General equilibrium in markets for lemons*

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Abstract. In an exchange economy, agents frequently have different abilities to distinguish goods (for example, a car in good condition from a car in bad condition). To study such a setting, it is useful to distinguish goods not only by their physical characteristics, but also by the agent that brings them to the market. Equilibrium is shown to exist, with agents receiving the cheapest bundle among those that they cannot distinguish from truthful delivery. An example is presented as an illustration.

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1 Introduction

Economic agents usually trade goods without having perfect knowledge of their characteristics. This applies to firms hiring workers with unknown productivity, to consumers buying used cars with unknown quality, and to financial institutions buying assets with unknown return. Each trader enters the market with specific prior knowledge and observation abilities concerning the characteristics of the goods being traded. This is a particular kind of asymmetric information (adverse selection), famous since the seminal contribution of Akerlof (1970).

General equilibrium models with adverse selection have been developed by Prescott and Townsend (1984a, 1984b), Gale (1992, 1996), Bisin and Gottardi (1999, 2006) and Rustichini and Siconolfi (2008). In these models, agents enter the market having private information about their type (endowments and preferences in each of the possible states of nature). Notably, all these works conclude that price-decentralization fails in economies with adverse selection. The main contribution of the present paper is a strong positive result.

Here, each agent's private information is described by a partition of the set of commodities, such that the agent can distinguish goods that belong to different sets of the partition.[1] This formalization, in the spirit of Akerlof (1970), follows the works of Minelli and Polemarchakis (2001) and of Meier, Minelli and Polemarchakis (2006).

In these latter works, there are only markets for classes of goods that everyone can distinguish. If there is an agent in the economy that does not distinguish red cherries from green cherries, then the other agents cannot trade red (or green)

[1]Frequently, the owner of a good has superior information about its characteristics. This is the case that is focused in this paper. Agents are assumed to have perfect information about their endowments, and only have differential information about the characteristics of the goods that are brought to the market by the other agents.

cherries among themselves. This seems very restrictive, and here an alternative framework is considered. Agents are allowed to buy any good. Nevertheless, an agent that buys red cherries, and is not able to distinguish red cherries from green cherries, may end up receiving green cherries instead of red cherries (or a particular mix of green and red cherries).

To understand the implications of private information (in this model), consider a market in which cherries of different quality are being offered. Some agents are able to observe the quality of the cherries, while others are not. Suppose that an agent who cannot distinguish the good from the bad cherries, decides, nevertheless, to buy 10 tons of good cherries. She may end up receiving 10 tons of good cherries (truthful delivery), 2 tons of good and 8 tons of bad cherries, or even 10 tons of bad cherries. Buying 10 tons of good cherries, all that an uninformed agent guarantees is the delivery of 10 tons of cherries (good or bad).[2]

Instrumental to this treatment of trade with adverse selection is the concept of *delivery rate*, adapted from the work of Dubey, Geanakoplos and Shubik (2005). If an agent that buys 10 tons of good cherries receives 2 tons of good cherries and 8 tons of bad cherries, the corresponding delivery rates are 0,2 and 0,8. It is assumed that agents take the delivery rates as given. In equilibrium, the anticipated delivery rates must coincide with the actual delivery rates.

As in the model of Prescott and Townsend (1984a, 1984b) and the subsequent literature, we assume that a profit maximizing firm is responsible for trade intermediation. Taking prices as given, the firm buys the endowments and resells them to the agents. The firm can find it in its interest to deliver bundles that are different from those that the agents ordered. In fact, to maximize profits, the

[2]An agent guarantees delivery of one out of a set of possibilities. This is closely related to what was termed as "uncertain delivery" by Correia-da-Silva and Hervés-Beloso (2008a, 2008b and 2009), in a series of papers that study ex-ante trade of contingent goods, with agents having different abilities to verify the occurrence of the exogenous states of nature.

firm delivers to each agent the cheapest possible alternative among those that the agent cannot distinguish from the truthful delivery.[3]

To sum up, the workings of the economy are summarized as follows. Goods are distinguished not only by their physical characteristics, but also by the agent that brings them to the market. There are prices and rates of delivery for each of these generalized goods, taken as given by the agents and by the intermediary firm. An equilibrium is composed by prices, delivery rates, orders and deliveries, such that: agents make orders that maximize the utility of the resulting deliveries (given the correctly anticipated delivery rates), which are feasible and maximize the profit of the intermediary firm (among those that the agents cannot distinguish from the truthful delivery).

The main result of this paper is the existence of equilibrium (Section 2), under very general conditions (the utility functions must be concave and strictly increasing). A characteristic of equilibrium is that agents receive the cheapest bundle among those that they cannot distinguish from truthful delivery.

To illustrate the main intuitions offered by the model, an example by Meier, Minelli and Polemarchakis (2006) is solved and explained (Section 3).

2 The model

We consider an exchange economy in which a finite number of agents, $i \in \mathcal{I} = \{1, ..., I\}$, trade a finite number of goods, $l \in \mathcal{L} = \{1, ..., L\}$.

