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Income Effects and the Cyclicality of Job Search Effort*

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Abstract

The canonical matching model predicts procyclical job search effort while recent evidence suggests that it is not. In this paper, we assess whether accounting for income effects in a model with matching frictions reconciles it with the evidence. We find that: (i) low income effects imply *procyclical search effort*; (ii) moderate income effects imply *acyclical search effort* but also *acyclical unemployment*; and (iii) high income effects imply *countercyclical search effort* but also *procyclical unemployment*. In our experiments, only fully rigid wages or excessively high replacement rate of unemployment benefits improves the predictions of the model. In short, the predictions of the model with income effects are sound under unsound assumptions.

JEL classification: E24; E32; J64.
Keywords: Matching Frictions; Job Search Effort; Income Effects.

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1. Introduction

Matching models are popular in modern macro labor analysis because they provide a compelling story and qualitatively accurate predictions for key labor market variables. Unemployment and vacancies are among these key variables, but job search effort is not. The canonical matching model predicts that job search effort is procyclical. Yet, most of the recent direct evidence on the cyclicality of job search effort suggests that it is either acyclical or countercyclical (DeLoach and Kurt, 2013; Leyva, 2017; Mukoyama, Patterson and Şahin, 2018; Shimer, 2004).[1]

Using US data from the American Time Use Survey (ATUS), DeLoach and Kurt (2013) and Leyva (2017) conclude that job search effort is acyclical due to two counteracting forces. In both studies, procyclical job-finding probability contributes to procyclical job search effort. This effect, however, is muted by wealth effects in DeLoach and Kurt and by a countercyclical value of a job in Leyva. Studying US data from the Current Population Survey, Shimer (2004) treats the number of job search methods as a proxy for job search effort. He concludes that there is no evidence of procyclical job search effort. Mukoyama, Patterson and Şahin (2018) extend the analysis of Shimer by combining the information from the CPS and the ATUS data and document that job search effort is countercyclical.

The cyclicality of job search effort is crucial for labor market dynamics. Because the job-filling rate increases in search effort, procyclical search effort amplifies the volatility of unemployment and vacancies, while countercyclical search effort dampens it. That is why, if search effort was counterfactually procyclical, it would help to overcome Shimer's (2005) critique (Gomme and Lkhagvasuren, 2015); and why Mukoyama, Patterson and Şahin (2018) conclude that if search effort was not countercyclical, we would observe a higher unemployment rate during the great recession. Thus, it is important to understand why job search effort is acyclical or countercyclical and how it links to other key labor market variables.

In this paper, we assess whether a dynamic general equilibrium model featuring matching frictions may predict acyclical or countercyclical search effort if it accounts for income effects. Income effects and, more broadly, wealth effects are important drivers of job search effort as suggested by the empirical literature. Faberman and Kudlyak (2016) find that those with the lowest returns to search tend to exert more job search effort. They argue that their evidence suggests that income effects dominate substitution effects in job search effort. Leyva (2017) argues that his evidence of countercyclical value of a job may be explained by the presence of income effects. Both

[1] An exception is Gomme and Lkhagvasuren (2015). Other studies report indirect evidence pointing to countercyclical job search effort. Ahn and Shao (2017) report that job search effort of *employed* workers is countercyclical. Faberman and Kudlyak (2016) find that workers tend to search less in areas with tighter labor markets, which suggests that job search effort is countercyclical. Hornstein and Kudlyak (2016) conclude that if the elasticity of the matching function with respect to vacancies is in the range of one-third to one, then job search effort is countercyclical.

DeLoach and Kurt (2013) and Mukoyama, Patterson and Şahin (2018) document that search effort is negatively correlated with wealth, suggesting that wealth plays a role in the dynamics of search effort.

We introduce income effects into the model by deviating from the assumption of risk-neutral household used in the canonical matching model. We employ the constant relative risk aversion utility specification and control the magnitude of income effects by controlling the households' attitude towards risk. A higher degree of risk aversion implies that the marginal utility of consumption falls at a higher rate, inducing stronger income effects. In expansions, higher income allows households to enjoy more consumption. In the presence of income effects, exerting more search effort to increase the job-finding probability becomes less desirable because of the diminishing marginal utility of consumption. As a result, the household may lower search effort to enjoy more leisure –which is valued higher in terms of goods. Both if and the extent to which search effort drops in expansions and increases in recessions depend on the magnitude of income effects.

For the remaining building blocks of the model, we draw on standard assumptions in the macro labor literature: (i) a Cobb-Douglas matching function determines the number of matches, (ii) Nash bargaining determines wages, and (iii) workers share consumption within households (Merz, 1995).

We find that the cyclical behavior of search effort depends on the magnitude of income effects. If income effects are absent or low, search effort is procyclical, at odds with the evidence. On the contrary, if income effects are moderate or high, search effort is, respectively, acyclical or countercyclical, consistent with the evidence. Yet, income effects also alter the behavior of other variables. If income effects are moderate, both unemployment and labor market tightness are acyclical. In this case, Shimer's (2005) critique is acute; and neither Hagedorn and Manovskii's (2008) calibration strategy nor Pissarides's (2009) fixed-matching-costs technique can overcome Shimer's critique. If income effects are high, unemployment is procyclical and vacancies are countercyclical, at odds with conventional wisdom and data.

Concisely, in our model, income effects improve the predicted cyclical behavior of search effort but worsen that of unemployment and vacancies. Mukoyama, Patterson and Şahin (2018) show that countercyclical search effort reduces the volatility of unemployment by amplifying the congestion externalities of matching frictions.[2] In our model, this channel is also at work; however, most of income effects propagate though the wage: a version of our model with *exogenous* search effort also predicts acyclical or procyclical unemployment (depending on the magnitude of income effects). Because of income effects, in expansions, workers demand higher wages to forgo their leisure, severely undermining job creation. In other words, income effects make the opportu-

[2] Mukoyama, Patterson and Şahin (2018) build a model with a generalized matching function and endogenous job search effort. They show that if the elasticity of job search effort with respect to labor market tightness is negative, the volatilities drop.

nity cost of working procyclical. And Chodorow-Reich and Karabarbounis (2016) show that a model with procyclical opportunity cost of working (as found in data) generates lower unemployment volatility relative to a model with acyclical opportunity cost.

If we depart from Nash bargaining and assume fully rigid wages, we close the channel through which income effects influence wages. In this case, income effects imply countercyclical search effort and strongly countercyclical unemployment, consistent with the evidence. But if we slightly deviate from full wage rigidity, the model's predictions are close to those under Nash bargaining: fully rigid wages are required to reconcile our model with data, which is implausible given data on the cyclicality of the wages of newly hired workers (Pissarides, 2009). Among several other robustness checks, only excessively high risk aversion and replacement rate of unemployment benefits improve the predictions of the model. In our experiments, thus, adding income effects to the model does not generate the cyclical patterns observed in the data unless we make unsound assumptions.

