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Endogenous Growth and Real Effects of Monetary Policy: R&D and Physical Capital Complementarities in a Cash-in-Advance Economy

Gustavo Iglésias
Pedro Mazeda Gil

Centro de Economia e Finanças da UP
www.fep.up.pt

Endogenous Growth and Real Effects of Monetary Policy: R&D and Physical Capital Complementarities in a Cash-in-Advance Economy*

Gustavo Iglésias[†]and Pedro Mazeda Gil[‡]

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We study the real long-run effects of inflation and of the structural stance of monetary policy in the context of a monetary model of R&D-driven endogenous growth complemented with physical capital accumulation. We look into the effects on a set of real macroeconomic variables that have been of interest to policymakers – the economic growth rate, the real interest rate, the physical investment rate, R&D intensity, and the velocity of money –, and which have been analysed from the perspective of different, separated, strands of the theoretical and empirical literature. Additionally, we analyse the theoretical predictions of our model as regards the effects of inflation on the effectiveness of real industrial policy shocks and on the market structure, assessed namely by the average firm size, and present novel cross-country evidence on the empirical relationship between the latter and the long-run inflation rate.

JEL Classification: O41, O31, E41

Keywords: Endogenous growth, R&D, physical capital, firm size, cash-in-advance, inflation, money.

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[†]University of Porto, Faculty of Economics

[‡]University of Porto, Faculty of Economics, and CEF.UP – Center for Economics and Finance at University of Porto. Corresponding author – Address: Rua Dr Roberto Frias, 4200-464, Porto, Portugal; e-mail: pgil@fep.up.pt.

1. Introduction

The real effects of inflation and of the structural stance of monetary policy over the long run are acknowledged by policymakers,[1] as well as confirmed by the empirical evidence gathered by academic economists. This paper looks into those real long-run effects in the framework of a monetary growth model of R&D and physical capital accumulation under a lab-equipment specification. The considered setup explicitly takes into account the close interrelation between physical and technological inputs empirically observed along growth processes (e.g., Dowrick and Rogers, 2002), by allowing physical capital accumulation and R&D to complement each other as engines of long-run growth (e.g., Howitt and Aghion, 1998 Howitt, 2000, Sedgley and Elmslie, 2013, Gil, Almeida, and Castro, 2017). This is a well-known analytical setup in the R&D-driven endogenous growth literature,[2] but which has not yet been used in the context of the monetary growth models.

We analyse, in this context, the effects of inflation (and of the monetary authority's policy variable) on the long-run economic growth rate, which has been one of the main concerns of the literature of monetary R&D-based growth models. However, we look into the effects on a number of other real macroeconomic variables – the real interest rate, the physical investment rate, and the R&D intensity – which recent empirical evidence suggests to be (alongside the economic growth rate) negatively related to inflation (e.g., Evers, Niemann, and Schiffbauer, 2007; Chu and Lai, 2013; Chu, Cozzi, Lai, and Liao, 2015; and empirical references in Gillman and Kejak, 2011). We also discuss two other topics emphasised by the empirical literature: the effects of inflation on the velocity of money (e.g., Palivos and Wang, 1995; Dotsey and Ireland, 1996; Rodríguez Mendizábal, 2006) and the non-linearity of the inflation effect on the economic growth rate (e.g., Burdekin, Denzau, Keil, Sitthiyot, and Willett, 2004; Gillman, Harris, and Mátyás, 2004; López-Villavicencio and Mignon, 2011). Both have also long been of interest to monetary policymakers (e.g., Bernanke and Mishkin, 1997).

Additionally, we analyse the theoretical predictions of our model as regards the effects of inflation on the effectiveness of real industrial policy shocks and on the market structure, assessed by the number of firms and average firm size. In particular as regards the market structure, we present cross-country evidence on the empirical relationship between firm size and long-run inflation (Figure 1, below, and Appendix A provide details on this) and show how the analytical mechanism uncovered in our model is capable of addressing it.

Following a recent literature on R&D-driven growth (e.g., Chu and Cozzi, 2014), we introduce money demand in the model by considering cash-in-advance (CIA) constraints

[1]For instance, the European Central Bank states that, over time, "price stability contributes to achieving high levels of economic activity and employment" (text on the "Benefits of price stability", at `https://www.ecb.europa.eu/mopo/intro/benefits/html/index.en.html`).

[2]Following Rivera-Batiz and Romer (1991), two polar cases can be considered as regards the specification of the R&D technology: the knowledge-driven case, which assumes human capital and knowledge are the only inputs to R&D activities; and the lab-equipment case, which assumes the technology for R&D is the same as the technology for final-good production, and thus human capital, raw labour, and capital goods are all productive in R&D.

on R&D investment and also on manufacturing of intermediate goods. However, since in our model the latter uses physical capital as an input, then the respective CIA constraint also affects the mechanism of physical capital accumulation.[3] Given the considered complementarity between physical capital accumulation and R&D along the growth process, this will imply a novel kind of interrelation between the two referred to CIA constraints, in comparison with the previous literature. This setup also allows us to discuss the impact of a decrease in the labour income share (as pointed out by recent empirical literature; e.g., Elsby, Hobijn, and Sahin, 2013; Karabarbounis and Neiman, 2014) in the magnitude and non-linearities of the real long-run effects of inflation shocks and related nominal variables, as well as of real industrial policy shocks.

Although under distinct analytical settings, our paper is close to Gillman and Kejak (2011), as we also focus on the long-run relationship between inflation and the economic growth rate, the real interest rate, and the physical investment rate (physical investment-to-output ratio). Gillman and Kejak study an endogenous growth model of human and physical capital accumulation, extended with endogenous labour supply and a banking sector that produces exchange credit. In particular, they address the empirical evidence of a negative effect of inflation on both the real interest rate and the investment rate, which, as pointed out by Gillman and Kejak, may be viewed as, respectively, a positive and a negative (long-run) Tobin effect. By exploring the interplay between R&D and physical capital, we uncover a different mechanism to address the seemingly opposite Tobin effects, while our results are not conditional on a specific interval of values for the inflation rate, in contrast to Gillman and Kejak (2011). Moreover, we expand the overall set of real variables that are analysed in a common general equilibrium setup (velocity of money, R&D intensity, and market structure variables).

In particular, we also contribute to the macro literature that combines a setup featuring CIA constraints and a velocity of money different from one, and theoretically corroborates a positive long-run relationship between inflation and the velocity of money (e.g., Rodríguez Mendizábal, 2006; Chen and Guo, 2008; Benk, Gillman, and Kejak, 2010).[4] To the best of our knowledge, our paper is the first one to explicitly address this topic in the context of an R&D-driven growth model.

Our paper also relates to Chu and Cozzi (2014), as referred to above. This paper develops a monetary model of R&D-driven growth under a knowledge-driven specification

[3]As Chu and Cozzi point out, empirical evidence clearly suggests that R&D investment is severely affected by liquidity requirements, even more so than physical investment (see, e.g., Brown, Martinsson, and Petersen, 2012; Falato and Sim, 2014; Brown and Petersen, 2015), thus backing up a growth model that features CIA requirements in both activities (R&D and physical investment). Other papers that have studied endogenous growth models with CIA constraints on R&D activities are Chu, Cozzi, Lai, and Liao (2015), Huang, Yang, and Cheng (2017), Chu, Ning, and Zhu (2017), and Chu, Cozzi, Furukawa, and Liao (2017). An alternative strand of the literature has focused on the effects of money and inflation in the context of (physical and/or human) capital-driven growth models with CIA constraints (e.g., Itaya and Mino, 2007 and Gillman and Kejak, 2011; see Gillman and Kejak, 2005a for a survey of earlier contributions on this topic).

[4]In the seminal cash-in-advance specification by Stockman (1981) and in many of the contributions that followed, money is required for all the transactions of consumption and investment goods. Therefore, the income velocity of money always equals one and hence is independent of changes in the nominal variables (the money growth rate, the inflation rate, etc.).

and with no physical capital accumulation. Differently from that paper, we obtain a negative relationship between inflation and the long-run economic growth rate, the real interest rate, and R&D intensity that is robust to the relative degree of CIA constrains on R&D vis-à-vis manufacturing.[5] However, and also differently from the existing literature, the real long-run effects of inflation in our model may be dampened in case a smaller labour income share is considered in line with recent empirical evidence.

Gillman and Kejak (2005a) look into the marginal non-linearity of the (negative) inflation effect on the economic growth rate from the point of view of alternative theoretical growth models in the literature, featuring physical capital accumulation, human capital accumulation, or both, possibly extended with endogenous labour supply and a credit sector. As already emphasised, we carry out this analysis in the context of a model that features both R&D and physical capital accumulation and explores their complementarity along the growth process. We obtain higher *elasticities* of the economic growth rate with respect to long-run inflation for higher levels of inflation, but which translate into *level* effects of inflation on economic growth that are marginally weaker as the inflation rate rises. Our model explores an alternative mechanism to those featured by the models reviewed by Gillman and Kejak, while eschewing the counter-empirical negative economic growth rates (e.g., Gillman, Harris, and Mátyás, 2004) that usually emerge for moderately high inflation rates (about 50%) in those models.[6]

As in Wu and Zhang (2001), our model allows for endogenous firm entry, while long-run growth is also endogenous and related to the mechanism of entry (through horizontal R&D), in line with Chu and Ji (2016). Wu and Zhang (2001) build a model with differentiated goods and endogenous entry in the intermediate-good sector, increasing returns to scale in the final-good-sector, and exogenous growth. They predict a negative relationship between both the number of firms and average firm size and inflation; this relationship hinges on the behaviour of the markup (which is posited to vary negatively with the number of firms in the model) and the fixed production cost in the intermediate-good sector. In turn, Chu and Ji (2016) explore a model of vertical R&D combined with endogenous entry that generates a negative correlation between the number of firms and inflation while firm size is independent of the latter in the long run, due to the specific mechanism of dilution of market-size effects considered in the model. Our setup, in contrast, highlights the role of the relative fraction of R&D and of manufacturing costs that are subject to a CIA constraint as regards the effects of inflation on the market structure. We obtain the usual negative relationship between the number of firms and inflation, but firm size, which is measured as physical capital per firm in our model, can vary negatively, positively or be independent of inflation. In case the degree of the

[5]Interestingly, Arawatari, Hori, and Mino (2017) build a knowledge-driven variety-expansion model with heterogeneous R&D abilities and household's worker-entrepreneur occupational choice, which, under CIA constraints on consumption and intermediate-good manufacturing (featuring both variable and fixed costs, measured in units of the final good), always generates a negative relationship between inflation and the long-run economic growth rate.

[6]More recently and under a different analytical setup, Arawatari, Hori, and Mino (2017) (see fn. 5) also focus on the non-linearity of the (negative) inflation effect on economic growth. However, their model generates stronger level effects of inflation on growth for higher inflation rates and obtains negative growth rates for even lower inflation rates (about 30%).