To capture the usual context in which the seller has superior information about the quality of the goods that she brings to the market, it is useful to consider

[3] It is assumed that the abilities to distinguish goods (information partitions) are commonly known, therefore, also known by the intermediary firm.

a generalized notion of a good, incorporating in its description the name of the agent that is endowed with the good. This allows us to study markets in which agents may not have the ability to distinguish good cars from bad cars in general, but are able to observe the quality of their own cars.

We refer to good l that is in the initial endowment of agent i as the generalized good $(l, i) \in \mathcal{L} \times \mathcal{I}$. The initial endowments of agent i, defined in terms of these generalized commodities, $f_i \in \mathbb{R}_+^{LI}$, relate to the usual definition of endowments, $e_i \in \mathbb{R}_+^L$, as follows:

$$f_i \in \mathbb{R}_+^{LI}, \quad \text{with} \quad f_i^{(l,i)} = e_i^l \quad \text{and} \quad f_i^{(l,j)} = 0, \ \forall l, i, j \neq i.$$

Similarly, the utility functions in terms of these generalized commodities, V_i, can be obtained from the usual utility functions, $U_i : \mathbb{R}_+^L \to \mathbb{R}$, as follows:

$$\begin{aligned} &V_i : \mathbb{R}_+^{LI} \to \mathbb{R}; \\ &V_i(x_i) = U(z_i), \quad \text{where} \quad z_i^l = \sum_{j \in \mathcal{I}} x_i^{(l,j)}. \end{aligned}$$

Agents wish to maximize their utility functions, $V_i(x_i)$, which are continuous, concave and strictly increasing[4].

Assumption 1 (Preferences).

The utility functions, V_i, are continuous, concave and strictly increasing.

Each agent has specific abilities to distinguish the different goods that are traded in the market. These observation abilities are described by a partition of the set of generalized goods, P_i, such that $(l', j') \in P_i(l, j)$ if and only if agent i cannot distinguish good (l', j') from good (l, j).

[4]By strictly increasing, it is meant that an increase in consumption of any of the goods is strictly desired by the agents: $x_i \geq x_i'$ and $x_i \neq x_i'$ implies that $V_i(x_i) > V_i(x_i')$.

The inability to distinguish between two goods, (l, j) and (l', j'), implies that an agent that buys certain quantities of (l, j) and (l', j'), say $y_i^{(l,j)}$ and $y_i^{(l',j')}$, may receive different quantities, $x_i^{(l,j)}$ and $x_i^{(l',j')}$, such that:

$$x_i^{(l,j)} + x_i^{(l',j')} = y_i^{(l,j)} + y_i^{(l',j')}.$$

More generally, when buying $y_i = (y_i^{(1,1)}, ..., y_i^{(1,I)}, y_i^{(2,1)}, ..., y_i^{(L,I)})$, agent i will receive $x_i = (x_i^{(1,1)}, ..., x_i^{(1,I)}, x_i^{(2,1)}, ..., x_i^{(L,I)})$ such that:

$$\begin{bmatrix} x_i^{(1,1)} \\ \vdots \\ x_i^{(1,I)} \\ x_i^{(2,1)} \\ \vdots \\ x_i^{(L,I)} \end{bmatrix} = \begin{bmatrix} k_i^{(1,1),(1,1)} & \cdots & k_i^{(1,1),(1,I)} & k_i^{(1,1),(2,1)} & \cdots & k_i^{(1,1),(L,I)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ k_i^{(1,I),(1,1)} & \cdots & k_i^{(1,I),(1,I)} & k_i^{(1,I),(2,1)} & \cdots & k_i^{(1,I),(L,I)} \\ k_i^{(2,1),(1,1)} & \cdots & k_i^{(2,1),(1,I)} & k_i^{(2,1),(2,1)} & \cdots & k_i^{(2,1),(L,I)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ k_i^{(L,I),(1,1)} & \cdots & k_i^{(L,I),(1,I)} & k_i^{(L,I),(2,1)} & \cdots & k_i^{(L,I),(L,I)} \end{bmatrix} \cdot \begin{bmatrix} y_i^{(1,1)} \\ \vdots \\ y_i^{(1,I)} \\ y_i^{(2,1)} \\ \vdots \\ y_i^{(L,I)} \end{bmatrix},$$

where $k_i^{(l',j'),(l,j)}$ denotes the number of units of good (l', j') that agent i receives for each unit of good (l, j) that she buys.

The delivery matrix, k_i, is endogenous (equilibrating variable). The delivery matrices that are compatible with the abilities of agent i to distinguish commodities are such that, for each (l, j):

$$\sum_{(l',j') \in P_i(l,j)} k_i^{(l',j'),(l,j)} = 1 \text{ and } k_i^{(l',j'),(l,j)} = 0, \; \forall (l', j') \notin P_i(l, j).$$

The set of matrices that satisfy these conditions is denoted K_i, and $K = \prod_{i \in \mathcal{I}} K_i$.[5]

It should be clear that if agent i is able to distinguish all the commodities, that is, if $P_i(l, j) = \{(l, j)\}$, $\forall (l, j)$, then K_i has a single element (the identity matrix). A perfectly informed agent receives exactly the bundle that she orders.

