls to other networks (off-net)."[21] While the German Competition Authority discontinued its investigations by the end of 2009, similar complaints against price differentiation between on-net and off-net calls have been made in other countries such as

[20] On second-degree price discrimination see, in particular, Katz (1984) and Stole (2007). However, the fact that there is no substitution between on-net and off-net calls makes the following analysis more reminiscent of models with third-degree price discrimination under oligopoly (cf. Corts, 1998 and Armstrong, 2008).

[21] See Bundeskartellamt (2010), Examination of Possible Abuse of a Dominant Position by T-Mobile and Vodafone by Charging Lower On-net Tariffs for Mobile Voice Telephony Services, online at: http://cms.bundeskartellamt.de/wEnglisch/download/pdf/Fallberichte/B07-170-07-engl.pdf.

Austria, Belgium (related to the German case), Italy, or Turkey. In 2011, the New Zealand Commerce Commission also expressed the concern that tariff-mediated network effects can be strategically used to stifle market competition and to secure market power.

In our setting, an immediate consequence of uniform pricing is that firms' joint profits are always equal to τ, irrespective of how the access charge is set. This is a standard result in Hotelling models, though formally it also follows immediately from expression (14) for $\bar{\Pi}^*$, after substituting $v_{ii} = v_{ij}$ and $r_{ij} = r_{ii}$ when $p_{ii} = p_{ij}$.[22] For each firm the optimal symmetric uniform price $p = p_{ii} = p_{ij}$ maximizes average perceived efficiency (i.e., taking account of the perceived cost of off-net calls rather than the real economic cost):

$$\frac{1}{2}\left[v(p) + q(p)(p - c)\right] - mq(p)L_{ij}(1/2). \tag{25}$$

While firms' profits are not affected by m, both total welfare and consumer surplus are maximized when $m = 0$ so that the optimal access charge is equal to cost: $a = c_T$, with resulting uniform price $p = c$. We now take these observations as a benchmark against which we compare our previous characterization for when firms can set different on-net and off-net prices.

When through separate regulation access charges are set equal to cost, it is immediate from the preceding observations that uniform pricing achieves the highest feasible welfare. Instead, from Proposition 1 we know that with price discrimination the optimal on-net price is always set inefficiently high, while the optimal off-net price is set inefficiently low when termination is set at cost. Thus the first best cannot be achieved under price discrimination with a non-uniform calling pattern.

It is more interesting to compare consumer surplus in the two cases. Denote the net surplus achieved per on-net call by

$$w_{ii} = v(p_{ii}) + q(p_{ii})(p_{ii} - c),$$

which thus abstracts from the fixed components u_0 and f, as well as from transportation costs. In addition, denote by $w^{\max}$ the maximum feasible surplus under uniform pricing, which is obtained when $m = 0$.

While price discrimination clearly reduces welfare, its impact on consumer surplus is more subtle. As we repeatedly observed, through creating tariff-induced network external-

[22] Recall that at symmetric market shares, which prevail in equilibrium, we have that $r_{ij} = (p_{ij} - c)q_{ij}$ does not directly depend on a.

ities, a difference between the price of on-net calls and that of off-net calls also affects the degree of competition. (In fact, recall from Proposition 2 that it affects *both* the costs and benefits of acquiring additional consumers.) Using expression (22) for consumer surplus under price discrimination, we obtain that it is strictly higher than maximal consumer surplus under uniform pricing when

$$\left(\frac{\mu}{2} - \hat{\mu}\right)(v_{ij} - v_{ii}) > w^{\max} - w_{ii}. \tag{26}$$

With uniform calling patterns, both the LHS and the RHS of the above expression are equal to zero when access is regulated at cost. Thus price discrimination and price uniformity lead to the same result both for welfare and for consumer surplus. This is no longer true for non-uniform calling patterns. At $m = 0$ we have from $p_{ij} < p_{ii}$ for $\mu < 1$ that $v_{ij} > v_{ii}$, i.e., a *necessary* condition for discrimination to improve consumer surplus is thus that $\hat{\mu} < \mu/2$. Interestingly, recall that this was also the condition for the equilibrium access charge a^* to exceed the welfare maximizing access charge a^W. While generally a higher concentration of calling patterns has thus an ambiguous effect on this condition, in our example $\hat{\mu} < \mu/2$ holds if and only if calling patterns are sufficiently concentrated (cf. condition (24)).

Proposition 7 *Suppose that the access charge is set equal to cost: $a = c_T$. Then a ban on price discrimination between on-net and off-net calls increases total welfare. However, it can decrease consumer surplus when condition (26) holds, for which it is a necessary requirement that $\hat{\mu} < \mu/2$.*

7 Concluding Remarks

We introduce a flexible model of network competition with non-uniform calling patterns. The model allows us to analyze the implications of non-uniform calling patterns ("calling circles") on equilibrium outcomes as well as profit-maximizing reciprocal access charges.