CIA constraint on R&D exceeds that on manufacturing, as suggested by the empirical literature on firm financing (e.g., Brown and Petersen, 2015), firm size and inflation exhibit a unequivocal positive relationship. This result is in line with the cross-country empirical evidence we gathered for a sample of OECD countries – Figure 1 depicts the positive (conditional) correlation between average firm size and long-run inflation, with firm size measured as the stock of capital per firm in the manufacturing sector (see Appendix A for a more complete analysis, including a description of the controls).[7]

[Figure 1 goes about here]

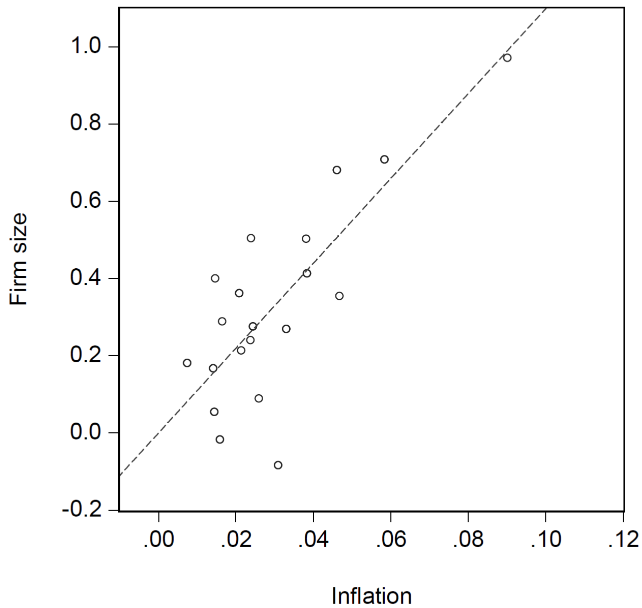

Figure 1: Average firm size versus long-run inflation rate – partial association from regression (6) of Table 3, Appendix A; the partial correlation is 0.86. Average firm size is measured as the stock of capital per firm in the manufacturing sector in 2008 and the inflation rate is measured as the average annual change rate of the GDP deflator in 1995-2007. Data for 21 OECD countries (see Appendix A for further details on the data, the list of controls, and the estimation results).

[7]By bringing the long-run inflation rate and the structural stance of the monetary policy into the picture, our analysis of the firm size-inflation relationship adds to the existing studies on the structural determinants of average firm size and firm size distribution from a macroeconomics point of view, most notably Poschke (2014) and Gomes and Kuehn (2017). Moreover, the diverse controls we use in our estimations suggest that a significant role is also played by the scale of the economy (measured as the population level), the overall economic development level (measured as GDPpc), human capital (captured by secondary versus tertiary educational attainment and by cognitive skills), institutional quality (proxied by political stability), financial development (proxied by several financial depth indicators), and overall market entry costs, as regards the cross-country distribution of the stock of capital per firm in manufacturing.

Finally, our model also allows us to look into the effects of industrial policy shocks – e.g., resulting from government subsidies to R&D activities or to intermediate-good manufacturing – from a new perspective, by relating the predicted impact on economic growth with the level of both long-run inflation and the labour income share. Our theoretical results indicate that industrial policy shocks have a smaller level effect on the growth rate when the long-run inflation rate is higher, whatever the type of subsidy. However, a decreased labour income share dampens the growth effect of subsidies to R&D activities and leverages that of subsidies to manufacturing. Although the former remains quantitatively more relevant under all considered calibrations, it is also noteworthy that our model with R&D and physical capital accumulation predicts a noticeably *smaller* growth effect of subsidies to R&D than a similar model without physical capital.

The remainder of the paper is as follows. Sections 2 and 3 present the model, derive the dynamic general equilibrium and study the interior long-run equilibrium in terms of its existence, uniqueness, and local dynamics properties. In Section 4, we study the comparative statics properties of the interior long-run equilibrium, while in Section 5 we analyse the effects of money and inflation on the long-run level of an array of real macroeconomic variables and compare with the empirical evidence. Section 6 concludes.

2. The model

We consider a version of the model of R&D and physical capital accumulation in Gil, Almeida, and Castro (2017), extended with a monetary sector, as in Chu and Cozzi (2014). This model can also be seen as an extension of Barro and Sala-i-Martin (2004, ch. 6) with physical capital accumulation and a monetary sector. This is a dynamic general-equilibrium endogenous growth model where a homogeneous final good can be used in consumption, accumulation of physical capital, and R&D activities – i.e., we allow for a lab-equipment R&D specification. The economy is populated by infinitely-lived (dynastic) households who consume and inelastically supply labour to final-good firms. The final good is produced using labour and a continuum of varieties of intermediate goods; in turn, the latter are produced using physical capital. Potential entrants into the intermediate-good sector devote resources to horizontal R&D, by which they increase the number of intermediate-good varieties, N, each produced by a specific industry. We incorporate money demand in the endogenous growth model via cash-in-advance (CIA) constraints on R&D activities and on manufacturing of intermediate goods,[8] whereas the monetary authority (the only form of government in the model) determines the money supply.

2.1. Production and price decisions

The final good is produced with a constant-returns-to-scale technology using labour and a continuum of intermediate goods indexed by $\omega \in [0, N]$,

[8] We abstract from the more conventional CIA constraint on consumption as we wish to focus our attention on the technology side of the model and its interaction with the monetary sector.

$$Y(t) = A \cdot L^{1-\alpha} \int_0^{N(t)} X(\omega,t)^\alpha d\omega, \;\; 0 < \alpha < 1, \tag{1}$$

where the measure of varieties of intermediate goods, $N(t)$, is allowed to vary over time t. A is the exogenous component of total factor productivity, L is the labour input (assumed as constant over time, for simplicity) with a share $1-\alpha$ in production, and $X(\omega,t)$ is the input of intermediate good ω. Final producers are price-takers; thus, they take wages, $w(t)$, and input prices, $p(\omega,t)$, as given and sell their output at a price also taken as given. All prices and wages are normalised by the price of the final good (thus, w and p are defined in real terms). From the profit maximisation conditions, the demand of intermediate good $\omega \in [0, N(t)]$ is $X(\omega,t) = L \cdot \left(\frac{A\cdot\alpha}{p(\omega,t)}\right)^{\frac{1}{1-\alpha}}$.

The intermediate good is produced in the manufacturing sector using physical capital. The sector uses the technology $\eta \cdot X(\omega,t) = K(\omega,t)$, where $K(\omega,t)$ is the input of capital in industry ω and $\eta > 0$ is a constant cost factor. We follow the literature and introduce a CIA constraint on manufacturing of intermediate goods by assuming that intermediate-good firms use money, borrowed from households subject to the nominal interest rate $i(t)$, to pay for a fraction $\Omega \in [0,1]$ of the capital input. Since firms cannot repay this amount to households until they earn revenue from production, households are effectively providing credit to these firms (Feenstra, 1986). Consequently, the cost of intermediate good ω has an operational component and a financial component, that is, $(1-\Omega)\cdot r(t)\cdot K(\omega,t) + \Omega\cdot(1+i(t))\cdot r(t)\cdot K(\omega,t) = K(\omega,t) + \Omega\cdot i(t)\cdot r(t)\cdot K(\omega,t)$, where $r(t)$ is the equilibrium market real interest rate and the cost of capital is the latter adjusted by the CIA constraint, i.e., $(1+\Omega\cdot i(t))\cdot r(t)$.[9] Thus, the intermediate good ω is produced with a cost function $(1+\Omega\cdot i(t))\cdot r(t)\cdot K(\omega,t) = (1+\Omega\cdot i(t))\cdot r(t)\cdot \eta\cdot X(\omega,t)$.[10]

The intermediate-good sector consists of a continuum $N(t)$ of industries, characterised by monopolistic competition at the sector level. The monopolist in intermediate-good industry ω chooses the price $p(\omega,t)$ in face of the demand curve $X(\omega,t)$. Profit in industry ω is thus $\Pi(\omega,t) = [p(\omega,t) - (1+\Omega\cdot i(t))\cdot r(t)\cdot\eta]\cdot X(\omega,t)$, and the profit maximising price is a markup over marginal cost, $p(\omega,t) \equiv p(t) = (1+\Omega\cdot i(t))\cdot\eta\cdot r(t)/\alpha$, which is constant across industries but possibly variable over time. Then, from the demand curve and the optimal price (markup), the optimal quantity produced of intermediate good ω is $X(\omega,t) = \tilde{X}(t) = L\cdot\left(\frac{A\cdot\alpha^2}{(1+\Omega\cdot i(t))\cdot\eta\cdot r(t)}\right)^{\frac{1}{1-\alpha}}$, which is also constant across industries but possibly variable over time.

In turn, equilibrium in the capital market requires $K(t) = \int_0^N K(\omega,t)d\omega = \int_0^N \eta\cdot X(\omega,t)d\omega = \eta\cdot\tilde{X}(t)\cdot N(t)$. In this setting, the technological-knowledge stock of the economy is measured by $N(t)$. By considering the optimal quantity $\tilde{X}(t)$ together with

[9]For sake of simplicity, we abstract from physical depreciation.

[10]In other words, we use Ω to parameterise the intensity of the CIA constraint on manufacturing of intermediate goods, where Ω is the share of the manufacturing cost that requires the borrowing of money from households. We will be interested in comparing the latter with the strength of the CIA constraint on R&D, to be introduced in Section 2.2, below.

the capital market equilibrium condition and solving with respect to $r(t)$, we get

$$r(t) = \frac{A \cdot \alpha^2 \cdot k(t)^{\alpha-1}}{(1+\Omega \cdot i(t)) \cdot \eta^{\alpha}}, \tag{2}$$

where $k(t) \equiv K(t)/(L \cdot N(t))$ is the physical capital-effective labour ratio. By using $\tilde{X}(t)$ and $r(t)$, we then get the optimal profit earned by the monopolist in ω, $\Pi(\omega,t) = \tilde{\Pi}(t) = \Pi_0 \cdot L \cdot \eta^{-\alpha} \cdot k(t)^{\alpha}$, with $\Pi_0 \equiv A \cdot \alpha \cdot (1-\alpha)$. From the above, we derive total optimal intermediate-good production, $X(t) = K(t)/\eta$, total optimal profits, $\Pi(t) = \Pi_0 \cdot L \cdot \eta^{-\alpha} \cdot k(t)^{\alpha} \cdot N(t)$, and total optimal final-good production,

$$Y(t) = A \cdot L \cdot \eta^{-\alpha} \cdot k(t)^{\alpha} \cdot N(t). \tag{3}$$

2.2. R&D decisions

We consider an R&D sector targeting horizontal innovation so that a new design pertains to a new variety of intermediate good. We modify the horizontal R&D sector in, e.g., Barro and Sala-i-Martin (2004, ch. 6) and Gil, Almeida, and Castro (2017) by introducing a CIA constraint on R&D activities. R&D is performed by (potential) entrants and successful R&D leads to the set-up of a new intermediate-good firm, with each new design being granted a perpetual patent. There is free entry, perfect competition, and constant returns to scale in the R&D business.

Horizontal innovation obtains according to $\dot{N}_e(t) = R_e(t)/(\zeta \cdot L)$, where $\dot{N}_e(t)$ is the contribution to the instantaneous flow of new varieties by potential entrant e, ζ is a constant (flow) fixed cost , and $R_e(t)$ is the flow of resources allocated to horizontal R&D by e at time t, measured in units of the final good. We also posit that there is an adverse complexity effect, so that the difficulty of introducing new varieties is proportional to the market size, which in turn is proportional to L (e.g., Dinopoulos and Thompson, 1999;Barro and Sala-i-Martin, 2004, ch. 6).[11] Then, $R = \sum_e R_e$ and $\dot{N}(t) = \sum_e \dot{N}_e(t)$, implying, at the aggregate,

$$\dot{N}(t) = \frac{1}{\zeta \cdot L} R(t). \tag{4}$$

The expected value of a new incumbent firm is $V(t) = \Pi_0 \cdot L \cdot \eta^{-\alpha} \cdot \int_t^{\infty} k(t)^{\alpha} \cdot e^{-\int_t^s r(v)dv} ds$, where $\Pi_0 \cdot L \cdot \eta^{-\alpha} = \tilde{\Pi}(t)/k(t)^{\alpha}$ is constant. As both physical capital accumulation and R&D investment represent foregone consumption (see Subsection 2.5, below), the real rate of return to R&D is equal to that for capital, r. We assume that the financing of R&D costs requires money borrowed from households, so that a CIA constraint on R&D activities also exists alongside that on manufacturing of intermediate goods. Therefore, the R&D cost has an operational and a financial component, that is, $(1-\beta) \cdot R(j) + \beta \cdot (1+i(t)) \cdot R(j) = R(j) + \beta \cdot i(t) \cdot R(j)$, where $\beta \in [0,1]$ is the share of the R&D

[11] As it is well known, these complexity costs offset the positive effect of scale on the (expected) profits of the successful innovator, thus delivering a long-run equilibrium without strong scale effects on growth, which are known to be contrafactual in modern economies.

cost that requires the borrowing of money from households. Given the CIA constraint, the free-entry condition in R&D is$\dot{N} \cdot V(t) = (1 + \beta \cdot i) \cdot R_n$, which, from (4), simplifies to $V(t) = (1 + \beta \cdot i) \cdot \zeta \cdot L$. Using the two expressions for $V(t)$ and applying time-differentiation yields the no-arbitrage condition facing a horizontal innovator

$$r(t) = \frac{\tilde{\Pi}(t)}{(1 + \beta \cdot i) \cdot \zeta \cdot L}. \tag{5}$$

From (4), total resources devoted to horizontal R&D are given by $R(t) = \zeta \cdot L \cdot \dot{N}(t)$.

