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Financial Intermediation in Economies with Investment Complementarities*

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Abstract

When individual returns are increasing in the aggregate level of investment, decentralized individuals fail to internalize the positive externality of their investment on the return of others. This paper shows how financial intermediation mitigates this coordination failure for individuals with private information. When providing

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financial products with low risk, intermediaries induce individuals with unfavorable private information to invest more. The increase in investment generates positive externalities, thereby raising social welfare and making banks socially desirable.

Keywords: Banking, Macroeconomics, Incomplete Information, Coordination, Complementarities, Externalities

JEL Classification Codes: G21, E44, D82, D62, C72

1 Introduction

Confidence and expectations are critical to determine equilibrium allocations in economies with production externalities since there is the possibility of coordination failure. Decentralized individuals do not internalize the positive externality of their investment on the return of others, thereby investing too little. Within this class of economies, how do financial intermediaries permit more effective coordination in the market? And how do intermediaries affect equilibrium allocations and social welfare?

To answer these questions, we consider an economy with production externalities in which individuals have private information about the underlying economic fundamentals. Our analysis builds on the model of Angeletos and Pavan (2004) in which the return to individual investment is increasing in the aggregate level of investment. Their framework has two main advantages. First, it is possible to compute social welfare explicitly and, second, results do not hinge on the volatility of the underlying economic fundamentals. We extend their model by adding financial intermediaries which transfer funds from a pool of investors to a pool of firms. Intermediaries are able to reduce the volatility of the underlying economic fundamentals of individual firms by monitoring them and, as a result, intermediaries are able to offer financial products which pay a relatively constant return across states.

The novel contribution of the paper is to show how financial intermediation alleviates the coordination failure which arises in economies with investment externalities. In a nutshell, the central argument of the paper is that intermediaries are able to create safe assets which entice investment by those individuals with unfavorable private information—a flight-to-quality effect by pessimistic individuals. Investment externalities raise individual returns across the economy, further stimulating investment and increasing social welfare.

In Section 2, we consider a market-based financial system in which individuals with homogenous expectations invest directly in firms, and show how coordination failures generate underinvestment. Better public information is beneficial for social welfare, since investors use public information to fine-tune their choices and take more efficient decisions.

In Section 3, we consider the existence of private information, which introduces heterogeneity in expectations about the underlying fundamentals. Heterogenous beliefs engender cross-sectional heterogeneity in investment choices, as pessimistic individuals (who received unfavorable private information) invest less than optimistic individuals.

We then consider the existence of financial intermediaries which alleviate the agency problems resulting from the relationship between investors and firms. One important problem arises if the manager of the firm must take some action to make proper use of the funds they have obtained from investors. For example, the manager may have the possibility to choose between two projects: one with high risk and private benefits and another with low risk and no private benefits. Investors cannot observe the manager's decisions, but the financial intermediary can observe the manager's actions by paying a monitoring cost. Hence, investors hire the intermediary to check what the manager is doing, and prevent him from choosing the riskiest project.[1] By monitoring the firm, financial intermediaries are able to transform risky investment projects into safe projects, thus enabling intermediaries to offer safe financial products to investors. Indeed, traditional banking activities transform risky investment in firms into safer financial assets, like time deposits.

We contrast the results between a financial system based exclusively on direct finance, and a financial system with coexistence between direct and intermediated finance. Pessimistic individuals prefer investing in safer financial products offered by financial intermediaries rather than investing directly in firms. Pessimistic individu-

[1] We assume that only the intermediation sector has access to the monitoring technology, or it is efficient to have a bank as a *delegated monitor*. In Germany and Japan, banks have large equity stakes in large corporations and perform a very important corporate governance role in large corporations.

als end up investing more than they would invest in a market-based financial system, thus raising aggregate investment. Individual returns increase as a result of investment complementarities, thereby inducing investment by optimistic investors. Aggregate investment and social welfare increase with coexistence between intermediated and direct finance.

Since it is possible to compute social welfare explicitly, we provide policy recommendations on how to mitigate coordination failures. Our results suggest stimulating financial intermediation when the degree of strategic complementarity between firms is large and there is substantial uncertainty regarding the fundamentals.

Our analysis has implications not only for economic policy but also for empirical work. Our model provides testable implications regarding the extent to which firms, industries, and regions can be expected to suffer from restrictions in intermediated finance or can be expected to benefit from government policies which boost indirect finance. In particular, changes in bank lending and credit policies will have the most impact where strategic complementarities are the most prevalent.

Review of the literature. Economies with production externalities are one example in which strategic complementarities play a prominent role. The individual firm's production function displays production externalities when the productivity of the individual firm increases with aggregate production. For instance, Cooper and John (1988) consider a model with technological complementarities among input suppliers to a shared production process of a public good, while Bryant (1983) shows that specialization and imperfect information lead to strategic complementarities among producers. In both frameworks, an increase in aggregate production will raise individual gains.

Another justification for the existence of production externalities is Alfred Marshall's concept of external scale economies. According to Marshall (1890), there are three sources of external scale economies at the firm level. First, there is the potential for more extensive interaction between suppliers and buyers, allowing for productivity gains resulting from vertical disintegration and supplier specialization. In a similar vein, Diamond's (1982) search model assumes that an increase in the number of potential trading partners makes trade easier, which in turn makes production more efficient. Second, there is the firm's ability to capture industry-specific knowledge and information spillovers which take place in related industries. Third, there are benefits from a larger pool of skilled labor associated with a stronger industry, and which favors the firm-worker matching process.

The endogenous growth literature has also provided several justifications for the existence of production externalities. According to Romer (1986) and Lucas (1988), capital includes both physical and human components, and two key assumptions generate technological complementarities. First, knowledge creation is a side effect of physical investment. A firm that increases its physical capital learns simultaneously how to produce more efficiently. Second, each firm's knowledge is a public good that any other firm can access at zero cost. Once discovered, a piece of knowledge spills over across the whole economy so that all firms can benefit from it. Alternatively, Barro (1990) shows that tax-financed government services are another possible source of production externalities. In this case, the government's choices determine the productivity in the economy.

External economies also play an important role in shaping the pattern of international trade, and are decisive in shaping the pattern of interregional trade. Researchers in international trade and economic geography have joined geographers and urban economists in investigating the relationship between production externalities and geographical agglomeration (see, for example, Krugman 1991a, 1991b). Below, we suggest using geographical agglomeration as a measure of production externalities.

A number of authors have embedded technological complementarities in general equilibrium models. Baxter and King (1991) and Benhabib and Farmer (1994) build dynamic stochastic general equilibrium models with technological complementarities and a representative agent, whereas Acemoglu (1993) considers a model with technological complementarities and heterogenous individuals.

Morris and Shin (2002) analyze an environment with strategic complementarities and heterogenous information. Since complementarities are present only at the private level, they find that more transparent public information can reduce welfare. Unlike Morris and Shin (2002), Angeletos and Pavan (2004) consider an economy in which complementarities are present at the social level, so that more precise public information necessarily increases welfare. Still, none of these authors consider changes in the volatility of the underlying fundamentals, as we do.

The finance literature has used bankruptcy as an instrument to identify channels for (negative) spillover effects among firms. Lang and Stulz (1992) and Ferris, Jayaraman, and Makhija (1997) document spillover effects of bankruptcy filings on investors of industry peers. Hertzel, Li, Officer, and Rodgers (2008) examine bankruptcy contagion effects along the supply chain of filing firms, while Boone and Ivanov (2012) define proximate non-filing firms as strategic alliance partners. Jorion and Zhang (2007) and Hertzel and Officer (2012) document bankruptcy contagion effects on industry capital providers. Addoum, Kumar, Le, Niessen-Ruenzi (2015) document that, following the bankruptcy of a geographically proximate firm, firms that are located geographically near the bankrupt firm reduce their investment expenditures. They document that local firms experience worse credit conditions if a local firm files for bankruptcy.

In a model with strategic complementarities and bank lending, Bebchuk and Goldstein (2011) show that firms are vulnerable to credit market freezes. Banks avoid lending to firms out of self-fulfilling fear (validated in equilibrium) that other banks would withhold loans to firms, thus causing their default. Like Bebchuk and Goldstein, we also suggest policies to mitigate the coordination problem, and point out a number of empirical implications. Still, there are important differences with our paper. First, we model explicitly the technology and the preferences in the economy. Second, we consider direct finance from households to operating firms and not just intermediated lending. Third, we take a broader view of strategic complementarities which enables us to make policy recommendations regarding business fluctuations, and not just extreme conditions such as credit market freezes. Fourth, and most importantly, our framework enables us to compute social welfare which in turn allows us to quantify the welfare implications of the policy measures.

