cef.up
Centro de Economia e Finanças da UPorto

cef.up working paper 2013-09

Volume Uncertainty in Construction Projects: a Real Options Approach

João Adelino Ribeiro
Paulo Jorge Pereira
Elísio Brandão

Centro de Economia e Finanças da UP
www.fep.up.pt

Volume Uncertainty in Construction Projects: a Real Options Approach

João Adelino Ribeiro,*[†] Paulo Jorge Pereira[‡] and Elísio Brandão[§]

May 27, 2013

Abstract

The levels of uncertainty surrounding construction projects are particularly high and construction managers should be aware that adequately managing the effects of the different types of uncertainty may lead to an increase in the project's final Net Present Value (NPV). The model proposed focus on the impact that a specific type of uncertainty - volume uncertainty - may produce in the project's expected NPV. Volume uncertainty is present in most construction projects since managers do not know, during the bid preparation stage, the exact volume of work that will be executed during the project's life cycle. Volume uncertainty leads to profit uncertainty and the model integrates a discrete-time stochastic variable, designated as "additional value", *i.e.*, the value that does not directly derive from the execution of the tasks specified in the bid documents, and which can only be quantified with precision by undertaking an incremental investment in human capital and technology. The model determines that, even only recurring to the skills of their own experienced staff, contractors will produce a more competitive bid, provided that the expected amount for the additional profit is greater than zero. However, construction managers often need to hire specialized firms and highly skilled professionals in order to quantify, with accuracy, the expected amount of additional value and, hence, the precise impact of such additional value in the optimal bidding price. Based on the option to sign the contract and to perform the project by the selected bidder, identified and evaluated by Ribeiro et al. (2013), the model's outcome is the threshold value for this incremental investment. A decision rule is then reached: construction managers should invest in human capital and technology provided that the cost of such incremental investment does not exceed the predetermined threshold value. The model also proposes new forms of reaching the optimal bidding price, considering solely the effects of the non-incremental investment and also considering the possible impact of the incremental investment in human capital and technology.

JEL Classification Codes: G31; D81.

Keywords: real options; construction projects; investment decisions; optimal bidding.

*PhD Student in Finance, Faculty of Economics, University of Porto, Portugal; email: 090427012@fep.up.pt

[†] João Adelino Ribeiro acknowledges the financial support provided by FCT (Grant nº SFRH/BD/71447/2010)

[‡] Assistant Professor at Faculty of Economics, University of Porto, Portugal; email: pjpereira@fep.up.pt

[§] Full Professor at Faculty of Economics, University of Porto, Portugal; email: ebrandao@fep.up.pt

Part I

Introduction

1 Uncertainty in Construction Projects in the Context of Bidding Competitions

Uncertainty surrounding construction projects is a crucial element that should be adequately managed since it may have a huge impact in construction companies overall performance. Construction companies or contractors are firms operating in the construction industry whose business reside in executing a set of tasks previously established by the client.[1] The amount of tasks to be performed constitutes a project, job or work. The vast majority of projects in the construction industry are assigned through what is known as "tender" or "bidding" processes (Christodoulou (2010); Drew et al. (2001)), this being the most popular form of price determination (Liu and Ling (2005); Li and Love (1999)). In a tender or bidding process, a certain number of contractors (bidders) compete to execute a project by submitting a single-sealed proposal until a specific date previously defined by the client. Potential bidders have access to a what is commonly known as the "bid package". This package contains a set of technical pieces (often also referred as "tender documents") which will serve as the basis for determining the price to include in the bid proposal. More specifically, the package includes plans and technical drawings, a proposal form, the "general conditions" covering procedures which are common to all construction contracts and the "special conditions" containing the procedures to be used and that are unique to the project in question (Halpin and Senior (2011)), including information about the type of contract that will be enforced.

The usual format of a tender or bidding process is based on the rule that - all other things being equal - the contract will be awarded to the competitor that submitted the lowest price (Christodoulou (2010); Cheung et al. (2008); Chapman et al. (2000)) or, which is

[1] For the purposes of this research, we exclude those situations where contractors execute their own projects, as it is often the case of construction companies operating in the real estate sector.

the same, the lowest bid. The bidder that proposed the lowest price will most likely be invited to sign the contract and, if the contract is actually signed, he or she will have to invest a substantial amount of money by incurring in the necessary direct costs to execute the project, *i.e.*, the construction costs.

Traditionally, construction management literature has been placing more emphasis on the negative effects of uncertainty, which means researchers seem to be more concerned with ways to deal with the risks involving the construction activities and how they may affect the project's expected Net Present Value (NPV) through a negative impact in the construction costs. In fact, few authors have been addressing uncertainty as a source of opportunity, as it is the case of Ford et al. (2002), Ng and Bjornsson (2004) and Yiu and Tam (2006). Ford et al. (2002) argued that construction projects may include specific sources of uncertainty that affect project value, but not necessarily just by reducing it. This argument is supported by Ng and Bjornsson (2004) when they state that, even though uncertainty can lead to cost over-runs and delays, it can also produce positive return if managed properly. Following the same line of thought, Yiu and Tam (2006) applied the real options approach to evaluate the intrinsic value of uncertainty and the managerial flexibility deriving from the options to defer and to switch modes of construction. Therefore, uncertainty can also be seen as a source of opportunities, rather than just an element that may cause undesirable effects during the construction stage - in clear opposition to the traditional view that "all uncertainty presumed loss" (Mak and Picken (2000)).

Ford et al. (2002) acknowledge the fact that many construction project conditions evolve over time and thus managerial choices for effective decision-making cannot be completely and accurately determined during the pre-project planning period. In fact, these authors observe that many aspects of construction projects are uncertain, such as input prices, the weather conditions, the length of some activities and the overall duration of the project, among others, meaning that the effects of some of these sources of uncertainty can only be recognized and properly managed as the project unfolds. This argument is also supported by Mattar and Cheah (2006) when they mention that contractors typically learn more about the value of the project as they invest over time and uncertainties are resolved.

Even though we recognize the fact that the possible consequences of some sources of uncertainty cannot be anticipated, we do believe that others can be predicted and accounted for during the bid preparation process. Moreover, we will try to demonstrate that is possible to establish a support decision model that accommodates the expected impact of a specific type of uncertainty - the uncertainty associated with the amount of work to be executed during the project's life cycle.

This piece of research builds on this crucial aspect by focusing on a specific source of uncertainty which may lead to a greater NPV by increasing the expected amount of work to be executed during the construction stage. This means we believe that this source of uncertainty is - at least - as decisive as the others in adding value to the project. Therefore, managers should recognize its importance by planning and strategically managing this element in a way that improves the project's NPV and, as we will demonstrate, their competitiveness in a bidding competition context.

Despite the fact that project value can be substantially increased by reducing costs, we would like to reinforce the idea that project value can also be increased by raising more income. As we will see, more income means the income that is generated through actually executing, during the construction phase, a certain amount of tasks which were not included in the tender documents. We are thus concerned with the uncertainty that may lead to more project value by increasing the amount of work to be performed by contractors, "vis-a-vis" with the amount of work contractors are contractually bound to execute. We will refer to this type of value as "hidden-value".[2]

Hidden-value should be captured and quantified in the pre-project stage while the bid proposal is being prepared, by carefully analyzing the portions of the project where it may be concealed. Ford et al. (2002) observed that hidden-value is present in the most uncertain portions of the project, enabling us to sustain that skilled engineers and experienced

[2]To the best of our knowledge, this designation was first adopted by Ford et al. (2002) and, in the context of their research, the definition encompasses other sources of hidden-value, rather than just those which may result in additional income.

managers - whose responsibility is to prepare the bid proposal - have a fairly good knowledge, based on their accumulated experience, of "where to look for". Chapman et al. (2000) stated that the bid preparation process begins with a preliminary assessment of the tender documents. We sympathize with this statement and argue that, in this preliminary assessment, is possible to recognize and quantify hidden-value and, more specifically, to stipulate a high-estimate and a low-estimate to this hidden-value and to attribute a probability of occurrence to each of the estimates just by undertaking a preliminary analysis of the tender documents.[3] However, the quantification of hidden-value with accuracy is, in most practical situations, a goal that can only be achieved by performing an exhaustive investigation of all the bid documents. For this purpose, many construction companies will have to invest in human capital and technology, which, in some cases, may imply hiring skilled technicians specialized in this type of tasks (Kululanga et al. (2001)), as well as contracting highly specialized firms possessing the necessary technology to carry out specific type of works with the purpose of supplying contractors with more accurate information concerning the project in hands. Kululanga et al. (2001) clearly support this idea when they state that an awareness of job factors, which may give rise for claiming extra-revenues due to extra-work to be executed is a skill that, generally, has to be specially acquired. Pinnell (1998) reinforces this argument when he mention that the individual (or the team) responsible for thoroughly analyzing the bid documents aiming to capture and quantify hidden-value during the bid preparation process may be a consultant expert or a team of consultant experts. Whether this incremental investment in hiring skilled consultants and contracting highly specialized firms aiming to supply contractors with more accurate information regarding the volume of work to be performed will be worthwhile constitutes the question we will address using the model proposed in Part II.

