Reaching an Optimal Mark-Up Bid through the Valuation of the Option to Sign the Contract by the Selected Bidder

João Adelino Ribeiro,[*][†] Paulo Jorge Pereira[‡] and Elísio Brandão[§]

Abstract

Reaching an optimal mark-up value in the context of bidding competitions has been a research topic since the pionner models of Friedman (1956) and Gates (1967) set the standards for future discussion. The model herein proposed is based on the existence of the option to sign the contract and perform the construction project by the selected bidder. This option constitues a real option because (i) the construction costs are uncertain, *i.e.*, the input prices vary stochastically from the moment the bid price is established and the bid results are publicly available; (ii) flexibility is present since the selected bidder may refuse to sign the contract and not execute the project if the construction costs, at the moment the contract has to be signed, are higher than the price included in the bid proposal, and (iii) construction costs are, at least, partially irreversible. Since this real option is only available to the selected bidder, its value must be weighted by the probability of winning the bid. A maximization problem that considers the value of the option to sign the contract and the probability of winning the bid is proposed and the model's outcome is the result of this maximization problem: to the highest value of the option to sign the contract weighted by the probability of winning the bid corresponds the optimal price (and, hence, the optimal mark-up bid). The model is later adapted in order to consider the existence of penalty costs, borne by the selected bidder if he or she refuses to sign the contract. Under these new conditions and in pure financial terms, the selected bidder should only exercise the option if the difference between the bid price and the construction costs is greater than the penalty costs, in the day the contract has to be signed. Results reached using a numerical example demonstrate that the optimal price is higher when penalty costs are present.

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Part I

Introduction

In this paper, we aim to reach an optimal profit margin in the context of a bidding contracting process applying the real options approach. The model herein proposed is a theoretical model whose purpose is to optimize the contractor's price through the valuation of the option to invest in performing the project. When a contractor presents a bid to the client and assuming that the probability of winning the bid is greater than zero, the option to sign the contract - and subsequently to invest in executing the project - does have value, as clearly established in the option pricing theory. The motivation behind the present work is also supported by the presence of uncertainty since the estimated costs of performing the project - the construction costs - will most likely vary from the moment the bidder computes them and defines the price to include in her or his bid proposal based on such estimate, closes the proposal, delivers the proposal to the client, and the moment the option is exercised or not, *i.e.*, the moment the selected bidder is invited by the client to sign the contract and decides to sign it or declines the invitation. In fact, and even though the bid price remains unchanged during this period, the uncertainty in construction costs will most likely lead to changes in the project's expected profit margin until the contract is eventually signed and the parties legally bounded. [1]

As far as the present research is concerned, contractors are firms operating in the construction industry whose business consists of executing a set of tasks previously defined by the client. The amount of tasks to be performed constitute a project, job or work. A significant amount of projects in the construction industry are assigned through what is known as "tender" or "bidding" processes (Christodoulou (2010); Drew et al. (2001)), being this the most popular form of price determination (Liu and Ling (2005); Li and Love (1999)). A bidding process

[1] We should mention that the risk deriving from the existence of uncertainty in the expected construction costs cannot be hedged since the bid participants do not know how the bidding process will end.

consists of a number of contractors competing to perform a particular job by submitting a sealed proposal until a certain date previously defined by the client. The usual format of a bidding process is based on the rule that - all other things being equal - the contract will be awarded to the competitor which submitted the lowest bid (Cheung et al. (2008); Chapman et al. (2000)), *i.e.*, the lowest price. Bearing this in mind, it is easy to conclude that the client's decision is very straightforward but the contractor's decision on what price to bid is more difficult to reach, being probably one of the most difficult decisions construction managers have to face during the bid preparation process (Li and Love (1999)).

The construction industry is known for featuring strong levels of price competitiveness (Chao and Liu (2007); Mochtar and Arditi (2001); Ngai et al. (2002)) and the competitive pressures are probably more intense than in any other industry (Drew and Skitmore (1997); Skitmore (2002)), which often leads contractors to lower their profit margins in order to produce a more competitive bid. Thus, it is not rare to see the winning bid include a near zero-profit margin (Chao and Liu (2007)). Moreover, under-pricing in the context of competitive bidding is a common phenomenon, namely explained by the need for work and penetration strategies (Drew and Skitmore (1997); Fayek (1998); Yiu and Tam (2006)), even tough bidding below cost does not necessarily guarantee a successful result to the bidder (Tenah and Coulter (1999)).

Contractors realize that bidding low when facing strong competition increases the chance of being chosen to perform the work but they are also aware of the opposite: if the price included in their proposal is higher, the likelihood of winning the bid will definitively be lower. This inverse relationship between the level of the profit margin (commonly known in the construction management literature as the "mark-up bid") and the probability of getting the contract is an accepted fact both in the construction industry and within the research community (see, for example, Christodoulou (2010); Kim and Reinschmidt (2006); Tenah and Coulter (1999); Wallwork (1999)).

Competitive bidding has been a subject of research since the important papers of Friedman (1956) and Gates (1967) set the standards for future discussion. Both models proposed a probabilistic approach to determine the most appropriate mark-up value and were supported by the definition of a relationship between the mark-up level and the probability of winning the bid. For that purpose, the two authors assumed the existence of previous bidding data - leading to the definition of the bidding patterns of potential competitors. Gates (1967) had the merit to extend the model built by Friedman (1956) and turned it into a general strategic model, with general applicability, setting the foundations for what is now commonly known as "Tendering Theory" (Runeson and Skitmore, 1999). Later attempts to establish a relationship between the probability of winning the contract and the level of the profit margin were based on previous bidding data – in line with the mentioned pioneer models. Carr (1982) proposed a model similar to Friedman's model but differing in the partitioning of the underlying variables: Friedman (1956) used a single independent variable, a composite "bid-to-cost" ratio, whereas Carr (1982) crafted his model around two distributions: one that standardizes the estimated cost of the analyzing bidder to that of all competitor bids, and another that standardizes the bids of an individual competitor against that of the analyzing bidder's estimated costs. More recently, Skitmore and Pemberton (1994) presented a multivariate approach by assuming that an individual bidder is not restricted to data for bids in which he or she has participated, as in the case of Friedman (1956) and Gates (1967) models, both based on bivariate approaches. Instead, the bidder is able to incorporate data for all auctions in which competitors and potential competitors have participated, regardless of the individual bidder's participation. This methodology had the merit of increasing the amount of data available for estimating the model's parameters. An optimal mark-up value is then reached against known competitors, as well as other types of strategic mark-ups.

Past research seems to suggest that it would be difficult to establish a link - with general applicability - between the mark-up level and the probability of winning the bid. Contractors may recur to previous bidding data and assume that bidders are likely to bid as they have done in the past in order to shape the relationship that best describes their specific situation. However, as Fayek (1998) stated, past bidding information is not always available. Chapman et al. (2000) argued that many managers consider that the information required by quantitative models is too difficult, too expensive or impossible to compile.To clearly understand what researchers mean by "past bidding information", we should distinguish between two different types of bidding data: (i) the one that is available to all contractors and comprises the estimates carried out by the client's engineers for the execution of each task or set of tasks, the price of each competitor for the execution of such task or set of tasks and, obviously, the final bid price of each competitor, *i.e.*, the price of executing all the tasks included in the bid package; (ii) the real empirical data each contractor (eventually) compiles concerning the results reached in past bidding competitions when a specific mark-up level was included in the bid proposal. This means that the first type of data is publicly available and allows researchers to reach, through the application of several models and methodologies, what is commonly known as "theoretical probabilities". These probabilities are computed based on data which is not real empirical data. Real empirical data is private information that contractors seldom share, meaning that this information is rarely observable and is, therefore, private knowledge of each contractor.[2]However, we recognize that assuming bidders are likely to bid as they have done in the past becomes inevitable, regardless of the type of data in question. In fact, utilizing past bidding information is only useful if one assumes that other bidders will decide in the future in the same way they have decided in the past. Still, and even though we agree that this assumption (which has been adopted since the pioneer works Friedman (1956) and Gates (1967)) may be considered some what restrictive, we sympathize with Crowley (2000) when this researcher states that bid models do not predict the future, but simply organize past

[2]Chapman et al. (2000) observed that some managers argue that collecting the necessary information to apply quantitative models is too difficult, to expensive or even impossible, which leads us to conclude that not all contractors actually compile data from previous bidding competitions.

bidding information in a way that is meaningful to current bid decisions.[3]