A number of recent papers focus on policy issues when there are strategic complementarities. In a coordination model akin to ours, Sákoviks and Steiner (2012) identify the optimal policy of investment subsidies. Subsidies should be targeted at those firms (*i*) whose investment has relatively large spillover effects on the economy (as we suggest), and (*ii*) which are relatively insensitive to the investment of others themselves (as they consider unlike us heterogeneous strategic complementarities across firms). Philippon and Schnabl (2013) analyze government interventions to recapitalize a banking system which suffers from coordination problems and restricts lending to firms (due to debt overhang). The efficient recapitalization policy injects capital in the banking system, thus alleviating the coordination problems among banks and augmenting firms' investment.

2 The Model

There is a continuum of investors indexed by i and uniformly distributed in interval $[0, 1]$. The utility of investor i equals

$$u_i = Ak_i - \frac{1}{2}k_i^2$$

where $k_i \in \mathbb{R}$ represents individual investment, A denotes the individual return to investment, and $k_i^2/2$ is the individual cost of investment. The aggregate level of investment is given by $K = \int_0^1 k_i di$. As in Benhabib and Farmer (1994), Acemoglu (1996), Romer (1996), and Angeletos and Pavan (2004), strategic complementarities are embodied in the return A, as the individual return is increasing in the aggregate level of investment. Formally,

$$A = (1 - \lambda)\theta + \lambda K.$$

The individual return A depends on the underlying exogenous economic fundamentals θ and on the aggregate level of investment K, while $\lambda \in \left[0, \frac{1}{2}\right)$ parametrizes the degree of strategic complementarity. Finally, social welfare equals

$$W = \int_0^1 u_i di = AK - \frac{1}{2}\int_0^1 k_i^2 di = (1 - \lambda)\theta K - (1 - 2\lambda)\frac{1}{2}K^2 - \frac{1}{2}\int_0^1 (k_i - K)^2 di.$$

As a result of strategic complementarities, social welfare depends both on the economic fundamentals and on aggregate investment. The term $\frac{1}{2}\int_0^1 (k_i - K)^2 di$ represents cross-sectional heterogeneity in investment decisions.

2.1 Market-based finance

Under market-based finance, k_i represents the direct investment of individual i in a representative firm. Individuals choose k_i to maximize their utility.

If θ were known, individuals would set $k_i = \theta$ and all investors would invest the same amount. Yet, the first-best prescribes setting a level of individual investment k_i larger than θ. Decentralized individuals do not internalize the positive externality of their investment on the return of others.[2]

[2] A Pigouvian corrective subsidy policy would implement the first-best allocation.

We now examine the cases in which the underlying economic fundamentals θ are uncertain. The exogenous return θ is not known at the time the investment decisions are made. Unlike Angeletos and Pavan (2004), we assume that the underlying economic fundamentals θ are a normal random variable with mean $\overline{\theta}$ and variance $\frac{1}{\gamma}$. Investor i maximizes its expected utility $E_i[u_i]$, so that optimal individual investment is given by

$$k_i = (1-\lambda)E_i[\theta] + \lambda E_i[K].$$

Individual investment is an increasing linear function of the expected economic fundamentals and the expected aggregate investment.

Proposition 1 *The equilibrium exists, is unique and given by $k_i = \overline{\theta}$. Ex ante social welfare is given by $E[W] = \frac{1}{2}\overline{\theta}^2$.*

Individual investment is constant, so that the volatility in the economic fundamentals has no impact on k_i. Social welfare does not depend on the volatility of the economic fundamentals. Again, there is an underinvestment problem as the first-best level of individual investment is larger than $\overline{\theta}$.

2.2 Market-based finance with public information

Consider that individuals receive an additional public signal z, such that

$$z = \theta + \frac{1}{\sqrt{\alpha}}\varepsilon$$

where ε is a standard normal random variable, independent of θ. The public signal z has precision α.

Proposition 2 *With public information, the equilibrium exists, is unique and given* *by* $k_i = \rho_1 \overline{\theta} + \rho_2 z$ *with* $\rho_1 = \frac{\gamma}{\gamma+\alpha}$ *and* $\rho_2 = \frac{\alpha}{\gamma+\alpha}$. *Ex ante social welfare is given by* $E[W] = \frac{1}{2}\overline{\theta}^2 + \frac{\alpha}{\gamma(\gamma+\alpha)}$.

The equilibrium investment k_i is a weighted average between the mean of the economic fundamentals $\overline{\theta}$ and the public signal z, with the weights depending on the variance of the fundamentals and on the precision of the public signal. All investors invest the same amount k_i, which varies with the public signal z.

It is efficient to set a high level of investment when the fundamentals are good and productivity is high. Increasing the precision of public information raises expected welfare, since the public signal z provides additional information about the fundamentals θ, thereby allowing investors to fine-tune their investment k_i to the exogenous return θ.

Such a fine-tuning effect provides a justification for promoting and regulating the disclosure of public information. It calls for stricter requirements regarding the disclosure of information by publicly traded companies, and demands incentives for the certification role by auditors or credit rating agencies.[3] It also entails increased transparency through disclosures from governments and other official institutions such as central banks.

Still, producing public information is not profitable. Financial intermediaries have no incentives to provide public information, if collecting information is costly. Intermediaries would be able to offer financial products identical to the ones already available to financial markets, but with lower return (since intermediaries would have to bear a cost to collect information and therefore charge fees to depositors). There is a free-rider problem, since everybody benefits from public information. For the rest of the paper, we do not consider the existence of a public signal z.

[3] A number of studies also suggests that bank loans provide public information to the market about the financial health of the firm (see, for example, James 1987). This is an additional channel through which financial intermediation is likely to have a positive effect on welfare.

3 Private information

We assume that each individual receives an additional piece of private information. Such information introduces heterogeneity in expectations about the fundamentals θ and may be understood as heterogeneity in the reading and interpretation of available information. With heterogenous beliefs about θ, there is cross-sectional heterogeneity in investment decisions, with optimistic individuals investing more than pessimistic individuals.

In Section 3.1 we investigate the case of market-based finance. Then, in Section 3.2 we consider the existence of a representative financial intermediary. The intermediary collects funds from individuals and invests these funds in firms. Monitoring enables the intermediary to offer an asset with less risk and lower expected return than direct investment in firms. Individuals can choose to invest directly in the firm, or they can choose to invest their funds through the financial intermediary. Pessimistic investors choose the safest option among the two investment alternatives, while optimistic investors choose market-based finance as it increases their potential gains.

In Section 3.3, we contrast the outcome in a market-based financial system with the outcome in a financial system with coexistence between intermediated and direct finance. Coexistence raises aggregate investment, thus increasing social welfare as a result of technological complementarities.

3.1 Market-based finance

There is a continuum of small firms financed directly by investors. Recall that the underlying economic fundamentals θ follow a normal random variable with mean $\overline{\theta}$ and variance $\frac{1}{\gamma}$. Consider that each investor receives a private signal

$$x_i = \theta + \frac{1}{\sqrt{\beta}}\varepsilon_i$$

where ε_i is standard normal, independent across investors and independent of θ, and β parametrizes the precision of private information.

Proposition 3 *With private information, equilibrium exists, is unique and given by* $k_i = \rho_3\overline{\theta} + \rho_4 x_i$ *with* $\rho_3 = \frac{\gamma}{\gamma+(1-\lambda)\beta}$ *and* $\rho_4 = \frac{(1-\lambda)\beta}{\gamma+(1-\lambda)\beta}$. *Ex ante social welfare is given by* $E[W] = (1-\lambda)\left[\overline{\theta}^2 + \frac{\rho_4}{\gamma}\right] - (1-2\lambda)\frac{1}{2}\left[\overline{\theta}^2 + \frac{\rho_4^2}{\gamma}\right] - \frac{1}{2}\frac{\rho_4^2}{\beta}$.

The functional form of equilibrium investment k_i is similar to the case with public information. Still, the weights of the two pieces of information in function k_i depend on the degree of strategic complementarity λ. If $\lambda = 0$, the two pieces of information would be given weights that are proportional to their precision (e.g., the private signal x_i would be given a weight equal to $\frac{\beta}{\gamma+\beta}$). The weights in the equilibrium strategy k_i deviate from these, so that the private signal is given relatively less weight. This property reflects the coordination motive arising from strategic complementarity in the actions of investors. It reflects the disproportionate influence of the public information embedded in the economic fundamentals, which individuals use to align their investment decisions.

What effects do the precision γ and the degree of strategic complementarity λ have on welfare? Expected welfare decreases with the precision of the fundamentals as individuals decrease the weight given to the private signal x_i and reduce the fine-tuning effect described above. The derivative of the expected social welfare $E[W]$ with respect to γ is negative, and this effect is illustrated in Figure 1(a) for specific values of $\overline{\theta}, \beta$, and λ.