Our model is thus based on the argument that uncertainty can add value to construction projects through the impact caused in the amount of work to be executed during the project's life cycle. This argument entails that contractors do not know, before the com-

[3]Our model will just contemplate a specific type of hidden-value: the one that may result in the creation of additional profit through the execution of more volume of work.

pletion of the project (or, at least, before the job begins), how much volume of work will effectively be executed. Hence, uncertainty is present concerning the volume of work, allowing us to designate this construction project-specific type of uncertainty from the contractor's perspective as "volume uncertainty". Volume uncertainty obviously leads to uncertainty about the project's final NPV since the execution of additional work implies receiving extra income (or extra revenues) and incurring in extra costs. We will designate, from now on, the difference between these extra revenues and these extra costs as "additional value". Additional value is, therefore, the value that may be generated because there is, at least, a specific source of uncertainty surrounding construction projects that may actually cause such effect. We now proceed to discuss this subject with more detail.

2 Recognizing and Quantifying Hidden-Value: The Concept of Additional Value

In many construction projects value is hidden in the most uncertain portions of the project, as we mentioned previously. After its detection and quantification, hidden-value becomes what we designate as additional value. To fully understand how hidden-value may be detected and properly quantified - and, hence, transformed into additional value - we must first know where hidden-value can be detected, which means we have to understand the nature of its sources.[4]

The construction management literature has been dedicating strong attention to a subject commonly known as "Claims". A construction claim can be defined as "a request by a contractor for compensation over and above the agreed-upon contract amount for additional work or damages supposedly resulting from events that were not included in the initial contract" (Adrian (1993)). This well-known definition implies that contractors can and should ask for a compensation when they execute works that were not considered in the initial contract.[5] Thomas (2001) argued that variations to the work are

[4]The pure detection of hidden-value does not necessarily result in the creation of additional value to the project. The increase in the project's expected NPV will only occur if the execution of the extra volume of work originates a profit. We will discuss this important aspect later.

[5]This definition also implies that there are other sources which may raise more income.

almost inevitable and Dyer and Kagel (1996) went even further when they stated that - inevitably (sic) - situations arise where clients actually deviate from the original construction scope, which means that, most likely, the initial scope will be increased. These statements strongly sustain our argument that, at least frequently, contractors do end up executing more work than the one deriving from what is established in the tender documents. Consequently, both statements also support the argument that contractors do not know, *ex-ante*, the precise amount of work they will be executing throughout the whole duration of the project.

Rooke et al. (2004) categorize construction claims in two different types: (i) proactive claims and (ii) reactive claims. Proactive claims are the ones that can be anticipated and, thus, planned for at the bid preparation stage. On the other hand, reactive claims are the ones which can only be recognized in the course of the project itself, in response to unforeseen events. Even though we are aware that reactive claims may have a considerable impact in the project's final NPV, we exclude them from our model precisely because they are, by definition, unforeseeable, which means that no acceptable estimate can be drawn. Therefore, our model incorporates estimates for those proactive claims which derive from the existence of uncertainty regarding the volume of work to be performed.[6]

2.1 Sources of Additional Value

There are two sources of uncertainty that may result in claims through the execution of more volume of work than the one directly deriving from the information contained in the bid package. We classify them as being of two different kinds: (i) extra quantities and (ii) additional orders.

[6] As we will carefully explain, depending on the contractual arrangements binding the parties, there is a specific type of volume uncertainty which does not necessarily generate more income.

2.1.1 Extra Quantities

Extra quantities occur when the contractor ends up executing, in the field, more quantities of a specific item than the ones specified in the tender documents. As we will see, if the type of contract allows such, the contractor will receive the unit price included in his or her proposal multiplied by the quantities he or she has actually executed and after being measured in the field by the client or the client's agent.[7] Under such contractual conditions, field quantities are the quantities that matter because they are the ones that will generate the income associated with the execution of each task included in the bid package. Ideally, from the client's point of view, field quantities should match the quantities included in the tender documents. However, frequently, discrepancies between the quantities estimated by the client and quantities actually executed in the field are observed. The literature refers that this inaccuracy is mainly due to the poor quality of the tender documents (see, for example, Laryea (2011); Rooke et al. (2004); Akintoye and Fitzgerald (2000)), meaning that the client´s estimates are not always accurate and, therefore, tender documents provided to the bidders often contain mistakes.

Bearing this in mind, most experienced contractors do not take for granted the accuracy of the information contained in the tender documents regarding the quantities to be performed when they are preparing the bid. On the contrary, if hidden-value is to be captured and quantified - since inaccuracies in the tender documents are likely to occur - mistakes can only be recognized if a proper measurement of all the technical drawings is performed. This is an important aspect we must stress: contractors will only know, with a strong degree of certainty, how many quantities they will be executing during the project's life cycle if a thorough and accurate measurement of all the technical drawings included in the bid package is undertaken. Moreover, as Rooke et al. (2004) stated, pricing a tender involves reading through bills of quantities often several inches thick, meaning that the quantities stated in the bill must be confronted with the quantities obtained after per-

[7] Such fact also implies that the contractor will receive less income if the quantities executed in the field are smaller than the ones specified in the bid documents.

forming a complete examination of all the drawings provided by the client. Rooke et al. (2004) also argued that most of the times - especially in the case of non-large contractors - companies do not have experts in this type of highly skilled job or, if they do, the amount of work in hands in a particular moment may imply the need for hiring external experts. This aspect is reinforced by the fact that contractors actually express concern over what they consider to be a short period of time that is normally allowed for bid preparation, as Laryea and Hughes (2008) observed.

2.1.2 Additional Orders

Additional orders, also known as "change orders", refer to a task or a set of tasks the contractor effectively performs during the project's life cycle, which possess a different nature from the ones specified in the bid package. This source of uncertainty that may give rise to additional work and extra profit is, thus, different from the one mentioned before, since change orders are related with varied work which is not of a similar character, or is not carried out under similar conditions than the one contained in the bill of quantities (Davinson (2003)).[8] However, we need to make clear that these tasks may include, for the purposes of their completion, the execution of an item or a set of items that actually were considered in the bill of quantities and previously priced by the bidder, since they were part of the project's initial scope. Hence, when contractors look for mistakes in the tender documents, they do not focus their attention merely in finding discrepancies that may lead to the execution of extra quantities solely associated with the tasks specified in the bid package. Instead, experienced engineers and skilled experts also search for possible tasks, which are likely to be executed and were not specified in the tender documents. By carefully analyzing all the plans and drawings provided by the client, it is possible to recognize that some parts of the project (or even the project "seen" as a whole) will not be properly completed if only the tasks included in the tender documents

[8]Change orders are also often designated as "increase in scope". The designation obviously acknowledges the fact that the scope of the original project becomes wider, which means the contractor will execute a certain number of tasks that were not included in the technical pieces that supported the initial contract.

are to be performed. Hence, additional orders can and should be considered as a potential source of additional value and our model will encompass this argument by assuming that contractors are able to stipulate a high-value estimate and a low-value estimate for additional orders, and also attribute, to each of these estimates, a probability of occurrence. For the purpose of defining the two estimates, we argue that contractors need to take into account: i) the amount of work the additional orders will generate in comparison to the amount established in the original contract; ii) the previous experience with the client as well as their history and frequency of placing new orders; iii) the bargaining skills of the client throughout the negotiation process.