[5]The sum of the elements of a column must be equal to 1, and those elements that correspond to goods that can be distinguished from the good in the diagonal must be equal to 0.

To simplify the problem, it is assumed that agents have perfect information about their own endowments.

Assumption 2 (Perfect information about own endowments).

$\forall(i,l): \ P_i(l,i) = \{(l,i)\}$.

Only goods that exist are priced and traded in the market. Such goods are those for which $\sum_{i\in\mathcal{I}} f_i^{(l,j)} > 0$, a condition which is equivalent to $f_j^{(l,j)} > 0$. Absence of some goods implies straightforward modifications of the spaces in which f_i, k_i, V_i and P_i are defined.

The set of goods that exist is denoted by $\mathcal{M} \subseteq \mathcal{L} \times \mathcal{I}$, and the number of goods that exist by $M \leq LI$. When the classical interiority assumption holds ($e_i \gg 0, \forall i \in \mathcal{I}$), all the goods are traded in the market and, therefore, $\mathcal{M} = \mathcal{L}\times\mathcal{I}$ and $M = LI$.

Taking prices, $p \in \Delta^M$, and delivery rates, $k_i \in K_i$, as given, agent i trades her initial endowments, $f_i \in \mathbb{R}_+^M$, for a bundle, $y_i \in \mathbb{R}_+^M$, that maximizes utility, $V_i(k_i y_i)$, among those that satisfy the budget restriction, $y_i \in B_i(p)$, with:

$$B_i(p) = \left\{ y_i \in \mathbb{R}_+^M : \ p \cdot y_i \leq p \cdot f_i \right\}.$$

Notice that Assumption 2 guarantees individual rationality of participating in the market. An agent can always “buy” her own endowments.

A profit maximizing firm is responsible for trade intermediation. Without loss of generality, it can be assumed that the firm buys the total endowments in the economy, as the firm can resell the goods at the same price. Furthermore, the firm can deliver a bundle that is different from the bundle that an agent orders. To maximize profits, the firm delivers the cheapest possible alternative among those that the agent cannot distinguish from truthful delivery.

Definition 1 (Equilibrium).

An equilibrium of the economy $\mathcal{E} \equiv \{f_i, V_i, P_i\}_{i\in\mathcal{I}}$ *is composed by a price system,* $p^* \in \Delta_+^M$*, individual choices,* $y^* = (y_1^*, ..., y_I^*) \in \mathbb{R}_+^{IM}$*, delivery rates,* $k^* = (k_1^*, ..., k_I^*) \in K$*, and the resulting allocation,* $x^* = (x_1^*, ..., x_I^*) \in \mathbb{R}_+^{IM}$*, which satisfy:*

$$y_i^* \in \underset{y_i \in B_i(p^*)}{\operatorname{argmax}} V_i(k_i^* y_i), \ \forall i \quad \textit{[utility maximization];}$$

$$k^* \in \underset{k \in K}{\operatorname{argmin}} \left\{ p^* \cdot \sum_{i\in\mathcal{I}} k_i y_i^* \right\} \quad \textit{[profit maximization];}$$

$$x_i^* = k_i^* y_i^*, \ \forall i \quad \textit{[resulting deliveries];}$$

$$\sum_{i\in\mathcal{I}} x_i^* \leq \sum_{i\in\mathcal{I}} f_i \quad \textit{[feasibility].}$$

Theorem 1 (Existence of equilibrium).

Under Assumptions 1 and 2, there exists an equilibrium of the economy.

Proof:

Consider, for now, bounded choice sets. For choices in the upper bound to imply aggregate excess delivery, let $E = \sum_{(l,j)\in\mathcal{M}} f_j^{(l,j)} + 1$ and define the following convex and bounded choice sets:

$$\overline{Y}_i = \{y_i \in \mathbb{R}_+^M : \ y_i^{(l,j)} \leq E, \forall (l,j)\}.$$

The budget set of agent i, in this bounded economy, is:

$$\overline{B}_i(p) = \left\{y_i \in \overline{Y}_i : \ p \cdot y_i \leq p \cdot f_i\right\}.$$

Let $\psi_i(y,p,k) = arg \max_{z_i \in \overline{B}_i(p)} \{V_i(k_i z_i)\}$.

The utility function, $V_i(k_i y_i)$, is continuous with respect to both k_i and y_i.

As long as $p \cdot f_i > 0$ (which is always the case for $p \gg 0$), the budget correspondence, $\overline{B}_i(p) = \{y_i \in \overline{Y}_i : \ p \cdot y_i \leq p \cdot f_i\}$, is continuous with non-empty compact values.

When this is the case, we know, from Berge's Maximum Theorem[6], that the demand correspondence, $\psi_i(y,k,p)$, is upper hemicontinuous with nonempty compact values. It is also convex-valued, because V_i is concave and k_i is constant.

Let $\Delta^M_\epsilon = \left\{p \in \Delta^M : \ p \geq \epsilon\right\}$. We will start by finding a fixed point with strictly positive prices, on Δ^M_ϵ, and then let $\epsilon \to 0$ to obtain a sequence of fixed points.

Let $\psi^\epsilon_p(y,p,k) = arg \max_{q\in\Delta^M_\epsilon} \left\{ q \cdot \sum_{i\in\mathcal{I}} (k_i y_i - f_i) \right\}$.