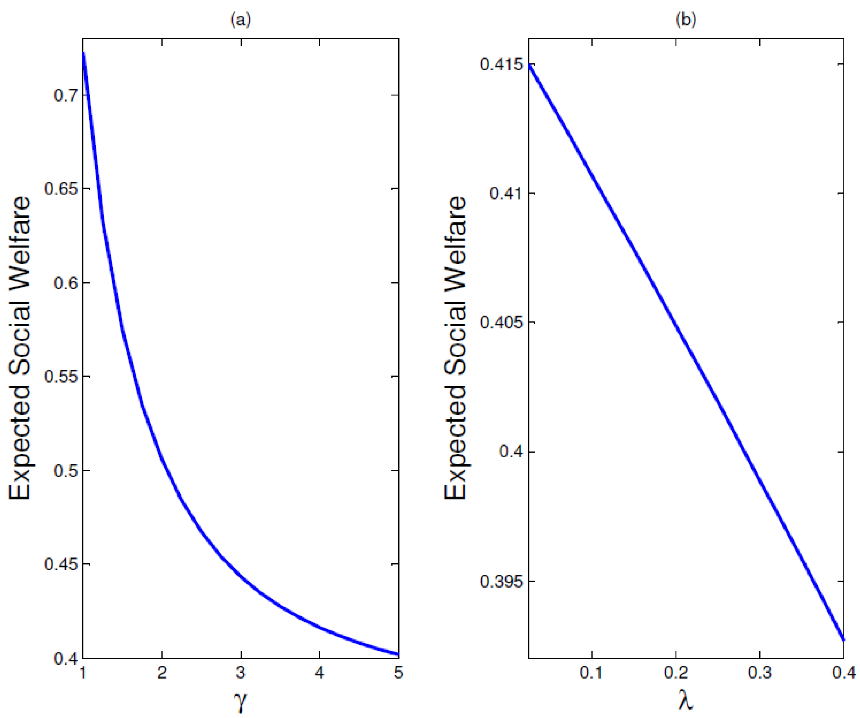

Figure 1: Expected social welfare with market-based finance as a function of γ and λ. Other parameters in this example: $\overline{\theta} = 0.85$ and $\beta = 4$; in (a) $\lambda = 0.25$ and in (b) $\gamma = 5$.

Investors reduce the weight placed on private information as strategic complementarities increase, thus reducing the social benefit of the fine-tuning effect. Figure 1(b) depicts a numerical example showing a negative relationship between expected welfare and the degree of strategic complementarity λ. We performed a set of numerical simulations using grids for parameters β and γ to investigate if the results were sensitive to the combination of these parameters, and we verified that results were robust to all settings.[4]

3.2 Coexistence between intermediated and market-based finance

We want to analyze now if a financial intermediary that comes between the investors and the firms can make it possible to increase investment. The main objective is to show that intermediaries raise individual investment by pessimistic investors, thus improving aggregate productivity as a result of strategic complementarities.

Consider the existence of a representative financial intermediary. Some investors deposit their funds with the intermediary, and the intermediary invests these funds in a pool of firms. Each firm is financed either by the intermediary or directly by investors. Each firm's manager has the possibility to choose between two products: one with high risk and private benefits, and other with low risk and no private benefits. By monitoring the firm, the financial intermediary forces its manager to implement the safest project, thus reducing the uncertainty in the returns of the firm and enabling the intermediary to offer a return with low risk to depositors. Firms that are directly financed by investors choose the riskiest project. This framework could be modelled along the lines of Holmström and Tirole (1997).

Alternatively, one could consider that the financial intermediary screens firms and issues securities which transfer risk from pessimistic to optimistic investors—in the

[4] Numerical simulations for the paper may be found in the webpage http://www.fep.up.pt/docentes/jjorge/, under the tab "Research".

spirit of Coval and Thakor (2005). Moral hazard or adverse selection are not indispensable assumptions. Rather, the only indispensable assumption is that the intermediary is able to reduce the risk of investment and therefore reduce the risk of the financial products being offered to their clients. The ability to offer products with little risk is a mild assumption, and a common result in the financial intermediation literature. For example, Allen and Gale (1997) document that financial intermediaries build up capital so as to offer an intertemporal smoothing of risk.

In our setup, the financial intermediary offers a financial product with the following features. Individuals who invest through the intermediary benefit from an individual return

$$\widehat{A} = (1-\lambda)\widehat{\theta} + \lambda\overline{K} - m$$

with

$$\widehat{\theta} = \overline{\theta} + \sqrt{\frac{\gamma}{\widehat{\gamma}}}(\theta - \overline{\theta}).$$

The random variable θ is a mean-preserving spread of $\widehat{\theta}$, so that $\sqrt{\frac{\gamma}{\widehat{\gamma}}} < 1$ with $1/\widehat{\gamma}$ being the variance of $\widehat{\theta}$. Reducing the volatility of the individual return implies a monitoring cost $m > 0$. Aggregate investment is equal to $\overline{K}$, and includes investment through the financial intermediary and direct investment in firms. Individuals investing through a financial intermediary have utility

$$\widehat{u}_i = \widehat{A}\widehat{k}_i - \frac{1}{2}\widehat{k}_i^2$$

where $\widehat{k}_i$ represents the individual investment in the intermediary. Individuals who invest directly in firms benefit from an individual return

$$A = (1-\lambda)\theta + \lambda\overline{K}$$

and have utility

$$\widetilde{u}_i = A\widetilde{k}_i - \frac{1}{2}\widetilde{k}_i^2$$

where $\widetilde{k}_i$ represents direct individual investment in firms. Individuals compare the expected return obtained in both investment alternatives, and invest exclusively in the alternative which yields the highest expected return.

Since θ is a mean-preserving spread of $\widehat{\theta}$, then $E_i[A] > E_i[\widehat{A}]$ for sufficiently high x_i and $E_i[A] < E_i[\widehat{A}]$ for sufficiently low x_i. As a result, there is a flight-to-quality effect by those individuals who receive low private signals about the economic fundamentals, whereas individuals with high private signals are not willing to invest in a financial product with low risk as it limits the upside potential.

There is a threshold $\overline{x}$ for the private signal at which individuals are indifferent between investing directly in the firms or via an intermediary. Investors with a private signal x_i lower than the indifference threshold $\overline{x}$ prefer to invest in the intermediary.

Lemma 1 *The indifference threshold is given by* $\overline{x} = \overline{\theta} + \frac{m(\widehat{\gamma}+\beta)(\gamma+\beta)}{(1-\lambda)\beta(\gamma-\widehat{\gamma})}$.

We call pessimistic to those investors who receive a private signal x_i below $\overline{x}$, and optimistic to those investors who receive a private signal $x_i > \overline{x}$. Only pessimistic investors are willing to switch from direct to intermediated finance.

If all pessimistic individuals invested through the financial intermediary, then the mass of investors in the intermediary would be variable. In this setting, the equilibrium investment decisions $\widetilde{k}_i$ and $\widehat{k}_i$ are not necessarily linear, in which case we are unable to obtain an analytical solution to the model.

In order to keep the analysis tractable enough to investigate the role of financial intermediation, we measure the marginal effect of adding a small financial intermediation system to the market equilibrium. To this end, we make two assumptions. First, we consider the limiting case when the financial intermediation sector is very small. Second, we restrict the set of those investors who can choose the financial intermediary to the less pessimistic investors. This is the most unfavorable setting for the impact of the financial intermediation, since the marginal effect would be stronger if other more pessimistic individuals could use the intermediary. This conservative approach is appropriate for our study, as we want to undoubtedly establish a role for financial intermediation.

In this simplified setting, the equilibrium investment decisions $\widetilde{k}_i$ and $\widehat{k}_i$ are linear. Order the individuals in the interval $[0,1]$ according to the size of their private signals, and denote the threshold investor who received signal $\overline{x}$ by $i(\overline{x})$. Define the set B of investors who are slightly less optimistic than individual $i(\overline{x})$,

$$B = \left]i(\overline{x}) - \xi, i(\overline{x})\right[$$

with small $\xi > 0$. We assume that only investors $i \in B$ have the option to choose the financial intermediary and, in equilibrium, these individuals invest through the intermediary.

For those investors who choose direct finance, individual investment equals

$$\widetilde{k}_i = E_i[(1-\lambda)\theta + \lambda \overline{K}]$$

while investment for those investors who choose intermediated finance equals

$$\widehat{k}_i = E_i[(1-\lambda)\widehat{\theta} + \lambda \overline{K}] - m.$$

Aggregate investment equals

$$\overline{K} = \int_{[0,1]\setminus B} \widetilde{k}_i di + \int_B \widehat{k}_i di.$$

The next result shows that there is a unique equilibrium with coexistence between direct and intermediated finance when the financial intermediation sector is sufficiently small.