In the next section, we present the model. The model is rather standard in the macro labor literature and features matching frictions and endogenous search effort. In Section 3., we present our calibration strategy. In Section 4., using the results of our numerical simulations, we evaluate the roles of income effects and of endogenous search effort. In the same section, we study the robustness of our results to alternative calibrations and specifications of the model. In Section 5., we offer some concluding remarks.

2. The Model

Time is discrete and denoted by t. There is a representative household and a representative firm. The representative household consists of employed and unemployed workers. Unemployed workers exert effort in searching for jobs, and when matched with a vacant job, they start working in the following period. Each period, a constant fraction of jobs, denoted by δ, are exogenously destroyed. The law of motion of employment is

$$n_{t+1} = (1-\delta)n_t + m_t, \tag{1}$$

where n_t is the employment rate and m_t is new matches. Vacancies, v_t, and total search effort, $\bar{e}_t u_t$, determine new matches via a matching function

$$m_t = \sigma v_t^{\eta}(\bar{e}_t u_t)^{1-\eta}, \tag{2}$$

where σ is a scale parameter and $0 < \eta < 1$ is the elasticity of the matching function with respect to vacancies. Total search effort is the product of the average job search effort, $\bar{e}_t$, and the unemployment rate, $u_t \equiv 1 - n_t$.

We define the labor market tightness as the vacancy-unemployment ratio

$$\theta_t \equiv \frac{v_t}{u_t}. \tag{3}$$

We write the job-finding probability per unit of search effort as

$$f(\bar{e}_t, \theta_t) = \sigma \left(\frac{\theta_t}{\bar{e}_t} \right)^{\eta}, \tag{4}$$

and the job-filling probability as

$$\mu(\bar{e}_t, \theta_t) = \sigma \left(\frac{\theta_t}{\bar{e}_t} \right)^{\eta - 1}. \tag{5}$$

2.1. The Household

The household derives utility from consumption and disutility from exerting effort in job search and from working in firms.[3] It pools consumption and discounts future utility by $0 < \beta < 1$. In maximizing the lifetime utility, the household takes the employment rate, the job-finding probability per unit of search effort, dividend income, d_t, and the wage, w_t, as given and chooses the paths of consumption, c_t, and job search effort, e_t. The optimization problem is

$$V_t = \max_{\{c_t, e_t\}} \left[\frac{c_t^{1-\gamma} - 1}{1 - \gamma} - \frac{\psi}{\zeta} e_t^{\zeta} (1 - n_t) - \chi n_t + \beta \mathrm{E}_t V_{t+1} \right],$$

subject to

$$c_t \leq w_t n_t + d_t,$$
$$n_{t+1} \leq (1 - \delta) n_t + e_t f(\bar{e}_t, \theta_t)(1 - n_t).$$

V_t is the lifetime utility of the household. The parameters ψ and $\zeta > 1$ measure the scale and the convexity of the disutility from search effort, respectively, and χ scales the disutility from working. The parameter $\gamma \geq 0$ is the coefficient of relative risk aversion and measures the curvature of the utility of consumption.

The first-order condition for search effort is

$$\psi e_t^{\zeta - 1} = \beta f(\bar{e}_t, \theta_t) \mathrm{E}_t V_{n,t+1}, \tag{6}$$

where $V_{n,t}$ is the value of an additional employed worker to the household. It is given by

$$V_{n,t} = w_t c_t^{-\gamma} + \frac{\psi}{\zeta} e_t^{\zeta} - \chi + (1 - \delta - e_t f(\bar{e}_t, \theta_t)) \beta \mathrm{E}_t V_{n,t+1}. \tag{7}$$

Equation (6) equates the marginal disutility from exerting effort to the expected discounted value of an additional employed worker to the household multiplied by the

[3] We follow Andolfatto (1996) and Merz (1995) by assuming working generates disutility because of forgone leisure. An equally valid assumption could be that workers also forgo unemployment benefits when they start working. We consider this case in our robustness checks in Section 4.3.

probability the worker finds a job. The value of an additional employed worker to the household is the sum of the continuation value of $V_{n,t}$ and three components: wage income of a worker valued at the marginal utility, the increase in the utility (because the worker stops searching), and the decrease in the utility (because the worker starts working).

If $\gamma > 0$, we deviate from the risk-neutrality assumption and introduce income effects into the model. In this case, income affects both the level and the marginal utility of consumption. In good times, due to the decreasing marginal utility of consumption, *ceteris paribus*, the household increases leisure by decreasing search effort. Thus, income effects render a negative correlation between consumption and effort possible.

2.2. The Firm

The firm produces a homogeneous good with the technology

$$y_t = z_t n_t, \tag{8}$$

where y_t is output and z_t is the productivity. The firm pays κ per vacancy. It takes the employment rate, the job-filling probability, and the wage as given and chooses the number of vacancies to maximize its value, J_t. The optimization problem is

$$J_t = \max_{v_t} \left(z_t n_t - w_t n_t - \kappa v_t + \beta \mathrm{E}_t \left[(c_{t+1}/c_t)^{-\gamma} J_{t+1} \right] \right),$$

subject to

$$n_{t+1} = (1-\delta) n_t + \mu(\bar{e}_t, \theta_t) v_t.$$

The first order condition for vacancies is

$$\beta \mu(\bar{e}_t, \theta_t) \mathrm{E}_t \left[(c_{t+1}/c_t)^{-\gamma} J_{n,t+1} \right] = \kappa, \tag{9}$$

where J_n is the value of an additional worker to the firm. It is given by

$$J_{n,t} = z_t - w_t + (1-\delta)\beta \mathrm{E}_t \left[(c_{t+1}/c_t)^{-\gamma} J_{n,t+1} \right]. \tag{10}$$

Equation (9) states that firms open vacancies until the increase in the continuation value of the firm equals the cost of opening vacancies. The value of an additional worker to the firm is the sum of output per worker net of the wage and the continuation value of $J_{n,t}$.