2.3. Households

The economy is populated by a constant number of dynastic identical families who consume and earn income from labour, L, inelastically supplied to final-good firms, and from investments in financial assets and money balances (the latter as in Chu and Cozzi, 2014). Households have perfect foresight and choose the path of consumption $\{C(t), t \geq 0\}$ in order to maximise discounted lifetime utility

$$U = \int_0^{\infty} \left(\frac{C(t)^{1-\theta} - 1}{1 - \theta} \right) \cdot e^{-\rho t} dt, \tag{6}$$

where $\rho > 0$ is the subjective discount rate and $\theta > 0$ is the inverse of the intertemporal elasticity of substitution in consumption. This maximisation is subject to the flow budget constraint

$$\dot{a}(t) + \dot{m}(t) = r(t) \cdot a(t) + w(t) \cdot L - C(t) + \tau(t) - \pi(t) \cdot m(t) + i(t) \cdot b(t), \tag{7}$$

where: $a(t)$ denotes the households' real financial assets holdings (equity); $m(t)$ is the households' real money balance; $\tau(t)$ denotes a lump-sum transfer/tax from the monetary authority; $\pi(t)$ is the inflation rate, which determines the cost of holding money; and $b(t)$ is the amount of money borrowed from households by incumbent intermediate-good firms and entrants to finance the manufacturing of intermediate-goods and R&D investment, respectively, and which return is $i(t)$. Thus, the CIA constraints imply that $b(t) \leq m(t)$. The initial level of the state variables, $a(0)$ and $m(0)$, is given. From standard dynamic optimisation,[12] we derive a no-arbitrage condition (this is the well-known Fisher equation and it establishes that $i(t)$ is, indeed, the nominal interest rate) and the optimal path of consumption,

$$i(t) = r(t) + \pi(t), \tag{8}$$

$$\dot{C}(t) = \frac{1}{\theta} \cdot (r(t) - \rho) \cdot C(t), \tag{9}$$

whereas the transversality conditions are $\lim_{t \to +\infty} e^{-\rho t} \cdot C(t)^{-\theta} \cdot a(t) = 0$ and $\lim_{t \to +\infty} e^{-\rho t} \cdot C(t)^{-\theta} \cdot m(t) = 0$.

[12] Appendix B provides further details on the derivation of the results.

2.4. Monetary authority

The monetary sector is considered as in Chu and Cozzi (2014). The nominal money supply is denoted by $M(t)$ and its growth rate is $\mu(t) \equiv \dot{M}(t)/M(t)$. The real money balance is $m(t) = M(t)/P(t)$, where $P(t)$ is the nominal price of the final good, and, thus, its growth rate is $\dot{m}(t)/m(t) = \mu(t) - \pi(t)$. The monetary authority implements a nominal interest rate peg by targeting the nominal level of the interest rate, so that $i(t) = i$. Given an i exogenously chosen by the monetary authority, the inflation rate, $\pi(t)$, is endogenously determined according to the Fisher equation (8), for each given $r(t)$. Then, given $\pi(t)$, the growth rate of the nominal money supply will ultimately be endogenously determined according to $\mu(t) = \dot{m}(t)/m(t) + \pi(t)$.[13] That is, the monetary authority will endogenously adjust the money growth rate to whatever level is needed for the targeted interest rate i to prevail. As usual in the literature, we consider that, to balance its budget, the monetary authority returns the seigniorage revenues to households as a lump-sum transfer, i.e., $\tau(t) = \dot{M}(t)/P(t) = \dot{m}(t) + \pi(t) \cdot m(t)$.

2.5. Macroeconomic aggregation and equilibrium capital accumulation

Consider the aggregate financial wealth held by all households, $a(t) = K(t) + \int_0^{N(t)} V(\omega, t) d\omega = K(t) + (1 + \beta \cdot i) \cdot \zeta \cdot L \cdot N(t)$.[14] Taking time derivatives and using (7), together with the lump-sum transfer, $\tau = \dot{m}(t) + \pi(t) \cdot m(t)$, and the real wage, $w(t) = (1 - \alpha) \cdot Y(t)/L$ (from the profit maximisation problem of the final-good firms), we get

$$[K(t) + (1 + \beta \cdot i) \cdot \zeta \cdot L \cdot N(t)] \cdot r(t) + (1-\alpha) \cdot Y(t) - C(t) + i \cdot b(t) = \dot{K}(t) + (1+\beta \cdot i) \cdot \zeta \cdot L \cdot \dot{N}(t).$$

Next, consider (2) and (3), to get $r(t) \cdot K(t) = \alpha^2 \cdot Y(t)/\left(1 + \Omega \cdot i\right)$, and (5) and optimal total profits, $\Pi(t) = \alpha \cdot (1 - \alpha) \cdot Y(t)$, to get $r(t) \cdot (1 + \beta \cdot i) \cdot \zeta \cdot L \cdot N(t) = \alpha \cdot (1 - \alpha) \cdot Y(t)$. Then, also recalling total R&D expenditure, $R(t) = \zeta \cdot L \cdot \dot{N}(t)$, and considering the amount of money lent by households as $b(t) = \beta \cdot \zeta \cdot L \cdot \dot{N}(t) + \Omega \cdot r(t) \cdot K(t)$, we obtain the product market equilibrium condition,

$$Y(t) = C(t) + \dot{K}(t) + R(t). \tag{10}$$

3. General equilibrium

The dynamic general equilibrium is defined by the allocation $\{X(\omega, t), \omega \in [0, N], t \geq 0\}$, the prices $\{p(\omega, t), \omega \in [0, N], t \geq 0\}$, and the aggregate paths $\{C(t), N(t), K(t), r(t), t \geq 0\}$, such that: (i) households, final-good firms and intermediate-good firms solve

[13]It will be shown below that the growth rate of real money balances is determined by the real growth rate of the economy in the long-run equilibrium.

[14]To see this, recall that $V(\omega, t) = V(t) = (1+\beta \cdot i) \cdot \zeta \cdot L$ and, thus, $\int_0^N V(\omega, t) d\omega = (1+\beta \cdot i) \cdot \zeta \cdot L \cdot N(t)$.

their problems; (ii) the innovation free-entry and no-arbitrage conditions are satisfied; and (iii) markets clear.

Firstly, we recall that, as both physical capital accumulation and R&D investment represent foregone consumption (see (10)), their real rates of return must be equalised in equilibrium. Therefore, we consider (2) and (5) and solve in order to $k(t)$, to get the equilibrium physical capital-effective labour ratio

$$k(t) = k^* = \frac{\alpha \cdot (1 + \beta \cdot i) \cdot \zeta}{(1 - \alpha) \cdot (1 + \Omega \cdot i)\, \eta^{\alpha}}. \tag{11}$$

It is noteworthy that k only depends on the nominal interest rate, i, and on structural parameters of the model. Given an i exogenously chosen by the monetary authority, k remains constant over time. Using (11) in (2) or (5) yields the equilibrium real interest rate also as a constant,

$$r(t) = r^* = \left(\frac{1}{\zeta}\right)^{1-\alpha} \cdot \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} \cdot \frac{A \cdot \alpha^2}{(1 + \Omega \cdot i)^{\alpha} \cdot (1 + \beta \cdot i)^{1-\alpha}\, \eta^{\alpha^2}}. \tag{12}$$

Secondly, by differentiating eq. (3) with respect to time, and considering $k(t) = k^*$ and L constant, we have $\dot{Y}(t)/Y(t) = \dot{N}(t)/N(t) = \dot{K}(t)/K(t)$. Then, using $R(t) = \zeta \cdot L \cdot \dot{N}(t)$ in (10) and solving in order to $\dot{N}$, we obtain

$$\frac{\dot{N}(t)}{N(t)} = \kappa \cdot \left\{ A \cdot \eta^{-\alpha} \cdot \left[\frac{\alpha \cdot (1 + \beta \cdot i) \cdot \zeta}{(1-\alpha) \cdot (1 + \Omega \cdot i) \cdot \eta^{\alpha}} \right]^{\alpha} - \frac{C(t)}{L \cdot N(t)} \right\}, \tag{13}$$

where

$$\kappa \equiv \frac{(1-\alpha) \cdot (1 + \Omega \cdot i) \cdot \eta^{\alpha}}{\{\alpha \cdot (1 + \beta \cdot i) + (1-\alpha) \cdot (1 + \Omega \cdot i) \cdot \eta^{\alpha}\} \cdot \zeta} > 0.$$

Recalling the Euler equation for consumption, (9),we use (12) to get

$$\frac{\dot{C}(t)}{C(t)} = g_C^* = \frac{1}{\theta} \cdot \left\{ \left(\frac{1}{\zeta}\right)^{1-\alpha} \cdot \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} \cdot \frac{A \cdot \alpha^2}{(1 + \Omega \cdot i)^{\alpha} \cdot (1 + \beta \cdot i)^{1-\alpha} \cdot \eta^{\alpha^2}} - \rho \right\}. \tag{14}$$

Considering $r(t) = r^*$, for all t , the model yields the growth rate of consumption also as a constant, g_C^*.

Finally, we note that a BGP as a representation of the dynamic long-run equilibrium associated with the dynamical system (13)-(14), where $\dot{K}(t)/K(t) = \dot{N}(t)/N(t)$, is the path $[C(t)^*, N(t)^*, K(t)^*, t \geq 0]$, along which the growth rates g_C^* and $g_N^* = g_K^*$ are constant. By considering eq. (10), $\dot{K}(t) = \eta \cdot \dot{N}(t) \cdot \tilde{X}$ and $R(t) = \zeta \cdot L \cdot \dot{N}(t)$, and the CIA constraint as an equality, i.e., $b(t) = m(t)$, a BGP only exists if: (i) the asymptotic growth rates of consumption, technological knowledge (the number of varieties), physical capital, real money balances, and final-good output are constant and equal to the real economic growth rate, $g_C = g_N = g_K = g_m = g_Y = g$; and (ii) the real interest rate is asymptotically trendless, $g_r = 0$. Under these conditions, $g_C = g_C^*$, $g_N = g_N^*$, $g_K = g_K^*$, $r = r^*$, and, from the Euler equation (14),

$$g_C^* = g_N^* = g_K^* = g_m^* = g_Y^* = g^* = \frac{1}{\theta} \cdot (r^* - \rho) \,. \tag{15}$$

With the above in mind, let $c(t) \equiv C(t)/(L \cdot N(t))$, with the property that, on the BGP, $\dot{c}(t) = 0$. Then, the dynamical system (13)-(14) is equivalent to

$$\frac{\dot{c}(t)}{c(t)} = \frac{\dot{C}(t)}{C(t)} - \frac{\dot{N}(t)}{N(t)} = \phi + \kappa \cdot c(t), \tag{16}$$

where $\phi \equiv r^*/\theta - \nu \cdot r^* - \rho/\theta$ and

$$\nu \equiv \frac{1 + \Omega \cdot i + \beta \cdot i + \Omega \cdot \beta \cdot i^2}{\alpha \cdot [\alpha \cdot (1 + \beta \cdot i) + (1 - \alpha) \cdot (1 + \Omega \cdot i) \cdot \eta^\alpha]}.$$