Proposition 4 *With private information and for a sufficiently small value of* ξ*, there is a unique equilibrium in which financial intermediaries coexist with market-based finance. Equilibrium is given by* $\widetilde{k}_i = \rho_5\overline{\theta} + \rho_6 x_i + \rho_7 m + \rho_8$ *and* $\widehat{k}_i = \rho_9\overline{\theta} + \rho_{10}x_i + \rho_{11}m + \rho_{12}$*, with* $\rho_5 \to \rho_3, \rho_6 \to \rho_4,\ \rho_7 \to 0, \rho_8 \to 0, \rho_9 \to \lambda\left(1 + \frac{\rho_4}{1-\lambda}\right)\frac{\gamma}{\gamma+\beta} + (1-\lambda)\frac{\widehat{\gamma}}{\widehat{\gamma}+\beta}, \rho_{10} \to \frac{(1-\lambda)\beta}{\widehat{\gamma}+\beta} + \frac{\lambda(1-\lambda)\beta^2}{[(\gamma+\beta)-\lambda\beta]\gamma+\beta}, \rho_{11} \to -1$ *and* $\rho_{12} \to 0$ *as* ξ *converges to* 0.

In the proof of Proposition 4 we compute the marginal effect of introducing financial intermediaries in the market-based economy described in Section 3.1. As in the market-based economy, individual investment depends on $\overline{\theta}$ and x_i. Regarding the investment of those individuals who choose direct finance, the weights ρ_5 and ρ_6 are near the values obtained in Proposition 3. As for the investment decisions of those individuals who invest through the financial intermediary, the weights ρ_9 and ρ_{10} now depend on the variance of $\widehat{\theta}$.

Individual investment decisions also depend on the monitoring cost m. Lower monitoring costs raise the individual investment from those individuals who invest through the financial intermediary. Productivity increases as a result of investment complementarities, thereby enticing individuals who invest directly in firms to raise their investment. As a result, the coefficient ρ_7 converges to zero from below.

3.3 Contrasting a financial system based exclusively on market-based finance with a financial system with coexistence

In this section we compare the equilibrium in Proposition 3 with the equilibrium in Proposition 4, and show that coexistence raises aggregate investment and social welfare.

In the next result, we compare individual and aggregate investment in a financial system based exclusively on marked-based finance (obtained in Proposition 3) with individual and aggregate investment in a financial system with coexistence (obtained in Proposition 4).

Proposition 5 *In equilibrium with private information and for sufficiently small values of* ξ *and* m, $\widetilde{k}_i > k_i$ *for* $i \in [0,1] \backslash B$, $\widehat{k}_i > k_i$ *for* $i \in B$, *and* $\overline{K} > K$.

The value invested by those individuals who choose the financial intermediary is higher than the value they would invest if they chose direct finance. As a result of investment complementarities, all individuals invest more in the case of coexistence, thus increasing the level of aggregate investment.

The social welfare with coexistence equals the sum of investors' welfare, the financial intermediary's profit, firms' profits and managers' private benefits. We assume that perfectly competitive capital markets drive the profits of the representative financial intermediary and of firms to zero. With private benefits arbitrarily small, social welfare converges to investors' welfare. The next proposition compares welfare in a financial system based exclusively on market-based finance with welfare in a financial system with coexistence.

Proposition 6 *In equilibrium with private information and for sufficiently small values of* ξ *and* m, *ex ante social welfare increases with coexistence.*

Financial intermediation is socially desirable. Decentralized individuals do not internalize the positive externality of their investment on the return of others. Financial intermediaries raise individual investment, thus generating positive externalities and raising social welfare. Although there is a monitoring cost associated to investment through the intermediary, the effect of strategic complementarities dominates.

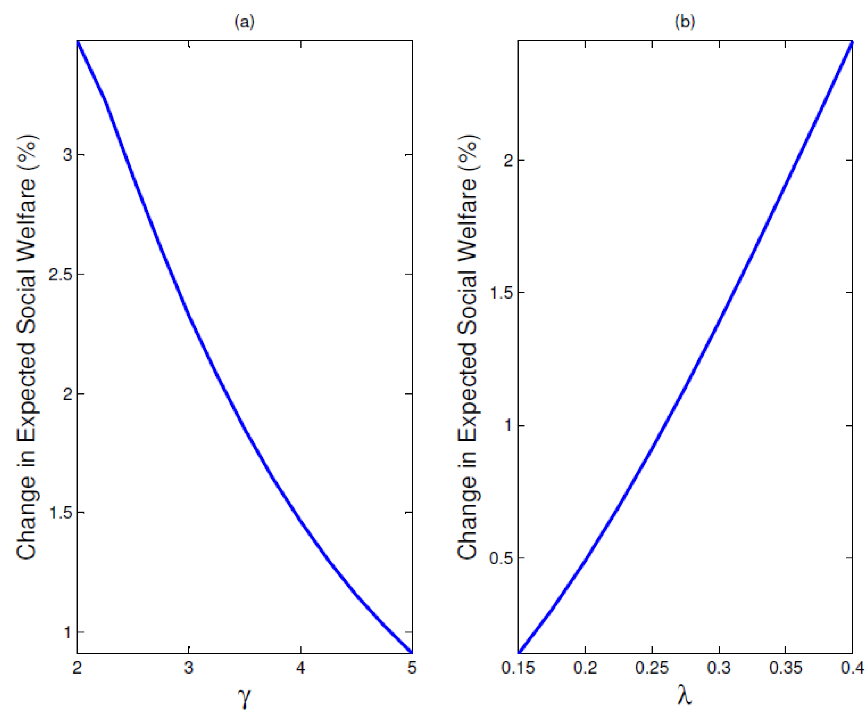

Figure 2: Change in expected social welfare from markets to coexistence when $\xi = 0.005\%$, as a function of γ and λ. Other parameters in this example: $\overline{\theta} = 0.85, \beta = 4, m = 10^{-5}, \widehat{\gamma} = 1.4\gamma$; in (a) $\lambda = 0.25$ and in (b) $\gamma = 5$.

The impact of financial intermediation on social welfare is less relevant for less volatile fundamentals. In this case, individuals place little weight on private information, so that there is little dispersion of individual investment (individuals set their investment close to $\overline{\theta}$) and financial intermediaries have little impact on investment decisions. Figure 2(a) depicts a numerical example showing the percentage increase in expected welfare as the economy shifts from a market-based system to a financial system with coexistence between intermediaries and markets, as a function of γ when the ratio $\frac{\widehat{\gamma}}{\gamma}$ is constant. The figure suggests that financial intermediaries become more relevant as the volatility of fundamentals increases.

Financial intermediation is relevant if and only if there are strategic complementarities. Without strategic complementarities, raising individual investment does not increase individual return and welfare. The numerical example depicted in Figure 2(b) suggests that financial intermediaries add more welfare as the degree of strategic complementarity increases.

We performed numerical simulations for various combinations of parameters $\xi, \overline{\theta}, \beta, m, \lambda, \widehat{\gamma}$, and γ, and obtained qualitative results identical the ones plotted in Figure 2.

3.4 Policy recommendations

The main contribution of our paper so far has been to show the possibility that financial intermediation adds to social welfare. Our results suggest that the size of the change in social welfare depends on the precision of the fundamentals γ and the degree of strategic complementarity λ, so that we shall now specify in which cases financial intermediation should be encouraged.

First, the welfare effect of financial intermediation increases when fundamentals are more volatile, thus suggesting that authorities should fine-tune their policies so as to incite intermediated finance to those industries which experience high uncertainty as a result of technological or regulatory shocks. In particular, credit lines targeted towards these industries would enhance investment and productivity. By the same token, those geographical areas which undergo periods of economic instability would benefit from favorable credit conditions.

At the aggregate level, our model prescribes raising the aggregate provision of intermediated finance in periods of macroeconomic uncertainty. For example, policy actions which influence the supply of bank credit will have an impact on investment if bank borrowers have no close substitutes to bank credit. Easy bank credit in periods of aggregate uncertainty would encourage intermediated loans so as to support firms' access to credit, fostering aggregate investment and returns, and raising aggregate welfare.

Second, intermediated finance is most useful in those industries with a substantial degree of strategic complementarity. Our results suggest that policy makers should promote specialized lending to those firms which benefit from external scale economies. Our model advises against subsidizing industries which do not benefit from strategic complementarities.

The equilibrium analysis in this section also provides a framework for analyzing and comparing specific government policies intended to promote lending.

(*i*) The infusion of capital into the banking system would raise the amount of intermediated funds, thus raising welfare. In our model, this would be equivalent to increasing the size of the representative financial intermediary.

(*ii*) Direct lending to firms would increase investment and welfare. In our model this would be equivalent to increasing the level of aggregate capital K. As in Bebchuk and Goldstein (2011), direct lending suffers from a disadvantage, as the government does not have the ability to monitor firms.