Most of the more recent contributions to the optimal mark-up bid debate have been concerned with the selection of factors construction managers should take into account when deciding what price to bid (Christodoulou, 2010). Research by authors such as Drew and Skitmore (1992), Shash (1993) and Drew et al. (2001) observed that different bidders apply different mark-up policies, which may be variable or fixed. These authors list a long set of factors aiming to explain the rationale behind mark-up bidding decision making: (1) amount of work in hands; (2) number and size of bids in hands; (3) availability of staff, including architects and other supervising officers; (4) profitability; (5) contract conditions; (6) site conditions; (7) construction methods and programme; (8) market conditions and (9) identity of other bidders, to name the ones they considered to be the most prominent. In general terms, factors are grouped in different categories and we sympathize with the 5 categories defined by Dulaimi and Shan (2002): (1) project characteristics; (2) project documentation; (3) contractor characteristics; (4) bidding situation and (5) economic environment. Following this line of thought, innovative research on the subject has been embracing more sophisticated methodologies. The paper by Li and Love (1999) manages to combine rule-based expert systems with Artificial Neural Networks (ANN) in the context of mark-up bid estimation, following previous research conducted by Li (1996), Moselhi et al. (1991), amongst others. In fact, the most recent and innovative models use ANN (as in Christodoulou (2010) and Liu and Ling (2005)) or Goal Programming Technique (Tan et al. (2009)), where those determinants (or attributes) provide the ground where models are built upon, thus recognizing the crucial importance of possessing a strong knowledge of the factors influencing the contractors bid mark-up decision for the purpose of identifying the optimal mark-up level (Dulaimi and Shan (2002)).

[3]We believe that past bidding information is, in fact, the best tool construction managers may use to create a perception as of how bidders will tend to act in the future. However, since each project has characteristics that distinguishes it from all other past projects, we believe construction managers should also consider the specific features of the current bid process, when establishing the mark-up bid.

Nevertheless, several studies suggest that decisions regarding the definition of the mark-up level are mainly supported using subjective judgment, gut feeling and heuristics (Hartono and Yap, 2011), hence acknowledging the fact that managers do have a perception in real-world situations as of how a specific mark-up level will affect the probability of winning the current bid. In fact, and even though, in general, construction managers do not support their mark-up bid decisions using some kind of mathematical expression linking the profit margin and the probability of winning the contract, they are aware that higher mark-up values will lead to lower chances of getting the job and - at least - do have a perception as of how their decision regarding the definition of the mark-up bid will affect the probability of being selected to perform the project. Bearing this in mind, we decided to include in our model a mathematical expression linking the mark-up level with the probability of winning the bid that (i) respects the generally accepted inverse relationship between these two variables; (ii) allows for flexibility and, thus, may be adapted to accommodate the unique circumstances that surround a particular bidding process.[4] Moreover, the mathematical relationship that we propose is very similar to the one that results from the application of the Gates (1967) model included in Skitmore et al. (2007) and, with a specific calibration, is graphically almost equal to the one these researchers have reached.[5] These authors used publicly available empirical data, which is included in a previous study carried-out by Schaffer and Micheau (1971).[6] In

[4]Chapman et al. (2000) state that, even though the probability of winning the bid may be difficult to determine, a relationship between the price and the probability of winning are implicit in any bidding process. These authors also argue that, usually, the persons involved in the pricing decision have their own implicit version of such relationship, which drives their decision making process. Moreover, Chapman et al. (2000) state that making these implicit perceptions explicit is an important part of arriving at an appropriate final bidding price. This line of thought reinforces our argument that managers - at least - do have a perception as of how a specific price will affect the probability of getting the contract and also underlines the importance of making explicit such implicit perception. Hence, it is straightforward to conclude that the purpose of making such implicit perceptions explicit can only be achieved if some sort of mathematical relationship between the two variables is adopted.

[5]The expression linking the mark-up level with the probability of winning the contract that we suggest comprises two parameters that can and should be calibrated with the purpose of accommodating contractor's past bidding data and their perception of how the mark-up decision affects the probability of winning the contract, considering the specific features of the current bid process.

[6]The empirical data used comprised the estimates done by the client's engineers in 50 bidding contracts and the prices presented by each of the bid participants.

their research piece, Skitmore et al. (2007) applied three different methodologies: the Gates (1967) model, the exponential model and the Weibull model.[7] The mathematical relationship we suggest may be calibrated to match the graphic representation of the results reached by using the Gates (1967) model and, still, is sufficiently flexible to be adapted in order to explicitly shape the perception construction managers have regarding the effect of the mark-up level in the probability of winning the current contract.

Even though some work has been developed consisting in the applying the real options approach to the construction management field (Espinoza (2011); Tseng et al. (2009); Yiu and Tam (2006); Mattar and Cheah (2006); Ng and Bjornsson (2004); Ng and Chin (2004)) and more specifically aiming to evaluate a set of real options in the context of large-scale investments (Pimentel et al. (2012); Couto et al. (2012)), there seems to be a lack of research contributing to the optimal mark-up debate using this methodology, motivating us to build up a model embracing the real options approach and aiming to reach the optimal mark-up level. This is achieved by evaluating the option to sign the contract and invest in performing the project and weighting the value of this option by the probability of winning the bid, since the option can only be exercised by selected bidder. According to our model, contractors should establish a price which corresponds to the highest value of the option to sign the contract, weighted by the probability of winning the bid. In financial terms and under the real options approach, this is the right perspective to follow: to the highest value of the option to invest - weighted by the probability of winning the contract - will correspond a certain level for the profit margin, this being the optimal mark-up bid.

The remainder of this paper unfolds as follows. In Part II, the two components of the model are described and the model's numerical solution is proposed. In Part III, a numerical example is presented and a sensitivity analysis is performed to the volatility of the option, its

[7]These authors stress that, for the Gates (1967) model to be considered valid in the context of their work, they had to assume that bids can be described using the proportional hazard family of statistical distributions. Please refer to Skitmore et al. (2007) for further details.

time to expiration and to the amount of the expected construction costs; we also assess the impact of changes, in each of the calibration parameters that integrate the proposed mathematical relationship between the mark-up value and the probability of winning the bid on the optimal price. We proceed to consider the existence of penalty costs if the selected bidder decides to decline the invitation to sign the contract and, consequently, does not undertake the project; we then adapt the model accordingly and present the new results based on the inputs considered in the numerical example. Finally, in Part IV, concluding remarks are given.

Part II

The Model

1 Introduction

Our model proposes a different approach regarding how the mark-up decision should be made, recognizing the real options approach as an effective methodology in addressing the optimal mark-up debate since the model herein presented (i) features uncertainty concerning the behavior of the construction costs, from the moment the bid price is established and the moment the client invites the selected bidder to sign the contract;[8] (ii) considers flexibility regarding the decision to sign the contract and invest in performing the project by the selected bidder and (iii) recognizes that the investment expenditures are, at least, partially irreversible since construction costs are "project-specific". The three characteristics the real options literature identifies as being essential for applying the real options approach to eval-

[8]Construction costs continue to behave stochastically after the selected bidder is invited to the sign the contract. However, in the context of the real option we have identified - which expires in the moment the selected bidder decides whether to sign the contract or not - such fact is not relevant.

uate investment decisions are, thus, present in the model we will describe.

2 Assumptions

In our model we assume that (i) each bidder decides what price to include in its proposal in isolation; (ii) each bidder prepares his or her proposal simultaneously with the other competitors; (iii) each bidder presents a single-sealed bid proposal to the client; (iv) each bidder has access to the available information concerning the project in hands and all documentation to support the cost estimation and the final bid decision, in line with all other potential bidders; (v) the bid package also contains information about the date when the bid results will be available to all participants; (vi) it is possible to establish an inverse relationship between the mark-up value and the probability of winning the bid; (vii) the selected bidder will only decide if he or she is going to invest in executing the project at the moment the contract needs to be signed and not before that date.

Our model is thus based on the existence of a single-sealed bid process where there is no interaction or contact of any kind with other bid participants. We will further assume that bidders have no information about the number of competitors until the bid results are publicly available. Firstly, we assume the absence of penalty costs in the case the selected bidder decides not to sign the contract. Later on, in Section 6, we will consider the presence of these costs, which may be of two different sorts: financial costs, legally enforced by the client and/or reputational costs, *i.e.,* costs that may be borne by the contractor in future bidding competitions as a result of declining the invitation to sign the present contract.

3 Model Description

The model aims to determine the optimal mark-up bid to be included in the contractor's proposal and depends on two different components: (i) the value of the option to sign the contract and invest in performing the project, which will be modeled as a contingent claim, adapting the exchange option model proposed by Margrabe (1978); (ii) since this option is only available to the selected bidder, the value of the option has to be weighted by the probability of winning the contract. As established by the option pricing theory, there is a positive relationship between the price included in the bid proposal and the value of the option to sign the contract and invest in executing the project. In fact, the option value increases when the corresponding "underlying asset", *i.e.*, the bid price assumes higher values. However, the higher the bid price the lower will be the probability of winning the contract, as we previously stated. Hence, the optimal bid price will be the solution of a maximization problem. We proceed to explain the two components separately.