And let $\psi_{k_i}(y,p,k) = arg \min_{d_i \in K_i} \{p \cdot d_i y_i\}$.

All these correspondences (ψ_i, ψ^ϵ_p and ψ_{k_i}) are upper hemicontinuous with nonempty compact and convex values. Therefore, the product correspondence, $\psi^\epsilon = \prod_{i=1}^I \psi_i \times \psi^\epsilon_p \times \prod_{i=1}^I \psi_{k_i}$, also is.

Applying the Theorem of Kakutani, we find that there exists a fixed point of ψ^ϵ, that we denote by $(y^\epsilon, p^\epsilon, k^\epsilon)$. Considering a sequence, $\{\epsilon^n\}_{n\in\mathbf{N}}$, that converges to zero, we obtain a sequence of fixed points, $\{(y^n, p^n, k^n)\}_{n\in\mathbf{N}}$. The sequence is contained in a compact set, therefore a subsequence converges to (y^*, p^*, k^*).

We want to consider this subsequence and verify that its limit is an equilibrium.

Suppose that the sequence of prices in the interior of the simplex, $\{p^n\}$, converges to a price on the border of the simplex. There is at least one agent whose income does not tend to zero ($p^* \cdot f_i > 0$), and therefore whose demand (which is u.h.c.) is driven to the bound of the choice set (recall that utility is strictly increasing). This implies that, for sufficiently large n and in the limit, there is aggregate excess delivery of at least some good:

$$\exists (l,j) : \sum_{i\in\mathcal{I}} x_i^{(l,j)*} = \sum_{i\in\mathcal{I}} \sum_{(l',j')} k_i^{(l,j),(l',j')*} y_i^{(l',j')*} > \sum_{i\in\mathcal{I}} f_i^{(l,j)}.$$

The budget restrictions imply that $p^n \cdot \sum_{i\in\mathcal{I}} y_i^n \leq p^n \cdot \sum_{i\in\mathcal{I}} f_i$, and the definition of ψ_{k_i} implies that $p^n \cdot \sum_{i\in\mathcal{I}} k_i^n y_i^n \leq p^n \cdot \sum_{i\in\mathcal{I}} y_i^n$ (the identity matrix is enough

[6] See, for example, Aliprantis and Border (2006).

for equality). In the limit:

$$p^* \cdot \sum_{i\in\mathcal{I}} x_i^* = p^* \cdot \sum_{i\in\mathcal{I}} k_i^* y_i^* \leq p^* \cdot \sum_{i\in\mathcal{I}} f_i.$$

To understand the idea of the proof, start by supposing that, in the limit, there is a single good, (l, j), with maximal excess delivery. From the definition of ψ_p, the price of this good tends to 1, and the remaining goods have vanishing prices. Therefore, the aggregate budget restriction becomes:

$$\sum_{i\in\mathcal{I}} y_i^{(l,j)*} \leq \sum_{i\in\mathcal{I}} f_i^{(l,j)}.$$

Given the definition of the ψ_{k_i} (the cheapest possibility is delivered):

$$\sum_{i\in\mathcal{I}} x_i^{(l,j)*} = \sum_{i\in\mathcal{I}} \sum_{(l',j')} k_i^{(l,j),(l',j')*} y_i^{(l',j')*} = \sum_{i\in\mathcal{I}} k_i^{(l,j),(l,j)*} y_i^{(l,j)*} \leq \sum_{i\in\mathcal{I}} y_i^{(l,j)*}.$$

Which implies that $\sum_{i\in\mathcal{I}} x_i^{(l,j)*} \leq \sum_{i\in\mathcal{I}} f_i^{(l,j)}$ (no excess delivery). Contradiction.

Now let's move on to the general case. Suppose that, in the limit, there is a set of goods, G, tied for the maximal excess delivery:

$$\forall (l,j) \in G: \sum_{i\in\mathcal{I}} x_i^{(l,j)*} = \sum_{i\in\mathcal{I}} \sum_{(l',j')} k_i^{(l,j),(l',j')*} y_i^{(l',j')*} > \sum_{i\in\mathcal{I}} f_i^{(l,j)}. \quad (1)$$

In the limit, the prices of these goods must add to 1, while the prices of the remaining goods become null. Aggregating the budget restrictions:

$$\sum_{i\in\mathcal{I}} \sum_{(l,j)\in G} p^{(l,j)*} y_i^{(l,j)*} \leq \sum_{i\in\mathcal{I}} \sum_{(l,j)\in G} p^{(l,j)*} f_i^{(l,j)}. \quad (2)$$

Consider the set of goods with highest price, $G_1 \subseteq G$. From the definition of the ψ_{k_i}, we know that buying one unit of a good in G_1 implies delivery of quantities of goods in G_1 that add to 1 unit or less, and that buying goods that do not belong to G_1 implies no delivery of goods in G_1. Therefore:

$$\sum_{i\in\mathcal{I}} \sum_{(l,j)\in G_1} x_i^{(l,j)*} \leq \sum_{i\in\mathcal{I}} \sum_{(l,j)\in G_1} y_i^{(l,j)*}.$$