2.3. The Wage

Workers and firms bargain over wages such that the bargained wage maximizes the Nash product. Namely,

$$w_t = \arg\max \left(V_{n,t}/c_t^{-\gamma}\right)^{\phi} J_{n,t}{}^{1-\phi}, \tag{11}$$

where the parameter $0 < \phi < 1$ measures the worker's bargaining power. The equilibrium wage is

$$w_t = \phi \left(z_t + e_t f(\bar{e}_t, \theta_t) \frac{\kappa}{\mu(\bar{e}_t, \theta_t)} \right) + (1 - \phi) \left(\chi - \frac{\psi}{\zeta} e_t^{\zeta} \right) \frac{1}{c_t^{-\gamma}}. \tag{12}$$

Thus, the wage increases with (i) the productivity of the match; (ii) the hiring costs because by accepting the match, the worker is providing savings for the firm; and (iii) the net disutility of working measured in goods terms.

2.4. Equilibrium

In equilibrium, each worker chooses the same job search effort and, thus, $e_t = \bar{e}_t$. To close the model, we write the resource constraint

$$y_t = c_t + \kappa v_t, \tag{13}$$

which states that output equals the sum of consumption and the costs of job creation.

3. Calibration

Table 1 summarizes our calibration choices. We calibrate the model to monthly data. In the benchmark calibration, we target an annual discount rate of 4.91%, implying that $\beta = 0.996$. Drawing on Shimer's (2012) measurement, we set the employment exit probability, δ, to 3.6%. Drawing on Petrongolo and Pissarides (2001), we set $\eta = 0.5$. We also set $\phi = 0.5$. We fix $\zeta = 2$, implying a quadratic disutility for job search effort (Gomme and Lkhagvasuren, 2015; Yashiv, 2000). Because the value of γ is central to our analysis, we assume a variety of values ranging from zero to four for γ.[4]

We pin down four parameters χ, ψ, σ, and κ using four steady-state targets. By dropping the time subscript t from variables to denote the steady-state values, Table 2 summarizes the four steady-state targets. Namely, we target (i) an unemployment rate of 5.7%, (ii) a search effort value of 1, (iii) a tightness value of 0.72 (Pissarides, 2009), and (iv) a ratio of total hiring costs to output of 1% (Christiano, Eichenbaum and Trabandt, 2016).[5]

[4]In one experiment in Section 4.3.4., we increase the upper bound of γ to 20.

[5]Our calibration implies that in steady state working gives higher disutility than searching, i.e., $\chi > \psi e^{\zeta}/\zeta$.

Table 1: Benchmark Calibration

Discount factor:	$\beta = 0.996$
Rate of job destruction:	$\delta = 0.036$
Matching function elasticity:	$\eta = 0.5$
Workers' bargaining power:	$\phi = 0.5$
Convexity of search effort disutility:	$\zeta = 2$
Relative risk aversion:	$\gamma \in [0, 4]$
Autocorrelation of productivity:	0.98
Standard deviation of productivity shock:	0.005

Table 2: Imposed Steady-State

Employment:	$n = 0.943$
Labor market tightness:	$\theta = 0.72$
Job search effort per worker:	$e = 1$
Total matching costs:	$\kappa v = 0.01y$

We study shocks to aggregate productivity, measured by z_t. For that, we assume that the aggregate productivity follows a first-order autoregressive process with a correlation coefficient of 0.98 and a standard deviation of the innovations of 0.005 (Shimer, 2010).

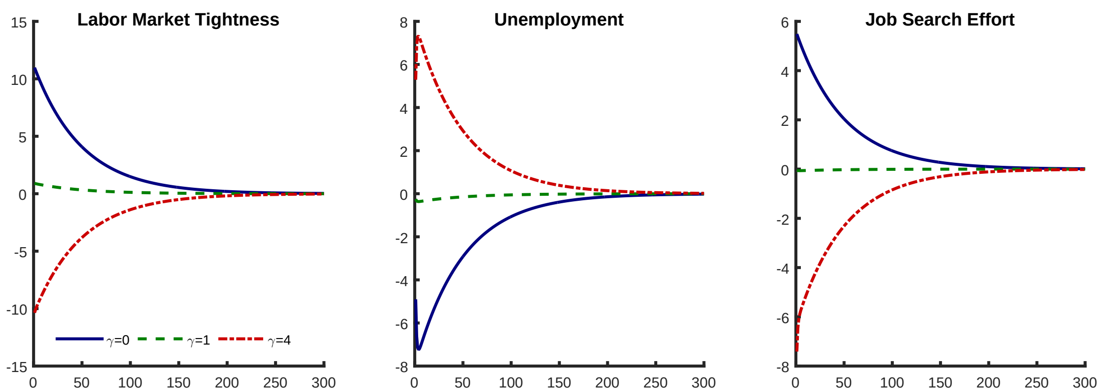
4. Results

Throughout this section, we use the log-linear version of the model to study how income effects alter the responses of key labor market variables to a positive productivity shock. We focus on the volatility and cyclicality of labor market tightness, unemployment, and job search effort. As measures of volatility and cyclicality, we use the magnitude and the sign of (cumulative) responses.

4.1. The Role of Income Effects

Figure 1 plots the responses of labor market tightness, unemployment, and job search effort to a positive productivity shock for three different levels of risk aversion, γ. First, as in the canonical matching model, we assume linear utility ($\gamma = 0$) and, thus, ignore income effects. In this case, the model predicts procyclical search effort, opposing the evidence reported in DeLoach and Kurt (2013), Leyva (2017), Mukoyama, Patterson and Şahin (2018), and Shimer (2004). The model also predicts procyclical tightness and countercyclical unemployment.

Figure 1: Impulse Response Functions



Note: The figure plots the impulse response functions. The horizontal axis measures time in months. The vertical axis measures the logarithmic/percentage deviation from the steady state. The impulse is a 1% increase in productivity. Solid lines represent the responses in the case of $\gamma = 0$, the dashed lines represent the responses in the case of $\gamma = 1$, and the dot-dashed lines represent the responses in the case of $\gamma = 4$.

In the model, after log-linearization, search effort is proportional to the difference between labor market tightness and the marginal utility of consumption:

$$\zeta \hat{e}_t = \hat{\theta}_t - \gamma \hat{c}_t, \tag{14}$$

where a hat denotes the log-deviation from the steady state.[6] Therefore, in the absence of income effects, $\gamma = 0$, procyclical labor market tightness entails procyclical search effort.

Next, as is standard in many macroeconomics models, we assume log-utility ($\gamma = 1$) and, thus, introduce somewhat moderate income effects. In this case, the model predicts acyclical search effort. The reason for acyclical search effort is particularly congruent with the evidence in DeLoach and Kurt (2013). In DeLoach and Kurt, job search effort is acyclical due to two counteracting forces: procyclical job finding probability and wealth effects. In the model, a positive productivity shock increases labor income and consumption thanks to higher wages and employment. Assuming moderate income effects ($\gamma = 1$), the fall in the marginal utility of consumption neutralizes the effect of higher job-finding probability, which generates acyclical job search effort (see equation (14)). Yet, the model also predicts acyclical labor market tightness and unemployment. Shimer (2005) shows that the volatilities of unemployment and labor market tightness in matching models fall short of their empirical counterparts. Income effects exacerbate this volatility problem.