2.2 Types of Contract

To fully understand the possible impact of the two sources of additional value on the project's expected NPV, we have to relate each of them with the type of contract that will bind the parties. Construction management literature addresses with more relevance two types of contracts (see, for example, Halpin and Senior (2011); Clough et al. (2000); Woodward (1997)): (i) the "unit-price" type of contract and (ii) the "lump-sum" type of contract.[9]

The unit-price contract allows for flexibility in meeting variations regarding the amount and quantity of work encountered during the construction stage. This means that, when this type of contract is adopted, the project is broken down into work items, which are characterized by units, such as cubic yards, linear and square feet, and piece numbers (Halpin and Senior (2011)). This fact implies that the contractor, during the bid preparation stage, will quote the price by units rather than as a single total contract price. Hence, if for some reason, the contractor effectively executes more quantities of one or more specific items included in the bill of quantities, he or she will be receiving the amount that results from multiplying the number of units executed by the unit price he or she has

[9]Other types of contract are mentioned in the literature, as the "cost-plus-fee" type and the "cost-reimbursement" type. However, unit-price and, especially, lump-sum contracts are the ones that are most commonly adopted, particularly in the context of bidding competitions.

included in the bid proposal.

If the type of contract enforced is the lump-sum, bidders are asked to price a specific task or item, regardless of the number of units that will actually be executed. Hence, if this type of contract is adopted, contractors will never receive more (less) income for executing more (less) quantities of an item or items clearly specified by the client than those he or she has predicted after analyzing the drawings and other technical documents contained in the bid package. The risk associated with the likelihood of performing more quantities than those that served as the basis for computing the corresponding global price for a specific task (or a group of tasks) is, thus, borne by the contractor. However, the contrary can also occur: contractors might actually perform less quantities in the field than those considered during the tender preparation stage, and which served as the basis for establishing the proposed price. Hence, and even though this specific type of uncertainty still exists when the parties are bounded by a lump-sum contract, it is not possible to account for its effects during the bid preparation stage since the contractor will only be aware if any additional value is actually raised through this mean after the task or tasks in question are executed, *i.e.*, as the project unfolds. Therefore, this source of uncertainty may affect the project's final NPV but can not be quantified before the work begins. Being so, in the presence of a lump-sum contract, additional value may only be obtained through the execution of additional orders whereas, if the contract assumes the unit-price type, both sources of uncertainty may create additional value by increasing the volume of work to be performed.[10]

Lump-sum contracts are the most common type of contracting, especially in the building sector (Rooke et al. (2004)). In the European Union, current legislation concerning public contracting virtually imposes the lump-sum form, which means that the unit-price type has had small to none application, due to the increasing effort european regulators have been exercising with the purpose of transferring the risk associated with possible

[10]Some projects encompass both types of contracts, meaning that some tasks should be priced using unit-prices and others applying the lump-sum form. In such cases, both types of uncertainty are present, in different parts of the project.

mistakes (also referred to, in technical language, as "errors and omissions") encountered in the technical pieces from the client to the contractor. This broad reality has compelled us to consider in our model only one of the two sources of additional value previously described: the additional value which may rise from the execution of additional orders. Being so, we will assume that the lump-sum type of contract is the one that actually binds the parties, which means that the possible execution of more quantities in the field than the ones eventually stated in the bid documents will not generate any additional revenues and, consequently, any additional value to the project. Obviously, this also implies that the costs associated with the possible execution of any extra quantities should be taken into account when determining the amount of constructions costs that will sustain the bid price.

3 How the Detection of Hidden-Value May Lead to Additional Value: Contractor's Opportunistic Bidding Behavior

Construction management literature clearly acknowledges the fact that the construction industry features strong levels of price competitiveness (see, for example, Chao and Liu (2007); Skitmore (2002); Ngai et al. (2002)) which may force bidders to lower their profit margin and, thus, increase the probability of winning the contract (Mohamed et al. (2011)). Consequently, it is not rare to see the winning bid include a near-zero profit margin (Chao and Liu (2007)) or even a price below cost. This intense competition encountered in bidding processes often leads to "under-pricing", a common phenomenon namely explained by the need for work and penetration strategies (Yiu and Tam (2006); Fayek (1998); Drew and Skitmore (1997)).[11]

In fact, contractors realize that bidding low when facing strong competition increases the

[11] Under-Pricing is not necessarily the same as bidding below cost; rather, we interpret this concept as the practice of including in the bid price a profit margin smaller than the one contractors would include in normal circumstances, *i.e.*, if the levels of competition in the construction industry were not generally recognized as being particularly intense.

chance of being selected to execute the project, but they are also aware of the opposite: if the profit margin included in their proposals is higher, the probability of getting the contract will be lower. This inverse relationship between the level of the profit margin and the probability of winning the bid is a generally accepted fact both in the construction industry and in the research community (*e.g.*, Christodoulou (2010); Kim and Reinschmidt (2006); Tenah and Coulter (1999); Wallwork (1999)). As we will detail, our model incorporates this crucial element and the mathematical expression which respects the inverse relationship between these two variables, proposed by Ribeiro et al. (2013), will be adopted.

Detecting hidden-value and executing more volume of work will only result in more value to the project if the difference between the extra revenues and the extra costs of performing the additional tasks is positive, *i.e.*, if contractors do actually generate a profit by executing them, which means that detecting and executing more volume of work than the one directly specified in the tender documents will not necessarily lead to more profit. However, experienced contractors that capture hidden-value ensure themselves that items where extra quantities are likely to be executed will be priced in a way that will lead to an increase in the project's expected NPV. By applying this practice during the bid preparation process, contractors increase their probability of winning the bid, by sacrificing the profit margin included in the bid proposal, knowing that they may recover such part of the profit in subsequent change orders or claims, as Tan et al. (2008) observed. This type of behavior is designated in the literature as "Opportunistic Bidding Behavior" (OBB).[12] Thus, following a proactive approach and, therefore, assuming that time is actually invested in detecting mistakes, which may lead to the likely execution of more (less) quantities, contractors will inflate (deflate) the unit price of the items where those mistakes were spotted. Over-charging (under-charging) those items can then be compensated by under-charging (over-charging) the unit-prices of some of the items whose quantities

[12]The literature identifies two different types of OBB: "front loading" and "claim loading". (see, for example, Arditi and Chotibhongs (2009); Yiu and Tam (2006)). Front loading consists in over-charging the tasks to be executed in the early stages of the project's life cycle and compensate such effect by under-charging the tasks to be performed in the last stages. We are not concerned with this type of behavior in our model since it does not derive from any detection of hidden-value.

contractors are certain to be accurately measured. Hence, this compensation mechanism allows contractors to maintain the previously defined overall price for executing the quantities specified in the tender documents and, still, leaving room for generating more profit through the likely execution of additional quantities.

Despite the fact that this behavior is potentially more effective in the presence of a unit-price form of contract, it may also produce positive effects when the contract type is the lump-sum. In fact, experienced contractors will most likely inflate prices of items they predict to be present in future additional orders since - and even though additional orders are subject to a specific process of price negotiation - it is likely that they will contain the execution of certain items which were contemplated in the original contract and, thus, whose price is already established between the parties. In these circumstances, the parties will agree that the unit price for such items will be the same. However, items that are different from the ones contained in the tender documents become a matter of negotiation between the contractor and the client or the client's agent, as Dyer and Kagel (1996) argued. This means that, unlike what happens with extra quantities, there is no predetermined form of pricing additional orders in its full extension. In fact, contractors do not have a way of predicting, with complete certainty, what price will be established and what profit will be generated if these additional orders are actually placed by the client.