The long-run equilibrium value of c (steady-state value) is obtained by setting $c^* = -\phi/\kappa$, so that $\dot{c} = 0$ with a non-null c. This geometrical locus represents an (interior) steady-state equilibrium with balanced growth in the usual sense, characterised by a constant and positive endogenous growth rate, as obtained in (15), and with the transversality condition in the households' optimisation problem satisfied with $\rho > (1 - \theta) \cdot g^*$. Let $\dot{c} = f(c)$ in (16). For a given exogenous nominal interest rate, i, the steady state is asymptotically stable if $f'(c^*) < 0$. Since $f'(c^*) = -\phi$, that condition is equivalent to $\phi > 0$. It can be shown that, given the usual numerical values assumed for the parameters and since i is assumed to be non-negative, this condition is never satisfied. Therefore, the steady state corresponds to $c^* > 0$ and is unstable. In order to eschew explosive growth trajectories and to satisfy all the optimality conditions, the households choose the path of consumption so that the economy satisfies $c = c^*$ and the corresponding BGP characterised by (15) for all $t \geq 0$. Hence, there is no transitional dynamics.[15]

As regards the behaviour of the economy in the long-run equilibrium, we underline that, as in Howitt and Aghion (1998) and others, physical capital accumulation – such that the ratio k is positive and constant in the long run – is necessary for growth to be sustained. Since capital is assumed to be a necessary factor of production, $k \to 0$ would drive final output, and hence the monopolists' profit in the intermediate-good sector, to zero in the long run (see (3)); consequently, the real interest rate would be driven to infinity (see (2)). Thus, the reward to innovation would fall to zero, making it impossible for the R&D free-entry condition or, equivalently, the no-arbitrage condition (5), to be satisfied along the BGP.[16] The reverse is also true, since, without innovation (growth of N), marginal diminishing returns would eventually bring capital accumulation to a halt.

[15]It is clear that the model features no transitional dynamics because of the equality between the rate of return of physical capital and the rate of return of R&D activities, which, in turn, pins down the ratio k as a strict function of the structural parameters of the model and the nominal interest rate, whatever time t. Therefore, in spite of the consideration of physical capital characterised by decreasing marginal returns (which is the typical mechanism that generates transitional dynamics in the baseline neoclassical growth model), the model displays no transition, just like the horizontal-R&D growth model without physical capital (see, e.g., Barro and Sala-i-Martin 2004, ch. 6).

[16]To put it another way, since the homogeneous final good is an input in R&D activities (i.e., a lab-equipment specification), final output driven to zero would imply no resources to allocate to R&D and, therefore, these activities would be shut down.

Hence, physical capital accumulation and innovation are complementary mechanisms in this setting.

4. Comparative statics analysis

4.1. Long-run real interest rate, economic growth rate, and physical capital-technology ratio

Straightforward comparative-statics techniques yield the following proposition concerning changes in the structural parameters and the exogenous monetary-policy variable.

Proposition 1. The long-run real interest rate, r^*, is independent of the discount rate, ρ, and of the inverse of the intertemporal elasticity of substitution, θ, and is decreasing in the intermediate-good cost factor, η, the fixed R&D cost, ζ, and the nominal interest rate, i. The long-run aggregate growth rate, g^*, is decreasing in ρ, θ, η, ζ, and i. The long-run physical capital-effective labour ratio, k^*, is independent of ρ and θ, is decreasing in η, and is increasing in ζ; k^* is increasing (decreasing) in i if $\beta > \Omega$ ($\beta < \Omega$) and is independent of i if $\beta = \Omega$.

Proof. Obtained by checking the partial derivatives of eqs. (11), (12), and (15) with respect to i and the relevant parameters of the model.

According to eqs. (11) and (12), in Section 3, the long-run real interest rate, r^*, and physical capital-effective labour ratio, k^*, depend only on the (structural) level of the nominal interest rate and on the parameters related to the technology and to the CIA constraints, and not on the preferences parameters. This contrasts with the results in the models that bring together R&D and physical capital accumulation but R&D is of the vertical type (e.g., Howitt and Aghion, 1998) or R&D is considered under a knowledge-driven specification (i.e., with labour as the R&D input; e.g., Romer, 1990). As shown above, in our model, the arbitrage between the returns to R&D and the returns to physical capital accumulation completely pins down the level of $k^*$and r^*.

The sign of the relationship between k^* and i depends on the strength of the CIA constraint on R&D activities, measured by β, vis-à-vis that on manufacturing of intermediate goods, measured by Ω. When β is larger than Ω, an increase in i raises the cost of R&D by more than the cost of intermediate-good production, therefore diverting resources from the latter to the former; since intermediate-good production uses physical capital as an input, the ratio of the stock of physical capital to the stock of technological knowledge, k^*, increases. The same mechanism explains why k^*varies negatively with the intermediate-good cost factor, η, and positively with the R&D fixed cost, ζ.

However, the ambiguous relationship between k^* and i translates into an unequivocal negative relationship between r^* and i, because the direct negative effect of i on r^* (due to the CIA constraints) always dominates the channel that commands the negative effect of i on k^* (which would indirectly increase r^*). Reflecting the unequivocal negative relationship between r^* and i, the model also yields a negative relationship between the long-run economic growth rate, $g*$, and i irrespective of the relationship between Ω and β. This result arises because, under a lab-equipment specification, as the one adopted in our model, the homogeneous final good is an input in R&D activities (see (10)), and thus labour and physical capital – by means of the intermediate goods – are ultimately all productive in R&D. In such a setting, an increase in i, by raising both the cost of intermediate-good production and the cost of R&D (whatever their relative magnitude) always increases the cost of R&D vis-à-vis manufacturing and, thereby, reduces the incentive to allocate resources to growth-enhancing activities. This contrasts with Chu and Cozzi (2014), who obtain a negative relationship between i and both $r^*$and g^* only when $\beta > \Omega$.[17] In their model, an increase in i also raises both the cost of intermediate-good production and the cost of R&D; however, since manufacturing and R&D compete for the same scarce resource (labour) in their knowledge-driven setup, the two CIA constraints have opposing effects and, hence, their relative strength determines the sign of the relationship between g^*, r^* and i . In our model, the same mechanism (complementarity between physical capital and R&D activities in a lab-equipment setup) explains why r^* and g^* depend negatively on both the cost parameters η and ζ.

Finally, we also note that, bearing in mind the Fisher equation (8) and the equilibrium relationships derived in Section 3, we have $\pi(t) = \pi^*$ and $i = r^* + \pi^* \Leftrightarrow \pi^* = i - r^*$. Since $r^*$is a decreasing function of i, as shown above, then an exogenous increase in i implies an increase in π^*, because of the movements both in i and in r^*; hence, $\Delta\pi^* > \Delta i$. Thus, one can extend all the above comparative-statics results pertaining to shifts in i also to shifts in the long-run inflation rate, π^*. Therefore, our theoretical results are in line with the recent empirical evidence suggesting that the economic growth rate and the real interest rate are negatively related to long-run inflation (e.g., Evers, Niemann, and Schiffbauer, 2007; Chu and Lai, 2013; Chu, Cozzi, Lai, and Liao, 2015)

4.2. Quantitative Analysis

In this section, we perform a numerical analysis considering the following set of baseline values for the parameters and the exogenous nominal (monetary policy) variable: $\rho = 0.02$; $\theta = 1.5$; $\alpha = 1/3$; $A = 0.74$; $\eta = 1$; $\zeta = 3.85$; $\beta = 1$; $\Omega = 0.5$; and $i = 0.075$. The values of ρ, θ, and α are standard in the growth literature (e.g., Barro and Sala-i-Martin, 2004), while we determine the values for A, ζ, η, and i in order to match the empirical data for the US in the last two decades regarding the long-run economic growth rate, inflation rate and real interest rate, that is, $g^* = 0.02$, $\pi^* = 0.025$, and $r^* = 0.05$ (see, e.g., Chu, Cozzi, Furukawa, and Liao, 2017). In turn, we will let α, β, and Ω take different values across the numerical exercises. The typical value considered for the capital share in production is $\alpha = 1/3$; however, given the recent empirical evidence on a sustained decrease of the labour income share, namely in the manufacturing sector (e.g., Oberfield and Raval, 2014), we will also consider $\alpha = 0.45$ in our numerical exercises. As for β and Ω, the empirical evidence suggests that R&D investment is even more severely affected by liquidity requirements than physical capital (e.g., Brown and Petersen, 2015), which

[17] In Chu and Cozzi's notation, when $\alpha > \beta$.

translates, in our model, into a degree of CIA constraint on R&D that is higher than on manufacturing, i.e., $\beta > \Omega$. Nevertheless, given the lack of direct evidence on the relative magnitude of the CIA parameters, we consider a number of alternative scenarios for β and Ω in our numerical exercises.

Growth effects of the structural monetary policy/inflation shocks First, we wish to analyse the elasticity of the BGP economic growth rate, g^*, with respect to changes in the (structural) level of the monetary policy variable, i, – and, consequently, of the long-run inflation rate, π^* – for different scenarios of the CIA constraints, parameterised by β and Ω, and capital income share, α. The elasticity is[18]

$$E_i^{g^*} = \frac{\partial g^*}{\partial i} \cdot \frac{i}{g^*} = -\left(1 + \frac{\rho}{\theta g^*}\right) \cdot \left[\frac{(1-\alpha)\cdot\beta}{1/i + \beta} + \frac{\alpha \cdot \Omega}{1/i + \Omega}\right] < 0 \tag{17}$$

Table 1, 4th column, depicts the results concerning $E_i^{g^*}$. We find that, for a given (pre-shock) g^*:[19]

- The elasticity becomes more negative as β (Ω) increases, for a given Ω (β) and α.
- However, ceteris paribus, the elasticity responds by more to an increase in β than in Ω (e.g., for $\alpha = 1/3$, when $(\beta, \Omega) = (1, 0)$, we get $E_i^{g^*} = -0.078$, and, when $(\beta, \Omega) = (0, 1)$, we get $E_i^{g^*} = -0.039$);
- The minimum value for $E_i^{g^*}$ in our exercise is achieved when the two CIA constraints are the most severe, i.e., $(\beta, \Omega) = (1, 1)$ (i.e., $E_i^{g^*} = -0.117$);
- A higher value of α makes $E_i^{g^*}$ less (more) negative, when $\beta > \Omega$ ($\beta < \Omega$); the elasticity is independent of α, when $\beta = \Omega$.

As an example, consider from Table 1 the elasticity values when $(\beta, \Omega) = (1, 0.5)$: $E_i^{g^*} = -0.0976$, if $\alpha = 1/3$, and $E_i^{g^*} = -0.0910$, if $\alpha = 0.45$. Then, a permanent increase in i from 0.075 to 0.10 implies a decrease in g^* of, respectively, 0.0651 and 0.0607 percentage points. Thus, the model predicts a significant negative effect on the long-run economic growth rates (the magnitude of the effect per percentage point of change in i is in line with that found in several other models in the literature; see a review in Gillman and Kejak, 2005a). But, on the other hand, as it is shown, a decrease in the labour income share as the one observed in recent empirical data implies a reduction in the magnitude of the real long-run effects of inflation on economic growth rate, if the CIA constraint on R&D is stronger than that on manufacturing – as it is indeed suggested by the empirical literature on firm financing (e.g., Brown and Petersen, 2015).

[Table 1 goes about here]

[18] See derivation in Appendix C.

[19] In this exercise, we calibrate the productivity parameter A so that we get $g^* = 0.02$ in all (pre-shock) scenarios for β, Ω, and α.