In fact, considering the set of goods with price higher or equal to some threshold, $t > 0$, denoted $G_t \subseteq G$, we have (from an analogous reasoning):

$$\sum_{i \in \mathcal{I}} \sum_{(l,j) \in G_t} x_i^{(l,j)*} \leq \sum_{i \in \mathcal{I}} \sum_{(l,j) \in G_t} y_i^{(l,j)*}.$$

Observing that $\sum_{i \in \mathcal{I}} y_i^*$ dominates $\sum_{i \in \mathcal{I}} x_i^*$ in the sense of Lorenz (for price inequality among goods instead of income inequality among agents), we obtain:

$$\begin{aligned}
\sum_{(l,j) \in G} \left\{ p^{(l,j)*} \sum_{i \in \mathcal{I}} x_i^{(l,j)*} \right\} \quad &\leq \quad \sum_{(l,j) \in G} \left\{ p^{(l,j)*} \sum_{i \in \mathcal{I}} y_i^{(l,j)*} \right\} \Rightarrow \\
\Rightarrow \sum_{i \in \mathcal{I}} \sum_{(l,j) \in G} p^{(l,j)*} x_i^{(l,j)*} \quad &\leq \quad \sum_{i \in \mathcal{I}} \sum_{(l,j) \in G} p^{(l,j)*} y_i^{(l,j)*}.
\end{aligned}$$

Which, together with (2), implies the following contradiction of (1):

$$\sum_{i \in \mathcal{I}} \sum_{(l,j) \in G} p^{(l,j)*} x_i^{(l,j)*} \leq \sum_{i \in \mathcal{I}} \sum_{(l,j) \in G} p^{(l,j)*} f_i^{(l,j)}.$$

There is not excess delivery of any good, therefore, x^* is feasible and $p^* \gg 0$. Existence of equilibrium in the bounded economy is established.

To check that this is an equilibrium when the bounds on the choice sets are removed, we must verify that individual choices remain unaltered.

Observe that the bound on the choice sets is large enough for the individual choices, y_i^*, to be in the interior of $\overline{Y}_i$ (otherwise we would not have feasibility). Since preferences are convex, we are sure that the bounds are not binding. If there were a strictly better choice outside $\overline{Y}_i$, then there would also be a strictly better choice in the frontier of $\overline{Y}_i$.

QED

We have established that, under general conditions, equilibrium exists.

3 Characteristics of equilibrium

A first property results directly from the definition of equilibrium. Each agent receives one of the cheapest bundles among those that she cannot distinguish from the truthful delivery.[7]

Proposition 1.

$$x_i^* \in \underset{k_i \in K_i}{\operatorname{argmin}} \{p^* \cdot k_i y_i^*\}.$$

In equilibrium, prices are strictly positive and there is no free disposal.

Proposition 2.

$$p^* \gg 0 \quad \textit{and} \quad \sum_{i \in \mathcal{I}} x_i^* = \sum_{i \in \mathcal{I}} f_i.$$

Proof:

In the proof of Theorem 1, it was established that $p \gg 0$.

The intermediary firm is unable to retain any goods because the agents are able to observe the quantities of each good that are delivered. To retain some quantity of a good, the firm would have to deliver an extra quantity of another good, and this is impossible because the firm has no endowments and there is no production. Therefore: $\displaystyle\sum_{i \in \mathcal{I}} x_i^* = \sum_{i \in \mathcal{I}} y_i^*$.

Feasibility, then, implies that: $\sum_{i \in \mathcal{I}} y_i^* \leq \sum_{i \in \mathcal{I}} f_i$. Since $p \gg 0$ and the budget restrictions are satisfied in equality, we must have: $\displaystyle\sum_{i \in \mathcal{I}} y_i^* = \sum_{i \in \mathcal{I}} f_i$.

QED

Each agent receives a bundle that costs the same as the bundle that was bought.

[7] It should be clear that this adverse delivery is anticipated by the agents.

Proposition 3.

$$\forall i \in \mathcal{I}: \ p^* \cdot x_i^* = p^* \cdot y_i^*.$$

Proof:

From Proposition 1, we know that an agent cannot receive something more expensive than what was bought. Then, from exact feasibility (Proposition 2) it follows that all agents receive a bundle that costs the same as the bundle that they bought. If some agent received something cheaper, another agent would have to be receiving something more expensive, which would be a contradiction.

QED

An agent who does not distinguish between two goods prefers to buy the cheapest. Therefore, two goods that the agents do not distinguish (except for the owners, who surely distinguish their endowments) and that are traded in the market (some agent other than the owner buys a positive quantity), are traded at the same price.

Proposition 4.

If $(l', j') \in P_i(l, j)$ *and* $y_i^{(l,j)*} > 0$*, then* $p^{(l,j)*} \leq p^{(l',j')*}$.[8]

Proof:

Suppose that $p^{(l,j)*} > p^{(l',j')*}$, and that $y_i^{(l,j)*} > 0$. Replacing good (l, j) by good (l', j') in the bundle that is bought by agent i, y_i^*, we construct a bundle that is cheaper and that is undistinguishable from y_i^*. In equilibrium, agent i must be receiving, therefore, a bundle that is cheaper than y_i^*. This contradicts Proposition 3.