Nevertheless, based on previous experiences and in current market prices, we believe that contractors can actually perform fair estimates on the final revenues to be generated by such orders and we also believe that, in the event these orders are placed and the additional work executed, a considerable profit will be made.[13]

The remainder of this paper unfolds as follows. In Part II, we begin by introducing the

[13] Dyer and Kagel (1996) conducted a study where a number of general contractors were interviewed and additional orders were frequently mentioned as being particularly profitable. It is generally accepted in the industry that the negotiation process leading to the price determination of such orders often develops in a very favorable manner to the contractor, mainly due to the client's awareness that the decision to switch to another contractor for merely executing those additional tasks will imply incurring in side costs and will also cause time delays.

model's basic numerical solution suggested by Ribeiro et al. (2013), which will enable us to reach the optimal price if no detection and quantification of hidden-value is considered. After listing the assumptions, we proceed to describe the model . In Part III, a numerical example is given, followed by a sensitivity analysis to some of the model's most important parameters. Finally, in Part IV, concluding remarks are given.

Part II

The Model

3.1 Introduction

The model herein proposed is based on the option to sign the contract and, consequently, invest in performing the project by the selected bidder. In a bidding competition, the contractor prepares the bid proposal and submits it until a certain date previously defined by the client. However, the client will only decide which bidder will be invited to sign the contract months later. Consequently, the estimated constructions costs, which served as the basis to establish the price included in the bid proposal, will most likely vary during this period, *i.e.*, from the moment the bid proposal is closed until the selected bidder is invited to sign the contract. On the contrary, the price established by the contractor and proposed to the client will remain unchanged during the same period. Recognizing these facts, Ribeiro et al. (2013) identified a specific real option: the option to sign the contract and, hence, to invest in performing the project by the selected bidder. This option constitutes a real option since the selected bidder has the right - but not the obligation - to sign the contract and, consequently, to invest in executing the job by incurring in the necessary costs to complete it - the construction costs. As the option pricing theory states, this real option has value and, we will apply the adapted version of the Margrabe (1978) exchange option pricing model proposed by Ribeiro et al. (2013) to evaluate the option, with the final purpose of reaching an optimal price. According to the model proposed by Ribeiro et al. (2013), the optimal price will be the one corresponding to the highest value of the option to invest. However, since this option is only available to the selected contractor, its value has to be weighted by the probability of winning the bid. These authors proposed a mathematical relationship linking the level of the profit margin (or the "mark-up", as it is commonly designated in construction parlance) and the probability of winning the contract. The proposed mathematical expression respects the generally accepted fact that there is an inverse relationship between the two variables. In fact, Ribeiro et al. (2013) argue that, even though construction managers seldom support their mark-up decisions using some sort of mathematical expression between the price and the probability of winning the bid, which means that the decision regarding the mark-up bid value is generally sustained in subjective judgments, gut feelings and heuristics, managers do have, at least, an implicit perception that the higher (lower) the profit margin he or she includes in the bid proposal the lower (higher) will be the probability of winning the bid. Thus, the numerical solution suggested by Ribeiro et al. (2013), which consists of a maximization problem, has two fundamental components: (i) the value of the option to sign the contract and, consequently, to invest by performing the project; (ii) the probability of winning the bid. We briefly present each of them below.

3.1.1 The Value of Option to Sign the Contract and Invest by Performing the Project in the Presence of Penalty Costs

According to Ribeiro et al. (2013), in some legal environments, a financial compensation may be imposed to the selected contractor if he or she declines the invitation to sign the contract. For instance, in the United States contractors are free to withdraw their bids without incurring in any penalties if that happens prior to the ending of the bidding period Halpin and Senior (2011). However, if the selected bidder decides to withdraw the proposal after that moment, a penalty equal to the difference between the second best proposal and the chosen bid may be legally imposed, even if the contract has not yet been signed. To accommodate these circumstances, Ribeiro et al. (2013) adapted the Margrabe (1978) exchange option pricing formula. According to these authors, the value of the option to invest, $F_g(P,K)$ in the presence of penalty costs, g will be given by the following equation:[14]

$$F_g(P,K) = [PN(d_1) - (1-g)KN(d_2) - gK(1-N(d_2)] \quad (1)$$

being (d_1) and (d_2):

$$d_1 = \frac{ln[P/(1-g)K] + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}} \quad (2)$$

$$d_2 = d_1 - (\sigma\sqrt{T-t}) \quad (3)$$

where P is the price, K the construction costs, $N(d_1)$ and $N(d_2)$ the probability density functions for the values that result from expressions (d_1) and (d_2), respectively, σ is the standard deviation and σ^2 is the variance, which, in this case, equals σ_K^2.[15] $T-t$ is the 'time to expiration', *i.e.*, the time between the moment the bid price is established and the moment the contractor is invited to sign the contract, and g is the expected value for the penalty costs. As in Ribeiro et al. (2013), we will assume that the value of the penalty costs, g is a percentage of K, the construction costs.[16]

[14] Penalty costs may also be seen as possessing a reputational nature, as Ribeiro et al. (2013) argue.

[15] Since $\sigma^2 = \sigma_P^2 - 2\sigma_P\sigma_K\rho_{PK} + \sigma_K^2$, where ρ_{PK} is the correlation coefficient between the price, P and the construction costs, K; since P remains unchanged during the life of the option, then σ_P^2 equals zero and so does $2\sigma_P\sigma_K\rho_{PK}$.

[16] In the absence of penalty costs, g equals zero and thus the Margrabe (1978) formula is reduced to its original form: $F(P,K) = [PN(d_1) - KN(d_2)]$, where:

$$d_1 = \frac{ln(P/K) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}$$

and d_2 remains unchanged.

3.1.2 The Probability of Winning the Bid

Ribeiro et al. (2013) suggest an inverse relationship linking the mark-up ratio and the probability of winning the bid, which is given by the following equation:

$$W(P,K) = e^{-b(P/K)^n} \tag{4}$$

where $W(P,K)$ is the probability of winning the bid, P/K is the mark-up ratio and 'n' and 'b' are parameters included in the equation for calibration purposes. Ribeiro et al. (2013) argue that managers should calibrate such parameters in order to best reflect each contractor's specific circumstances and the conditions surrounding the bid in hands. Parameter 'b' sets the probability of winning the bid if the price equals the construction costs, *i.e.*, if the mark-up ratio equals 1. Parameter 'n' is responsible for shaping the function's concavity and convexity. Our model embraces this functional relationship and a specific calibration for each of the parameters will be specified in the numerical example presented in Part III.

3.1.3 The Base Price

We will designate the "base price", in the context of the present model, as being the optimal price that results from the maximization problem suggested by Ribeiro et al. (2013). We will adopt that same numerical solution to determine this price. The base price is the price contractors should include in their bid proposal if no detection and quantification of hidden-value, which may lead to the generation of additional profit through the execution of additional orders is to be undertaken, in accordance with the assumptions listed below, in Section 4. Hence, the base price is the optimal price without considering those effects or, which is the same, the optimal price that derives from considering the value solely generated through the execution of the tasks included in the bid documents.

Assuming that penalty costs, g are present, let P_b designate the base price. P_b will be the outcome of the maximization problem proposed by Ribeiro et al. (2013) and reproduced below:

$$\begin{aligned} V(P_b, K_b) &= \max_{P_b} \quad \{[P_b N(d_1) - (1-g)K_b N(d_2) - gK_b(1 - N(d_2))] \\ &\qquad [e^{-b(P_b/K_b)^n}]\} \end{aligned} \quad (5)$$

where K_b is the amount of costs the contractor will have to incur in order to exclusively perform the amount of work which derives from what is established in the initial contract (the base construction costs). Thus, the option value for each mark-up level $V_g(P_b, K_b)$ will be given by the outcome of the adapted Margrabe (1978) formula, $F_g(P_b, K_b)$ weighted by the probability of winning the bid, $W(P_b, K_b)$.