3.1 The Option to Sign the Contract and Invest in Performing the Project

The Margrabe (1978) exchange option model builds on the Black and Scholes (1973) model, used to evaluate a typical european *call* option and considers the existence of only one stochastic variable: the price of the "underlying asset", whereas the Margrabe (1978) formula incorporates two "underlying assets", being the model's outcome the value of an european *call* option to exchange one asset for another. We adapt the Margrabe (1978) exchange option model to accommodate the fact that only the exercise price is uncertain, *i.e.*, the construction costs.[9] Let P denote the price included in the bid proposal and K the expected amount for the construction costs computed during the bid preparation stage. K follows a stochastic process

[9]We could have decided to use an adapted version of the Black and Scholes (1973) model, where the "underlying asset" would be the profit margin and the "exercise price" the construction costs. However, if we have adopted this model, we would be considering that the construction are constant and known during the life of the option. Since in our model the construction costs vary stochastically throughout the life of the option and the "underlying price", *i.e.*, the bid price remains constant during the same period, we believe that it is theoretically more correct to adopt the Margrabe (1978) exchange option model and adapted it to accommodate the fact that only the construction costs are uncertain during the life of the real option in question. Nevertheless, we recognize that, if we have applied an adapted version of the Black and Scholes (1973) model and considered, in the formula, the risk-free interest rate to be zero, we would have reached the same results for the option value.

known as geometric Brownian motion, given by the following equation:[10]

$$dK = \alpha K dt + \sigma K dz \tag{1}$$

where α is the drift parameter, dt is the time interval, σ is the standard deviation (volatility parameter) and dz is the increment of a standard Wiener process. The Margrabe (1978) formula (F) becomes:

$$F(P,K) = PN(d_1) - KN(d_2) \tag{2}$$

being (d_1) and (d_2):

$$d_1 = \frac{ln(P/K) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}} \tag{3}$$

$$d_2 = d_1 - (\sigma\sqrt{T-t}) \tag{4}$$

$N(d_1)$ and $N(d_2)$ are the probability density functions for the values resulting from expressions (d_1) and (d_2), respectively and:

- σ^2 is the variance which, in our model, equals σ_K^2. [11]
- $T-t$ is the time between the moment the bid price is established and the moment the contract has to be signed.

[10] As, in our case, the price, P remains unchanged from the moment the bid proposal is closed until the contract is signed, then $dP = 0$.

[11] Since $\sigma^2 = \sigma_P^2 - 2\sigma_P\sigma_K\rho_{PK} + \sigma_K^2$, where ρ_{PK} is the correlation coefficient between the price, P and the construction costs, K; since P remains unchanged, then σ_P^2 equals zero and so does $2\sigma_P\sigma_K\rho_{PK}$.

3.2 The Probability of Winning the Bid

Based on our previous considerations, we propose an inverse relationship linking the mark-up ratio, (P/K) and the probability of winning the bid, $W(P,K)$ which is given by the following equation:

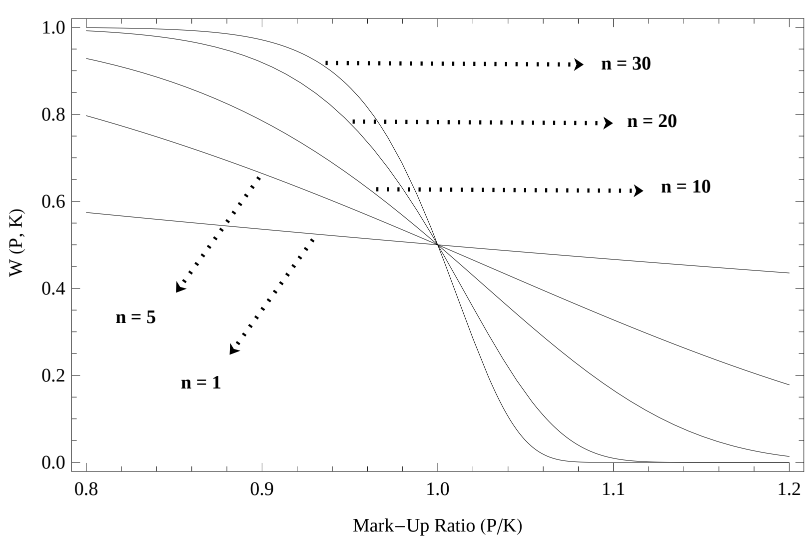
$$W(P,K) \quad = \quad e^{-b(P/K)^n} \tag{5}$$

where 'n' and 'b' are parameters that should be used to calibrate the expression linking the mark-up level and the probability of winning the contract in order to best reflect each contractor specific circumstances, as we previously argued. We will show how each of these parameters affect the graphical representation of equation (2.5).

Parameter 'n'

Parameter 'n' is responsible for shaping the graphical configuration of equation (2.5) in terms of its concavity and convexity. Assuming parameter 'b' equals $ln(1/0.5)$, Figure 1 shows the impact caused by parameter 'n', considering five different values.

Figure 1: graphical representation of the equation linking the mark-up ratio with the probability of winning the bid, considering five different values for parameter 'n'

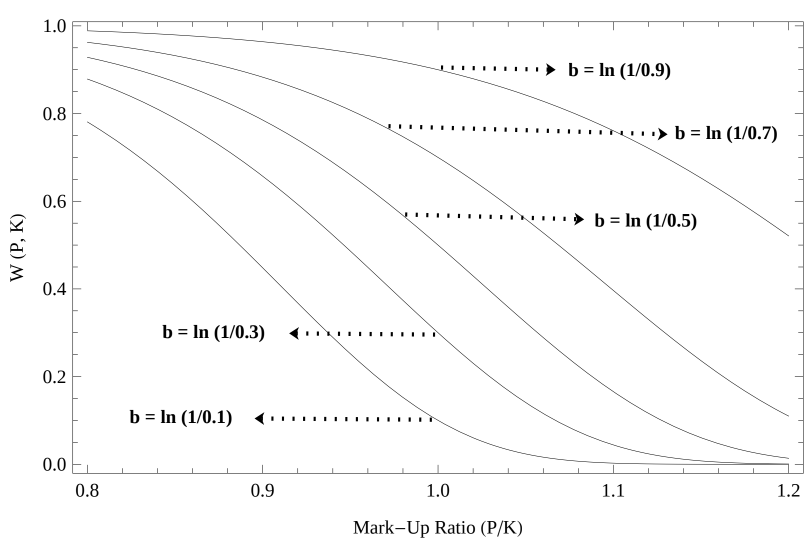


The figure above shows that the curve becomes more pronounced as parameter 'n' assumes higher values. In its concave region, higher mark-up values cause increasing variations in the probability of winning the contract whereas, in the convex region, higher mark-up values lead to decreasing variations in the probability of winning. This effect assumes more importance as the curve stretches out in response to greater values assumed by parameter 'n'. For lower values assumed by parameter 'n', the relationship between the mark-up ratio and the probability of winning the contract becomes (almost) linear. In fact, the function shrunks towards the center, gradually showing both weaker concavity and convexity and, thus, traducing lower sensitivity to changes in the mark-up values. In real-world situations, managers should calibrate this parameter and establish the existence and the pace at which this effect takes place.

Parameter 'b'

Assuming parameter 'n' equals 10, Figure 2 shows the impact on the configuration of equation (5) as a result of considering five different possible values for parameter 'b'.