(*iii*) Government guarantees which provide funds to operating firms when they have low returns would increase individual investment and social welfare. These guarantees enable firms to offer stable returns to investors without wasting so much government resources as direct lending. Public guarantees would benefit from the advantages of monitoring, if these guarantees were channeled through specialized financial intermediaries.

3.5 Empirical implications

In addition to providing a framework for analyzing and evaluating government-supported mechanisms, our analysis also has substantial implications for empirical investigation.

First, Figure 2(b) suggests that financial intermediation is more important when strategic complementarities are most prevalent, so that a contraction in intermediated finance should have different impact across industries and geographical areas. A sharp test of our model would compare the impact of shocks on bank credit across industries and geographical areas with various degrees of strategic complementarity. One would expect industry clusters and regions with intense complementarities to be more sensitive to credit rationing. Bebchuk and Goldstein (2011) also suggest that sectors with large strategic complementarities are more vulnerable to credit freezes.[5]

Second, policies which stimulate the supply of intermediated finance should have more impact on those industries and geographical areas where strategic complementarities are most prevalent. To the extent that bank lending depends on central banks' actions, monetary policy should have a differential impact across regions and across industries. Using data for the US, Carlino and DeFina (1998, 1999) document that monetary policy has a differential impact across regions, and some sectors of the economy, such as manufacturing, are more sensitive to monetary policy shocks than other sectors, such as services and retail. Yet, it remains to be shown that industry clusters and regions with intense complementarities are more sensitive to monetary policy shocks.

Finally, our results suggest that supporting financial intermediation is likely to raise welfare significantly, and some of the responses to recent crises seem to conform to this belief. Using data for the Japanese banking crisis, Giannetti and Simonov (2013) show how bank bailouts had a positive effect on operating firms. Tong and Wei (2011) analyze 192 interventions for 15 countries from September 2008 to July 2010, and show that unconventional monetary interventions aimed at inducing banks to be more willing to lend had a positive effect on non-financial firms.

Overall, our model highlights the need for and the value of empirical research which

[5]Still, Bebchuk and Goldstein (2011) highlight a different channel. They suggest that banks may refuse to lend to firms in sectors which benefit from strategic complementarities.

identifies the role of strategic complementarities in the relationship between financial intermediation and welfare.

Measuring strategic complementarities. Agglomeration is widely recognized as a source and result of external scale economies. The literature has developed a number of location coefficients which quantify those external scale economies that result from the spatial concentration of firms of a particular industry in a given region and that are internalized by firms of that particular industry (see Ellison and Glaeser 1997, and Guimarães, Figueiredo, and Woodward 2007). Their basic principle is to measure the discrepancy between the distribution of regional employment in a particular industry against the regional distribution of the overall employment. Examples of industries with high geographic concentration are high-tech industries in Silicon Valley, the auto industry in Detroit, the entertainment industry in Hollywood, or investment banking in London.

Our paper suggests that industries with high geographic concentration should be more sensitive to variations in the supply of intermediated finance than industries with lower geographic concentration. A simple test of the theory would be to use the difference-in-differences estimator to gauge the significance of a shock in the supply of credit in geographically concentrated industries.

4 Conclusion

We offer a stylized view of financial intermediation—our intermediary is rather similar to an institution which monitors and holds equity positions in firms—but one that is adequate for our purposes and is consistent with the results in the literature on financial intermediation.

We examine the welfare effects of introducing financial intermediaries in economies with investment complementarities. Decentralized individuals do not internalize the positive externality of their investment on the return of others, thereby investing too little. By monitoring firms and offering low risk financial products, intermediaries induce pessimistic individuals to invest more. Increased investment raises returns due to strategic complementarities, thus inducing more investment across the economy. In this way, intermediaries help to overcome the coordination failure among decentralized individuals, thus raising social welfare and making financial intermediation socially desirable.

Three extensions to the model may provide additional insights that have not been captured in the paper. First, we have assumed that financial intermediaries are special because they possess a monitoring technology which enables them to offer safe securities. Instead, one could consider that intermediaries have access to the same technology as other investors and assume that intermediaries use their capital to hedge the risk in the underlying economic fundamentals. Capital enables intermediaries to offer safe financial products by averaging risks across states. This involves depleting capital if the returns to financial intermediaries' assets are low, and accumulating gains if returns are high. Intermediaries can thus offer financial products which pay a relatively constant amount across states. Allen and Gale (1997) use a multiperiod model to describe how intermediaries build up their capital. They suggest that intermediaries can perform intertemporal smoothing in individual welfare, by averaging risks over time. This entails intermediaries building up reserves of safe assets when the returns on intermediaries' assets are high, and reducing them when returns are low.

Second, we have restricted the set of contracts available to investors. Investors and financial intermediaries have an equity stake in the firm and appropriate the whole surplus. Our *qualitative* results carry over to a less restrictive set of contracts, as long as financial intermediaries offer contracts with low risk. Still, enlarging the set of available contracts opens the debate on the *quantitative* significance of our results. Since the most common forms of intermediated finance—bank credit and bank deposits—have less risk than equity and reinforce the risk absorption by financial intermediaries, the existence of credit and deposit contracts is likely to strengthen the effects described in the paper.

Finally, we have performed our analysis for the particular case in which the financial intermediation system is small. In a model with strategic complementarities, Corsetti, Dasgupta, Morris and Shin (2004) show that a large player exercises a disproportionate influence on the behavior of small players. Extending our analysis to the case in which the intermediation system is large would allow for a more complete policy analysis.

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A Appendix

A.1 Proof of Proposition 1

We guess $k_i = \overline{\theta}$, so that $K = \int_0^1 k_i di = \int_0^1 \overline{\theta} di = \overline{\theta}$. Hence, $k_i = (1-\lambda)E_i[\theta] + \lambda E_i[K] = (1-\lambda)\overline{\theta} + \lambda\overline{\theta} = \overline{\theta}$ and the initial guess is verified. Equilibrium is unique as in Angeletos and Pavan (2004).

Expected social welfare is given by

$$E[W] = (1-\lambda)E\left[\theta K\right] - (1-2\lambda)\frac{1}{2}E\left[K^2\right] - \frac{1}{2}E\left[\int_0^1 (k_i - K)^2 di\right] = \frac{1}{2}\overline{\theta}^2$$

as $\int_0^1 (k_i - K)^2 di = 0$.■

A.2 Proof of Proposition 2

We guess $k_i = \rho_1\overline{\theta} + \rho_2 z$, so that $K = \int_0^1 k_i di = \int_0^1 (\rho_1\overline{\theta} + \rho_2 z) di = \rho_1\overline{\theta} + \rho_2 z$. Hence,

$$\begin{aligned} k_i &= (1-\lambda)E_i[\theta] + \lambda E_i[K] = (1-\lambda)\frac{\gamma\overline{\theta} + \alpha z}{\gamma + \alpha} + \lambda E_i[\rho_1\overline{\theta} + \rho_2 z] = \\ &= (1-\lambda)\frac{\gamma\overline{\theta} + \alpha z}{\gamma + \alpha} + \lambda(\rho_1\overline{\theta} + \rho_2 z) \\ &= \left((1-\lambda)\frac{\gamma}{\gamma+\alpha} + \lambda\rho_1\right)\overline{\theta} + \left((1-\lambda)\frac{\alpha}{\gamma+\alpha} + \lambda\rho_2\right) z \end{aligned}$$

and the initial guess is verified with

$$\begin{aligned} \rho_1 &= (1-\lambda)\frac{\gamma}{\gamma+\alpha} + \lambda\rho_1 \Leftrightarrow \rho_1 = \frac{\gamma}{\gamma+\alpha} \\ \rho_2 &= (1-\lambda)\frac{\alpha}{\gamma+\alpha} + \lambda\rho_2 \Leftrightarrow \rho_2 = \frac{\alpha}{\gamma+\alpha}. \end{aligned}$$

Expected social welfare is given by

$$E[W] = (1-\lambda)E\left[\theta K\right] - (1-2\lambda)\frac{1}{2}E\left[K^2\right] - \frac{1}{2}E\left[\int_0^1 (k_i - K)^2 d_i\right] =$$
$$= (1-\lambda)E_i\left(\theta\frac{\gamma\overline{\theta}+\alpha z}{\gamma+\alpha}\right) - (1-2\lambda)\frac{1}{2}\left(Var\left[K\right] + (E\left[K\right])^2\right) =$$
$$= (1-\lambda)\left(\overline{\theta}^2 + \frac{\alpha}{\gamma(\gamma+\alpha)}\right) - (1-2\lambda)\frac{1}{2}\left(\frac{\alpha}{\gamma(\gamma+\alpha)} + \overline{\theta}^2\right)$$
$$= \frac{1}{2}\overline{\theta}^2 + \frac{\alpha}{\gamma(\gamma+\alpha)}.\blacksquare$$