4 Assumptions

In our model we will assume that (i) each bidder decides what price to bid without engaging in any kind of interaction or contact with other bid participants; (ii) each bidder prepares his or her proposal simultaneously with the other competitors; (iii) each bidder presents a single sealed proposal to the client; (iv) each bidder has access to the information contained in the bid package, allowing him or her to establish the base price, determined applying the maximization problem expressed by equation (5) above; (v) the selected bidder will only decide if he or she is going to perform the project when the contract has to be signed and not before that date; (vi) it is possible to establish a mathematical relationship between the mark-up level and the probability of winning the bid, and the expression linking these two variables is given by equation (4) above; (vii) penalty costs are present if the selected contractor decides not to sign the contract and, consequently, not invest in executing the project; (viii) the parties are bound by a lump-sum contract, which means that no additional income is generated if extra quantities are executed; (ix) by only using the skills of their own experienced staff, contractors are able to stipulate a high-value estimate and a low-value estimate for the expected profit to be generated through the execution of additional orders, and also to attribute a probability of occurrence to each of them, during the bid preparation stage; (x) once the true estimate is known, contractors will adjust the price (extra revenues) during the negotiation process to compensate for any variations that may occur in the estimated extra costs, which means that any changes observed in the necessary costs to perform the additional orders will lead to an adjustment in the price requested to the client, with the purpose of maintaining the expected profit at the level previously established during the bid preparation period.

5 Model Description

The present model is motivated by the fact that, in most construction projects, the volume of work to be executed is not known with precision during the bid preparation stage. Hence, uncertainty is present concerning the level of profit to be generated. Even though we have identified two different sources of volume uncertainty, we will only focus on the one deriving from the possible execution of additional orders to be placed by the client, since we assume that the parties are bound by a lump-sum contract - thus preventing contractors from generating extra revenues by merely executing extra quantities of items included in the bid package.

As previously mentioned, we assume that experienced contractors are be able to stipulate a high-value estimate and a low-value estimate to the additional orders and to attribute a probability of occurrence to each of them just by undertaking a preliminary assessment of the tender documents, meaning this goal can be achieved without the need to incur in any additional costs associated with hiring skilled professionals and contracting specialized firms, *i.e.*, without any incremental investment in human capital and technology. Let C_1 designate this non-incremental level of investment contractors will undertake using only the skills of their experienced staff. By spending the amount C_1, contractors will thus

(i) define a high-value estimate and a low-value estimate for the price (revenues) to be obtained through the execution of additional orders; (ii) stipulate a high-value estimate and a low-value estimate for the necessary costs to successfully perform these orders; (iii) attribute a probability of occurrence to each of the estimates.

Hence, by investing the amount C_1, contractors will be establishing a discrete-time stochastic variable, which we designate as "additional value", with two possible outcomes: the high-value estimate and the low-value estimate and affecting a probability of occurrence to each of them.

5.1 The Impact of the Non-Incremental Investment

The base price, P_b represents the amount of income to be received due to the execution of the volume of work included in the bid package: this is the price resulting from expression (5) presented above and - again - we stress that this is the optimal price contractors should include in their proposals if no detection and quantification of any additional value resulting from the execution of additional orders is undertaken. By investing the amount C_1, contractors will most likely detect and quantify hidden-value, which may result in the creation of additional value to the project. Hence, we now need to consider the additional income that will be received, assuming that additional orders will be executed during the project's life cycle and, also, the necessary costs to successfully perform the additional work. Let p_A represent the additional income (extra revenues) deriving from the possible executions of additional orders, and k_A the amount of costs the contractor will have to incur to perform the additional orders. Finally, π represents the amount of profit (or additional value) generated by executing the additional orders, *i.e.*, the difference between p_A and k_A.

Also, let (i) p_A^H designate the high-value estimate for the revenues associated with the execution of the additional orders; (ii) p_A^L designate the low-value estimate for such revenues; (iii) k_A^H designate the high-value estimate for the costs associated with the execution of the additional orders; (iv) k_A^L designate the low-value estimate for such costs; (v) θ represent the probability associated with p_A^H and k_A^H; hence $(1-\theta)$ will represent the probability associated with p_A^L and k_A^L. Finally, let π^H and π^L denote the additional profit for the high-value and the low-value estimates, respectively, and which are given by the following equations:

$$\pi^H = p_A^H - k_A^H \tag{6}$$

$$\pi^L = p_A^L - k_A^L \tag{7}$$

Thus, the expected value for the additional profit, $E(\pi)$ will be given by equation (8) below:

$$E(\pi) = \pi^H\theta + \pi^L(1-\theta) \tag{8}$$

Let P_1 designate the optimal price in the present conditions, *i.e.,* the price that incorporates the effect of the expected value for the additional profit, $E(\pi)$, given by equation (3.8). Hence, we adapt the numerical solution suggested in Chapter II, with the purpose of incorporating the effects caused by the expected value for the additional profit, $E(\pi)$. The optimal price, P_1 will be the outcome for the following maximization problem:

$$\begin{aligned} V(P_1, K_b) = \max_{P_1}\{[(P_1 + E(\pi))N(d_1) - (1-g)K_bN(d_2) - \\ gK_b(1-N(d_2))][e^{-b(P_1/K_b)^n}]\} \end{aligned} \tag{9}$$

The price, P_1, which results from the maximization problem expressed by equation (9) is smaller than the one resulting from the maximization problem expressed by equation (5), *i.e.*, the base price, P_b since the former is the optimal price in the absence of any recognition and quantification of hidden-value generating more profit through the execution of additional orders, whereas the later reflects the optimal price considering the expected impact of the additional orders to be performed, at this stage, by investing the amount C_1. This means that, under these conditions, P_1 is the price contractors should include in their bid proposals because it is the optimal price if no incremental investment is undertaken.

We also need to make clear that equation (9) is a function of K_b - the direct costs of solely performing the tasks included in the tender documents, rather than the total estimated costs the contractor will incur if he or she wins the contract and performs the project. This is due to the fact that the extra costs are already considered in the amount of the expected profit which, as the same equation shows, integrates the maximization problem given by equation (9).

Thus, the probability of winning the contract adopting the optimal price, P_1 is given by:

$$W(P_1, K_b) \quad = \quad e^{-b(P_1/K_b)^n} \tag{10}$$

The outcome of equation (10) is greater than the outcome of equation (4). In fact, the probability of winning the contract considering the effects of investing C_1 will always be greater than the probability considering the base price, P_b because P_1 - and assuming that some hidden-value is captured and quantified at this stage - is smaller than the base price,P_b *i.e.,* the optimal price if no quantification of hidden-value is considered, as we have mentioned. Thus, just by investing the amount C_1, contractors will produce a more competitive bid price, provided that some hidden-value leading to the generation of additional profit through the execution of additional orders has actually been captured and quantified.

5.1.1 The Value of the Option to Sign the Contract and Perform the Project

Bearing this in mind, the value of the option to invest in such conditions, $V(P_1, K_b)$ will be given by the following equation:

$$\begin{aligned} V(P_1,K_b) &= \{[(P_1+E(\pi))N(d_1)-(1-g)K_bN(d_2)- \\ &\quad gK_b(1-N(d_2))][e^{-b(P_1/K_b)^n}]\} \end{aligned} \quad (11)$$

being:

$$d_1 = \frac{ln[(P_1+E(\pi))/(1-g)K_b]+\frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}} \quad (12)$$

and

$$d_2 = d_1-(\sigma\sqrt{T-t}) \quad (13)$$

$N(d_1)$ and $N(d_2)$ the probability density functions for the values resulting from expressions (d_1) and (d_2), respectively, σ is the standard deviation and σ^2 is the variance which, as we have mentioned, equals σ_K^2. $T-t$ is the 'time to expiration', *i.e.*, the time between the moment the price included in the bid proposal is established and the moment the contractor is invited to sign the contract and, finally, g is the estimated level of penalty costs.

Equation (11) determines the value of the option to invest if the expected value for the additional profit to be generated by executing the additional orders, $E(\pi)$ is, in fact, the true value. Thus, if no incremental investment is put through, the outcome of equation (11) is the value of the option to sign the contract and perform the project after the amount C_1 has been invested and assuming that some hidden-value, resulting in more volume of work, was captured and quantified as a direct result of such investment.[17]

5.2 The Impact of the Incremental Investment in Human Capital and Technology

Let C_2 represent the incremental investment in human capital and technology, which will allow the contractor to eliminate the uncertainty concerning the true value of the additional work to be performed and the extra profit to be generated through the execution of this additional work. Hence, after investing the amount C_2, the contractor may face two different scenarios since this investment will reveal if the true value is the high estimate or the low estimate previously established during the bid preparation stage. Being so, we first need to determine the optimal price and the corresponding value of the option to invest, for each scenario.