Figure 2: graphical representation of the equation linking the mark-up ratio with the probability of winning the bid, considering five different values for parameter 'b'



This parameter enables contractors to calibrate the functional relationship between the mark-up ratio and and the probability of winning the contract, given by equation (2.5), by setting the probability of winning the contract when the price includes a zero profit margin (*i.e.*, when the mark-up ratio equals 1). Figure 2 shows that the greater the probability of winning the contract considering a zero-profit margin (curves are designed for 10%, 30%, 50%, 70% and 90% probability of winning the bid with a zero-profit margin) the more shifted up and to the right the curve is. Also, the greater the probability the less pronounced the convexity region is, reflecting that variations in the mark-up ratio for values situated in this area will cause smaller decreasing impacts in the probability of winning the contract. On the contrary, as the curve shifts up and to the right, the concave region becomes more pronounced and variations in the mark-up ratio located in this area will lead to greater increasing variations in the probability of winning the bid.[12]

3.3 The Optimal Price

The optimal price will be the one that maximizes the value of the option to sign the contract and invest in performing the project weighted by the probability of winning the bid. Thus, the model's outcome is the solution for the maximization problem given by equation (6) below:

$$V(P,K) = \max_{P} \left\{ [PN(d_1) - KN(d_2)][e^{-b(P/K)^n}] \right\} \tag{6}$$

which means that the option value for each mark-up level will be given by the outcome of the adapted Margrabe (1978) formula weighted by the probability of winning the bid. To the highest value of the option weighted by the probability of being awarded the contract will correspond a specific price, P and, therefore, a specific mark-up value, $M = P - K$ and the corresponding mark-up ratio, P/K. This will be the optimal price, P^*, the optimal margin,

[12] If we calibrate equation (5) with parameter 'b' approximately equal to ln(1/0.225) and parameter 'n' equal to 12 and also consider the mark-up ratio ranging from 0.75 to 1.15, the graphical representation of equation (5) almost matches the one Skitmore et al. (2007) reached by applying the Gates (1967) model.

$M^* = P^* - K$ and the optimal mark-up ratio P^*/K, as we illustrate in the following numerical example.

Part III

Numerical Example

4 The Base Case

Table 1 includes information about the inputs used in our numerical example.

Table 1: inputs: description and values

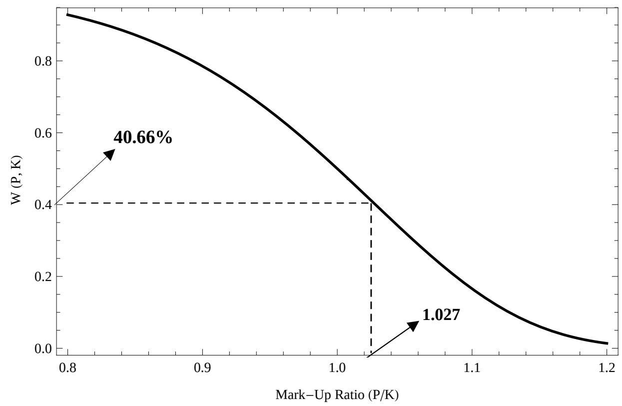
input	description	value
K	construction costs	USD 50,000,000
σ	standard deviation	25%
$T-t$	time from the moment the price is established until the contract is awarded	0.5 (years)
n	parameter for calibrating the relationship between P/K and W	10
b	parameter for calibrating the relationship between P/K and W	ln(1/0.5)

Considering the values included in Table 1, the relationship between the mark-up ratio and the probability of winning the bid will be given by the following equation:

$$W(P,K) = e^{-ln(1/0.5)(P/K)^{10}} \tag{7}$$

We are thus assuming that there is a 50% probability of winning the bid if the contractor defines a zero-profit margin, *i.e.*, $P/K = 1$, and also that parameter 'n' equals 10. Figure 3 below shows the configuration that results from this specific calibration, considering that the mark-up ratio ranges from 0.8 to 1.2.

Figure 3: graphical representation of the equation linking the mark-up ratio with the probability of winning the bid, considering that parameter 'b' equals $ln(1/0.5)$ and parameter 'n' equals 10



This inverted S-shaped curve respects the generally accepted inverse relationship between the mark-up ratio and the probability of winning the bid and typically comprises two different regions: a concave region until the mark-up ratio equals, approximately 1.027, corresponding to a probability of winning the contract of 40.66%, and a convex region onwards. In its concave region, changes in the mark-up ratio lead to increasing variations in the probability of winning the job, whereas in the convex region changes in the mark-up ratio will lead to decreasing variations in the probability of winning the contract. The following table includes a set of representative values for both variables, using equation (7):

Table 2: illustrative values for the mark-up ratio and the probability of winning the bid (for: b=ln (1/0.5); n = 10)

P/K	0.800	0.850	0.900	0.950	1.000	1.050	1.100	1.150	1.200
$W(\%)$	0.928	0.872	0.785	0.660	0.500	0.323	0.166	0.061	0.014

The next table includes results for a set of different prices, P, the corresponding mark-up values, M and mark-up ratios, P/K.

Table 3: different results for the option value, considering different price levels

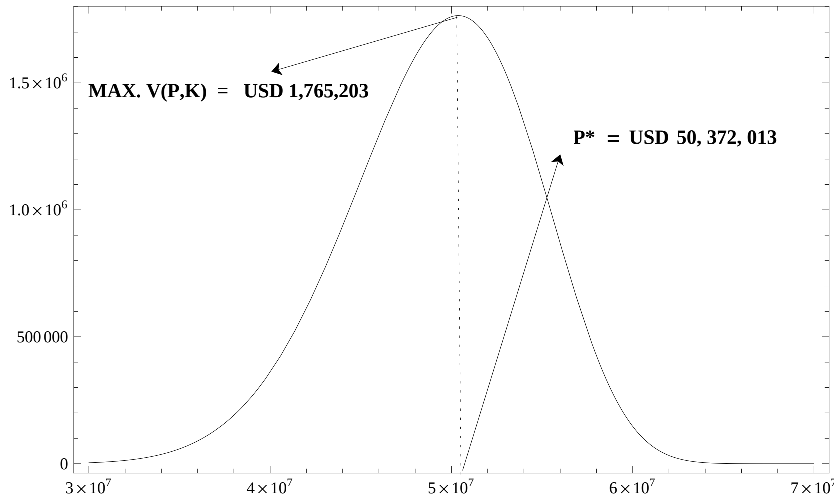
(for: K = USD 50,000,00; b = ln (1/0.5); n = 10; σ = 0.25; $T-t$ = 0.5 years)

P **(USD)**	P/K	$M(P,K)$ **(USD)**	$F(P,K)$ **(USD)**	$W(P,K)$ **(%)**	$V(P,K)$ **(USD)**
40,000,000	0.800	-10,000,000	388,726	92.83	360,845
45,000,000	0.900	-5,000,000	1,420,579	78.53	1,115,585
50,000,000	1.000	0,000,000	3,521,599	50.00	1,760,799
50,372,013	**1.0074**	**372,013**	**3,723,805**	**47.40**	**1,765,203**
55,000,000	1.100	5,000,000	6,720,607	16.57	1,113,305
60,000,000	1.200	10,000,000	10,757,755	1.37	147,171

Results included in Table 3 demonstrate that the higher the value of the "underlying asset" the higher the value of $F(P,K)$ (the value of the option to sign the contract increases as higher mark-up levels or prices are considered) and the lower the value of $W(P,K)$, since this probability decreases as the profit margin (or price) assumes greater values. As a result, $V(P,K)$ increases until reaching its maximum value: USD $1,765,203$. To this maximum value of the option, weighted by the probability of winning the bid, corresponds the optimal mark-up value, M^*= USD $372,013$ and the corresponding optimal mark-up ratio, $P/K^* = 1.0074$. Thus, the optimal price is P^*= USD $50,372,013$, the price that should be included in the bid proposal.

Figure 4 below illustrates the relationship between the price, P and the option value, $V(P,K)$, considering the inputs included in Table 1.

Figure 4: relationship between the price and the option value



The Figure above graphically expresses the maximization problem given by equation (6). The function increases until it reaches the maximum value for the option, *i.e.*, $V(P,K)$ =USD 1,765,203, to which corresponds the optimal price, P^* =USD 50,372,013. From this price onwards, the option value, $V(P,K)$ decreases and tends to zero, as the price tends to infinity.

5 Sensitivity Analysis

We first perform a sensitivity analysis to two of the parameters that influence the option value, $V(P,K)$ - the volatility parameter and the 'time to expiration' parameter - and the impact of considering three different values for each of them on the optimal price, P^*. We also perform a sensitivity analysis to the expected amount of direct costs of executing the project, aiming to conclude if a scale-effect is present in the model. Finally, we present the results of the impact caused by considering different values for each of the parameters included in equation 5, *i.e.*, parameter 'b' and parameter 'n', on the option value and on the optimal price.

5.1 Volatility

The next Table includes the results of the impact produced by different volatility levels, σ on the option value, $V(P,K)$ and on the optimal price, P^*.