QED

[8] If we also have $y_i^{(l',j')*} > 0$, then $p^{(l,j)*} = p^{(l',j')*}$.

In the case in which buyers do not distinguish the original owners of the goods, the law of one price is verified (if the same commodity is sold by two agents, then it is sold at the same price).

Proposition 5.

If $\exists i, i', j, j'$ *and* l *such that* $(l, j') \in P_i(l, j) \cap P_{i'}(l, j)$, $y_i^{(l,j)*} > 0$ *and* $y_{i'}^{(l,j')*} > 0$, *then:* $p^{(l,j')*} = p^{(l,j)*}$.

4 An example

In this section, an example is presented as an illustration.[9] Here the notation is simpler than in the previous section, because it will not be necessary to deal with generalized goods in a formal way.

In the first case, an agent that does not distinguish two goods is not able to consume the high quality good, in spite of being willing to pay any price for a small quantity of this good. In the second case, the two goods are traded at the same equilibrium price. The uninformed agent receives both goods, with the delivery rates resulting from the "leftovers" of the informed agents.

4.1 Cherry picking

Three individuals, $\mathcal{I} = \{1, 2, 3\}$, trade three commodities, $\mathcal{L} = \{0, r, g\}$, that we can think of as 'money', 'red cherries' and 'green cherries'.

[9]The example is adapted from one presented by Meier, Minelli and Polemarchakis (2006). The equilibrium concept used here leads to qualitatively different solutions.

Agent 1 is endowed with 'money', agent 2 with 'red cherries' and agent 3 with 'green cherries':

$$\begin{cases} e_1 = \{12, 0, 0\}; \\ e_2 = \{0, 12, 0\}; \\ e_3 = \{0, 0, 12\}. \end{cases}$$

All agents prefer 'green cherries' to 'red cherries'. Agent 2 does not like 'red cherries' at all. Preferences are described by the following utility functions:[10]

$$\begin{cases} U_1(x_1) = ln(x_1^0) + ln(x_1^r) + 2ln(x_1^g); \\ U_2(x_2) = ln(x_2^0) + 2ln(x_2^g); \\ U_3(x_3) = ln(x_3^0) + ln(x_3^r) + 2ln(x_3^g). \end{cases}$$

There is asymmetric information because agent 1 cannot distinguish 'red cherries' from 'green cherries' while agents 2 and 3 are able to distinguish the three commodities:

$$\begin{cases} P_1 = \{\{0\}, \{r, g\}\}; \\ P_2 = \{\{0\}, \{r\}, \{g\}\}; \\ P_3 = \{\{0\}, \{r\}, \{g\}\}. \end{cases}$$

Agents 2 and 3 are perfectly informed, and therefore receive exactly what they buy. On the other hand, agent 1 can buy either 'red cherries' or 'green cherries', but must accept whatever cherries are delivered (because agent 1 cannot distinguish the two goods).[11]

Since agent 2 will not buy 'red cherries', her budget restriction is (notice that the price of 'money' is normalized to $p^0 = 1$):

$$p^0 x_2^0 + p^r x_2^r + p^g x_2^g = p^r e_2^r \Rightarrow x_2^0 + p^g x_2^g = 12p^r.$$

[10] In rigor, we will consider that agent 1 has lexicographic preferences. First, the agent maximizes $U_1(x_1) = ln(x_1^0) + ln(x_1^r) + 2ln(x_1^g)$. Then, in case of a tie (which occurs among all bundles with $x_1^g = 0$), agent 1 is assumed to maximize $v_1(x_1) = ln(x_1^0) + ln(x_1^r)$.

[11] Observe that there is an infinite number of possibilities for the delivery of 10 'cherries' (10 'red' and 0 'green', 5 'red' and 5 'green', 7 'red' and 3 'green', etc.).

The optimality condition implies equality between the ratios between marginal utility and price, for each good demanded:

$$x_2^0 = 0.5 p^g x_2^g.$$

From the budget restriction and the optimality condition, we find the demand of agent 2:

$$(x_2^0, x_2^r, x_2^g) = \left(4p^r, 0, 8\frac{p^r}{p^g}\right).$$

Similarly, we can obtain the demand function of agent 3:

$$\begin{cases} x_3^0 + p^r x_3^r + p^g x_3^g = 12p^g \\ x_3^0 = p^r x_3^r = 0.5 p^g x_3^g \end{cases} \Rightarrow (x_3^0, x_3^r, x_3^g) = \left(3p^g, 3\frac{p^g}{p^r}, 6\right).$$

Looking at the demand of agents 2 and 3 for 'green cherries', we find that $p^r < p^g$, otherwise there would be excess demand. More precisely:

$$8\frac{p^r}{p^g} + 6 \leq 12 \Rightarrow p^r \leq 0.75 p^g.$$

Suppose that agent 1 buys a quantity y_1^{rg} of 'cherries' (guarantees delivery of 'red cherries' and 'green cherries' such that $x_1^r + x_1^g = y_1^{rg}$). If 'red cherries' are cheaper than 'green cherries', then the agent should receive only 'red cherries', and no 'green cherries'.