5.2.1 The Optimal Price and the Value of the Option to Invest Considering the High-Value Estimate

If the investment in C_2 reveals that the true value for the additional profit, π is given by its high-estimate, then the optimal price in such conditions, P_2^H will be the outcome of the maximization problem given by equation (14).

$$\begin{aligned} V(P_2^H, K_b) &= \max_{P_2^H}\{[(P_2^H + \pi^H)N(d_1) - (1-g)K_bN(d_2) - \\ &\quad gK_b(1 - N(d_2))][e^{-b(P_2^H/K_b)^n}]\} \end{aligned} \tag{14}$$

where:

$$d_1 = \frac{ln[(P_2^H + \pi^H)/(1-g)K_b] + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}} \tag{15}$$

[17] This implies that the expected value for the additional profit, $E(\pi)$ is greater than zero.

and:

$$d_2 = d_1 - (\sigma\sqrt{T-t}) \quad (16)$$

The value of the option to invest, assuming the true value for the additional profit equals the high-value estimate, is given by the following equation:

$$\begin{aligned} V(P_2^H, K_b) &= \{[(P_2^H + \pi^H)N(d_1) - (1-g)K_bN(d_2) - \\ &\quad gK_b(1 - N(d_2))][e^{-b(P_2^H/K_b)^n}]\theta\} \end{aligned} \quad (17)$$

5.2.2 The Optimal Price and the Value of the Option to Invest Considering the Low-Value Estimate

However, if the true value revealed for π is given by its low-estimate, then the optimal price, P_2^L will be the outcome of the following maximization problem:

$$\begin{aligned} V(P_2^L, K_b) &= \max_{P_2^L}\{[(P_2^L + \pi^L)N(d_1) - (1-g)K_bN(d_2) - \\ &\quad gK_b(1 - N(d_2))][e^{-b(P_2^L/K_b)^n}]\} \end{aligned} \quad (18)$$

And the value of the option to invest, in these conditions, is given by equation (3.19), presented below:

$$\begin{aligned} V(P_2^L, K_b) &= \{[(P_2^L + \pi^L)N(d_1) - (1-g)K_bN(d_2) - \\ &\quad gK_b(1 - N(d_2))][e^{-b(P_2^L/K_b)^n}](1-\theta)\} \end{aligned} \quad (19)$$

being:

$$d_1 = \frac{ln[(P_2^L + \pi^L)/(1-g)K_b] + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}} \quad (20)$$

and:

$$d_2 = d_1 - (\sigma\sqrt{T-t}) \quad (21)$$

5.3 The Threshold Value for the Incremental Investment

By adding up the outcome of equations (17) with the outcome (19), we then reach the value of the option to invest considering the two scenarios, $V(P_2, Kb)$, *i.e.*, the one where the high-estimate is the true value and the one where the low-estimate is the true value. Thus:

$$V(P_2, K_b) = V(P_2^H, K_b) + V(P_2^L, K_b) \quad (22)$$

The outcome of equation (11) is the value of the option to invest considering solely the effects produced by the non-incremental investment, C_1. Equation (22) determines the value of the option considering the effects of the incremental investment, C_2, for both scenarios. Hence, the difference between the outcome of equation (22) and the outcome of equation (11) is the exact amount of investment, C_2^*, below which any level of investment will add value to the project and will increase its expected NPV. For incremental investment levels above C_2^*, the project's expected NPV will be reduced. The threshold value for the incremental investment, C_2^* will thus be given by the following equation:

$$C_2^* = V(P_2, K_b) - V(P_1, K_b) \quad (23)$$

The ratio between the incremental investment threshold and the base construction costs is given by the following equation:

$$RC_2^* = \frac{C_2^*}{K_b} \qquad (24)$$

Part III

Numerical Example

5.4 The Base Case

The following table includes information about the inputs used in the present numerical example:

Table 1: inputs: description and values

Inputs	**Description**	**Values**
K_b	base construction costs	USD 100,000,000
σ	standard deviation	0.25
$T-t$	"time to expiration"	0.5 (years)
n	calibration parameter of W(P,K)	10
b	calibration parameter of W(P,K)	ln(1/0.5)
g	penalty costs	0.02
p_A^H	high-value estimate for the additional revenues	USD 30,000,000
k_A^H	high-value estimate for the additional costs	USD 15,000,000
p_A^L	low-value estimate for the additional revenues	USD 9,000,000
k_A^L	low-value estimate for the additional costs	USD 6,000,000
θ	probability of occurrence of the high-value estimate	0.5
$(1-\theta)$	probability of occurrence of the low-value estimate	0.5

Using the inputs listed above and applying the model described in Part II, we have reached the following outputs:

Table 2: outputs: description, values and corresponding equations

Outputs	Description	Values	Equation
P_b	base price	USD 101,756,344	(5)
π^H	additional profit considering the high-value estimate	USD 15,000,000	(6)
π^L	additional profit considering the low-value estimate	USD 3,000,000	(7)
$E(\pi)$	expected value for the additional profit	USD 9,000,000	(8)
P_1	optimal price after investing C_1	USD 98,743,325	(9)
$W(P_1, K_b)$	probability of winning with price P_1	54.29%	(10)
$V(P_1, K_b)$	value of the option after investing C_1	USD 6,773,641	(11)
P_2^H	optimal price considering the high-value estimate	USD 96,955,641	(14)
$V(P_2^H, K_b)\theta$	option value considering the high-value estimate	USD 4,767,489	(17)
P_2^L	optimal price considering the low-value estimate	USD 100,700,095	(18)
$V(P_2^L, K_b)(1-\theta)$	option value considering the low-value estimate	USD 2,257,852	(19)
$V(P_2, K_b)$	option value considering both estimates	USD 7,025,341	(22)
C_2^*	incremental investment threshold value	USD 251,700	(23)
RC_2^*	ratio "investment threshold / base construction costs"	0.25%	(24)

5.5 Sensitivity Analysis

5.5.1 Is There a Scale-Effect?

Considering that the size of a construction project is given by the dimension of its base construction costs, K_b, we performed a sensitivity analysis with the purpose of verifying if a scale-effect is present, *i.e.*, if the investment threshold ratio, RC_2^* assumes different values in response to variations in the level of the base construction costs. We defined two alternative scenarios, where (i) the amount of the base construction costs are twice as great and four times as great as in the base case, *i.e.*, equal to USD $200,000,000$ and USD $400,000,000$, (ii) the level for the high-value estimate and the low-value estimate of the additional profit respects this same proportion and (iii) the corresponding probabilities of occurrence remain unchanged. We reached the following results for the incremental investment threshold value, C_2^* and for the incremental investment threshold ratio, RC_2^*:

Table 3: sensitivity analysis: scale-effect

(for: $\sigma = 0.25$; $T - t = 0.5$; $n = 10$; $b = ln(1/0.5)$; $g = 0.02$)

	base case	**alternative scenario 1**	**alternative scenario 2**
K_b	**USD 100,000,000**	**USD 200,000,000**	**USD 400,000,000**
π^H	USD 15,000,000	USD 30,000,000	USD 60,000,000
π^L	USD 3,000,000	USD 6,000,000	USD 12,000,000
θ	50%	50%	50%
$(1-\theta)$	50%	50%	50%
$\varepsilon(\pi)$	USD 9,000,000	USD 18,000,000	USD 36,000,000
C_2^*	USD 251,700	USD 503,400	USD 1,006,800
RC_2^*	**0.25%**	**0.25%**	**0.25%**

Results included in Table 3 clearly demonstrate that the investment threshold is proportional to the amount of the base construction costs, which means that no scale-effect is present. In fact, for the three different dimensions considered, the ratio between the threshold and the base construction costs remains constant and equal to 0.25%. Thus, these results express the existence of a linear relationship between the incremental investment threshold value and the base construction costs.