[6]See Appendix A1. for the derivation of equation (14).

To see the consequences of strong income effects, we then consider an arbitrarily large coefficient of relative risk aversion, $\gamma = 4$. In this case, the model has an aberrant behavior. It predicts countercyclical search effort, in line with the evidence in Mukoyama, Patterson and Şahin (2018). But it also predicts procyclical unemployment and countercyclical tightness, at odds with the evidence and conventional wisdom. To see the reason for procyclical unemployment and countercyclical tightness note that the equilibrium wage,

$$\hat{w}_t = \frac{\phi}{w}\left(z\hat{z}_t + \frac{(\zeta - 1)\kappa\theta}{\zeta}\hat{\theta}_t\right) + \gamma\frac{(1-\phi)\chi c^{\gamma}}{w}\hat{c}_t, \tag{15}$$

is increasing in income effects.[7] A positive productivity shock shifts wages upwards, thereby increasing labor income and consumption. If $\gamma = 0$, the increase in consumption has no effect on wages. If $\gamma > 0$, however, the marginal utility of consumption decreases with the level of consumption. Accordingly, the worker demands higher wages to trade leisure for consumption. In the case of strong income effects ($\gamma = 4$), the upward pressure in wages is so high that firms decrease vacancies. It follows that the labor market tightness becomes countercyclical and unemployment procyclical.

The magnitude of income effects determines the behavior of the labor market. To illustrate this claim, in Figure 2, we plot the impact of γ on the 8-year-cumulative responses of θ_t, u_t, and e_t to a positive productivity shock. Low income effects (low γ) imply higher volatility, but also procyclical search effort. A small interval of γ implies qualitatively-reasonable but also dull responses. High income effects (high γ) imply countercyclical job search effort, but also qualitatively-unreasonable responses of unemployment and tightness. Concisely, in our model, income effects improve the predicted cyclical behavior of search effort but worsen that of unemployment and vacancies.

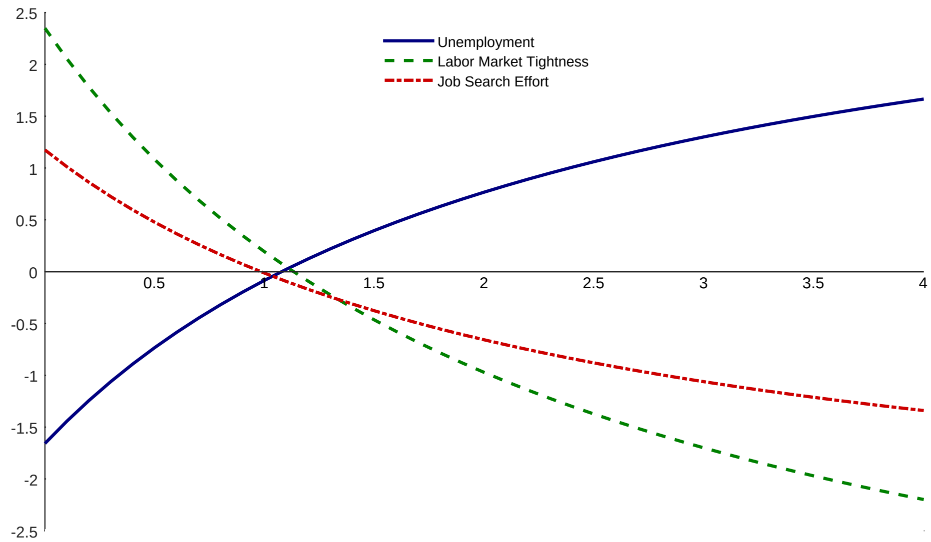
4.2. The Role of Search Effort

Because endogenous search effort is not a standard feature of matching models, we now consider a variant of the model with exogenous search effort.[8] Figure 3 plots the 8-year-cumulative responses of unemployment and labor market tightness as functions of γ in the models with endogenous and exogenous search effort. As before, the impulse is a positive productivity shock. The responses in both models coincide in the neighborhood of $\gamma = 1$, i.e., in models assuming log-utility, search effort plays an insignificant role. In the case of linear utility, the response of unemployment triples and the response of labor market tightness doubles by endogenizing search effort. As γ increases, unemployment and tightness first become less cyclical and then become

[7] Equation (15) is in log-linear form. Recall that variables without a time subscript denote values in the steady state.

[8] Appendix B1. outlines the model with exogenous search effort.

Figure 2: The Role of Income Effects



Note: The horizontal axis measures the degree of income effects, $\gamma \in [0, 4]$. The vertical axis measures the 8-year cumulative responses. The solid line represents unemployment; the dashed line represents labor market tightness; and the dot-dashed line represents job search effort.

qualitatively-inconsistent with data. Consequently, the finding in Gomme and Lkhagvasuren (2015) that endogenous search effort amplifies the responses of labor market variables only applies to the case of weak income effects.

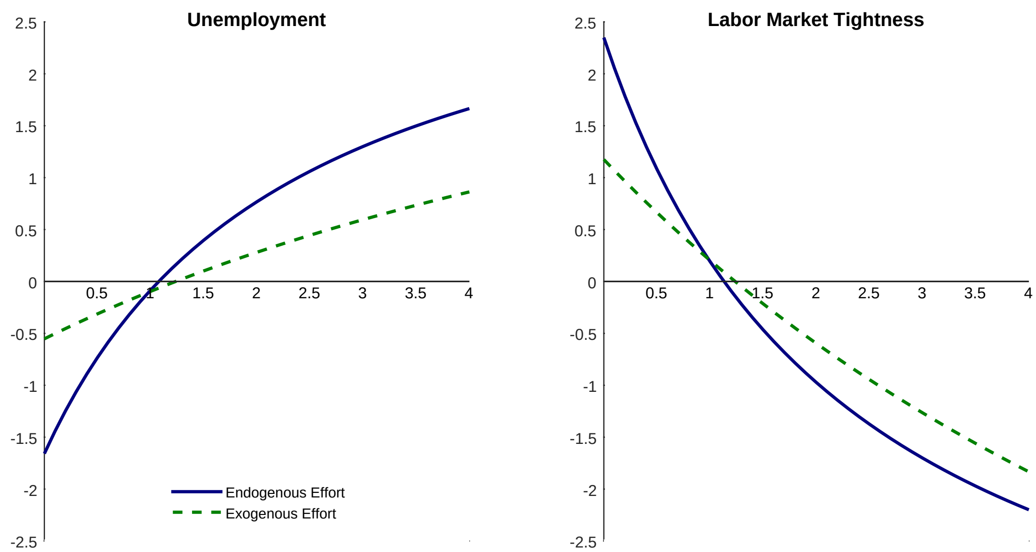
Figure 3 highlights another important finding. The counterfactual results – that unemployment (tightness) is procyclical (countercyclical)– are not induced by endogenous search effort but by income effects. The reason is that the wage still positively depends on income effects even if search effort is exogenous.[9] Therefore, if γ is large enough, firms open less vacancies when productivity rises. Endogenous search effort just slightly reduces the threshold value of γ that generates procyclical unemployment and countercyclical labor market tightness. Intuitively, and as shown by Mukoyama, Patterson and Şahin (2018), countercyclical search effort implies that unemployed workers search less in good times. This causes the job-filling probability to fall; as a result, firms lower vacancies.