If the agent received some 'green cherries', then one could wonder why someone is delivering these 'green cherries', instead of trading them in the market for 'red cherries' plus 'money' and delivering the 'red cherries' while keeping the 'money' (there would be an arbitrage opportunity).

Following this reasoning, since $p^r < p^g$, then $x_1^g = 0$ and $x_1^r = y_1^{rg}$. Assuming that agent 1 is aware that of this (delivery rates are anticipated and taken as given), we can find her demand function:[12]

$$\begin{cases} x_1^0 + p^r x_1^r = 12 \\ x_1^0 = p^r x_1^r \end{cases} \Rightarrow \begin{cases} x_1^0 + x_1^0 = 12 \Rightarrow x_1^0 = 6; \\ p^r x_1^r + p^r x_1^r = 12 \Rightarrow x_1^r = \frac{6}{p^r}. \end{cases}$$

For demand to equal supply:

$$\begin{cases} x_1^0 + x_2^0 + x_3^0 = 12 \Rightarrow 4p^r + 3p^g = 6; \\ x_1^r + x_2^r + x_3^r = 12 \Rightarrow \frac{6}{p^r} + 3\frac{p^g}{p^r} = 12; \\ x_1^g + x_2^g + x_3^g = 12 \Rightarrow 8\frac{p^r}{p^g} = 6 \Rightarrow p^r = 0.75p^g. \end{cases}$$

These equations allow the determination of equilibrium prices:

$$p^* = (p^0, p^r, p^g) = (1; 0.75; 1).$$

The allocation is, therefore ($y^* = x^*$ yields truthful deliveries):

$$\begin{cases} x_1^* = (6, \frac{6}{p^r}, 0) = (6, 8, 0); \\ x_2^* = (4p^r, 0, 8\frac{p^r}{p^g}) = (3, 0, 6); \\ x_3^* = (3p^g, 3\frac{p^g}{p^r}, 6) = (3, 4, 6). \end{cases}$$

4.2 Modified cherry picking

Is it always the case that agent 1 does not consume 'green cherries'? In this modified example, we find that agent 1 can consume 'green cherries' if the incentives for agents 2 and 3 to deliver only 'red cherries' disappear.

[12] Agent 1 prefers any interior bundle ($x_1 \gg 0$) to a bundle that is in the frontier of the consumption set. But she cannot get any 'green cherries' and, therefore, U_1 becomes infinitely negative. Recall that among bundles with $x_1^g = 0$ (in the frontier of the consumption set), the preferences of agent 1 are described by $v_1(x_1) = ln(x_1^0) + ln(x_1^r)$.

This occurs if 'green cherries' become much more abundant than 'red cherries'. Let's double the endowments of agent 3 (green cherries):

$$\begin{cases} e_1 = \{12, 0, 0\}; \\ e_2 = \{0, 12, 0\}; \\ e_3 = \{0, 0, 24\}. \end{cases}$$

Demand of agent 2 remains unaltered:

$$\begin{cases} x_2^0 + p^g x_2^g = 12p^r \\ x_2^0 = 0.5p^g x_2^g \end{cases} \Rightarrow (x_2^0, x_2^r, x_2^g) = (4p^r, 0, 8\frac{p^r}{p^g}).$$

While demand of agent 3 doubles:

$$\begin{cases} x_3^0 + p^r x_3^r + p^g x_3^g = 24p^g \\ x_3^0 = x_3^r p^r = 0.5x_3^g p^g \end{cases} \Rightarrow x_3^* = (6p^g, 6\frac{p^g}{p^r}, 12).$$

Now the demand of agents 2 and 3 for 'green cherries' does not exceed supply as long as $p^r \leq 1.5p^g$:

$$x_2^g + x_3^g = 8\frac{p^r}{p^g} + 12 \leq 24 \Rightarrow p^r \leq 1.5p^g.$$

There are three possibilities: (a) the price of 'green cherries' is higher than the price of 'red cherries' and thus agent 1 only consumes 'red cherries' (as in the previous example); (b) the price of 'red cherries' is higher than the price of 'green cherries' and thus agent 1 only consumes 'green cherries'; (c) the prices of 'green cherries' and 'red cherries' coincide.

In case (a), there would be excess supply of 'green cherries', as aggregate consumption is lower than 20:

$$x_1^g + x_2^g + x_3^g = 0 + 8\frac{p^r}{p^g} + 12 < 20.$$

In case (b), there would be excess supply of 'red cherries', as aggregate consumption is lower than 6:

$$x_1^r + x_2^r + x_3^r = 0 + 0 + 6\frac{p^g}{p^r} < 6.$$

Thus, in equilibrium we must have case (c): $p^r = p^g = p^{rg}$. Thus:

$$\begin{cases} x_2 = (4p^{rg}, 0, 8) \\ x_3 = (6p^{rg}, 6, 12) \end{cases} \Rightarrow x_2 + x_3 = (10p^{rg}, 6, 20).$$

The only candidate for an equilibrium allocation gives agent 1 the following consumption bundle: $x_1 = (12 - 10p^{rg}, 6, 4)$.