5.5.2 The Impact of Different Levels of Probabilities Associated with the High/Low Value Estimates

The results concerning the impact of considering different levels for the probabilities associated with the high-value and the low-value estimates in the model's outcome are included in Table 4.

Table 4: sensitivity analysis: probabilities associated with high/low value estimates

(for information about the inputs considered, please refer to Table 1)

θ	$(1-\theta)$	$E(\pi)$	$V(P_1, K_b)$	$V(P_2, K_b)$	C_2^*	RC_2^*
99%	1%	USD 14,880,000	USD 9,475,025	USD 9,484,785	USD 9,760	0.01%
90%	10%	USD 13,800,000	USD 8,943,931	USD 9,033,050	USD 89,119	0.09%
80%	20%	USD 12,600,000	USD 8,345,927	USD 8,531,123	USD 185,196	0.19%
70%	30%	USD 11,400,000	USD 7,819,379	USD 8,029,196	USD 209,817	0.21%
60%	40%	USD 10,200,000	USD 7,286,518	USD 7,527,268	USD 240,750	0.24%
50%	**50%**	**USD 9,000,000**	**USD 6,773,641**	**USD 7,025,341**	**USD 251,700**	**0.25%**
40%	60%	USD 7,800,000	USD 6,280,981	USD 6,523,414	USD 242.433	0.24%
30%	70%	USD 6,600,000	USD 5,808,732	USD 6,021,487	USD 212.755	0.21%
20%	80%	USD 13,800,000	USD 5,357,042	USD 5,519,559	USD 162.517	0.16%
10%	90%	USD 5,400,000	USD 4,926,015	USD 5,017,632	USD 91,617	0.09%
1%	99%	USD 4,200,000	USD 4,555,803	USD 4,565,898	USD 10,095	0.01%

Results included in Table 4 clearly show that, the closer the probabilities are to the upper limit or the lower limit, the smaller is the investment threshold value. This is due to the fact that, the closer the probability levels are to 100% or to 0%, the lower is the uncertainty regarding which will be the true value, meaning that the level of incremental investment needed that is needed to eliminate that level of uncertainty is smaller. The two more extreme scenarios clearly reflect such: when parameter θ equals 99% or 1%, the investment threshold assumes very low values (USD $9,760$ and USD $10,095$, respectively). On the contrary, as probabilities tend to 50%, the higher is the threshold value, C_2^*. Thus, threshold value reaches its maximum when the level of uncertainty is the highest, *i.e.*, when the probabilities associated with the two estimates assume the same value.

5.5.3 The Impact of Variations in the Difference Between the Two Estimates

Table 5 includes values concerning three different scenarios and the results reached by changing the difference between the high-value and low-value estimates but assuming that both the expected profit, $E(\pi)$ and the probabilities of occurrence,θ and $(1-\theta)$, remain unchanged.

Table 5: sensitivity analysis: difference between the high and the low value estimates

(for: $\sigma = 0.25$; $T - t = 0.5$; $n = 10$; $b = ln(1/0.5)$; $g = 0.02$)

	base case	**alternative scenario 1**	**alternative scenario 2**
K_b	USD 100,000,000	USD 100,000,000	USD 100,000,000
π^H	USD 15,000,000	USD 17,000,000	USD 13,000,000
π^L	USD 3,000,000	USD 1,000,000	USD 5,000,000
$(\pi^H - \pi^L)$	**USD 12,000,000**	**USD 16,000,000**	**USD 8,000,000**
θ	50%	50%	50%
$E(\pi)$	USD 9,000,000	USD 9,000,000	USD 9,000,000
$V(P_1, K_b)$	USD 6,773,641	USD 6,773,641	USD 6,773,641
$V(P_2, K_b)$	USD 7,025,341	USD 7,219,649	USD 6,885,769
C_2^*	**USD 251,700**	**USD 446,008**	**USD 112,128**
RC_2^*	**0.25%**	**0.45%**	**0.11%**

In the alternative scenario 1, the difference between the two estimates is greater than in the base case. Results show that, when this difference increases from USD $12,000,000$ (the difference considered in the base case) to USD $16,000,000$, the incremental investment threshold also increases (from USD $251,700$ to USD $446,008$). The explanation resides in the fact that contractors face more uncertainty concerning which of the two estimates will become the true value and, hence, the incremental investment assumes a higher importance. In fact, the corresponding threshold value is higher since the increase in the value of the option to invest considering both estimates, $V(P_2, K_b)$ assumes now a higher value, whereas the value of the option to invest considering solely the effects of the non-incremental investment, $V(P_1, K_b)$ remains unchanged. On the contrary, for a smaller difference between the two estimates (USD $8,000,000$), as the results reached for the alternative scenario 2 clearly reflect, the investment threshold is smaller: USD $112,128$, compared to USD $251,700$, in the base case. The level of uncertainty associated with the two estimates is now lower and this lower level of uncertainty is reflected in the value of option to invest considering both estimates, $V(P_2, K_b)$. The value of this option is, in the alternative scenario 2, smaller than in the other two cases and thus closer to the value of the option to invest considering only the investment in C_1, $V(P_1, K_b)$, whose value does not depend upon the differences between the two estimates, since the expected value for the additional profit, $E(\pi)$ remains unchanged.

We thus conclude that, the higher (lower) the difference between the high-value estimate and the low-value estimate, the higher (lower) will be the uncertainty concerning which estimate will become the true value, and the higher (lower) will be the value of the option to invest considering the two estimates, $V(P_2, K_b)$. Consequently - since the value of the option to invest considering only the effects of the non-incremental investment, $V(P_1, K_b)$ remains unchanged - the higher (lower) will be the value for investment threshold, C_2^*, and the higher (lower) will be the value of the ratio, RC_2^*.

Part IV

Concluding Remarks

Several types of uncertainty surround construction projects and construction companies should proactively manage the effects they may produce in the project's final NPV. We approached a specific type of uncertainty and designated it as "volume uncertainty". This type of uncertainty is critical since, at least frequently, managers do not know with precision the amount of work they will be executing throughout the project's life cycle and, consequently, the expected final profit the project will generate. To assess the impact of volume uncertainty on the project's final NPV, we defined a discrete-time stochastic variable and designated it as "additional value". Additional value is the value that is hidden in the the most uncertain parts of the project and, in the context of the present research, is defined as the one that does not derive from merely executing the tasks specified in the bid documents.

To capture and quantify this type of hidden-value, construction companies need to invest. Initially, by only using the skills of his or her own experienced staff, construction managers are able to define a high-value estimate and a low-value estimate for the additional profit, and to stipulate a probability of occurrence to each of the estimates. Based on the numerical solution proposed by Ribeiro et al. (2013), we suggested a model which led us to conclude that managers will produce a more competitive bid even if no incremental investment is undertaken, since the optimal price that results from the expected profit determined just by undertaking a preliminary analysis of the bid documents is greater than the base price, *i.e.*, the optimal price in the absence of any recognition and quantification of hidden-value. However, in order to resolve the uncertainty concerning which of the two estimates will become the true value for the expected additional profit, contractors often need to invest in human capital and technology and, hence, hire specialized firms and highly skilled professionals. The outcome of our model is the threshold value for this incremental investment. Therefore, managers may apply a simple decision rule: hire external services with the purpose of resolving the uncertainty concerning which of the two estimates previously established is the true value, provided that the cost of the incremental investment in human capital and technology does not exceed the threshold value previously determined. Any amount paid for external services, which is lower than the threshold value, will increase the project's expected NPV and, the lower this cost is, the higher will be the increase in the project's expected NPV. On the contrary, if the amount actually invested exceeds the predetermined threshold value, then the project's expected NPV will be reduced. The model also model determines the optimal bid price in the case the true value reached by undertaking the incremental investment equals the high-estimate for the additional value and also in the case the true value equals the low-estimate, both previously established.