4.3. Robustness Checks

We conduct several experiments to assess the robustness of our results. Recall that search effort is acyclical only if Shimer's (2005) critique is rather acute. For this reason,

[9]In the case of exogenous search effort, the wage is given by $\hat{w}_t = \phi \left[z\hat{z}_t + \kappa\theta\hat{\theta}_t \right] / w + \gamma(1-\phi)\chi c^{\gamma}\hat{c}_t / w.$

Figure 3: Endogenous vs Exogenous Search Effort



Note: The horizontal axis measures the degree of income effects, $\gamma \in [0,4]$. The vertical axis measures the 8-year cumulative responses. The solid (dashed) lines represent the responses in the model with endogenous (exogenous) search effort.

we start by experimenting with solutions shown in the literature that help to overcome Shimer's critique. In particular, we consider fixed matching costs a la Pissarides (2009), the alternative calibration proposal of Hagedorn and Manovskii (2008), and wage rigidity.[10] Then, we study the impact of introducing unemployment benefits and capital into the model. As a summary statistic, we use the 8-year cumulative responses of θ_t, u_t, and e_t to a positive productivity shock. Table 3 summarizes the results of each experiment for three levels of relative risk aversion: $\gamma = \{0, 1, 4\}$.

4.3.1. Fixed Matching Costs

Pissarides (2009) shows that the addition of fixed matching costs raises the elasticity of labor market tightness. We embed this idea into our model by rewriting the value of the firm, J_t, as

$$J_t = \max_{v_t} \left(z_t n_t - w_t n_t - (\kappa + \mu(\bar{e}_t, \theta_t) H) v_t + \beta \mathrm{E}_t \left[(c_{t+1}/c_t)^{-\gamma} J_{t+1} \right] \right),$$

[10]See Ljungqvist and Sargent (2017) for a detailed analysis of these and other reconfigurations.

Table 3: Sensitivity Analysis

	$\gamma = 0$			$\gamma = 1$			$\gamma = 4$		
	θ	u	e	θ	u	e	θ	u	e
Benchmark	2.35	-1.66	1.17	0.19	-0.08	-0.01	-2.20	1.67	-1.34
$v\mu(e,\theta)H = 0.005y$	2.83	-1.90	1.21	0.25	-0.11	-0.02	-2.35	1.71	-1.27
$v\mu(e,\theta)H = 0.0075y$	3.23	-2.11	1.24	0.30	-0.13	-0.02	-2.46	1.73	-1.23
$\phi = 0.05$	24.39	-17.21	12.19	0.11	-0.02	-0.05	-3.59	2.60	-1.93
$v(\kappa + \mu(e,\theta)H) = 0.002y$	11.75	-8.29	5.87	0.12	-0.04	-0.05	-3.25	2.36	-1.77
$\rho = 0.99$	2.67	-1.81	1.17	0.47	-0.20	-0.05	-2.01	1.61	-1.41
$\rho = 1$	26.65	-13.09	1.17	23.32	-9.76	-2.58	18.81	-5.24	-7.67
$b = 0.4$	2.35	-1.66	1.17	0.98	-0.63	0.37	-1.09	0.92	-0.87
Model with Capital	2.35	-1.66	1.17	0.92	-0.59	0.34	-0.77	0.68	-0.69

Note: The table gives the 8-year cumulative responses of θ_t, u_t, and e_t to a positive productivity shock. The panels only differ in the degree of risk aversion, $\gamma = \{0, 1, 4\}$. The first row presents the results under the benchmark calibration. The second and third rows present the results for the case of Pissarides' fixed matching costs: if $H = 0.14$, fixed matching costs represent 50% of total hiring costs; and if $H = 0.21$, fixed matching costs represent 75% of total hiring costs. The fourth and fifth rows present the results for the case of Hagedorn and Manovskii's calibration strategy: $\phi = 0.05$ implies a very low value of workers' bargaining power; and $\kappa v = 0.005y$ implies an augmented value of leisure. The fifth and sixth rows present the results with wage rigidity. The seventh row presents the results with a replacement rate of unemployment benefits slightly above 40%. The last row presents the results of the model with capital.

in which we assume that firms pay κ per vacancy and H per match.[11] Following Pissarides, to accommodate H, we adjust κ such that we keep the steady-state value of hiring costs, $v(\kappa + \mu(e,\theta)H)$, unchanged.[12] With lower κ and higher H, hiring costs are less responsive to market conditions, thus inducing firms to open more vacancies when productivity rises. We experiment with two values of H. Consistent with Pissarides, we find that fixed matching costs enhances the volatilities of labor market variables in the model. Nevertheless, the model continues to improve the cyclical behavior of job search effort only if it severely deteriorates the cyclical behavior of unemployment and labor market tightness.

[11] Accordingly, the free-entry condition, Eq. (9), is replaced by

$$\beta \mathrm{E}_t \left[(c_{t+1}/c_t)^{-\gamma} J_{n,t+1} \right] \mu(\bar{e}_t, \theta_t) = \kappa + \mu(\bar{e}_t, \theta_t) H;$$

the wage equation, Eq. (11), is replaced by

$$w_t = \phi \left[z_t + e_t f(\bar{e}_t, \theta_t) \left(\frac{\kappa}{\mu(\bar{e}_t, \theta_t)} + H \right) \right] + (1 - \phi) \left(\chi - \frac{\psi}{\zeta} e_t^{\zeta} \right) \frac{1}{c_t^{-\gamma}},$$

where we assume, as Pissarides (2009), that H is sunk at the time of bargaining; and finally the resource constraint, Eq. (13), is replaced by

$$y_t = c_t + (\kappa + \mu(\bar{e}_t, \theta_t) H) v_t.$$

[12] In the way indicated, the steady-state values of labor market tightness and the wage remain the same.