Table 4: sensitivity analysis: volatility parameter

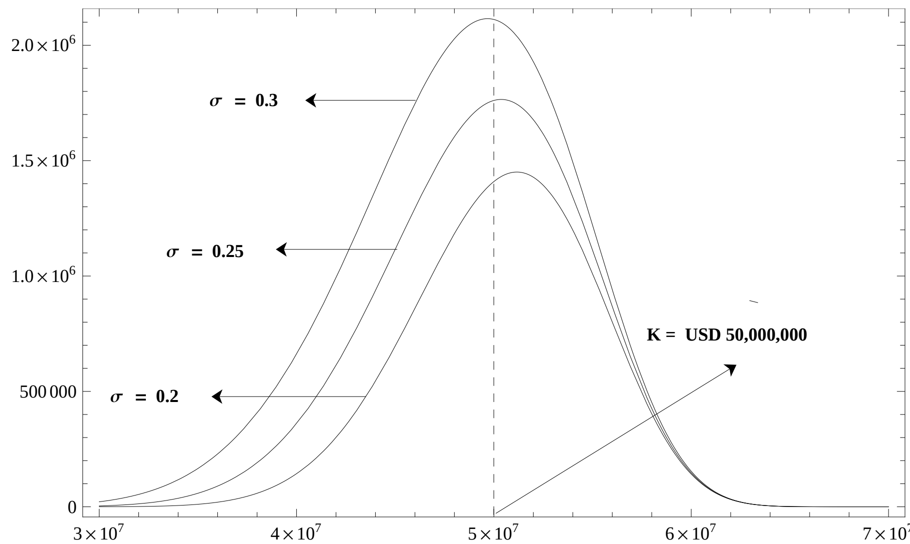
(for: K = USD 50,000,000; b = ln (1/0.5); n = 10; $T-t$ = 0.5 years)

Volatility (σ)	**Option Value** $V(P,K)$ **(USD)**	**Optimal Price** (P^*) **(USD)**
0.15	1,181,914	52,067,212
0.175	1,310,133	51,604,571
0.2	1,450,541	51,166,446
0.225	1,602,813	50,755,633
0.25	**1,765,203**	**50,372,013**
0.275	1,936,357	50,014,191
0.3	2,115,193	49,680,279
0.325	2,300,821	49,368,271
0.35	2,492,498	49,076,224

The results clearly reflect that, the higher the volatility level the higher the maximum value of the option to invest, as the option pricing theory states. However, the optimal price - the one which corresponds to the highest value of the option - decreases since contractors will present a lower bid as a consequence of holding a more valuable option. Moreover, it should be stressed that, according to these results, contractors will establish a price below cost if the volatility associated with the construction costs assumes values approximately above 27.5%.

Figure 5 shows the different configurations that result from considering a set of three different levels of volatility: $\sigma = 0.2$; $\sigma = 0.25$ and $\sigma = 0.3$.

Figure 5: sensitivity analysis: volatility parameter



5.2 Time to Expiration

The next Table includes the results of the impact produced by considering three different values for the 'time to expiration' parameter $(T-t)$ on the option value, $V(P,K)$ and on the optimal price, P^*.

Table 5: sensitivity analysis: 'time to expiration' parameter

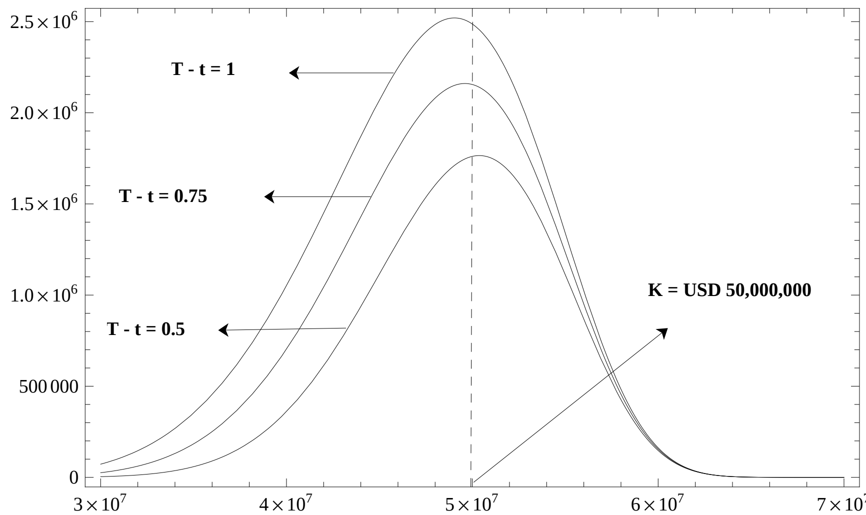
(for: K = USD 50,000,000; b = ln (1/0.5); n = 10; σ = 0.25)

Time to Expiration, $(T-t)$ (Years)	**Option Value, $V(P,K)$ (USD)**	**Optimal Price, (P^*) (USD)**
0.50	**1,765,203**	**50,372,013**
0.75	2,160,525	49,601,111
1.00	2,520,195	49,036,227

The results are consistent with the ones we have reached in the last analysis: the longer the life of the option the greater the maximum value for the option to invest (as established by the option pricing theory) and the lower the optimal price, as we have explained above.

The next Figure shows the different configurations that result from considering each of the three values for the 'time to expiration' parameter, included in Table 6.

Figure 6: sensitivity analysis: 'time to expiration' parameter



5.3 Construction Costs

The next Table includes the results of the impact caused by a set of possible values for the expected construction costs, K on the option value, $V(P,K)$, on the optimal price, P^* and on the optimal mark-up ratio, P^*/K.

Table 6: sensitivity analysis: construction costs

(for: b = ln (1/0.5); n = 10; σ = 0.25; $T - t$ = 0.5 years)

K **(USD)**	$V(P,K)$ **(USD)**	P^* **(USD)**	(P^*/K)
12,500,000	441,301	12,593,003	1.0074
25,000,000	882,601	25,186,006	1.0074
50,000,000	**1,765,203**	**50,372,013**	**1.0074**
75,000,000	2,647,804	75,558,019	1.0074
100,000,000	3,530,405	100,744,025	1.0074
125,000,000	4,413,007	125,930,032	1.0074

The results demonstrate that no scale-effect is present. Regardless of the level of construction costs, the optimal mark-up ratio remains unchanged, which means that the model's outcome responds linearly to variations in the project dimension.

5.4 Calibration Parameters

5.4.1 The Impact of Changes in Parameter 'b' on the Option Value and on the Optimal Price

The next Table exhibits the results produced by considering a set of different values for parameter 'b' on the option value after being weighted by the probability of winning the contract, $V(P^*,K)$, on the probability of winning the bid, $W(P^*,K)$ and, finally, on the optimal price, P^* and on the optimal mark-up ratio, P^*/K. We set parameter 'n' to 10.

Table 7: the impact of changes in parameter 'b' on the option value and on the optimal price, setting parameter 'n' to 10

	$ln(1/0.3)$	$ln(1/0.4)$	$ln(1/0.5)$	$ln(1/0.6)$	$ln(1/0.7)$	$ln(1/0.8)$	$ln(1/0.9)$
$V(P^*,K)$ (USD)	1,142,685	1,425,043	1,765,203	2,202,200	2,810,469	3,768,774	5,712,657
$W(P^*,K)$	43.61%	45.48%	47.40%	49.50%	51.94%	55.06%	59.82%
P^*(USD)	48,173,914	49,250,298	50,372,013	51,624,536	53,134,736	55,168,960	58,584,904
P^*/K	0.9635	0.9850	1.0074	1.0325	1.0627	1.1034	1.1717

Not surprisingly, the results demonstrate that the higher the probability of winning the contract with a zero-profit margin (which is exactly what parameter 'b' establishes) the higher the option value, $V(P^*,K)$ and the higher the optimal price, P^* and the optimal mark-up ratio, P^*/K. Positive variations in this parameter, therefore, produce positive variations in the option value before being weighted by the probability of winning the bid (as a consequence of considering higher optimal prices), positive variations in the probability of winning the contract and also positive variations in the optimal price and in the optimal mark-up ratio.

5.4.2 The Impact of Changes in Parameter 'n' on the Option Value and on the Optimal Price

The next Table includes the results of considering several possible values for parameter 'n' and the corresponding results in the option value, $V(P^*,K)$, in the optimal price, P^* and in the optimal mark-up bid, P^*/K. Results illustrating the impact on the value of the option before being weighted by the probability of winning the contract, $F(P^*,K)$ and on the probability of winning the bid, $W(P^*,K)$ are also shown.