α	β	Ω	$E_i^{g^*}$	$E_\zeta^{g^*}$	$E_\eta^{g^*}$
1/3	0	0	0.000	-1.111	-0.185
	0	0.5	-0.020	-1.111	-0.185
	0	1	-0.039	-1.111	-0.185
	0.5	0	-0.040	-1.111	-0.185
	0.5	0.5	-0.060	-1.111	-0.185
	0.5	1	-0.079	-1.111	-0.185
	1	0	-0.078	-1.111	-0.185
	1	0.5	-0.098	-1.111	-0.185
	1	1	-0.116	-1.111	-0.185
0.45	0	0	0.000	-0.917	-0.338
	0	0.5	-0.027	-0.917	-0.338
	0	1	-0.052	-0.917	-0.338
	0.5	0	-0.033	-0.917	-0.338
	0.5	0.5	-0.060	-0.917	-0.338
	0.5	1	-0.085	-0.917	-0.338
	1	0	-0.064	-0.917	-0.338
	1	0.5	-0.091	-0.917	-0.338
	1	1	-0.116	-0.917	-0.338

Tabela 1: The elasticity of the BGP economic growth rate, g^*, with respect to the monetary policy variable, i, and the technological parameters ζ and η, for different scenarios of CIA constraints, parameterised by β and Ω, and of the physical capital income share, α. Decreases in the technological parameters ζ and η may be interpreted as government subsidies to, respectively, R&D activities and intermediate-good production. $\rho = 0.02$; $\theta = 1.5$; $\eta = 1$; $\zeta = 3.85$; $i = 0.075$, $g^* = 0.02$, $\pi^* = 0.025$, and $r^* = 0.05$.

i	π^*	g^*	$E_i^{g^*}$	$E_\zeta^{g^*}$	$E_\eta^{g^*}$
0.0750	0.0250	0.0200	−0.0976	−1.1109	−0.1851
0.1000	0.0509	0.0194	−0.1291	−1.1255	−0.1876
0.1250	0.0769	0.0188	−0.1603	−1.1405	−0.1901
0.1500	0.1027	0.0182	−0.1911	−1.1559	−0.1927
0.1750	0.1286	0.0176	−0.2216	−1.1717	−0.1953
0.2000	0.1544	0.0171	−0.2520	−1.1879	−0.1980

Tabela 2: The elasticity of the BGP economic growth rate, g^*, with respect to the monetary policy variable, i, and the technological parameters ζ and η, for different levels of the nominal interest rate, i, and of long-run inflation, π^*. $\rho = 0.02$; $\theta = 1.5$; $A = 0.74$; $\eta = 1$;$\zeta = 3.85$; $\alpha = 1/3$; $\beta = 1$; $\Omega = 0.5$.

We also wish to analyse the effect of the (structural) level of the monetary policy variable/long-run inflation rate on the elasticity $E_i^{g^*}$, to check if our model predicts some sort of non-linearity as regards the inflation effects on the economic growth rate, as discussed by the empirical literature. Table 2, 3rd column, illustrates the impact of the level of i (and, thus, of π^*) on $E_i^{g^*}$, with higher values of i implying more negative values of $E_i^{g^*}$. A reasonably high value of i (e.g., $i = 0.20$, corresponding to π^* of about 0.15) takes the elasticity to a significant negative value; e.g., when $(\beta, \Omega) = (1, 0.5)$, we get $E_i^{g^*} = -0.252$, which compares with $E_i^{g^*} = -0.0976$ when $i = 0.075$.[20]

However, in light of these numbers, it is also of interest to compare the impact of a given *level* change in i on the long-run economic growth rate g^*, also in terms of *level* changes. For example, considering from Table 2 the elasticity value $E_i^{g^*} = -0.252$, with $i = 0.20$, then an increase in i to 0.225 implies a decrease in $g^*$of 0.054 percentage points. This is smaller than the already referred to decrease of 0.065 percentage points for an increase in i from 0.075 to 0.10, when $E_i^{g^*} = -0.0976$.

[Table 2 goes about here]

Thus, in spite of obtaining higher *elasticities* of the economic growth rate for higher levels of i, the long-run effects of a higher i on economic growth, in *levels*, are marginally weaker for higher levels of i (and thus of π^*). This is in line with the empirical evidence as reported in, e.g., Burdekin, Denzau, Keil, Sitthiyot, and Willett (2004) and Gillman, Harris, and Mátyás (2004). In Gillman and Kejak (2005b)´s review of the theoretical mechanisms that may be able to generate this sort of non-linearities in the inflation-growth effects – pertaining to a number of (physical and/or human) capital-driven growth models with CIA constraints –, they emphasise the role played by the consideration of a banking sector that supplies credit to avoid the "inflation tax" on consumers (see Gillman and Kejak, 2005b, Section 5.3). This way, the interest elasticity of money demand increases as inflation also increases, thereby reducing the inflation-growth effect

[20]In this exercise, we do not keep the value of g^*fixed at 0.02 because i is a policy variable and not a structural parameter. The impact of changes in i on g^* are central to the mechanism at work (see, e.g., Gillman and Kejak, 2005b).

for higher inflation levels. In our model, we get a similar quantitative result without considering a banking sector but instead positing a CIA constraint in two sectors (R&D and manufacturing of intermediate goods) that play complementary roles as regards the long-run growth process. Analytically, the non-linearity is a consequence of the fact that $\partial g^{*}/\partial i$ decreases (in modulus) as g^{*}also decreases in response to a rise in i (see Appendix C), as a direct consequence of the referred to CIA constraints. In particular when $\beta = \Omega = 1$, our model approximates the BGP growth equation and the quantitative results for the non-linearities of the inflation-growth effect of a simple AK model with a CIA constraint in the consumer and the investment-good sectors (e.g., Dotsey and Sarte, 2000; Gillman and Kejak, 2005a, Section 3.2). However, in contrast to the latter, our model generates other empirically relevant results (as further explained in Section 5, below), namely regarding the real interest rate, the velocity of money, and R&D intensity; these are either constants in the AK model or absent from it. Importantly, our results are also robust to a very wide range of inflation rates, in particular by predicting a positive long-run economic growth rate even for an inflation rate as high as 255%, while in, e.g., Dotsey and Sarte (2000)'s model, a counter-empirical negative growth rate emerges for an inflation rate of about 50% (other models surveyed by Gillman and Kejak, 2005a, generate even shorter admissible ranges for the inflation rate; see also Arawatari, Hori, and Mino, 2017).[21]

For the sake of comparison with the set of quantitative (theoretical) results gathered by Gillman and Kejak (2005a, Table 9), we show in Table 4, Appendix D, the magnitude of the BGP growth effects in our model when i is raised from 0 to 0.10 and from 0.10 to 0.20, for given selected values of the capital share, α, and the CIA parameters (β, Ω).

Growth effects of real industrial policy shocks Now, we wish to look into the elasticity of the BGP economic growth rate with respect to the technological parameters ζ and η, again for different scenarios of β, Ω, and α. We interpret ζ and η as industrial policy parameters. A decrease in these technological parameters may be seen as equivalent to a proportional government subsidy to, respectively, R&D activities and intermediate-good production.[22] The elasticities are[23]

[21]The recent empirical literature on the inflation-growth relationship has generally confirmed its negative sign but also pointed out that this relationship is influenced by the inflation level itself. However, the nature of this non-linearity is a subject of debate, with the literature focusing on the existence either of marginal non-linearities or of thresholds and/or discontinuities regarding the inflation-growth effects (López-Villavicencio and Mignon, 2011). The robustness of these results has been questioned as they tend to depend on the model specification, the sample used, and the methodology applied (Hineline, 2007). Moreover, it is not clear how some of the identified non-linearities can be made compatible with the evidence that indicates the economic growth rates do not become negative even for reasonably large inflation levels (Gillman, Harris, and Mátyás, 2004). As already noted, our theoretical model is fit to address the marginal non-linearities of the inflation-growth effects, while eschewing the counter-empirical negative economic growth rates that emerge for moderately high inflation rates in other models in the literature.

[22]The usual simplifying assumption underlying this type of exercise is that the government balances its budget every period by levying the necessary amount of lump-sum taxes.

[23]See derivation in Appendix C.

$$E_\zeta^{g^*} = \frac{\partial g^*}{\partial \zeta} \cdot \frac{\zeta}{g^*} = -\left(1 + \frac{\rho}{\theta g^*}\right) \cdot (1-\alpha) < 0 \tag{18}$$

$$E_\eta^{g^*} = \frac{\partial g^*}{\partial \eta} \cdot \frac{\eta}{g^*} = -\left(1 + \frac{\rho}{\theta g^*}\right) \cdot \alpha^2 < 0 \tag{19}$$

Table 1, 5th and 6th column, depicts the results regarding $E_\zeta^{g^*}$ and $E_\eta^{g^*}$. We find that, for a given (pre-shock) g^*:

- The elasticities are independent of β and Ω;
- A higher value of α makes $E_\zeta^{g^*}$ ($E_\eta^{g^*}$) less (more) negative, whatever β and Ω.

The fact that ζ is a cost parameter pertaining to the R&D sector and η to the intermediate-good sector (and thereby to physical capital accumulation) explains the contrasting impact of an increase in the share of physical capital, α, in $E_\zeta^{g^*}$ and $E_\eta^{g^*}$. The cost ζ interacts with physical capital through the no-arbitrage condition that equalises the real rate of return of R&D investment and of physical capital accumulation; in turn, this connects to the marginal productivity of capital, which implies thatζ impacts g^* with an elasticity $\alpha - 1$. The cost η relates to physical capital directly through its share in production, which implies that it impacts g^* with an elasticity $-\alpha$ (see (14)).

As an illustration, consider from Table 1 the elasticity values when $\alpha = 1/3$, $E_\zeta^{g^*} = -1.111$ and $E_\eta^{g^*} = -0.185$, and when $\alpha = 0.45$, $E_\zeta^{g^*} = -0.917$ and $E_\eta^{g^*} = -0.338$. Then, a government subsidy that implies a permanent reduction in ζ and η of 10% induces an increase in g^* of, respectively, 0.2222 and 0.0370 percentage points, if $\alpha = 1/3$, and of, respectively, 0.1834 and 0.0676 percentage points, if $\alpha = 0.45$. These results make clear the quantitative relevance of the subsidies to R&D investment vis-à-vis those to the production of intermediate goods, in spite of the complementarity of the two activities in the growth process, even in a context of a rising income share of capital. However, it should be also noted that our model with R&D and physical capital accumulation predicts a noticeably *smaller* growth effect of subsidies to R&D (i.e., of a reduction in ζ) than a similar model but without physical capital (see, e.g., Barro and Sala-i-Martin, ch. 6). In that case, for a similar calibration and fixing g^* at 0.02, we would get $E_\zeta^{g^*} = -1.666$, implying a significantly more elastic response of g^* to ζ.

In turn, Table 2 , 4th and 5th column, illustrates the impact of the level of i (and, thus, of π^*) on $E_\zeta^{g^*}$ and $E_\eta^{g^*}$, which allows us to analyse the predicted effect of the (structural) level of the monetary policy variable/long-run inflation rate on the effectiveness of real industrial policy shocks. We find that higher values of i imply more negative values of the elasticities $E_\zeta^{g^*}$ and $E_\eta^{g^*}$, although this effect is significantly less intense in the latter than in the former. However, again, it is important to look into the implication of this elasticity values in terms of *level* effects on g^*. In light of the numbers in Table 2, we see that, for example, considering the elasticity values for $i = 0.20$, $E_\zeta^{g^*} = -1.1879$ and $E_\eta^{g^*} = -0.1980$, a government subsidy that implies a permanent reduction in ζ and η of 10% leads to an increase in $g^*$of, respectively, 0.2031 and 0.0339 percentage points. Both numbers are smaller than those obtained for $i = 0.075$ (increase of 0.2222 and 0.0370 percentage points). So despite the *elasticities* being larger in modulus for a higher i , the fact that a higher i also decreases g^* induces smaller *level* effects of the industrial policy shocks on g^*. Therefore, according to these results, an industrial policy shock of given magnitude has a smaller impact (is less effective) on the level of long-run growth when the long-run inflation rate is higher.