4.3.2. Hagedorn and Manovskii's Calibration Strategy

In the next two experiments, we focus on the workers' bargaining power and on the value of non-market activity. Hagedorn and Manovskii (2008) show that decreasing workers' bargaining power and increasing the value of non-market activities generates volatile labor market variables. By decreasing workers' bargaining power, ϕ, the wage becomes less sensitive to changes in productivity and labor market tightness, rendering labor costs also less sensitive and increasing the incentives to open vacancies. In our model, the value of non-market activities indirectly corresponds to the disutility of working, χ.[13] By increasing χ, the equilibrium value of an additional worker to the household, V_n, falls. Because of Nash bargaining, a lower V_n implies a lower J_n (the equilibrium value of an additional worker to the firm). Therefore, small changes in productivity have large effects on J_n, increasing the incentives to open vacancies.

We find that the logic above only applies to the case of small income effects.[14] Interestingly, in the case of log-utility, $\gamma = 1$, Hagedorn and Manovskii's proposal decreases the volatilities of unemployment and labor market tightness even further. And if income effects are stronger, unemployment is even more procyclical than in the benchmark. The reason is that by reducing ϕ and increasing χ, income effects has a higher weight on wages (recall that the last term on the right hand side of equation (15), the wage equation, is $\gamma(1-\phi)\chi c^{\gamma}\hat{c}_t/w$). If $\gamma = 0$, income effects vanish and, thus, the calibration of Hagedorn and Manovskii generates highly volatile and procyclical unemployment. But if $\gamma > 0$, their calibration increases the weight of income effects in the wage dynamics. Therefore, wages tend to increase more and reduce job creation.

4.3.3. Wage Rigidity

We have shown that income effects play a determinant role in the evolution of wages. If income effects are high, workers demand higher wages to exchange their leisure for additional consumption, thereby undermining the value of job creation. Yet, this follows from Nash bargaining. In this section, we deviate from this assumption and instead assume rigid wages. In particular, as in Shimer (2010), wages are a weighted average of lagged wages and the wage obtained through Nash bargaining, w_t^*:

$$w_t = \rho w_{t-1} + (1-\rho) w_t^*, \tag{16}$$

[13] In our calibration, we use χ to target relative hiring costs, $\kappa v/y$. A lower weight implies a higher χ.

[14] Chodorow-Reich and Karabarbounis (2016) show that the calibration strategy of Hagedorn and Manovskii (2008) only improves the predictions of the canonical matching model because the opportunity cost of working is constant in the model. In our model, income effects imply that the opportunity cost of working is procyclical, consistent with the evidence documented by Chodorow-Reich and Karabarbounis.

where $0 < \rho \leq 1$ governs the degree of rigidity.[15] Fully rigid wages amplify the volatility of unemployment and labor market tightness (as in, for example, Shimer, 2005 and Hall, 2005). We also find that if $\gamma = 1$ or higher, job search effort is countercyclical without compromising the qualitative responses of unemployment and tightness.[16] Thus, if wages are fully rigid, the model is in line with the evidence.

The high volatility of unemployment and vacancies follows from the increase in incentives to open vacancies. If wages are constant, following an increase in productivity, firms do not have to share the increased surplus with workers. The natural response of firms is, thus, to open more vacancies than under flexible wages, leading to lower unemployment. To understand the dynamics of search effort under wage rigidity, recall that in equilibrium the marginal disutility of search effort equals the expected discounted value of an additional employed worker to the household multiplied by the probability the worker finds a job (equation 6). The job-finding probability is highly procyclical. But, in good times, the value of an additional worker to the household tends to decrease because the same wage evaluated in utility terms falls. If income effects are strong enough, the second effect dominates the first.

Hence, rigid wages allows the model with income effects to be reconciled with the data. The drawback is that the model requires wages to be constant ($\rho = 1$). For example, even if $\rho = 0.99$, the results are not much different from the benchmark. As shown in the literature surveyed by Pissarides (2009), the wage of new hires –those that determine unemployment fluctuations in the model– changes almost one to one with productivity, whereas $\rho = 1$ implies the wage of new hires is invariant.

4.3.4. Unemployment Benefits

In our benchmark model, we assume that the opportunity cost of working is only forgone leisure (which is in utility terms). Now we assume that, as in Nakajima (2012) and Chodorow-Reich and Karabarbounis (2016), the opportunity cost of working also includes forgone unemployment benefits (which are in goods terms). To see why this can be relevant for our results, let τ_t denote lump-sum taxes and b denote unemployment benefits.[17] The households' budget constraint is then

$$c_t \leq w_t n_t + d_t + b(1 - n_t) - \tau_t,$$

[15] In this case, the wage equation, Eq. (11), is replaced by

$$w_t^* = \phi \left(z_t + e_t f(\bar{e}_t, \theta_t) \frac{\kappa}{\mu(\bar{e}_t, \theta_t)} \right) + (1 - \phi) \left(\chi - \frac{\psi}{\upsilon} e_t^{\upsilon} \right) \frac{1}{c_t^{-\gamma}} +$$
$$+ (1 - \delta_n - e_t f(\bar{e}_t, \theta_t)) \beta \mathrm{E}_t \left[(c_{t+1}/c_t)^{-\gamma} \left(w_{t+1}^* - w_{t+1} \right) \right].$$

[16] Job search effort is procyclical if $\gamma < 0.26$.

[17] We assume that the government runs a balanced budget every period, implying $\tau_t = b_t(1 - n_t)$.

implying that the wage is given by

$$w_t = \phi\left(z_t + e_t f(\bar{e}_t,\theta_t)\frac{\kappa}{\mu(\bar{e}_t,\theta_t)}\right) + (1-\phi)b + (1-\phi)\left(\chi - \frac{\psi}{\zeta}e_t^{\zeta}\right)\frac{1}{c_t^{-\gamma}}. \tag{17}$$