Table 8: the impact of changes in parameter 'n' on the option value and on the optimal price, fixing parameter 'b' to ln(1/0.5)

	1	5	10	11	15	20	30
$F(P^*,K)$ (USD)	72,134,741	7,129,958	3,723,805	**3,514,893**	3,061,423	2,852,161	2,755,647
$W(P^*,K)$	18.40%	30.93%	47.40%	**50.96%**	58.93%	66.70%	76.13%
$V(P^*,K)$ (USD)	13,268,446	2,205,641	1,765,203	**1,760,805**	1,804,013	1,902,298	2,097,916
P^*(USD)	122,134,741	55,550,219	50,372,013	**49,987,465**	49,106,393	48,674,585	48,469,189
P^*/K	2.4427	1.1110	1.0074	**0.9997**	0.9821	0.9735	0.9694

The results we have reached are not so straighforward to interpret as in the previous case. The option value, $V(P^*,K)$ decreases until parameter 'n' equals 11 and increases when this parameter assumes values higher than 11. In fact, until 'n' equals 11, the impact produced by the optimal price, P^* in the value of the option before being weighted by the probability of winning the contract, $F(P^*,K)$ is stronger than the impact of opposite nature the same optimal price produces in the probability of winning the bid, $W(P^*,K)$. Hence, the combined effect of this two opposite effects result in a lower option value, $V(P^*,K)$.[13] For values of 'n' higher than 11, the opposite occurs: the effect that changes in this parameter produce in the probability of winning the bid is stronger than the effect produced in the value of the option before being weighted, $F(P^*,K)$, hence resulting in higher option values, $V(P^*,K)$.

If we confront these results with the graphical representation of the impact caused by considering different values for parameter 'n' on the probability of winning the contract - which are included in Figure 1 - we conclude that the consideration of low values for this parameter in equation 2.5 ('n' $= 1$ and 'n' $= 5$, in Figure 1) lead to an almost linear relationship between the mark-up ratio and the probability of winning the bid. Hence, in the presence of low values for 'n', changes in this parameter (say, from $n = 1$ to $n = 2$) lead to less strong positive variations in the probability of winning the bid, $W(P^*,K)$ than the stronger negative

[13]Please note that $V(P^*,K) = F(P^*,K) * W(P^*,K)$.

impact that is produced by considering lower optimal prices for the value of the option before being weighted by the probability of winning the contract, $F(P^{*},K)$. On the other hand, when parameter 'n' assumes higher values, changes in its value (say, from 'n' = 15 to 'n' = 30) will lead to a higher impact on the probability of winning the bid, $W(P^{*},K)$ than on the value of the option before being weighted, $F(P^{*},K)$ since, as we have observed, variations in the optimal mark-up ratio situated in the curve's concave (convex) region will produce more (less) than proportional variations in the probability of winning the contract. Hence, when we consider higher values for parameter 'n', the impact that changes in this parameter produce on the probability of winning the bid is stronger than the impact caused on the value of the option before being weighted, $F(P^{*},K)$, hence resulting in greater levels for the option value, $V(P^{*},K)$.

We thus conclude that the option value and the optimal price are highly sensitive to this parameter 'n' when it assumes very low levels, *i.e.*, in the case the relationship between the mark-up value and the probability of winning the bid, given by equation (5), becomes closer to a linear relationship. However, as 'n' assumes higher levels (15, 20, 30, in our example), the impact of changes is much less strong. In fact, the negative variations observed in the optimal price for values of 'n' greater than 10 are not substantial, as the results included in Table 9 clearly reflect.

6 Considering the Existence of Penalty Costs

In the previous Sections we have not considered the existence of penalty costs, which means that we have been assuming that the selected bidder will not bear any type of costs if he or she decides to decline the invitation to sign the contract . Yet, in some legal environments, the selected contractor may have to pay a legal compensation if the option to sign the contract is not exercised. According to Halpin and Senior (2011), in the United States contractors are free to withdraw their bids without incurring in any penalties if that happens prior to the ending of the bidding period. However, if a contractor decides to withdraw the bid after that moment - and assuming that he or she is the selected bidder - a penalty equal to the difference between the second best proposal and the selected bid is legally imposed, even if the contract has not yet been signed. According to Halpin and Senior (2011), "this may occur in the event that the selected bidder realizes that he or she has underbid the project and that pursuing the work will result in a financial loss" (p.44).[14] In these circumstances, the client may exercise the legal right of receiving the difference between the two bid prices. If, say, a contractor included a price of USD $5,000,000$ in the bid and refuses to enter into contract and the next low bid is USD $5,100,000$, the client is thus damaged in the amount of USD $100,000$. Hence a legal compensation equal to the difference between these two values may be enforced, *i.e.*, USD $100,000$.

For the sake of convenience but also because - from the contractor's perspective - the expected penalty costs can be seen as a percentage of the construction costs, we will assume that g, the amount of the penalty costs, is estimated as being a percentage of K, the construction costs.[15] Thus, the payoff (at maturity) of the option to sign the contract, under these new conditions, will be:

$$Max[P - K_T; -gK_T] \tag{8}$$

Expression (8) entails that the contractor will chose to pay the legal penalty, "gK_T" if this cost is smaller than the financial loss given by the difference between P and K_T, *i.e.*, the expected profit the project will generate, at the moment the contract needs to be signed, T. On the contrary, if this difference is smaller than the amount of the legal compensation, gK_T, than the contractor will prefer to sign the contract and execute the job. Considering these new

[14] We believe the authors use the word "underbid" to express the fact that the selected bidder realizes, in the day the contract has to be signed, that the expected construction costs are higher than the bid price.

[15] As we previously mentioned, penalty costs may also assume the nature of reputational costs.

conditions, we again adapted the Margrabe (1978) formula. Equation (9) below includes two important changes when compared with equation (2):

1) The exercise price is now equal to “$K - gK$” . This is due to the fact that, if the contractor declines the invitation to sign the contract, he or she incurs in a penalty cost equal to the amount “gK”. On the contrary, if the contract is signed and the project undertaken, the contractor will invest the amount “K”. Thus, seen in incremental terms, "$K - gK$" becomes the exercise price if the contractor exercises the option to execute the project in the presence of these costs.

2) $N(d_2)$ is the risk-neutral probability that the option will be exercised at maturity (Nielsen, 1992) in the original Black and Scholes (1973) formula or, in other words, the probability that the option will finish “in-the-money” in a risk-neutral world (Smith, 1976). This also holds the same meaning in the Margrabe (1978) model. Hence, $[1 - N(d_2)]$ expresses the probability that the option will not be exercised at the maturity since it will not finish “in-the-money”. In these circumstances the contractor will prefer to incur in the legal cost, “gK“ and not sign the contract. Thus, the adapted version of Margrabe (1978) formula, $F_g(P_g, K)$ becomes:

$$F_g(P_g, K) = [P_g N(d_1) - (1-g)KN(d_2) - gK(1 - N(d_2))] \tag{9}$$

where:

$$d1 = \frac{ln[P_g/(1-g)K)] + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}} \tag{10}$$

and

$$d2 = d1 - (\sigma\sqrt{T-t}) \tag{11}$$

Equation (5) still holds. Hence, under these new conditions, the model's outcome is the solution for the following maximization problem:

$$V_g(P_g,K) \quad = \max_{P_g} \quad \left\{ [P_g N(d_1) - (1-g)KN(d_2) - gK(1-N(d_2))][e^{-b(P_g/K)^n}] \right\} \tag{12}$$

Using the same inputs considered in Table 1, Table 10 includes results for $F_g(P_g,K)$, $W(P,K)$ and $V_g(P_g,K)$ considering a set of different prices, P_g, the corresponding mark-up values, M_g and mark-up ratios, P_g/K , and assuming that the penalty costs, g equal 2% of the construction costs, K.

Table 9: the impact of different prices on the option value and on the probability of winning the bid, in the presence of penalty costs

(for: K = USD 50,000,000; b = ln (1/0.5); n = 10; σ = 0.25; $T-t$ = 0.5 years; g = 0.02)

P_g **(USD)**	P_g/K	$M_g(P_g,K)$ **(USD)**	$F_g(P_g,K)$ **(USD)**	$W(P_g,K)$ **(%)**	$V_g(P_g,K)$ **(USD)**
40,000,000	0.800	-10,000,000	-405,117	92.83	-376,060
45,000,000	0.900	-5,000,000	970,174	78.53	761,881
50,000,000	1.000	0,000,000	3,519,395	50.00	1,759,698
50,878,172	**1.018**	**878,172**	**4,084,993**	**43.82**	**1,790,233**
55,000,000	1.100	5,000,000	7,128,665	16.57	1,180,902
60,000,000	1.200	10,000,000	1,145,375	1.37	156,693

The value of the option to sign the contract, $V_g(P,K)$ assumes negative values for low levels of P_g, in line with the interpretation of expression (8). The option value, $V_g(P_g,K)$ becomes positive for price levels above USD $41,943,879$ and reaches its maximum value when $V_g(P_g,K)$ =USD $1,790,233$, to which corresponds a price, $P_g =$ USD $50,878,172$. This is the optimal price, P_g^* in the presence of penalty costs, a slightly higher value than in the base case, where penalty costs are not considered. Figure 7 illustrates the relationship between the price, P_g and the option value, $V_g(P_g,K)$ for an estimated level of penalty costs, $g = 0.02$.