5. Long-run velocity of money, R&D intensity, investment rate, and market structure

This section focuses on the long-run effect of changes in the (structural) level of the monetary policy variable, i, (or, equivalently, in the long-run inflation rate, π^*) on a set of relevant real macroeconomic variables – the physical investment rate, R&D intensity, and the velocity of money –, whose long-run empirical relationship with the inflation rate (alongside the economic growth rate and the real interest rate) has been emphasised by different strands of the literature. According to the empirical evidence, there tends to be, on the long run:

- A positive relationship between **i**nflation and velocity of money (e.g., Palivos and Wang, 1995; Dotsey and Ireland, 1996; Rodríguez Mendizábal, 2006);
- A negative relationship between inflation and both R&D intensity and the economic growth rate (e.g., Evers, Niemann, and Schiffbauer, 2007; Chu and Lai, 2013; Chu, Cozzi, Lai, and Liao, 2015);
- A negative relationship between inflation and both the real interest rate and physical investment rate (e.g., Gillman and Kejak, 2011 and references therein).

Based on own empirical evidence for OECD countries, we add to the list above:

- A positive relationship between inflation and average firm size in manufacturing, measured as the stock of capital per firm (see Table 3, Appendix A).

In order to associate our results with these empirical facts, we analyse the theoretical counterparts to the long-run velocity of money, R&D intensity and physical investment rate in our model, respectively, v^*, $(R/Y)^*$, and $\left(\dot{K}/Y\right)^*$. We use eq. (3), $R(t) = \zeta \cdot L \cdot \dot{N}(t)$, and $\dot{K}(t)/K(t) = \dot{N}(t)/N(t)$, to get

$$v^* = (Y/m)^* = \frac{A \cdot (k^*)^{\alpha}}{\eta^{\alpha} \cdot (\beta \cdot \zeta \cdot g^* + \Omega \cdot r^* \cdot k^*)}, \tag{20}$$

$$\left(\frac{R}{Y}\right)^* = \frac{\zeta}{A \cdot \eta^{\alpha}} \cdot (k^*)^{-\alpha} \cdot g^*, \tag{21}$$

$$\left(\frac{\dot{K}}{Y}\right)^* = \frac{1}{A} \cdot \eta^\alpha \cdot (k^*)^{1-\alpha} \cdot g^*. \tag{22}$$

We also wish to focus on the effects on the market structure along the BGP, measured as the number of firms, $N(t)^*$, and average firm size, $(K/Y)^*$.[24] From (15) and from $k(t) \equiv K(t)/(L \cdot N(t))$, we get

$$N^*(t) = N_0 \cdot e^{g^* t} \tag{23}$$

$$\left(\frac{K}{N}\right)^* = k^* \cdot L. \tag{24}$$

The following proposition summarises the relationship between the (structural) level of the monetary policy variable/long-run inflation rate and the real macroeconomic variables of interest.

Proposition 2. Assuming everything else constant, a higher nominal interest rate, i (and, thus, a higher long-run inflation rate, π^*) implies: (i) smaller long-run R&D intensity, $(R/Y)^*$, and physical investment rate, $\left(\dot{K}/Y\right)^*$; (ii) a higher long-run velocity of money, v^*; (iii) a smaller long-run number of firms $N^*(t)$, for each given t, but an ambiguous effect on the long-run average firm size $(K/N)^*$– the latter is increasing (decreasing) in i if $\beta > \Omega$ ($\beta < \Omega$) and is independent of i if $\beta = \Omega$.

Proof. Obtained by checking the partial derivatives of eqs. (20)-(24) with respect to i.

According to Proposition 2, the sign of the theoretical relationships between the monetary policy variable/long-run inflation rate and the long-run physical investment rate, R&D intensity, and the velocity of money – together with the real interest rate and the economic growth rate, already addressed in Proposition 1 (Section 4.1) – correspond to the patterns identified by the cited empirical literature.

As regards part (i) of Proposition 2, it is noteworthy that we obtain a negative relationship between inflation and the long-run R&D intensity that is robust to the relative degree of CIA constrains on R&D vis-à-vis manufacturing, in contrast to Chu and Cozzi (2014). The underlying mechanism in our model is the same that explains the relationship between inflation and the economic growth rate and the real interest rate, as laid out in Section 4.1, above.

Interestingly, we also obtain a negative relationship between inflation and the long-run physical investment rate, jointly with the already referred to negative relationship between inflation and the real interest rate. Although being in line with the empirical evidence, this dual negative effect of inflation amounts to seemingly opposite theoretical Tobin effects, as noted by Gillman and Kejak (2011). The effect of inflation on

[24] Alternatively, average firm size can be measured as $(X/N)^* = k^* L/\eta$; the resulting analytical expression is identical to $(K/N)^*$, up to a factor $1/\eta$.

physical investment rate decomposes into the (ambiguous) effect on the physical capital-effective labour ratio and the (negative) effect on the economic growth rate (see (22) and Proposition 1). The latter effect always dominates the former. Although under a distinct analytical setting, the growth effect also dominates in Gillman and Kejak (2011), thereby leading to an overall negative inflation-investment rate effect. But, in contrast, our results are not conditional on a specific interval of values for the inflation rate, while being compatible with inflation effects on other real macroeconomic variables also in line with the empirical evidence.

As regards part (ii) of Proposition 2, the clear-cut positive relationship between the velocity of money and the nominal interest rate (and inflation) in our model is driven by the fact that the demand for money emanates from the CIA constraints on R&D and manufacturing activities, whereas no CIA applies to consumption (see fn. 8, above). However, it can be shown that a version of our model with a CIA constraint also applying to consumption, with a parameter ξ controlling for its magnitude, is also able to generate a positive relationship as long as the coefficients β and Ω are sufficiently large relative to ξ. On the other hand, the fact that the presence of a CIA constraint on consumption would weaken the negative relationship between the velocity of money and the nominal interest rate (and inflation) addresses the empirical evidence emphasised by Rodríguez Mendizábal (2006), according to which a relatively low correlation between the velocity of money and inflation may occur in some samples reflecting diverse transaction technologies across countries. In our model, that diversity would arise as differences in the fractions of consumption expenditure and of R&D and manufacturing costs that must be financed by the households' cash balances across countries, and which may lead to a weakened relationship between the velocity of money and the nominal variables.

Finally, by allowing for endogenous firm entry, simultaneously with endogenous physical capital and knowledge accumulation, our model addresses the effects of inflation on the long-run market structure, assessed by both the number of firms and average firm size. We uncover an alternative analytical mechanism (and results) to those explored by Wu and Zhang (2001) and Chu and Ji (2016), by highlighting the role of the relative fraction of R&D and of manufacturing costs that are subject to a CIA constraint. We confirm the negative relationship between the number of firms and inflation obtained by those papers, but, in contrast, average firm size can vary negatively, positively or be independent of inflation, depending on the strength of the CIA constraint on R&D activities vis-à-vis that on manufacturing of intermediate goods. Wu and Zhang (2001) predict a negative relationship, while Chu and Ji (2016) predict average firm size is independent of inflation, with firm size measured as employment per firm. In our model, long-run average firm size is measured as capital per firm and, thus, inherits the features of the relationship between the long-run physical capital-effective labour ratio, k^*, and inflation (see Section 4.1, above). Thus, when the CIA constraint on R&D is stronger than that on manufacturing, as suggested by the empirical literature on firm financing referred to above, firm size and inflation display a unequivocal positive relationship. This result is in line with the empirical evidence concerning a set of OECD countries, as described earlier.

6. Concluding remarks

This paper studies the real long-run effects of inflation and of the structural stance of monetary policy in the context of a monetary model of R&D-driven endogenous growth complemented with physical capital accumulation. We study the effects on a large set of relevant real macroeconomic variables – the economic growth rate, the real interest rate, the physical investment rate, R&D intensity, the velocity of money –, and on market-structure variables. These effects have been analysed from the perspective of different strands of the theoretical and empirical literature over time.

Following a recent literature on R&D-driven growth, we introduce money demand in the model by considering CIA constraints on R&D investment and on manufacturing of intermediate goods. Our setup implies that the latter also impacts physical capital accumulation. Given the complementarity between physical capital accumulation and R&D along the growth process, this implies a novel kind of interrelation between the two considered CIA constraints, in comparison with the previous literature.

Our paper obtains a negative relationship between long-run inflation and the economic growth rate, real interest rate, physical investment rate, and R&D intensity, that is robust to the relative degree of CIA constrains in R&D vis-à-vis manufacturing, in contrast to previous models. In all cases (including the velocity of money), the sign of the theoretical relationships is in line with recent empirical evidence. This setup also allows us to discuss the impact of a decrease in the labour income share (as pointed out by recent empirical literature) in the magnitude of the real long-run effects of inflation and related nominal variables.

As regards our quantitative results, we highlight that: (i) the model predicts a significant negative effect of inflation on the long-run economic growth rates (the magnitude of the effect per percentage point of change in i is in line with that found in several other models in the literature); (ii) however, a decrease in the labour income share as the one observed in recent empirical data may imply a not unnoticeable reduction in the magnitude of the real long-run effects of inflation, if the liquidity constraints on R&D are relatively large (as seems to be the case empirically); (iii) we obtain higher *elasticities* of the economic growth rate with respect to long-run inflation for higher levels of inflation; however, when taking into account a given *level* change in the long-run inflation and its *level* effects on economic growth, the latter are marginally weaker for higher levels of the inflation rate, which corroborates the non-linearity of the inflation-growth effects emphasised by some strands of the empirical literature; (iv) a decrease in the labour income share may also imply a reduction in the degree of this non-linearity, again if the liquidity constraints on R&D are relatively large; (v) real industrial policy shocks (e.g., in the form of a government subsidy to R&D or manufacturing of intermediate goods) have a smaller impact on long-run growth when the long-run inflation rate is higher, thus implying that inflation reduces the effectiveness of industrial policy shocks.

Finally, our setup also underlines the role of the relative fraction of R&D and of manufacturing costs that are subject to a CIA constraint as regards the effects of inflation on the long-run market structure. We obtain the usual negative relationship between the number of firms and inflation, while average firm size varies positively with inflation, in case the liquidity constraints are relatively large on R&D activities. As a complement, we present novel cross-country evidence on the empirical relationship between the average firm size (measured as the stock of capital per firm) and the long-run inflation rate, which indicates a positive correlation when a number of macroeconomic, financial, and regulation and governance variables are used as controls. Our analysis of the firm size-inflation relationship adds to the existing macro literature on the structural determinants of average firm size and firm size distribution (e.g., Poschke, 2014 and Gomes and Kuehn, 2017). However, for simplicity, our theoretical model assumes homogeneity at the firm level. It would be certainly insightful to examine this issue in more detail by explicitly allowing for endogenous firm heterogeneity and, thereby, non-degenerate firm size distribution. This could be done along the lines of, e.g., Gil and Figueiredo (2013), who derive and study a heavy-tailed and asymmetric long-run firm size distribution in the context of an endogenous growth model where, by allowing both horizontal and vertical R&D, incumbent firm heterogeneity arises as vertical innovation follows a firm-level Poisson process.

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A. Average firm size and inflation: data and estimation results

In this appendix, we investigate the relationship between average firm size and long-run inflation using OECD cross-country data. Average firm size is measured as the stock of capital per firm in the manufacturing sector in 2008 and the inflation rate is measured as the average annual change rate of the GDP deflator in 1995-2007, both available from the OECD online database at `http://stats.oecd.org/`. We measure firm size this way because this is the empirical counterpart to the theoretical variable of firm size in the model, K/N (see eq. (24)).