A.3 Proof of Proposition 3

We guess $k_i = \rho_3\overline{\theta} + \rho_4 x_i$, so that $K = \int_0^1 k_i di = \int_0^1 (\rho_3\overline{\theta} + \rho_4 x_i)d_i = \rho_3\overline{\theta} + \rho_4\theta$. Hence,

$$k_i = (1-\lambda)E_i[\theta] + \lambda E_i[K] = (1-\lambda)\frac{\gamma\overline{\theta}+\beta x_i}{\gamma+\beta} + \lambda E_i[\rho_3\overline{\theta} + \rho_4\theta]$$
$$= (1-\lambda)\frac{\gamma\overline{\theta}+\beta x_i}{\gamma+\beta} + \lambda(\rho_3\overline{\theta} + \rho_4\frac{\gamma\overline{\theta}+\beta x_i}{\gamma+\beta})$$
$$= \left((1-\lambda)\frac{\gamma}{\gamma+\beta} + \lambda\rho_3 + \lambda\rho_4\frac{\gamma}{\gamma+\beta}\right)\overline{\theta} + \left((1-\lambda)\frac{\beta}{\gamma+\beta} + \lambda\rho_4\frac{\beta}{\gamma+\beta}\right)x_i$$

and the initial guess is verified with

$$\rho_3 = (1-\lambda)\frac{\gamma}{\gamma+\beta} + \lambda\rho_3 + \lambda\rho_4\frac{\gamma}{\gamma+\beta} \Leftrightarrow \rho_3 = \frac{\gamma}{\gamma+(1-\lambda)\beta}$$
$$\rho_4 = (1-\lambda)\frac{\beta}{\gamma+\beta} + \lambda\rho_4\frac{\beta}{\gamma+\beta} \Leftrightarrow \rho_4 = \frac{(1-\lambda)\beta}{\gamma+(1-\lambda)\beta}$$

Expected social welfare is given by

$$
\begin{aligned}
E[W] &= (1-\lambda)E\left[\theta K\right] - (1-2\lambda)\frac{1}{2}E\left[K^2\right] - \frac{1}{2}E\left[\int_0^1 (k_i - K)^2 di\right] \\
&= (1-\lambda)E\left[\theta K\right] - (1-2\lambda)\frac{1}{2}\left(Var\left[K\right] + \left(E\left[K\right]\right)^2\right) - \frac{1}{2}E\left[\int_0^1 (k_i - K)^2 d_i\right] \\
&= (1-\lambda)\left(\overline{\theta}^2 + \frac{(1-\lambda)\beta}{\gamma\left(\gamma + (1-\lambda)\beta\right)}\right) \\
&\qquad - (1-2\lambda)\frac{1}{2}\left(\overline{\theta}^2 + \frac{\left((1-\lambda)\beta\right)^2}{\gamma\left(\gamma + (1-\lambda)\beta\right)^2}\right) - \frac{1}{2}\frac{\left((1-\lambda)\beta\right)^2}{\beta\left(\gamma + (1-\lambda)\beta\right)^2} \\
&= (1-\lambda)\left(\overline{\theta}^2 + \frac{\rho_4}{\gamma}\right) - (1-2\lambda)\frac{1}{2}\left(\overline{\theta}^2 + \frac{\rho_4^2}{\gamma}\right) - \frac{1}{2}\frac{\rho_4^2}{\beta}. \blacksquare
\end{aligned}
$$

A.4 Proof of Lemma 1

The threshold $\overline{x}$ is the signal at which the investor is indifferent between investing in the market or in the bank, that is $E_i[A] = E_i[\widehat{A}]$. Hence,

$$
E_i[(1-\lambda)\theta + \lambda\overline{K}] = E_i[(1-\lambda)\widehat{\theta} + \lambda\overline{K} - m] \Leftrightarrow x_i = \overline{\theta} + \frac{m(\widehat{\gamma} + \beta)(\gamma + \beta)}{(1-\lambda)\beta(\gamma - \widehat{\gamma})}. \blacksquare
$$

A.5 Proof of Proposition 4

We guess $\widetilde{k}_i = \rho_5\overline{\theta} + \rho_6 x_i + \rho_7 m + \rho_8$ and $\widehat{k}_i = \rho_9\overline{\theta} + \rho_{10}x_i + \rho_{11}m + \rho_{12}$, so that

$$
\begin{aligned}
\overline{K} &= \int_{[0,1]\setminus B} \widetilde{k}_i di + \int_B \widehat{k}_i di \\
&= \int_{[0,1]\setminus B} \left(\rho_5\overline{\theta} + \rho_6 x_i + \rho_7 m + \rho_8\right) di + \int_B \left(\rho_9\overline{\theta} + \rho_{10}x_i + \rho_{11}m + \rho_{12}\right) di \\
&= \int_{[0,1]\setminus B} \left(\rho_5\overline{\theta} + \rho_7 m + \rho_8\right) di + \int_B \left(\rho_9\overline{\theta} + \rho_{11}m + \rho_{12}\right) di + \rho_6 \int_{[0,1]\setminus B} x_i di + \rho_{10}\int_B x_i di \\
&= (1-\xi)\left(\rho_5\overline{\theta} + \rho_7 m + \rho_8\right) + \xi\left(\rho_9\overline{\theta} + \rho_{11}m + \rho_{12}\right) + \rho_6(1-\xi)\theta + \rho_{10}\xi\overline{\overline{x}} \\
&= (1-\xi)\left(\rho_5\overline{\theta} + \rho_6\theta + \rho_7 m + \rho_8\right) + \xi\left(\rho_9\overline{\theta} + \rho_{10}\overline{\overline{x}} + \rho_{11}m + \rho_{12}\right)
\end{aligned}
$$

with $\overline{\overline{x}} = \frac{\int_B x_i di}{\xi}$. Hence,

$$\begin{aligned}
\widetilde{k}_i &= E_i[(1-\lambda)\theta + \lambda\overline{K}] \\
&= (1-\lambda)\frac{\gamma\overline{\theta} + \beta x_i}{\gamma+\beta} + \lambda E_i\left[(1-\xi)\left(\rho_5\overline{\theta} + \rho_6\theta + \rho_7 m + \rho_8\right) + \xi\left(\rho_9\overline{\theta} + \rho_{10}\overline{\overline{x}} + \rho_{11}m + \rho_{12}\right)\right] \\
&= (1-\lambda)\frac{\gamma\overline{\theta} + \beta x_i}{\gamma+\beta} + \lambda\left[(1-\xi)\left(\rho_5\overline{\theta} + \rho_6\frac{\gamma\overline{\theta} + \beta x_i}{\gamma+\beta} + \rho_7 m + \rho_8\right) + \xi\left(\rho_9\overline{\theta} + \rho_{10}\overline{\overline{x}} + \rho_{11}m + \rho_{12}\right)\right] \\
&= \left((1-\lambda)\frac{\gamma}{\gamma+\beta} + \lambda(1-\xi)\rho_5 + \lambda(1-\xi)\rho_6\frac{\gamma}{\gamma+\beta} + \lambda\xi\rho_9\right)\overline{\theta} + \left((1-\lambda)\frac{\beta}{\gamma+\beta} + \lambda(1-\xi)\rho_6\frac{\beta}{\gamma+\beta}\right)x_i \\
&+ \left[\lambda(1-\xi)\rho_7 + \lambda\xi\rho_{11}\right]m + \lambda(1-\xi)\rho_8 + \lambda\xi\left(\rho_{10}\overline{\overline{x}} + \rho_{12}\right)
\end{aligned}$$

and

$$\begin{aligned}
\widehat{k}_i &= E_i\left[(1-\lambda)\widehat{\theta} + \lambda\overline{K} - m\right] = (1-\lambda)\frac{\widehat{\gamma}\overline{\theta} + \beta x_i}{\widehat{\gamma}+\beta} + \lambda E_i[\overline{K}] - m \\
&= (1-\lambda)\frac{\widehat{\gamma}\overline{\theta} + \beta x_i}{\widehat{\gamma}+\beta} + \lambda E_i[(1-\xi)\left(\rho_5\overline{\theta} + \rho_6\theta + \rho_7 m + \rho_8\right) + \xi\left(\rho_9\overline{\theta} + \rho_{10}\overline{\overline{x}} + \rho_{11}m + \rho_{12}\right)] - m \\
&= (1-\lambda)\frac{\widehat{\gamma}\overline{\theta} + \beta x_i}{\widehat{\gamma}+\beta} + \lambda\left[(1-\xi)\left(\rho_5\overline{\theta} + \rho_6\frac{\gamma\overline{\theta} + \beta x_i}{\gamma+\beta} + \rho_7 m + \rho_8\right) + \xi\left(\rho_9\overline{\theta} + \rho_{10}\overline{\overline{x}} + \rho_{11}m + \rho_{12}\right)\right] - m \\
&= \left((1-\lambda)\frac{\widehat{\gamma}}{\widehat{\gamma}+\beta} + \lambda(1-\xi)\rho_5 + \lambda(1-\xi)\rho_6\frac{\gamma}{\gamma+\beta} + \lambda\xi\rho_9\right)\overline{\theta} + \left((1-\lambda)\frac{\beta}{\widehat{\gamma}+\beta} + \lambda(1-\xi)\rho_6\frac{\beta}{\gamma+\beta}\right)x_i \\
&\quad + \left[\lambda(1-\xi)\rho_7 + \lambda\xi\rho_{11} - 1\right]m + \lambda(1-\xi)\rho_8 + \lambda\xi\left(\rho_{10}\overline{\overline{x}} + \rho_{12}\right).
\end{aligned}$$