Figure 7: relationship between the price and the option value, considering the existence of penalty costs

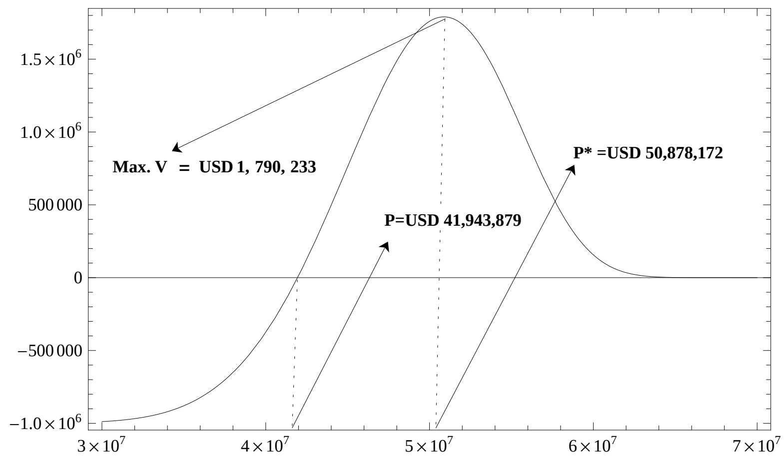
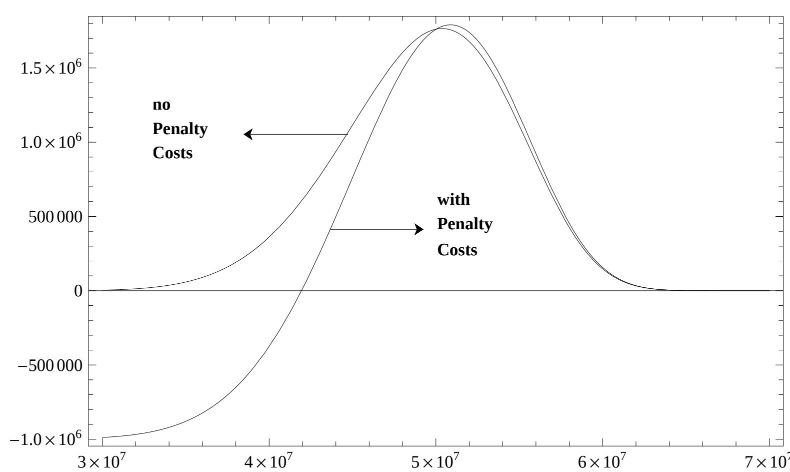


Figure 8 compares the relationship between the price, P and the value of the option to sign the contract, $V(P,K)$ for both cases, *i.e.*, considering the base case where $g = 0$ and the case where penalty costs are considered and estimated to be $g = 0.02$.

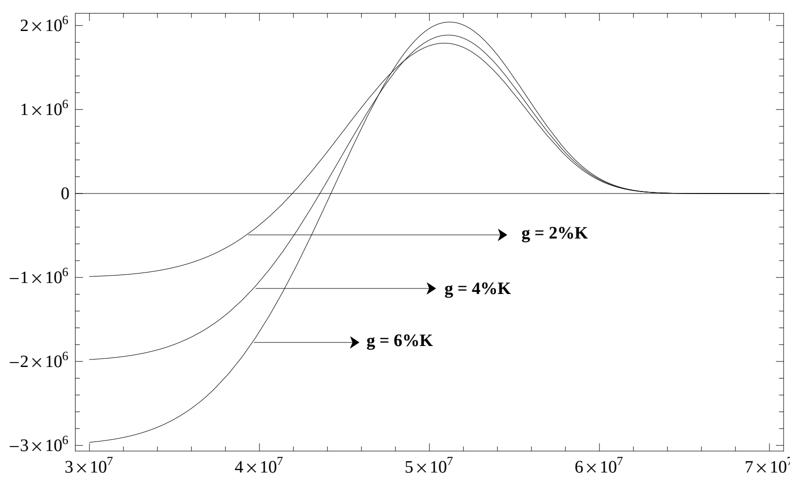
Figure 8: relationship between the price and the option value, with and without the consideration of penalty costs



In the ascending part of the two curves, the option value is higher in the absence of penalty costs for a specific price value. This difference becomes less important as the price increases, until the maximum value for the option is reached, in both scenarios. In their descending part, the curves feature very similar configurations, with the option value - in the presence of penalty costs - reaching a slightly greater value for a specific price level, until both curves converge when higher price levels are considered.

Figure 9 shows that the value of the option to invest, $V_g(P,K)$ increases as the level of penalty costs, g assumes greater levels. Consequently, the positive difference between the optimal price in the presence of penalty costs and the optimal price when these costs are not considered becomes greater.

Figure 9: relationship between the price and the option value, considering different levels of penalty costs



The next Table includes the values for the optimal price, P_g^* which result from considering a set of three different levels for the penalty costs, $g = 0.02$, $g = 0.04$ and $g = 0.06$.

Table 10: the impact of different levels of penalty costs on the option value and on the optimal price

(for: K = USD 50,000,000; b = ln (1/0.5); n = 10; σ = 0.25; $T - t$ = 0.5 years)

g (%K)	g **(USD)**	$V_g(P_g, K)$ **(USD)**	P_g^***(USD)**
2%	**100,000**	**1,790,233**	**50,878,172**
4%	200,000	1,886,377	51,114,430
6%	300,000	2,041,853	51,181,377

The results reveal that the option value, $V_g(P_g, K)$ increases as the penalty costs, g assume greater levels, which means that the option to invest is more valuable when penalty costs are higher. The explanation resides in the fact that, the higher the level of the penalty costs the lower the exercise price, and the lower the exercise price the higher the option value is, as stated by the option pricing theory. The optimal price, P_g^* increases with the consideration of greater levels of penalty costs since, clearly, contractors need to establish higher bid prices to compensate for the growing impact produced by this type of costs.

Part IV

Concluding Remarks

The theoretical model herein presented aims to underline the importance of considering the existence of uncertainty and the presence of flexibility, at the bid preparation stage. We have identified an option that is only available to the selected bidder - the option to sign the contract and invest in executing the project. The selected bidder has flexibility concerning the decision of whether to sign the contract and execute the project and, as clearly stated in the option pricing theory, flexibility does have value. The option to sign the contract and perform the project constitutes a real option and, when there are no penalty costs involved, the option should only be exercised if the construction costs, at the time the contract has to be signed, are lower than the price included in the bid proposal. However, when penalty costs are present, the selected bidder should only exercise the option if the difference between the bid price and the construction costs is greater than the penalty costs, in the day the contract has to be signed.

A numerical solution, which consists of a maximization problem, is proposed. This solution determines that - to the highest value of the option to execute the project weighted by the probability of winning the contract - corresponds the optimal price. According to the approach adopted, to this optimal price corresponds the optimal mark-up bid contractors should include in their proposals.

We performed sensitivity analysis to three of the parameters that influence the value of the option and also assessed the impact of each of the calibration parameters included in the suggested mathematical relationship between the price and the probability of winning the bid. Results revealed that the maximum value of the option is higher and the optimal price is smaller in response to positive variations in the volatility associated with the construction costs, as well as to positive variations in the life of the option. Results also showed that the model's outcome responds linearly to variations in the construction costs, which means that the optimal mark-up ratio remains unchanged for any dimension the investment may assume.

Sensitivity analysis performed to the parameters included in the equation linking the price and the probability of winning the contract revealed that the option value and the optimal price are highly sensitive to changes in each of them. The impact produced by considering higher values for parameter 'b' on the optimal price is strong, and the higher the value assumed by this calibration parameter the higher the option value, and the higher the optimal price. However, when we examined the impact of changes in the other calibration parameter, 'n', we concluded that the value of the option to sign the contract and perform the project decreases until a specific value assumed by this parameter, and increases from then onwards. When very low values for parameter 'n' are considered, the optimal price is highly sensitive to variations in this parameter and is much less sensitive to variations when 'n' assumes higher values. Therefore, construction managers must consider the strong sensitivity that the model's outcome, *i.e.*, the optimal price exhibits in response to changes in this parameter, particularly when very low levels are considered. In fact, for low levels of 'n', any small change in the value considered to this parameter will produce a very strong impact on the optimal price.

Finally, based on the input values considered in the numerical example, we concluded that, when penalty costs are present the optimal price is greater, corresponding to a greater maximum value of the option to execute the project, when compared with the optimal price in the base case, where penalty costs are not present. Moreover, the optimal price increases in response to positive variations in the level of the penalty costs. Higher expected values of penalty costs lead construction managers to establish higher mark-up bids. This increase in the mark-up bids is the compensation managers demand for supporting the presence of higher levels of penalty costs.