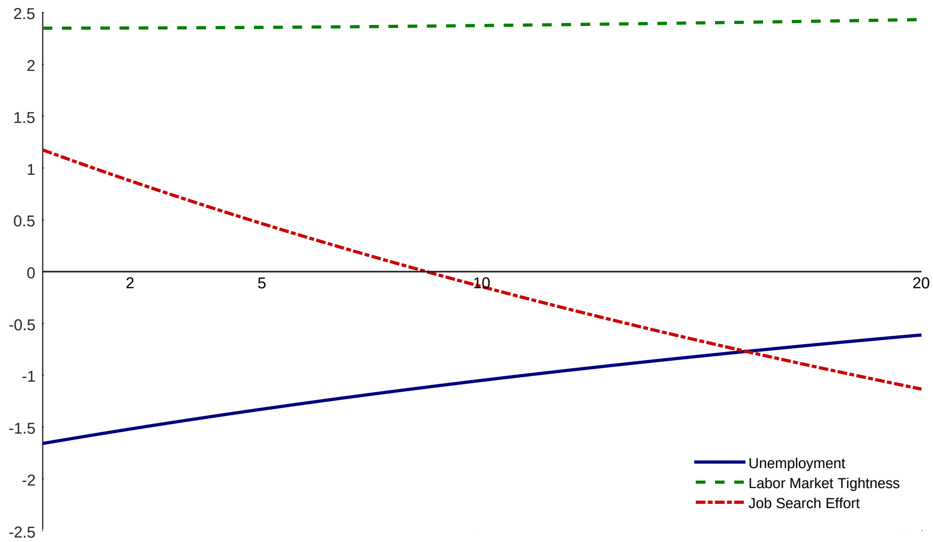
Unemployment benefits, b, reduce the required value of χ that pins down relative hiring costs in our calibration ($\kappa v/y = 0.01$ in our benchmark), which, in turn, reduces the relevance of income effects for the dynamics of wages (the last term in equation (15)). By reducing wage fluctuations relative to the benchmark, introducing unemployment benefits into the model increases the incentives for job creation. At the same time, the equation driving search effort (equation (14)) remains unchanged. Thus, unemployment benefits may reconcile the model with the evidence.

We conduct two experiments with unemployment benefits. In the first one, we assume that $b = 0.4$, implying a replacement rate of unemployment benefits just slightly above 40% (which is close to the one in Shimer, 2005). In the case of $\gamma = 1$, assuming that $b = 0.4$ qualitatively changes the behavior of search effort: effort rises when productivity rises, at odds with the evidence. In other cases, the models with $b = 0$ (benchmark) and $b = 0.4$ behave similarly. Assuming that $b = 0.4$ only increases the thresholds after which search effort becomes countercyclical and after which unemployment becomes procyclical.

In our second experiment, we consider the extreme that the opportunity cost of working only includes forgone unemployment benefits. In this case, $\chi = 0$ and we pin down the value of b to target relative hiring costs. Thus, income effects do not play any direct role for wage dynamics. We present the implications of this experiment in Figure 4, where we plot the impact of γ on the 8-year-cumulative responses of θ_t, u_t, and e_t to a positive productivity shock. We find that the range of values that generates acyclical or countercyclical search effort without compromising the cyclical behavior of unemployment is extremely large: search effort becomes countercyclical if $\gamma > 8.8$ and unemployment only becomes procyclical if $\gamma > 41.5$. Furthermore, the 8-year-cumulative response of labor market tightness is almost unresponsive to the degree of risk aversion.[18] Thus, if $\chi = 0$, the model's predictions are much more in line with the evidence. The problem, however, is that this only occurs if γ is very high (around nine) and the replacement rate of unemployment benefits exceeds 90%, much higher than its empirical counterpart.

[18] In this extreme case of $\chi = 0$, the calibration strategy of Hagedorn and Manovskii (2008) magnifies the volatility of unemployment and labor market tightness. As explained in Section 4.3.2., decreasing ϕ or the relative hiring costs $\kappa v/y$ increases the relevance of income effects for the wage dynamics. But, if $\chi = 0$, this channel is muted.

Figure 4: The Case $\chi = 0$



Note: The horizontal axis measures the degree of income effects, $\gamma \in [0, 20]$. The vertical axis measures the 8-year cumulative responses. The solid line represents unemployment; the dashed line represents labor market tightness; and the dot-dashed line represents job search effort.

4.3.5. The Model with Capital

In our last experiment, to make our model closer to the standard real business cycle model, we introduce capital.[19] The results resemble those of the model with unemployment benefits assuming $b = 0.4$. Introducing capital only increases the thresholds after which search effort becomes countercyclical and after which unemployment becomes procyclical. In expansions households increase their capital stock by delaying consumption. This, in turn, reduces the change in the marginal utility of consumption (relative to the benchmark) and, consequently, the relevance of income effects both for the wage and job search effort.

5. Concluding Remarks

In this paper, we study the role of income effects in a general equilibrium model extended with matching frictions. In particular, we assess whether the model generates the cyclical patterns observed in data: acyclical or countercyclical search effort and, at the same time, strongly countercyclical unemployment and strongly procyclical labor market tightness.

[19] Appendix B2. outlines the model with capital.

We find that income effects dictate the behavior of the model. In the baseline model, depending on the magnitude of income effects, one of three possibilities emerge. If income effects are absent or low, search effort is procyclical, which is at odds with the evidence. If income effects are moderate, search effort is acyclical but Shimer's (2005) critique is acute because the volatilities of the other labor market variables are extremely low. If income effects are high, search effort is countercyclical but unemployment is procyclical and labor market tightness is countercyclical, a result at odds with conventional wisdom and data.

In our robustness checks, we only find two ways to improve the predictions of the model. One is to assume a fully rigid wage. The other is to assume excessively high risk aversion and replacement rate of unemployment benefits. We conclude that adding income effects to the model does not generate the cyclical patterns observed in the data unless we make unsound assumptions.

In our model, income effects can be broadly interpreted as wealth effects because household's wealth is mainly the present discounted value of future income. This allows us to induce a broader implication from our results: as long as both job search effort and working reduce leisure, income and, more generally, wealth effects worsen the predictions of the model. Chodorow-Reich and Karabarbounis (2016) reach a similar conclusion. After documenting that the opportunity cost of working is procyclical in US data, they show that a model with procyclical opportunity cost of working generates lower unemployment volatility relative to a model with acyclical opportunity cost. Thus, income and wealth effects may make job search effort acyclical or countercyclical as suggested by the empirical literature. But in the presence of income and wealth effects, the opportunity cost of working becomes procyclical, which renders the wage more procyclical and job creation less procyclical or even countercyclical.

The volatility problem of models with matching frictions is even more acute than suggested by Shimer (2005). Chodorow-Reich and Karabarbounis (2016) show that procyclical opportunity cost of working reduces labor market volatilities. Mukoyama, Patterson and Şahin (2018) show that countercyclical job search effort also reduces labor market volatilities. Our paper reveals that these two forces reinforce each other, increasing the challenge for macro labor economists. We hope that our paper further stimulates the search for alternative specifications that overcome Shimer's (2005) critique and, at the same time, generate acyclical or countercyclical job search effort.

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A Derivation of Effort and Wage Equations

In this appendix, we present the derivation of equations (14) and (15) that we use in Section 4.