To check whether this is an equilibrium, we need to find the demand of agent 1. A problem that we face is that agent 1, through her demand, may influence the quality of the cherries (the proportion between red and green cherries).

It is assumed that she takes the proportions of delivered red and green cherries as given. In equilibrium, this proportion must be fulfilled (otherwise it would not be an equilibrium). Since we already have a single candidate for the equilibrium consumption of agent 1, $x_1 = (12 - 10p^{rg}, 6, 4)$, we must assume that agent 1 expects to receive 60% red cherries and 40% green cherries.

The utility and the maximization condition of agent 1 are (with $x_1^{rg} = x_1^r + x_1^g$):

$$u_1(x_1) = ln x_1^0 + ln(0.6x_1^{rg}) + 2ln(0.4x_1^{rg}) \Rightarrow x_1^0 = \frac{1}{3}p^{rg}x_1^{rg}.$$

Putting this together with the budget restriction, demand is obtained:

$$\begin{cases} x_1^0 + p^{rg}x_1^{rg} = 12; \\ x_1^0 = \frac{1}{3}p^{rg}x_1^{rg} \end{cases} \Rightarrow (x_1^0, x_1^r, x_1^g) = (3, 0.6\frac{9}{p^{rg}}, 0.4\frac{9}{p^{rg}}).$$

For demand of 'money' to equal supply:

$$x_1^0 + x_2^0 + x_3^0 = 12 \Rightarrow 3 + 4p^{rg} + 6p^{rg} = 12 \Rightarrow p^{rg} = 0.9.$$

Equilibrium prices are, therefore, $p^* = (1; 0.9; 0.9)$, and the allocation is:

$$\begin{cases} x_1^* = (3; 6; 4); \\ x_2^* = (3.6; 0; 8); \\ x_3^* = (5.4; 6; 12). \end{cases}$$

In this case, agent 1 consumes both 'red cherries' and 'green cherries', which are traded at the same price. Agents 2 and 3 optimize by delivering 3.6 units of 'red cherries' and 5.4 units of 'green cherries' to agent 1.

The quantities of red and green cherries that agent 1 buys (y_1^r and y_1^g) are irrelevant, given that they add to 10. If $y^* = x^*$, there is truthful delivery. In any case, the delivery rates adjust to be such that delivery is surely that calculated above: $x_1^r = 6$ and $x_1^g = 4$.

Observe that the increase in the endowment of 'green cherries' from 12 to 24 units led to a qualitatively different solution. Increasing from 12 to 13 diminishes the price of the 'green cherries' and increases the price of the 'red cherries', but the price of the 'green cherries' remains higher and, therefore, the uninformed agent is still only able to consume 'red cherries'. By straightforward calculations, it can be found that as the endowment of 'green cherries' hits 16, the prices of the two kinds of cherries become equal, but the uninformed agent still receives only 'red cherries'. The solution, in this case, is:

$$\begin{cases} p^* = (1; 0.75; 0.75) \\ x_1^* = (6; 8; 0); \\ x_2^* = (3; 0; 8); \\ x_3^* = (3; 4; 8). \end{cases}$$

Then, as the endowment of 'green cherries' increases further, 'red cherries' become more valuable than 'green cherries' because the scarcity effect overweights the quality effect. The uninformed agent becomes able to consume some 'green cherries' (those that the informed agents leave for her). This is what occurs in the above example (in which $e_3^g = 24$). The equilibrium price of both kinds of 'cherries' is higher (0.9 instead of 0.75) because of the increased quality of the mix received by the uninformed agent.

Eventually, if the endowment of 'green cherries' increases enough, the price of 'green cherries' becomes lower than the price of 'red cherries', and the uninformed agent consumes only 'green cherries' and no 'red cherries'. Such bifurcation occurs for an endowment of 'green cherries' equal to 48, the solution becoming:

$$\begin{cases} p^* = (1; 0.6; 0.6) \\ x_1^* = (6; 0; 16); \\ x_2^* = (2.4; 0; 8); \\ x_3^* = (3.6; 12; 24). \end{cases}$$

5 Concluding remarks

In an economy in which agents trade goods with uncertain quality, the ability to observe the quality of the good is very useful. To study markets such as the used car market (Akerlof, 1970), it is natural to assume that agents know the quality of the goods that they bring to the market, but not the quality of the goods brought by the other agents.

To model this kind of information asymmetry, we have considered a generalized notion of a good, incorporating in its description the agent that is endowed with this good. This allowed us to study economies in which agents may not have the ability to distinguish good cars from bad cars, but are able to observe the quality of their own cars.

Equilibrium is shown to exist, and characterized by the fact that agents always receive the cheapest delivery that is consistent with their observation abilities (that is, that they cannot distinguish from truthful delivery).

It is found that if two agents sell goods that are undistinguished by the others, then they sell them at the same price. However, the price of the same good may vary across sellers (if, for example, we allow agents to know the quality of their brother's car, but not the quality of another agent's car that is exactly in the same condition).[13]

It is left for future work the case in which agents have imperfect information about their own endowments.

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[13]In any case, the solution concept rules out price discrimination (the price of a good does not depend on the buyer).

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