We show how equilibrium tariffs (on- and off-net call prices, and fixed fees) vary depending on the calling pattern. Concentrated calling patterns can also help to explain the (on-net to off-net) imbalance ratios that are observed in practice. Our main focus, however, is to analyze when networks would choose reciprocal access charges above cost or below cost in order to dampen competition. With uniform calling patterns, it is known that this is achieved through setting access charges below cost. We show that this result is reversed if calling patterns are sufficiently concentrated: Profit-maximizing access charges are set above cost, which sustains high off-net prices. Our results on above-cost access charges also imply that, contrary to other results in the literature, at the profit-maximizing reciprocal access charge on-net prices can be below off-net prices. We analyze how these different results are obtained from the interaction of two effects: Competition is dampened either when it becomes relatively more expensive to capture the marginal customer or when having the marginal customer is less profitable. We show how the strength of either effect changes when calling patterns become more concentrated.

Furthermore, information obtained on the concentration of calling patterns should inform optimal regulation. We explore this issue with respect to both access charge regulation and a prohibition of discriminating between on-net and off-net calls. In particular, we derive conditions for when the welfare maximizing access charge is strictly lower than the one prevailing in an unregulated market, and when an obligation of uniform pricing can increase welfare and/or consumer surplus.

As in much of the literature on network competition, we restricted consideration to a model with only two networks. This has the additional benefit of making our findings comparable to extant results. Also, we are able to offer a simple definition of our concept of more concentrated calling patterns. A consumer is identified by his ("brand") preference over networks, which for two networks we chose to model in a familiar Hotelling fashion, and by his preference to call particular other consumers. These two preferences are, in the case of increasingly more concentrated calling patterns, ordered so that consumers prefer to call those with similar brand preferences. We motivated this, for instance, by the brand positioning that different networks choose, according to targeted consumer groups, as well as by networks' regional coverage.

Our modelling ideas readily extend to more general settings involving, in particular, more than two competing networks. The advantage of the Hotelling setting, where two-dimensional brand preferences are folded into a single dimension, can be carried over if it is assumed that consumers' preference space follows the generalized Hotelling model of Hoernig (2010) and that a consumer's "club calls" are limited to his own Hotelling line. More generally, in the spirit of discrete choice models, each consumer could be identified, first, by a vector of (potentially independently drawn) brand preferences for each network and, second, by a calling distribution function that would assign "more-than-uniform probability" to consumers whose preferences are closer to his own preferences. We leave a generalization in this direction to future work.

8 Appendix: Omitted Proofs

Proof of Proposition 1. Given constant market shares, network 1's marginal customer is determined by the condition $V_1(\hat{x},\hat{x}) = \overline{V}$, which can be restated as $F_1 = v_1(\hat{x},\hat{x}) + u_0 - \tau\hat{x} - \overline{V}$. Substituting $\hat{L}_{11}(\hat{x}) = \hat{x}G(\hat{x}|\hat{x})$ and $\hat{L}_{12}(\hat{x}) = \hat{x}(1 - G(\hat{x}|\hat{x}))$ into network 1's profits leads to

$$\begin{aligned}\overline{\Pi}_1(\hat{x}) &= \hat{L}_{11}(\hat{x})v(p_{11}) + L_{11}(\hat{x})(p_{11} - c)q(p_{11}) \\ &\quad + \hat{L}_{12}(\hat{x})v(p_{12}) + L_{12}(\hat{x})(p_{12} - c - m)q(p_{12}) + const.,\end{aligned}$$

where the last term on the right-hand side does not depend on p_{11} or p_{12}. We obtain from the maximization of the relevant terms with respect to p_{11} the first-order condition

$$(p_{11} - c)q'(p_{11}) + \left(1 - \frac{\hat{L}_{11}(\hat{x})}{L_{11}(\hat{x})}\right) q(p_{11}) = 0,$$

which solves for the expression (4) presented in the proposition. The result for the off-net price is derived similarly.

Finally, we state a sufficient condition that allows us to ignore the participation constraint of all subscribers with location $x < \hat{x}$, i.e., when indeed, as presumed in the relaxed program, $V_1(x,\hat{x}) \geq V_2(x,\hat{x})$ for all $x \leq \hat{x}$. We have

$$V_1(x,\hat{x}) - V_2(x,\hat{x}) = [v_1(x,\hat{x}) - v_2(x,\hat{x})] + \tau(1-2x) - [F_1 - F_2],$$

where

$$\begin{aligned}v_1(x,\hat{x}) - v_2(x,\hat{x}) &= \{G(\hat{x}|x)v(p_{11}) + [1 - G(\hat{x}|x)]\,v(p_{12})\} \\ &\quad - \{[1 - G(\hat{x}|x)]\,v(p_{22}) + G(\hat{x}|x)v(p_{21})\}.\end{aligned}$$

A sufficient condition for $V_1(x,\hat{x}) \geq V_2(x,\hat{x})$ holding for all $x < \hat{x}$ is that

$$\frac{\partial}{\partial x}[V_1(x,\hat{x}) - V_2(x,\hat{x})] \leq 0,$$

which is equivalent to

$$\frac{\partial G(\hat{x}|x)}{\partial x}\left[v(p_{11}) - v(p_{12}) + v(p_{22}) - v(p_{21})\right] \leq 2\tau. \tag{27}$$