References

Black, F. and M. Scholes: (1973), 'The Pricing of Options and Corporate Liabilities'. *Journal of Political Economy* **Vol. 81, Nº 3**, 637–654.

Carr, R.: (1982), 'General Bidding Model'. *ASCE Journal of the Construction Division* **Vol. 108, Nº 4**, 639–650.

Chao, L. and C. Liu: (2007), 'Risk Minimizing Approach to Bid-Cutting Limit Determination'. *Construction Management and Economics* **Vol. 25, Nº 8**, 835–843.

Chapman, C., S. Ward, and J. Bennell: (2000), 'Incorporating Uncertainty in Competitive Bidding'. *International Journal of Project Management* **Vol.18, Nº 5**, 337–347.

Cheung, S. O., P. Wong, A. Fung, and W. Coffey: (2008), 'Examining the Use of Bid Information in Predicting the Contractor Performance'. *Journal of Financial Management of Property and Construction* **Vol. 13, Nº 2**, 111–122.

Christodoulou, S.: (2010), 'Bid Mark-Up Selection Using Artificial Neural Networks and an Entropy Metric'. *Engineering Construction and Architectural Management* **Vol. 17, Nº 4**, 424–439.

Couto, G., C. Nunes, and P. Pimentel: (2012), 'High-Speed Rail Transport Valuation and Conjecture Shocks'. *The European Journal of Finance* **Vol. 18, Nº 2**, 167–183.

Crowley, L.: (2000), 'Friedman and Gates - Another Look'. *Journal of Construction Engineering and Management* **4**, 306–312.

Drew, D. and M. Skitmore: (1992), 'Competitiveness in Bidding: a Consultant Perspective'. *Construction Management and Economics* **Vol. 10, Nº 3**, 227–247.

Drew, D. and M. Skitmore: (1997), 'The Effect of Contract Type and Contract Size on Competitiveness in Construction Contract Bidding'. *Construction Management and Economics* **Vol. 15, Nº 5**, 469–489.

Drew, D., M. Skitmore, and H. Lo: (2001), 'The Effect of Client and Size and Type of Construction Work on a Contractor Bidding Strategy'. *Building Envirnoment* **Vol. 36, Nº 3**, 393–406.

Dulaimi, M. and H. Shan: (2002), 'The Factors Influencing Bid Mark-Up Decisions of Large and Medium-Size Contractors in Singapore'. *Construction Management and Economics* **Vol. 20, Nº 7**, 601–610.

Espinoza, R. D.: (2011), 'Contingency Estimation Using Option Pricing Theory: Closing the Gap between Theory and Practice'. *Construction Management and Economics* **Vol. 29, Nº 9**, 913–927.

Fayek, A.: (1998), 'Competitive Bidding Strategy Model and Software System for Bid Preparation'. *Journal of Construction Engineering and Management* **Vol. 124, Nº 1**, 1–10.

Friedman, L.: (1956), 'A Competitive Bidding Strategy'. *Operations Research* **Vol. 4, Nº 1**, 104–112.

Gates, M.: (1967), 'Bidding Strategies and Probabilities'. *ASCE Journal of the Construction Division* **Vol. 93, Nº 1**, 75–107.

Halpin, D. and B. Senior: (2011), *Construction Management*. New Jersey: John Wiley & Sons, Inc., 4th edition.

Hartono, B. and C. Yap: (2011), 'Understanding Risky Bidding: A Prospect-Contingent Perspective'. *Construction Management and Economics* **Vol. 29, Nº 6**, 579–593.

Kim, H. and K. Reinschmidt: (2006), 'A Dynamic Competition Model for Construction Contractors'. *Construction Management and Economics* **Vol. 24, Nº 9**, 955–965.

Li, H.: (1996), 'Neural Network Models for Intelligent Support of Mark-Up Estimation'. *International Journal of Engineering Construction and Architectural Management* **Vol. 3, Nº 1**, 69–81.

Li, H. and P. Love: (1999), 'Combining Rule-Based Expert Systems and Artificial Neural Networks for Mark-Up Estimation'. *Construction Management and Economics* **Vol. 17, Nº 2**, 169–176.

Liu, M. and Y. Ling: (2005), 'Modeling a Contractor Mark-Up Estimation'. *Journal of Construction Engineering and Management* **Vol. 131, Nº 4**, 391–399.

Margrabe, W.: (1978), 'The Value of an Option to Exchange One Asset for Another'. *The Journal of Finance* **Vol. 33, Nº 4**, 177–186.

Mattar, M. and C. Cheah: (2006), 'Valuing Large Engineering Projects under Uncertainty: Private Risk Effects and Real Options'. *Construction Management and Economics* **Vol. 24, Nº 8**, 847–860.

Mochtar, K. and D. Arditi: (2001), 'Pricing Strategy in the US Construction Industry'. *Construction Management and Economics* **Vol. 19, Nº 4**, 405–415.

Moselhi, O., T. Hegazy, and P. Fazio: (1991), 'Neural Networks as Tools in Construction'. *Journal of Construction Engineering and Management* **Vol. 117, Nº 4**, 606–625.

Ng, F. and H. Bjornsson: (2004), 'Using Real Options and Decision Analysis to Evaluate Investments in the Architecture, Construction and Engineering Industry'. *Construction Management and Economics* **Vol. 22, Nº 5**, 471–482.

Ng, F. and S. Chin: (2004), 'Valuing a Price Contract for Material Procurement as a Real Option'. *Construction Management and Economics* **Vol. 22, Nº 2**, 141–150.

Ngai, S., D. Drew, H. Lo, and M. Skitmore: (2002), 'A Theoretical Framework for Determining the Minimum Number of Bidders in Construction Bidding Competitions'. *Construction Management and Economics* **Vol. 20, Nº 6**, 473–482.

Nielsen, L. T.: (1992), 'Understanding N(d1) and N(d2): Risk-Adjusted Probabilities in the Black-Sholes Model'. *INSEAD Working Paper (available at http://www.ltnielsen.com)* pp. 1–16.

Pimentel, P. M., J. Azevedo-Pereira, and G. Couto: (2012), 'High-Speed Rail Transport Valuation'. *The European Journal of Finance* **Vol. 18, Nº 2**, 167–183.

Runeson, G. and M. Skitmore: (1999), 'Tendering Theory Revisited'. *Construction Management and Economics* **Vol. 17, Nº 3**, 285–296.

Schaffer, L. R. and T. W. Micheau: (1971), 'Bidding with Competitive Strategy Models'. *Journal of Construction Division* **Vol. 97, Nº 1**, 113–126.

Shash, A.: (1993), 'Factors Considered in Tendering Decisions by Top UK Contractors'. *Construction Management and Economics* **Vol. 11, Nº 2**, 111–118.

Skitmore, M.: (2002), 'Predicting the Probability of Winning Sealed Bid Auctions: A Comparison of Models'. *The Journal of the Operational Research Society* **Vol. 53, Nº 1**, 47–56.

Skitmore, M. and J. Pemberton: (1994), 'A Multivariate Approach to Construction Contract Bidding Mark-Up Strategies'. *The Journal of the Operational Research Society* **Vol. 45, Nº 11**, 1263–1272.

Skitmore, M., A. Pettitt, and R. McVinish: (2007), 'Gates Bidding Model'. *Journal of Construction Engineering and Management* **Vol. 133, Nº 11**, 855–863.

Smith, C.: (1976), 'Option Pricing: A Review'. *Journal of Financial Economics* **Vol.3, Nº 1**, 3–51.

Tan, Y., L. Shen, W. Lu, and Q. Shen: (2009), 'Multiple-Objective Bidding Strategy Using Goal Programming Technique'. *Management Decision* **Vol. 46, Nº 4**, 666–672.

Tenah, K. and C. Coulter: (1999), 'Bid Mark-Up Metodologies'. *Cost Engineering* **Vol. 41, Nº 11**, 39–44.

Tseng, C.-L., T. Zhao, and C. Fu: (2009), 'Contingency Estimation Using a Real Options Approach'. *Construction Management and Economics* **Vol. 27, Nº 11**, 1073–1087.

Wallwork, J.: (1999), 'Is There a Right Price in Construction Bids?'. *Cost Engineering* **Vol. 40, Nº 2**, 41–43.

Yiu, C. Y. and C. S. Tam: (2006), 'Rational Under-Pricing in Bidding Strategy: A Real Options Model'. *Construction Management and Economics* **Vol. 24, Nº 5**, 475–484.