A1. The Effort Equation

The Nash bargaining problem set in equation (11) implies

$$V_{n,t} = \frac{\phi}{1-\phi} J_{n,t} c_t^{-\gamma}. \tag{A.1}$$

First, we lead this equation one period and take period t expectation. Then, we use the result to substitute out $\mathrm{E}_t V_{n,t+1}$ from the first order condition for effort ($\psi e_t^{\zeta-1} = \beta f(\bar{e}_t, \theta_t) \mathrm{E}_t V_{n,t+1}$). This gets

$$\psi e_t^{\zeta-1} = \beta f(\bar{e}_t, \theta_t) \frac{\phi}{1-\phi} E_t \left[J_{n,t+1} c_{t+1}^{-\gamma} \right]. \tag{A.2}$$

Then, using the free-entry condition ($\beta \mathrm{E}_t [J_{n,t+1} c_{t+1}^{-\gamma}] = \kappa c_t^{-\gamma} / \mu(\bar{e}_t, \theta_t)$) in equation (A.2) implies that

$$\psi e_t^{\zeta-1} = f(\bar{e}_t, \theta_t) \frac{\phi}{1-\phi} \frac{\kappa c_t^{-\gamma}}{\mu(\bar{e}_t, \theta_t)}.$$

This further simplifies to

$$\psi e_t^{\zeta} = \theta_t \frac{\phi}{1-\phi} \kappa c_t^{-\gamma}, \tag{A.3}$$

because $f(\bar{e}_t, \theta_t) = \frac{\theta_t}{\bar{e}_t} \mu(\bar{e}_t, \theta_t)$ and $\bar{e}_t = e_t$. Log-linearizing equation (A.3) yields equation (14) in the main text.

B Two Variants of the Model

In this appendix, we outline two variants of the baseline model in Section 2., the model with exogenous search effort and the model with capital.

B1. Variant with Exogenous Search Effort

To generate Figure 3, we employ a variant of the model with exogenous search effort. Assuming exogenous effort changes the matching function, the job-finding probability, and the job-filling probability:

$$\begin{aligned} m_t &= \sigma v_t^{\eta} (u_t)^{1-\eta}, \\ f(\bar{e}_t, \theta_t) &\equiv f(\theta_t) = \sigma \left(\theta_t\right)^{\eta}, \\ \mu(\bar{e}_t, \theta_t) &\equiv \mu(\theta_t) = \sigma \left(\theta_t\right)^{\eta-1}. \end{aligned}$$

Assuming exogenous effort also changes the optimization problem of the household:

$$V_t = \max_{\{c_t\}} \left[\frac{c_t^{1-\gamma} - 1}{1-\gamma} - \chi n_t + \beta \mathrm{E}_t V_{t+1} \right],$$

subject to

$$n_{t+1} \leq (1-\delta) n_t + f(\theta_t)(1-n_t).$$

This change implies that equation (6) is dropped from the model and that the value of an additional employed worker for the household is

$$V_{n,t} = w_t c_t^{-\gamma} - \chi + (1 - \delta - f(\theta_t)) \beta \mathrm{E}_t V_{n,t+1}.$$

Finally the wage evolves according to

$$w_t = \phi \left(z_t + \theta_t \kappa \right) + (1-\phi) \frac{\chi}{c_t^{-\gamma}}.$$

B2. Variant with Capital

The introduction of capital, k_t, into the model requires rewriting the household's maximization problem, the firm's maximization problem, and the resource constraint.

B2.1. Household's Maximization Problem

In the model with capital, the household maximizes lifetime utility taking the employment rate, the job-filling probability, the wage, and the rental rate of capital, r_t, as given and choosing the paths of consumption, capital, and job search effort. The optimization problem is

$$V_t = \max_{\{c_t, k_{t+1}, e_t\}} \left[\frac{c_t^{1-\gamma} - 1}{1-\gamma} - \frac{\psi}{\zeta} e_t^{\zeta} (1-n_t) - \chi n_t + \beta \mathrm{E}_t V_{t+1} \right],$$

subject to

$$c_t + k_{t+1} \leq w_t n_t + k_t (1-\delta_k) + k_t r_t,$$
$$n_{t+1} \leq (1-\delta) n_t + e_t f(\bar{e}_t, \theta_t)(1-n_t),$$

where δ_k is the rate of depreciation of capital. The first-order condition for effort (equation (6)) and the value of an additional employed worker to the household (equation (7)) are unchanged. The first-order condition for capital is

$$c_t^{-\gamma} = \beta \mathrm{E}_t \left[c_{t+1}^{-\gamma} (1 - \delta_k + r_{t+1}) \right],$$

which equates cost and benefit, in utility terms, of increasing capital stock by a unit.

B2.2. Firm's Maximization Problem

We assume a Cobb-Douglas production function:

$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha},$$

where α is the elasticity of output with respect to capital. The firm takes the employment rate, the job-filling probability, the wage, and the rental rate of capital as given. To maximize its value, the firm chooses the number of vacancies to open and the amount of capital to rent. The optimization problem is

$$J_t = \max_{v_t, k_{t+1}} \left(z_t k_t^{\alpha} n_t^{1-\alpha} - r_t k_t - w_t n_t - \kappa v_t + \beta \mathrm{E}_t \left[\left(c_{t+1}/c_t \right)^{-\gamma} J_{t+1} \right] \right),$$

subject to

$$n_{t+1} = (1-\delta) n_t + \mu(\bar{e}_t, \theta_t) v_t.$$

The free-entry condition (equation (9)) is unchanged and the value of an additional worker to the firm is given by

$$J_{n,t} = (1-\alpha)\frac{y_t}{n_t} - w_t + (1-\delta)\beta \mathrm{E}_t \left[\left(c_{t+1}/c_t \right)^{-\gamma} J_{n,t+1} \right].$$

The first-order condition for capital implies that the marginal cost of capital equals its marginal product:

$$r_t = \alpha \frac{y_t}{k_t}.$$

In our experiments, we set $\alpha = 0.33$.

B2.3. The Wage

Wage bargaining in the model with capital implies that the wage is given by

$$w_t = \phi \left((1-\alpha)\frac{y_t}{n_t} + e_t f(\bar{e}_t, \theta_t) \frac{\kappa}{\mu(\bar{e}_t, \theta_t)} \right) + (1-\phi) \left(\chi - \frac{\psi}{\zeta} e_t^{\zeta} \right) \frac{1}{c_t^{-\gamma}}.$$

B2.4. Resource Constraint

Adding capital implies that the resource constraint is

$$y_t = c_t + k_{t+1} - (1-\delta_k) k_t + \kappa v_t.$$