Considering data availability on firm size, the list of countries in the (full) sample is: Australia, Austria, Belgium, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, South Korea, Luxembourg, Mexico, Netherlands, Norway, Poland, Portugal, Slovakia, Slovenia, Sweden, the United Kingdom, the United States, and Lithuania (non-OECD country).

The controls that we include in our cross-country regressions of average firm size on long-run inflation follow from close the macro literature on the structural determinants of firm size (see Gomes and Kuehn, 2017, and the references therein). The list of controls and respective data source is the following:

- GDP per capita, 1995, from the OECD online database at `http://stats.oecd.org/`. This variable captures the overall economic development level, which may potentially correlate positively with firm size. Considering the *initial* GDP per capita also allows us to account for possible catching-up effects in the inflation rate in those countries initially further away from the technological frontier;

- Population, 2007, from the OECD online database at `http://stats.oecd.org/`. This is typically used as measure of the scale of the economy, or, more concretely, as a proxy for the size of the domestic market, and it is also expected to correlate positively with firm size;

- Educational attainment of the population over the age of 15, 2005, from the Barro-Lee dataset, available at `www.barrolee.com/` and described in Barro and Lee (2013). The selected indicators are: the fraction of the population whose highest level attained is secondary education (completed or not); the fraction of the population whose highest level attained is tertiary education (i.e., college education, completed or not); average years of total schooling. As indicators of the human capital stock, these are expected to correlate positively with firm size, in as much as a higher endowment of human capital tends to raise firm productivity as well as the number of entrepreneurs with higher managerial ability;

- Cognitive skills measured by the average standardised scores on international student achievement tests in math and science, primary through the end of secondary school, available at `www.cesifo.de/woessmann#data` and described in Hanushek and Woessmann (2012, App. B). This intends to be a direct measure of educational achievement and, as such, an alternative indicator of human capital.[25] This variable is also expected to correlate positively with firm size;

- Total market entry costs as a percentage of output per worker, from Bento (2014). These costs are composed of regulatory costs (broken down into monetary regulatory start up costs and wage costs of both startup delays and the post-start up time required to do taxes, all based on the World Bank Doing Business Survey)

[25]The argument goes that the studies using average years of schooling, or other quantitative indicator of schooling, as a measure of human capital implicitly assume that a year of schooling delivers the same increase in skills and competences regardless of the education system, as well as that formal schooling is the primary source of education. In contrast, this strand of the literature concentrates directly on the role of cognitive skills, by using measures of educational achievement derived from the international student achievement tests (e.g., the OECD PISA scores; see Hanushek and Kimko, 2000, for an earlier significant contribution along these lines).

and nonregulatory startup costs. Being a proxy for the level of barriers to market entry, this variable may potentially correlate positively with firm size;

- Financial depth indicators, 2007, from the Global Financial Development database, available at www.worldbank.org/financialdevelopment, and described in Cihák, Demirgüç-Kunt, Feyen, and Levine (2013). The eight selected indicators are: deposit money banks' assets to GDP; liquid liabilities to GDP; domestic credit to private sector to GDP; stock market capitalization to GDP; gross portfolio equity liabilities to GDP; gross portfolio debt liabilities to GDP; bank deposits to GDP; and number of listed companies per 1,000,000 people. These variables work as measures of financial development and, as such, may potentially correlate with firm size, although with an ambiguous sign (higher financial development may facilitate firm entry but also post-entry firm growth);

- Governance indicators, 2007, from the World Bank Worldwide Governance Indicators, available at http://info.worldbank.org/governance/wgi/, as measures of institutional quality. The six indicators are: "Control of corruption"; "Government effectiveness"; "Political stability and absence of violence/terrorism"; "Regulatory quality"; "Rule of law"; and "Voice and accountability". These variables also potentially correlate with firm size, although again with an ambiguous sign;

- Government debt-to-GDP ratio, 2007, and Government balance-to-GDP ratio, average 1995-2007, from the OECD online database at http://stats.oecd.org/. These indicators are also interpreted as proxies of institutional quality.

As our baseline exercise, we run a cross-country OLS regression of the average firm size on the inflation rate, to estimate the unconditional correlation between the two variables, and then compare the result with the estimates of the conditional (partial) correlation obtained by also considering a number of controls, as described above. Table 3 reports the details on the baseline regression, column (1), and on a selection of regressions that include controls, columns (2)-(6).[26] Comparing regressions (1) and (2) suggests that the negative unconditional correlation obtained in the first regression essentially reflects a catching-up effect in the inflation rate, as those countries that are initially less developed tend to feature relatively higher average inflation rates (usually paralleled by also relatively larger economic growth rates) and smaller average firm sizes than the already more developed countries. The correlation becomes positive as soon as initial GDP per capita is added to the regression, with this control featuring a positive and significant coefficient. Overall, when additional control variables are included, the coefficient for the inflation rate remains positive and significant. We attest the crucial role played by initial GDP per capita by re-estimating regressions (2)-(6) without this control variable (not shown in the table) and observing that the coefficient for the inflation rate either remains positive but is no longer significant or becomes negative (either significant or non-significant).

[26]Regression (5) includes 25 observations, because there is no available data on "Liquid liabilities" for Estonia. Regression (6) includes 21 observations, because of missing data on "Total entry costs" for Czech Republic, Denmark, Estonia, Slovenia, and Lithuania.

[Table 3 goes about here]

We also carry out robustness checks where we test for all the controls listed above, besides those included in Table 3.[27] Below we summarise our results as regards the set of controls (besides GDP per capita) and draw a brief comparison with those obtained by Gomes and Kuehn (2017), as reported in their online Appendix – though, when comparing the results, one should bear in mind that Gomes and Kuehn measure firm size as the number of persons employed per firm, consider the Total Industry and Market Services and, in general, use a larger sample of countries than ours (despite also considering a sub-sample of OECD countries in one of their robustness exercises):

- Population always displays a positive coefficient in our exercises, although in some regressions it appears as non-significant. Gomes and Kuehn (2017) estimate both positive and negative coefficients for population in their different exercises. For a sub-sample of OECD countries, the authors estimate a negative, non-significant, coefficient for population.

- The fraction of the population with secondary education consistently appears with a negative significant coefficient, while the fraction with tertiary education displays a positive coefficient. Replacing the latter with the "Cognitive skills" variable maintains the signs while allowing for more precise estimates overall and, in particular, as regards the coefficient on inflation. In contrast, Gomes and Kuehn (2017) consistently get a positive and significant impact of secondary education. For a sub-sample of OECD countries, the authors find both secondary and tertiary have a positive impact, although only the former remains significant after the inclusion of other variables in the regression. These results suggest that the type of human capital that favours large firms from the point of view of capital per firm (via tertiary education) is different from the type that favours large firms as regards the number of persons employed per firm (secondary education).

- Total entry costs as a percentage of output per worker display a positive and significant coefficient when also considered with the fraction of the population with tertiary education or with the "Cognitive skills" variable. In contrast, Gomes and Kuehn (2017) use the variable "Business start-up costs" in one regression and get a negative and significant coefficient.

- As regards the eight financial depth indicators, only liquid liabilities to GDP, gross portfolio equity liabilities to GDP, gross portfolio debt liabilities to GDP, and bank deposits to GDP appear with positive and significant coefficients, although bank deposits to GDP looses significance when the "Political stability" variable is included. In turn, the number of listed companies per 1,000,000 people has a negative and significant coefficient when Government debt-to-GDP and Government balance-to-GDP ratios are also included in the regression. In Gomes and Kuehn

[27] Summary statistics for all variables and additional estimation results are available from the authors upon request.

	(1)	(2)	(3)	(4)	(5)	(5)'	(6)
Constant	1.187^{***}	-0.964^{***}	-0.923^{***}	-3.976^{***}	-4.050^{***}	-5.067^{***}	-7.970^{***}
(s.e.)	(0.180)	(0.287)	(0.317)	(1.413)	(1.375)	(1.257)	(1.086)
Inflation rate	-12.536^{***}	5.744^{*}	5.056	11.395^{***}	11.964^{***}	13.729^{***}	10.984^{***}
(s.e.)	(3.079)	(2.817)	(3.079)	(4.181)	(2.776)	(2.799)	(2.947)
GDP pc	-	1.936^{***}	1.817^{***}	2.033^{***}	1.193^{***}	1.408^{***}	1.761^{***}
(s.e.)		(0.287)	(0.342)	(0.393)	(0.338)	(0.294)	(0.218)
Population	-	-	0.180^{**}	0.187^{***}	0.406^{***}	0.395^{***}	0.552^{***}
(s.e.)			(0.086)	(0.062)	(0.074)	(0.071)	(0.079)
Secondary educ.	-	-	-	-0.009^{**}	-0.010^{***}	-0.011^{***}	-0.010^{***}
(s.e.)				(0.004)	(0.003)	(0.003)	(0.003)
Cognitive skills	-	-	-	3.195^{**}	3.215^{**}	4.112^{***}	5.828^{***}
(s.e.)				(1.272)	(1.310)	(1.251)	(1.134)
Liquid liabilities	-	-	-	-	0.004^{***}	0.004^{***}	0.002^{**}
(s.e.)					(0.0008)	(0.0007)	(0.0008)
Political stability	-	-	-	-	0.387^{**}	0.273^{*}	0.652^{***}
(s.e.)					(0.165)	(0.159)	(0.187)
Total entry costs	-	-	-	-	-	-	0.339^{***}
(s.e.)							(0.104)
Observations	26	26	26	26	25	21	21
R^2	0.213	0.725	0.765	0.807	0.883	0.900	0.946
Adj.R^2	0.188	0.701	0.733	0.759	0.834	0.846	0.910

Table 3: Partial association between average firm size, measured as the stock of capital per firm in manufacturing in 2008 (dependent variable), and the inflation rate, measured as the average annual change rate of the GDP deflator in 1995-2007, both from the OECD online database, `http://stats.oecd.org/`. The controls are: "Population" (population size, 2007, OECD online database); "Secondary educ." (the fraction of the population whose highest level attained is secondary education, 2007, Barro-Lee dataset, `http://www.barrolee.com/`); "Cognitive skills" (average standardised scores on international student achievement tests in math and science, `www.cesifo.de/woessmann#data`); "Liquid liabilities" (liquid liabilities to GDP, 2007, Global Financial Development database, `www.worldbank.org/financialdevelopment`); "Political stability" (Political Stability and Absence of Violence/Terrorism indicator, 2007, Worldwide Governance Indicators, `http://info.worldbank.org/governance/wgi/`); "Total entry costs" (regulatory and nonregulatory entry costs to output per worker, Bento, 2014). Standard errors (s.e.) are heteroskedasticity consistent. *** indicates significance at the 1% level, ** indicates significance at the 5% level, and * indicates significance at the 10% level.

(2017), all the considered financial development indicators are non-significant (although their financial indicators do not entirely coincide with the ones we use in our study).[28]

- As for the six governance indicators, only the variable "Political stability" is significant, appearing with a positive coefficient in our regressions. In Gomes and Kuehn (2017), "Political stability" appears with a positive and significant coefficient (in one regression), while "Voice and accountability" appears with a negative and significant coefficient (in another regression).

- Government debt-to-GDP and Government balance-to-GDP ratios both appear with negative coefficients, although without significance in several equations (in particular when the financial depth indicators are also included). The observed pattern suggests that a lower debt ratio is associated with larger average firm size and a higher balance ratio is associated with a smaller average firm size. These variables are not used as controls in Gomes and Kuehn (2017).