The initial guesses are verified with

$$\begin{aligned}
\rho_5 &= (1-\lambda)\frac{\gamma}{\gamma+\beta} + \lambda(1-\xi)\rho_5 + \lambda(1-\xi)\rho_6\frac{\gamma}{\gamma+\beta} + \lambda\xi\rho_9 \\
\rho_6 &= (1-\lambda)\frac{\beta}{\gamma+\beta} + \lambda(1-\xi)\rho_6\frac{\beta}{\gamma+\beta} \\
\rho_7 &= \lambda(1-\xi)\rho_7 + \lambda\xi\rho_{11} \\
\rho_8 &= \lambda(1-\xi)\rho_8 + \lambda\xi\left(\rho_{10}\overline{\overline{x}} + \rho_{12}\right)
\end{aligned}$$

and

$$\begin{aligned}
\rho_9 &= (1-\lambda)\frac{\widehat{\gamma}}{\widehat{\gamma}+\beta} + \lambda(1-\xi)\rho_5 + \lambda(1-\xi)\rho_6\frac{\gamma}{\gamma+\beta} + \lambda\xi\rho_9 \\
\rho_{10} &= (1-\lambda)\frac{\beta}{\widehat{\gamma}+\beta} + \lambda(1-\xi)\rho_6\frac{\beta}{\gamma+\beta} \\
\rho_{11} &= \lambda(1-\xi)\rho_7 + \lambda\xi\rho_{11} - 1 \\
\rho_{12} &= \lambda(1-\xi)\rho_8 + \lambda\xi\left(\rho_{10}\overline{\overline{x}} + \rho_{12}\right)
\end{aligned}$$

so that

$$\rho_7 = -\frac{\lambda\xi}{1-\lambda}, \rho_{11} = \rho_7 - 1$$
$$\rho_6 = \frac{(1-\lambda)\beta}{(\gamma+\beta) - \lambda(1-\xi)\beta}$$
$$\rho_{10} = \frac{1}{(1-\lambda\xi)}\left[\frac{(1-\lambda)\beta}{\widehat{\gamma}+\beta} + \frac{\lambda(1-\xi)(1-\lambda)\beta^2}{[(\gamma+\beta)-\lambda(1-\xi)\beta]\,\gamma+\beta}\right]$$
$$\rho_8 = \rho_{12} = \frac{\lambda\xi\overline{\overline{x}}}{1-\lambda}\rho_{10}$$
$$\rho_5 = \frac{1-\lambda+\lambda\,(1-\xi)\,\rho_6\left(1+\frac{\lambda\xi}{1-\lambda}\right)+\lambda^2\xi\,(1-\xi)}{1-\lambda\,(1-\xi)}\frac{\gamma}{\gamma+\beta} + \lambda\xi\frac{\widehat{\gamma}}{\widehat{\gamma}+\beta}$$
$$\rho_9 = \lambda\,(1-\xi)\left(1+\frac{\rho_6}{1-\lambda}\right)\frac{\gamma}{\gamma+\beta} + [1-\lambda\,(1-\xi)]\,\frac{\widehat{\gamma}}{\widehat{\gamma}+\beta}.$$

When $\xi \to 0$,

$$\rho_7 = 0, \rho_{11} = -1$$
$$\rho_6 = \rho_4$$
$$\rho_{10} = \left[\frac{(1-\lambda)\beta}{\widehat{\gamma}+\beta} + \frac{\lambda(1-\lambda)\beta^2}{[(\gamma+\beta)-\lambda\beta]\,\gamma+\beta}\right]$$
$$\rho_8 = \rho_{12} = 0$$
$$\rho_5 = \rho_3$$
$$\rho_9 = \lambda\left(1+\frac{\rho_4}{1-\lambda}\right)\frac{\gamma}{\gamma+\beta} + (1-\lambda)\,\frac{\widehat{\gamma}}{\widehat{\gamma}+\beta}. \blacksquare$$

A.6 Proof of Proposition 5

We proceed by steps. First, we compare the level of investment for those individuals who invest through the bank ($i \in B$) with what they would invest if there were no banks. Second, we compare the level of investment of those individuals who choose direct finance when there are banks ($i \in [0,1] \backslash B$), with their investment when there are no banks. Finally, we compare the level of aggregate investment and the individual investment decisions with and without coexistence.

Step 1. To show that $\int_B \widehat{k}_i di > \int_B k_i di$ we use the following Lemma.

Lemma 2 *The integral* $\int\limits_B \widehat{k}_i di$ *converges to*

$$(1-\lambda)\int\limits_B E_i[\widehat{\theta}]di + \lambda(1-\lambda)\int\limits_B\int\limits_B E_i\left[E_j[\widehat{\theta}]\right]djdi + \lambda^2(1-\lambda)\int\limits_B\int\limits_B\int\limits_B E_i\left[E_j\left[E_\iota[\widehat{\theta}]\right]\right]d\iota djdi + ...$$

$$-\int\limits_B mdi - \lambda\int\limits_B\int\limits_B mdjdi - \lambda^2\int\limits_B\int\limits_B\int\limits_B md\iota djdi - ... + \lambda\int\limits_B\int\limits_{[0,1]\setminus B} E_i\left[\widetilde{k}_j\right]djdi$$

$$+\lambda^2\int\limits_B\int\limits_B\int\limits_{[0,1]\setminus B} E_i\left[E_j\left[\widetilde{k}_\iota\right]\right]d\iota djdi + \lambda^3\int\limits_B\int\limits_B\int\limits_B\int\limits_{[0,1]\setminus B} E_i\left[E_j\left[E_\iota\left[\widetilde{k}_\tau\right]\right]\right]d\tau d\iota djdi + ...$$

Proof. We have $\int\limits_B \widehat{k}_i di = \int\limits_B E_i[(1-\lambda)\widehat{\theta} + \lambda\overline{K} - m]di = \int\limits_B \left((1-\lambda)E_i[\widehat{\theta}] + \lambda E_i[\overline{K}] - m\right)di$

$$= \int\limits_B \left((1-\lambda)E_i[\widehat{\theta}] + \lambda E_i\left[\int\limits_{[0,1]\setminus B}\widetilde{k}_j dj + \int\limits_B \widehat{k}_j dj\right] - m\right)di$$

$$= (1-\lambda)\int\limits_B E_i[\widehat{\theta}]di + \lambda\int\limits_B\int\limits_{[0,1]\setminus B} E_i\left[\widetilde{k}_j\right]djdi + \lambda\int\limits_B\int\limits_B E_i\left[\widehat{k}_j\right]djdi - \int\limits_B mdi$$

$$= (1-\lambda)\int\limits_B E_i[\widehat{\theta}]di + \lambda\int\limits_B\int\limits_{[0,1]\setminus B} E_i\left[\widetilde{k}_j\right]djdi + \lambda\int\limits_B\int\limits_B E_i\left[(1-\lambda)E_j[\widehat{\theta}] + \lambda E_j[\overline{K}] - m\right]djdi - \int\limits_B mdi$$

$$= (1-\lambda)\int\limits_B E_i[\widehat{\theta}]di + \lambda\int\limits_B\int\limits_{[0,1]\setminus B} E_i\left[\widetilde{k}_j\right]djdi + \lambda(1-\lambda)\int\limits_B\int\limits_B E_i\left[E_j[\widehat{\theta}]\right]djdi + \lambda^2\int\limits_B\int\limits_B E_i\left[E_j[\overline{K}]\right]djdi - \lambda\int\limits_B\int\limits_B mdjdi - \int\limits_B mdi\ .$$