Sensitivity analysis revealed that the threshold value responds linearly to variations in the investment size, which means that no scale-effect is present in the model. Sensitivity analysis also showed that, the closer to 50% is the probability of occurrence associated with the estimates, the greater the threshold value needs to be, since undertaking the incremental investment assumes a higher importance in response to the presence of higher levels of uncertainty regarding which of the two estimates will become the true value. Finally, sensitivity analysis performed to the distance between the two estimates demonstrated that, the more distant the two estimates defined for the additional value are from each other, the greater the incremental investment threshold value needs to be, since contractors face more uncertainty over which of the two estimates will be the true value. The decision of undertaking the incremental investment becomes, therefore, more important and such higher importance is reflected in a greater threshold value. On the contrary, if the two estimates are closer to each other, then the threshold value is lower, since the level of uncertainty surrounding the two estimates is, now, also lower. Hence, the decision of undertaking the incremental investment assumes less importance and this smaller importance is reflected in a lower value for the incremental investment threshold.

References

Adrian, J. J.: (1993), *Construction Claims: A Quantitative Approach*. New Jersey: Prentice-Hall.

Akintoye, A. and E. Fitzgerald: (2000), 'A Survey of Current Cost Estimating Practices'. *Construction Management and Economics* **Vol. 18, Nº 2**, 161–72.

Arditi, D. and R. Chotibhongs: (2009), 'Detection and Prevention of Unbalanced Bids'. *Construction Management and Economics* **Vol. 27, Nº 8**, 721–732.

Chao, L. and C. Liu: (2007), 'Risk Minimizing Approach to Bid-Cutting Limit Determination'. *Construction Management and Economics* **Vol. 25, Nº 8**, 835–843.

Chapman, C., S. Ward, and J. Bennell: (2000), 'Incorporating Uncertainty in Competitive Bidding'. *International Journal of Project Management* **Vol.18, Nº 5**, 337–347.

Cheung, S. O., P. Wong, A. Fung, and W. Coffey: (2008), 'Examining the Use of Bid Information in Predicting the Contractor Performance'. *Journal of Financial Management of Property and Construction* **Vol. 13, Nº 2**, 111–122.

Christodoulou, S.: (2010), 'Bid Mark-Up Selection Using Artificial Neural Networks and an Entropy Metric'. *Engineering Construction and Architectural Management* **Vol. 17, Nº 4**, 424–439.

Clough, R., G. Sears, and K. Sears: (2000), *Construction Project Management*. New York: John Wiley & Sons, Inc.

Davinson, P.: (2003), *Evaluating Contract Claims*. Oxford, UK: Blackwell Publishing.

Drew, D. and M. Skitmore: (1997), 'The Effect of Contract Type and Contract Size on Competitiveness in Construction Contract Bidding'. *Construction Management and Economics* **Vol. 15, Nº 5**, 469–489.

Drew, D., M. Skitmore, and H. Lo: (2001), 'The Effect of Client and Size and Type of Construction Work on a Contractor Bidding Strategy'. *Building Envirnoment* **Vol. 36, Nº 3**, 393–406.

Dyer, D. and J. Kagel: (1996), 'Bidding in Common Value Auctions: How the Commercial Construction Industry Corrects for the Winners Curse'. *Management Science* **Vo. 42, Nº 10**, 1463–1475.

Fayek, A.: (1998), 'Competitive Bidding Strategy Model and Software System for Bid Preparation'. *Journal of Construction Engineering and Management* **Vol. 124, Nº 1**, 1–10.

Ford, D., D. Lander, and J. Voyer: (2002), 'A Real Options Approach to Valuing Strategic Flexibility in Uncertain Construction Projects'. *Construction Management and Economics* **Vol. 20, Nº 4**, 343–351.

Halpin, D. and B. Senior: (2011), *Construction Management*. New Jersey: John Wiley & Sons, Inc., 4th edition.

Kim, H. and K. Reinschmidt: (2006), 'A Dynamic Competition Model for Construction Contractors'. *Construction Management and Economics* **Vol. 24, Nº 9**, 955–965.

Kululanga, G. K., W. Kuotcha, R. McCaffer, and F. Edum-Fotwe: (2001), 'Construction Contractors Claim Process Framework'. *Journal of Construction Engineering and Management* **Vol.127, Nº 4**, 309–314.

Laryea, S.: (2011), 'Quality of Tender Documents: Case Studies from the U.K.'. *Construction Management and Economics* **Vol. 29, Nº 3**, 275–286.

Laryea, S. and W. Hughes: (2008), 'How Contractor Price Risk in Bids: Theory and Practice'. *Construction Management and Economics* **Vol. 26, Nº 9**, 911–924.

Li, H. and P. Love: (1999), 'Combining Rule-Based Expert Systems and Artificial Neural Networks for Mark-Up Estimation'. *Construction Management and Economics* **Vol. 17, Nº 2**, 169–176.

Liu, M. and Y. Ling: (2005), 'Modeling a Contractor Mark-Up Estimation'. *Journal of Construction Engineering and Management* **Vol. 131, Nº 4**, 391–399.

Mak, S. and D. Picken: (2000), 'Using Risk Analysis to Determine Construction Project Contingencies'. *Journal of Construction Engineering and Management* **Vol. 126, Nº 3**, 130–136.

Margrabe, W.: (1978), 'The Value of an Option to Exchange One Asset for Another'. *The Journal of Finance* **Vol. 33, Nº 4**, 177–186.

Mattar, M. and C. Cheah: (2006), 'Valuing Large Engineering Projects under Uncertainty: Private Risk Effects and Real Options'. *Construction Management and Economics* **Vol. 24, Nº 8**, 847–860.

Mohamed, K., S. Khoury, and S. Hafez: (2011), 'Contractors Decision for Bid Profit Reduction Within Opportunistic Bidding Behavior of Claims Recovery'. *International Journal of Project Management* **Vol. 29, Nº 1**, 93–107.

Ng, F. and H. Bjornsson: (2004), 'Using Real Options and Decision Analysis to Evaluate Investments in the Architecture, Construction and Engineering Industry'. *Construction Management and Economics* **Vol. 22, Nº 5**, 471–482.

Ngai, S., D. Drew, H. Lo, and M. Skitmore: (2002), 'A Theoretical Framework for Determining the Minimum Number of Bidders in Construction Bidding Competitions'. *Construction Management and Economics* **Vol. 20, Nº 6**, 473–482.

Pinnell, S. S.: (1998), *How To Get Paid for Construction Changes: Preparation, Resolution Tools and Techniques*. New York: McGraw-Hill.

Ribeiro, J. A., P. J. Pereira, and E. Brandão: (2013), 'Reaching an Optimal Mark-Up Bid through the Valuation of the Option to Sign the Contract by the Selected Bidder'. *Available at SSRN: http://ssrn.com/abstract=2265862* pp. 1–38.

Rooke, J., D. Seymour, and R. Fellows: (2004), 'Planning for Claims: an Ethnography of Industry Culture'. *Construction Management and Economics* **Vol. 22, Nº 6**, 655–662.

Skitmore, M.: (2002), 'Predicting the Probability of Winning Sealed Bid Auctions: A Comparison of Models'. *The Journal of the Operational Research Society* **Vol. 53, Nº 1**, 47–56.

Tan, Y., L. Shan, A. Khlaled, and S. Song: (2008), 'An Examination of the Factors Affecting Contractors Competition Strategy: a Hong Kong Sudy'. *International Journal of Project Organisation and Management* **Vol. 1, Nº1**, 4–23.

Tenah, K. and C. Coulter: (1999), ‘Bid Mark-Up Metodologies’. *Cost Engineering* **Vol. 41, Nº 11**, 39–44.

Thomas, R.: (2001), *Construction Contract Claims*. New York: Palgrave, 2nd Edition.

Wallwork, J.: (1999), ‘Is There a Right Price in Construction Bids?’. *Cost Engineering* **Vol. 40, Nº 2**, 41–43.

Woodward, J. F.: (1997), *Construction Project Management: Getting It Right First Time*. ,London: Thomas Telford.

Yiu, C. Y. and C. S. Tam: (2006), ‘Rational Under-Pricing in Bidding Strategy: A Real Options Model’. *Construction Management and Economics* **Vol. 24, Nº 5**, 475–484.