B. Household's dynamic optimisation problem

Following the standard Optimal Control Theory, the maximisation of intertemporal utility (6) requires the consideration of the auxiliary Hamiltonian function[29]

$$\mathcal{H} = \left(\frac{C(t)^{1-\theta}-1}{1-\theta}\right)\cdot e^{-\rho t}+v(t)\cdot(r(t)\cdot a(t)+w(t)\cdot L-C(t)+\tau(t)-\pi(t)\cdot m(t)+i(t)\cdot b(t))+$$

$$+\lambda(t)\cdot(b(t)-m(t))$$

were a and m are the state variables, v and λ are the costate variables, and C and b are the control variables. Then, the necessary conditions under the Maximum Principle are:

a) $\partial\mathcal{H}/\partial C(t)=0\Leftrightarrow e^{-\rho t}\cdot C(t)^{-\theta}=\nu(t)$

b) $\partial\mathcal{H}/\partial b(t)=0\Leftrightarrow \nu(t)\cdot i(t)+\lambda(t)=0$

c) $\partial\mathcal{H}/\partial a(t)=-\dot{\nu}(t)\Leftrightarrow \nu(t)\cdot r(t)=-\dot{\nu}(t)$

d) $\partial\mathcal{H}/\partial m(t)=-\dot{\nu}(t)\Leftrightarrow -\nu(t)\cdot\pi(t)-\lambda(t)=-\dot{\nu}(t)$

e) $\partial\mathcal{H}/\partial v(t)=\dot{a}(t)+\dot{m}(t)$

f) $\partial\mathcal{H}/\partial\lambda(t)=0$

g) $\lim_{t\to+\infty}\nu(t)\cdot a(t)=0;\ \lim_{t\to+\infty}\nu(t)\cdot m(t)=0$

Using b), c) and d) yields $\nu(t)\cdot r(t)=-\nu(t)\cdot\pi(t)+\nu(t)\cdot i(t)$. Then, by dividing both sides of the equation by $\nu(t)$ and rearranging terms, we get the non-arbitrage equation (8) in the text. Considering a) and b), applying logarithms and deriving with respect to time gives us the consumption Euler equation (9). Finally, using a) together with g) yields the transversality conditions in the text.

[28] Also note that, in their robustness exercises, Gomes and Kuehn (2017) do not report results considering simultaneously controls for institutional quality and for financial development as we do.

[29] In doing so, we follow the usual approach in the literature and consider the (static) CIA constraint is binding, i.e., $b(t)=m(t)$.

C. Derivation of the growth rate elasticities

Recalling eqs. (14) and (15), we get the BGP growth rate

$$g^* = \frac{1}{\theta} \cdot \left\{ \left(\frac{1}{\zeta}\right)^{1-\alpha} \cdot \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} \cdot \frac{A \cdot \alpha^2}{(1+\Omega \cdot i)^{\alpha} \cdot (1+\beta \cdot i)^{1-\alpha} \cdot \eta^{\alpha^2}} - \rho \right\}.$$

Firstly, we derive the elasticity of g^* with respect to i, $E_i^{g^*} = \frac{\partial g^*}{\partial i} \cdot \frac{i}{g^*}$. Consider

$$\frac{\partial g^*}{\partial i} = w_0 \cdot \left[\frac{1}{(1+\Omega \cdot i)^{\alpha} \cdot (1+\beta \cdot i)^{1-\alpha}} \right] \cdot \left[\frac{(\alpha-1) \cdot \beta}{1+\beta \cdot i} - \frac{\alpha \cdot \Omega}{1+\Omega \cdot i} \right],$$

where $w_0 \equiv \frac{1}{\theta} \cdot \left(\frac{1}{\zeta}\right)^{1-\alpha} \cdot \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} \cdot A \cdot \alpha^2 \cdot \eta^{-\alpha^2}$. By noting that, then,

$$g^* = w_0 \cdot \left[\frac{1}{(1+\Omega \cdot i)^{\alpha} \cdot (1+\beta \cdot i)^{1-\alpha}} \right] - \frac{\rho}{\theta},$$

we get

$$\frac{\partial g^*}{\partial i} = -\left(g^* + \frac{\rho}{\theta}\right) \cdot \left[\frac{(1-\alpha) \cdot \beta}{1+\beta \cdot i} + \frac{\alpha \cdot \Omega}{1+\Omega \cdot i} \right],$$

and thus

$$E_i^{g^*} = -\left(g^* + \frac{\rho}{\theta}\right) \cdot \left[\frac{(1-\alpha) \cdot \beta}{1+\beta \cdot i} + \frac{\alpha \cdot \Omega}{1+\Omega \cdot i} \right] \cdot \frac{i}{g^*},$$

which is equivalent to (17) in the text.

Secondly, we derive the elasticity of g^* with respect to ζ, $E_i^{g^*} = \frac{\partial g^*}{\partial \zeta} \cdot \frac{\zeta}{g^*}$. Following the same steps as before, consider

$$\frac{\partial g^*}{\partial \zeta} = w_1 \cdot \left(\frac{1}{\zeta}\right)^{1-\alpha} \cdot (\alpha - 1) \cdot \frac{1}{\zeta},$$

where $w_1 \equiv \frac{1}{\theta} \cdot \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} \cdot \frac{A \cdot \alpha^2}{(1+\Omega \cdot i)^{\alpha} \cdot (1+\beta \cdot i)^{1-\alpha} \cdot \eta^{\alpha^2}}$. Then, noticing that

$$g^* = w_1 \cdot \left(\frac{1}{\zeta_n}\right)^{1-\alpha} - \frac{\rho}{\theta},$$

we have

$$\frac{\partial g^*}{\partial \zeta} = \left(g^* + \frac{\rho}{\theta}\right) \cdot \frac{\alpha - 1}{\zeta},$$

and thus

$$E_{\zeta}^{g^*} = \left(g^* + \frac{\rho}{\theta}\right) \cdot \frac{\alpha-1}{\zeta} \cdot \frac{\zeta}{g^*} = -\left(g^* + \frac{\rho}{\theta}\right) \cdot \frac{1-\alpha}{g^*},$$

which is equivalent to (18) in the text.

Finally, we derive the elasticity of g^* with respect to η, $E_i^{g^*} = \frac{\partial g^*}{\partial \eta} \cdot \frac{\eta}{g^*}$. Consider first that

$$\frac{\partial g^*}{\partial \eta} = -w_2 \cdot \frac{1}{\eta^{\alpha^2}} \cdot \alpha^2 \cdot \frac{1}{\eta},$$

with $w_2 \equiv \frac{1}{\theta} \cdot \left(\frac{1}{\zeta_n}\right)^{1-\alpha} \cdot \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} \cdot \frac{A \cdot \alpha^2}{(1+\Omega \cdot i)^{\alpha} \cdot (1+\beta \cdot i)^{1-\alpha}}$. Given that we also have

$$g^* = w_2 \cdot \frac{1}{\eta^{\alpha^2}} - \frac{\rho}{\theta},$$

then

$$\frac{\partial g^*}{\partial \eta} = -\left(g^* + \frac{\rho}{\theta}\right) \cdot \frac{\alpha^2}{\eta},$$

and

$$E_\eta^{g^*} = -\left(g^* + \frac{\rho}{\theta}\right) \cdot \frac{\alpha^2}{\eta} \cdot \frac{\eta}{g^*} = -\left(g^* + \frac{\rho}{\theta}\right) \cdot \frac{\alpha^2}{g^*},$$

which allows us to get (19) in the text.

D. Non-linear BGP growth effects of changes in the monetary policy variable

In this appendix, we compare our quantitative results with those gathered by Gillman and Kejak (2005a, Table 9) from a number of growth models in the literature. Table 4 shows the magnitude of the BGP growth effects in our model when i is raised from 0 to 0.10 and from 0.10 to 0.20, for given selected values of the capital share, α, and the CIA parameters (β, Ω). Our quantitative results concerning the degree of marginal non-linearity of the BGP growth effects are in line with those listed by Gillman and Kejak. Also notice that, similar to the results depicted by Table 1, in the text, the level change in g^* is smaller (larger) when α is larger if $\beta > \Omega$ ($\beta < \Omega$) and is independent of α if $\beta = \Omega$. Interestingly, Table 4 shows that the degree of non-linearity of the BGP growth effects relates to α and (β, Ω) in exactly the same way.

[Table 4 goes about here]

E. Optimality of the Friedman rule and of R&D and physical investment rates

The intertemporal utility function (6) can be equivalently rewritten as

$$U = \frac{1}{1-\theta} \int_0^\infty C(0)^{1-\theta} \cdot e^{[(1-\theta) \cdot g^* - \rho] \cdot t} - e^{-\rho t} dt,$$

where g^* is given by (15). By normalising $C(0)$ to unity and solving the integral, we get

Δ Nominal interest rate, i	Δ Growth rate, g^*	Δ Real interest rate, r^*	Δ Growth rate, g^*	Δ Real interest rate, r^*
	$\beta = 1; \Omega = 0.5;\ \alpha = 1/3$		$\beta = 1; \Omega = 0.5;\ \alpha = 0.45$	
$0.00 \to 0.10$	-0.0027	-0.0041	-0.0025	-0.0038
$0.10 \to 0.20$	-0.0023	-0.0035	-0.0022	-0.0033
Degree of non-linearity:	14.7%		13.9%	
	$\beta = 1; \Omega = 1;\ \alpha = 1/3$		$\beta = 1; \Omega = 1;\ \alpha = 0.45$	
$0.00 \to 0.10$	-0.0033	-0.0049	-0.0033	-0.0049
$0.10 \to 0.20$	-0.0027	-0.0041	-0.0027	-0.0041
Degree of non-linearity:	16.7%		16.7%	
	$\beta = 0.5; \Omega = 1;\ \alpha = 1/3$		$\beta = 0.5; \Omega = 1;\ \alpha = 0.45$	
$0.00 \to 0.10$	-0.0022	-0.0033	-0.0024	-0.0036
$0.10 \to 0.20$	-0.0019	-0.0029	-0.0021	-0.0031
Degree of non-linearity:	12.3%		13.2%	

Table 4: Marginal non-linear BGP growth effects of changes in the monetary policy variable, i, for selected values of α and (β, Ω). $\rho = 0.02$; $\theta = 1.5$; $\eta = 1$;$\zeta = 3.85$. The baseline is $g^* = 0.02$, $\pi^* = 0.025$, and $r^* = 0.05$, for $i = 0.075$ and $A = 0.74$. The degree of non-linearity is measured as $|(\Delta_{0.1\to0.2}g^* - \Delta_{0.0\to0.1}g^*)/\Delta_{0.0\to0.1}g^*|$. The three models in Gillman and Kejak (2005a, Table 9) that feature quantitatively relevant marginal non-linearities in the growth effects display degrees of non-linearity of, respectively, 7.1%, 16.3% and 19.1%.

$$U = -\frac{1}{(1-\theta)\cdot[(1-\theta)\cdot g^* - \rho]} - \frac{1}{\rho\cdot(1-\theta)}.$$

Then, consider the derivative

$$\frac{\partial U}{\partial i} = \frac{\partial g^*/\partial i}{[(1-\theta)\cdot g^* - \rho]^2},$$

where, from Appendix C, we have

$$\frac{\partial g^*}{\partial i} = -\left(g^* + \frac{\rho}{\theta}\right)\cdot\left[\frac{(1-\alpha)\cdot\beta}{1+\beta\cdot i} + \frac{\alpha\cdot\Omega}{1+\Omega\cdot i}\right] < 0.$$

Therefore, we also get $\partial U/\partial i < 0$. Given the non-negativity constraint on i, $i \geq 0$, we conclude that the households' intertemporal utility is maximised when $i = 0$, which implies that, under the considered analytical setup, the Friedman rule of a zero nominal interest rate is optimal, a usual result in this literature (e.g., Chu and Cozzi, 2014).

However, since it can also be shown that $\partial(R/Y)^*/\partial i < 0$ and $\partial(\dot{K}/Y)^*/\partial i < 0$ (see Section 5, in the text), this implies that, at the (optimal) Friedman rule ($i = 0$), the long-run R&D intensity, $(R/Y)^*$, and physical investment rate, $(\dot{K}/Y)^*$, are suboptimal, with underinvestment in both R&D and physical capital in the corresponding long-run equilibrium.