While condition (27) is not stated in terms of the primitives alone, as the terms $v(\cdot)$ depend on the respective prices, note that τ does not enter call prices. Hence, holding all else constant, we can always choose the degree of horizontal differentiation τ (and jointly the fixed utility from participation u_0) large enough, ensuring that all consumers participate and that (27) holds everywhere (cf. also Laffont et al., 1998b). (Note that this uses also that over the compact support $[0,1]^2$ the continuous function $\partial G(\hat{x}|x)/\partial x$ is bounded.) **Q.E.D.**

Proof of Proposition 2. Since calls are balanced with $L_{12}(\hat{x}) = L_{21}(\hat{x})$, and framing the first-order condition in terms of the market share rather than the fixed fee, we have

$$\frac{d\overline{\Pi}_1(\hat{x})}{d\hat{x}} = F_1 - f + \hat{x}\frac{dF_1}{d\hat{x}} + L'_{11}(\hat{x})r_{11} + L'_{12}(\hat{x})r_{12} = 0.$$

Note next that $L'_{11}(\hat{x}) + L'_{12}(\hat{x}) = 1$, which yields the first-order condition at a symmetric equilibrium candidate

$$F^* - f + r_{ii} + \frac{1}{2}\left.\frac{dF_1}{d\hat{x}}\right|_{\hat{x}=1/2} + L'_{ij}(1/2)(r_{ij} - r_{ii}) = 0.$$

Using $L'_{ij}(1/2) = 0$ and substituting finally for $dF_1/d\hat{x}$ from (11) yields expression (13).

To obtain equilibrium profits, in a symmetric equilibrium, and after substituting for F^* from (13), profits (3) become

$$\begin{aligned}\bar{\Pi}^* &= \frac{1}{2}\left(F^* - f + r_{ii}\right) + L_{ij}(1/2)\left(r_{ij} - r_{ii}\right) \\ &= \frac{1}{2}\left[\tau - \hat{\mu}(v_{ii} - v_{ij})\right] + \frac{\mu}{4}(r_{ij} - r_{ii}).\end{aligned}$$

Q.E.D.

Proof of Proposition 3. When differentiating profits in (14) w.r.t. m, note first that at a symmetric equilibrium $\hat{x} = 1/2$ does not change. Moreover, from Proposition 1 only off-net but not on-net prices change with m. Note further that $dp_{ij}/dm > 0$ and that profits do not directly depend on m. Therefore we can equivalently maximize $\bar{\Pi}^*$ over p_{ij}.

We obtain[23]

$$2\frac{d\bar{\Pi}^*}{dp_{ij}} = -\hat{\mu}q_{ij} + \frac{\mu}{2}\left[q_{ij} + (p_{ij} - c)\, q'_{ij}\right]$$
$$= \left[\frac{\mu}{2}\left(1 - \frac{p_{ij} - c}{p_{ij}}\eta(p_{ij})\right) - \hat{\mu}\right] q_{ij} = 0,$$

or

$$\frac{p_{ij} - c}{p_{ij}} = \frac{1}{\eta(p_{ij})}\left(1 - \frac{2\hat{\mu}}{\mu}\right). \tag{28}$$

This condition can be solved for p_{ij} as

$$p_{ij} = c\frac{\mu\eta(p_{ij})}{\mu\left(\eta(p_{ij}) - 1\right) + 2\hat{\mu}}.$$

On the other hand, from (8) we obtain

$$p_{ij} = (c + m)\,\frac{\mu\eta(p_{ij})}{\mu\left(\eta(p_{ij}) - 1\right) + 1}. \tag{29}$$

Equating and solving for m leads to the result stated in the Proposition. **Q.E.D.**

Proof of Proposition 4. Equating p_{ij} in (28) and p_{ii} from (8) leads to $1 - 2\hat{\mu}/\mu = (1-\mu)/(2-\mu)$, or $\hat{\mu} = \mu/(4 - 2\mu)$. For larger $\hat{\mu}$ we will have $p_{ij} < p_{ii}$, due to (28). **Q.E.D.**

Proof of Proposition 5. Substituting for CS and concentrating only on the off-net elements, we obtain

$$W = 2\bar{\Pi}^* + CS = \frac{\mu}{2}\left(r_{ij} + v_{ij}\right) + const. \tag{30}$$

Thus the socially optimal off-net price continues to be $p_{ij} = c$ even for general calling patterns, as should be expected. Equating to (29) and solving for a leads to the result in the text. The profit-maximizing access charge is equal to the socially optimal one if and only if the relative weights in the objective functions (14) and (30) on r_{ij} and v_{ij} are equal, or if $\hat{\mu} = \mu/2$. **Q.E.D.**

[23] It is easy to show that the sufficient second-order condition for a strict local maximum holds for a constant demand elasticity, which implies also that profits are quasi-concave in p_{ij}.

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