Reorder the terms, replace $\overline{K}$ and iterate again,

$$= (1-\lambda)\int\limits_B E_i[\widehat{\theta}]di + \lambda(1-\lambda)\int\limits_B\int\limits_B E_i\left[E_j[\widehat{\theta}]\right]djdi - \lambda\int\limits_B\int\limits_B mdjdi - \int\limits_B mdi + \lambda\int\limits_B\int\limits_{[0,1]\setminus B} E_i\left[\widetilde{k}_j\right]djdi + \lambda^2\int\limits_B\int\limits_B E_i\left[E_j\left[\int\limits_{[0,1]\setminus B}\widetilde{k}_\iota d\iota + \int\limits_B \widehat{k}_\iota d\iota\right]\right]djdi$$

$$= (1-\lambda)\int\limits_B E_i[\widehat{\theta}]di + \lambda(1-\lambda)\int\limits_B\int\limits_B E_i\left[E_j[\widehat{\theta}]\right]djdi - \lambda\int\limits_B\int\limits_B mdjdi - \int\limits_B mdi + \lambda\int\limits_B\int\limits_{[0,1]\setminus B} E_i\left[\widetilde{k}_j\right]djdi + \lambda^2\int\limits_B\int\limits_B E_i\left[E_j\left[\int\limits_{[0,1]\setminus B}\widetilde{k}_\iota d\iota\right]\right]djdi + \lambda^2\int\limits_B\int\limits_B E_i\left[E_j\left[\int\limits_B \widehat{k}_\iota d\iota\right]\right]djdi\ .$$

Iterating n times and letting $n \to \infty$, the term in $\widehat{k}$ vanishes and we obtain the result. ■

Apply Lemma 2 to obtain

$$\int_B k_i di = (1-\lambda)\int_B E_i[\theta]di + \lambda(1-\lambda)\int_B\int_B E_i\left[E_j[\theta]\right]djdi + \lambda^2(1-\lambda)\int_B\int_B\int_B E_i\left[E_j\left[E_\iota[\theta]\right]\right]d\iota djdi + ...$$

$$+\lambda\int_B\int_{[0,1]\setminus B} E_i\left[k_j\right]djdi + \lambda^2\int_B\int_B\int_{[0,1]\setminus B} E_i\left[E_j\left[k_\iota\right]\right]d\iota djdi$$

$$+\lambda^3\int_B\int_B\int_B\int_{[0,1]\setminus B} E_i\left[E_j\left[E_\iota\left[k_\tau\right]\right]\right]d\tau d\iota djdi + ...$$

Compute

$$\int_B \widehat{k}_i di - \int_B k_i di =$$

$$(1-\lambda)\int_B E_i[\widehat{\theta}-\theta]di + \lambda(1-\lambda)\int_B\int_B E_i\left[E_j[\widehat{\theta}-\theta]\right]djdi + \lambda^2(1-\lambda)\int_B\int_B\int_B E_i\left[E_j\left[E_\iota[\widehat{\theta}-\theta]\right]\right]d\iota djdi + ...$$

$$-\int_B mdi - \lambda\int_B\int_B mdjdi - \lambda^2\int_B\int_B\int_B md\iota djdi - ... + \lambda\int_B\int_{[0,1]\setminus B} E_i\left[\widetilde{k}_j - k_j\right]djdi$$

$$+\lambda^2\int_B\int_B\int_{[0,1]\setminus B} E_i\left[E_j\left[\widetilde{k}_\iota - k_\iota\right]\right]d\iota djdi + \lambda^3\int_B\int_B\int_B\int_{[0,1]\setminus B} E_i\left[E_j\left[E_\iota\left[\widetilde{k}_\tau - k_\tau\right]\right]\right]d\tau d\iota djdi + ...$$

For sufficiently low ξ,

$$\int_B \widehat{k}_i di - \int_B k_i di \to$$

$$(1-\lambda)\int_B E_i[\widehat{\theta}-\theta]di + \lambda(1-\lambda)\int_B\int_B E_i\left[E_j[\widehat{\theta}-\theta]\right]djdi + \lambda^2(1-\lambda)\int_B\int_B\int_B E_i\left[E_j\left[E_\iota[\widehat{\theta}-\theta]\right]\right]d\iota djdi + ...$$

Since $E_i\left[\widehat{\theta}\right] > E_i[\theta]$ for $i \in B$, then $\int_B \widehat{k}_i di - \int_B k_i di > 0$.

Step 2. To show $\int_{[0,1]\setminus B} \widetilde{k}_i di > \int_{[0,1]\setminus B} k_i di$, apply Lemma 2 to both integrals to obtain

$$\int_{[0,1]\setminus B} \widetilde{k}_i di - \int_{[0,1]\setminus B} k_i di =$$

$$\int_{[0,1]\setminus B} \left(\widetilde{k}_i - k_i\right)di = \lambda\int_{[0,1]\setminus B}\int_B E_i\left[\widehat{k}_j - k_j\right]djdi + \lambda^2\int_{[0,1]\setminus B}\int_{[0,1]\setminus B}\int_B E_i\left[E_j\left[\widehat{k}_\iota - k_\iota\right]\right]d\iota djdi + ...$$

The terms in $\widetilde{k}$ vanish as a result of Lemma 2. Since we proved in step1 that $\int_B \widehat{k}_i di > \int_B k_i di$, then

$$\int_{[0,1]\setminus B} \widetilde{k}_i di - \int_{[0,1]\setminus B} k_i di > 0 \Leftrightarrow \int_{[0,1]\setminus B} \widetilde{k}_i di > \int_{[0,1]\setminus B} k_i di.$$

Step 3. By steps 1 and 2, $\overline{K} > K$ and for a sufficiently low m, we obtain $\widehat{k}_i = (1-\lambda)E_i[\widehat{\theta}]+\lambda E_i[\overline{K}]-m > (1-\lambda)E_i[\theta]+\lambda E_i[K] = k_i$ for $i \in B$, and $\widetilde{k}_i = (1-\lambda)E_i[\theta]+\lambda E_i[\overline{K}] > (1-\lambda)E_i[\theta]+\lambda E_i[K] = k_i$.■

A.7 Proof of Proposition 6

For $i \in B$, individuals choose $\widehat{k}_i$ to maximize their expected utility. Hence,

$$\widehat{u}_i = E_i\left[\left[(1-\lambda)\widehat{\theta}+\lambda\overline{K}-m\right]\widehat{k}_i - \frac{1}{2}\widehat{k}_i^2\right] \geq E_i\left[\left[(1-\lambda)\widehat{\theta}+\lambda\overline{K}-m\right]k_i - \frac{1}{2}k_i^2\right]$$

where k_i is given in Proposition 3. It follows that

$$E_i\left[\left[(1-\lambda)\widehat{\theta}+\lambda\overline{K}-m\right]k_i - \frac{1}{2}k_i^2\right] > E_i\left[\left[(1-\lambda)\theta+\lambda K-m\right]k_i - \frac{1}{2}k_i^2\right]$$

since $E_i\left[\widehat{\theta}\right] > E_i\left[\theta\right]$ and $\overline{K} > K$. Moreover,

$$E_i\left[\left[(1-\lambda)\theta+\lambda K-m\right]k_i - \frac{1}{2}k_i^2\right] = E_i\left[u_i - mk_i\right],$$

so that $E_i\left[\widehat{u}_i\right] > E_i\left[u_i - mk_i\right]$ and $E_i\left[\widehat{u}_i\right] > E_i\left[u_i\right]$for sufficiently low m.

For $i \in [0,1]\setminus B$, individuals choose $\widetilde{k}_i$ to maximize their expected utility. Hence,

$$E_i\left[\widetilde{u}_i\right] = E_i\left[\left[(1-\lambda)\theta+\lambda\overline{K}\right]\widetilde{k}_i - \frac{1}{2}\widetilde{k}_i^2\right] \geq E_i\left[\left[(1-\lambda)\theta+\lambda\overline{K}\right]k_i - \frac{1}{2}k_i^2\right]$$

where k_i is given in Proposition 3. Since $\overline{K} > K$, $E_i\left[\widetilde{u}_i\right] > E_i\left[u_i\right]$.

Ex ante expected social welfare with coexistence is given by $E\left[\int\limits_{[0,1]\setminus B} E_i\left[\widetilde{u}_i\right]di + \int\limits_{B} E_i\left[\widehat{u}_i\right]di\right]$ which is larger than $E\left[\int\limits_{[0,1]\setminus B} E_i\left[u_i\right]di + \int\limits_{B} E_i\left[u_i\right]di\right]$, where u_i is the equilibrium utility obtained in Proposition 3. Since $E\left[\int\limits_{[0,1]\setminus B} E_i\left[u_i\right]di + \int\limits_{B} E_i\left[u_i\right]di\right] = E\left[\int_0^1 E_i\left[u_i\right]di\right] = E\left[\int_0^1 u_i di\right]$ by the law of iterated expectations, we obtain the result.■